

# Study of Nonlinear Behavior of DC Motor Using Modeling and Simulation

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**Abstract-** DC motors have been widely used in the electromechanical systems due to its simple structure, ease of implementing variable speed control and low cost. In high accuracy servo control system, high control performance of DC motor is needed. DC motors have traditionally been modelled as 2nd order linear system, which ignores the dead nonlinear zone of the motor. Unfortunately, the dead zone caused by the nonlinear friction would bring great effect to servo systems. This paper studies the non linear model of the DC motor and the effects of different types of friction in determining motor dynamics at low speeds using modelling and simulation with MATLAB simulink.

**Index Terms-** DC Motor Non linearity, Friction, Modeling, MATLAB Simulink

## I. INTRODUCTION

Most mechanical systems used in industry are composed of masses moving under the action of position and velocity dependent forces. These forces exhibit nonlinear behaviour in certain regions of operation. For a multi-mass rotational system, the nonlinearities, like Coulomb friction and dead zone, significantly influence the system operation.

A sound knowledge of the system characteristics is of primary importance in any industrial application [1]. For process control and automation purposes, a detailed analysis of the plant with its every component should be performed. Especially, control selection and design for high performance electromechanical systems such as weigh feeders requires the availability of detailed information about the components of the plant, with its linear and nonlinear, static and dynamic characteristics [1,2]. For such systems, the assumption that the nonlinearities are negligible leads to loss in the generality of the results obtained [1].

DC motors, as components of electromechanical systems, are widely used as actuating elements in industrial applications for their advantages of easy speed and position control and wide adjustability range [3]. Consequently, examination of DC motor behaviour constitutes a useful effort for analysis and control of many practical applications [1-3]. In modelling a DC motor connected to a load via a shaft, the general approach is to neglect the nonlinear effects and build a linear transfer function representation for the input-output relationship of the DC motor and the load it drives [1,4]. This assumption is satisfactorily accurate and valid as far as conventional control problems are concerned.

However, when the DC motor operates at low speeds and rotates in two directions, or when it has a wide range of operation and high precision control is needed for the application, the assumption that the nonlinear effects on the system are negligible may lead to intolerably high modelling errors and result in poor control performance [1,3].

This paper basically focuses on nonlinear modelling and proposes an innovative MATLAB model to study of dynamic response of DC motors in open loop for changes in load torque and armature voltage to identify the effects of Coulomb friction and dead zone nonlinearities. The results of the MATLAB model shall prove to be very useful in designing the control strategy for applications involving DC motors

## II. NON LINEAR MODEL OF DC MOTOR

Most Electromechanical systems driven by DC motor such as weigh belt feeder exhibits nonlinear behaviour because of motor friction, motor saturation, and quantization noise in the measurement sensors. The dynamics of the system are dominated by the motor. In presence of this non-linear behaviour, it is difficult to tune a PI controller as the friction effects are difficult to predict and vary with the plant load. Modelling is difficult, and control strategies that work in "textbook" cases often fail to work in the real world. Two of the factors that often contribute to this difficulty are friction and backlash. These effects are highly nonlinear, difficult to model and analyze even with a 'fully nonlinear' model, yet cannot be ignored.

The DC motor which is typically employed in electromechanical systems such as weigh feeder for low speed control also has significant friction. Figure below depicts the electrical and mechanical model of a typical shunt wound, separately excited DC motor.

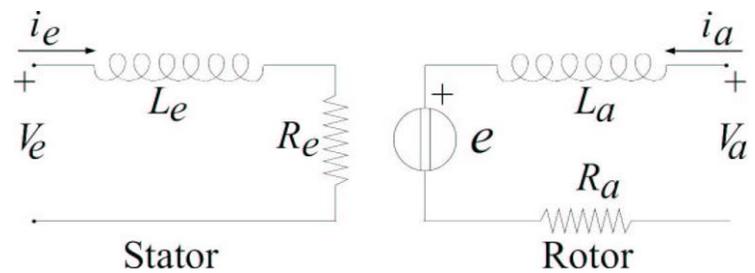


Fig 1: Equivalent circuit of Shunt wound separately excited DC Motor

The equations associated with electrical circuits are given by:

$$V_e(t) = L_e \frac{di_e}{dt} + R_e i_e \quad \text{for the stator side}$$

$$V_a(t) = L_a \frac{di_a}{dt} + R_a i_a \quad \text{for the rotor side}$$

The motor exerts a torque, due to voltages supplied on the stator and on the rotor. The following two equations hold for the back EMF  $e$  and the torque  $T_M$ :

$$T_M = K i_e i_a$$

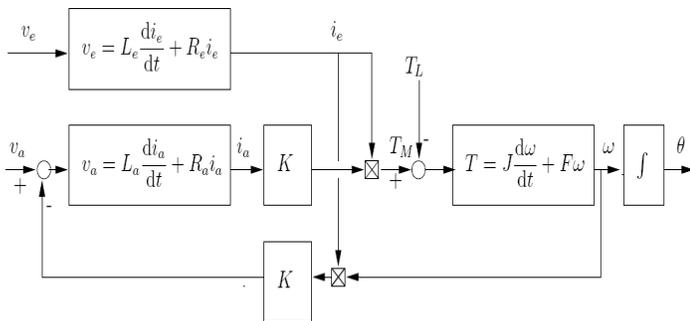
$$e = K i_e \omega, \text{ where } K \text{ is the torque constant of the motor}$$

This torque acts on the mechanical structure, which is characterized by the rotor inertia  $J$  and the **viscous friction coefficient  $F$** . It has also to be taken into account that in any operating environment a load torque is exerted on the motor; then, if  $T_L$  is the load torque, the following equation may be written:

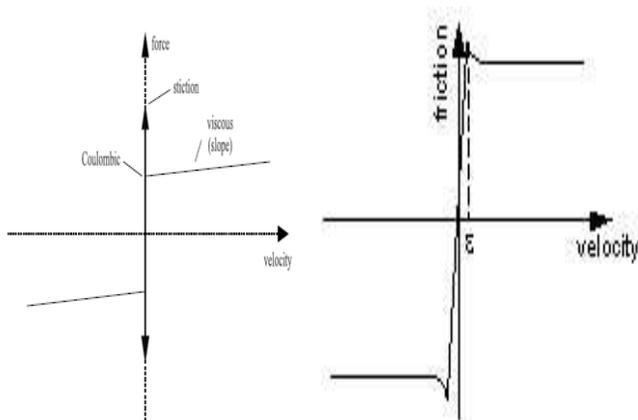
$$T_M - T_L = J \frac{d\omega}{dt} + F \omega$$

where  $\omega$  is the angular velocity of the motor rotor

By implementing the above equations in a nonlinear block diagram, the result shown in Figure 8 is obtained. In the block diagram, the variable  $\theta$  represents the rotor angular position. The nonlinear model results in a two-input, one-output map, having a disturbance input  $T_L$



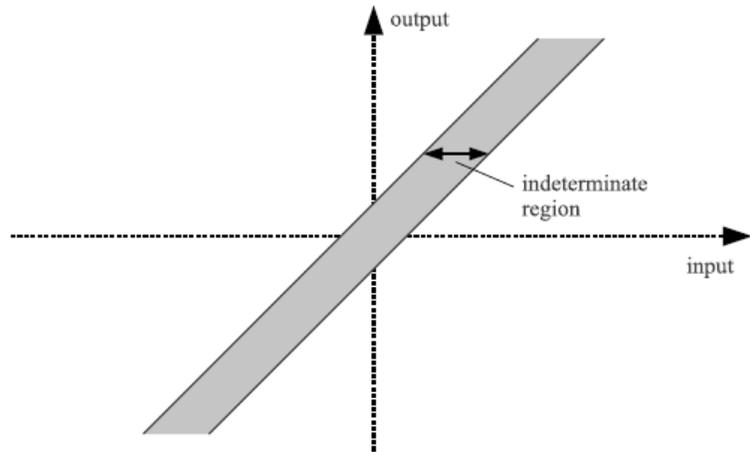
**Fig 2: Non Linear Dynamic model of a Shunt wound separately excited DC Motor**



**Fig 3: Force Vs Velocity Plot for Friction with  $\xi$  representing small threshold value indicating discontinuity**

Figure above shows the main characteristics of a mechanical interface with friction. The stiction (or starting friction) is the amount of force required to break the interface loose. Most interfaces with friction also exhibit stiction (or starting friction). Stiction is the effect where, if the interface has remained still for any length of time, the amount of force required to start the relative motion is greater than the amount required to sustain it. The Coulombic (or “dry”) friction is that portion of the running friction that is dependent only on the direction of motion but has constant magnitude. Finally most mechanisms with friction also display some viscous drag that is more or less proportional to velocity.

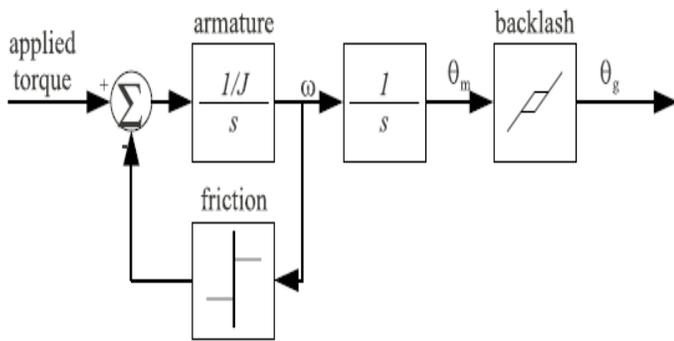
Further, the speed required by the load is too low as compared to the nominal speed of the motor. In such cases, gears are introduced between the motor and the load, thus reducing by a factor  $n$  the angular velocity of the load itself. When a gear is inserted in an actuator system, backlash is experienced on its output due to the coupling between the cogwheels of the gear. This gives rise to nonlinearities and discontinuities in the force/velocity relationships.



**Figure 4: Input Vs Output displacement for an element with Backlash**

Figure above shows that backlash presents problems to the system model that are nearly as severe as those presented by friction. In the case of backlash there is a hidden state – the difference between the input and output positions, where there is no change in output for a change in input.

Figure below depicts the nonlinear model in presence of different types of friction and backlash in the drive system consisting of motor and gearbox.



**Figure 5: Non linear model with friction and backlash**

Typically, in an electromechanical system with DC Motor the friction effects are dominant at low motor speeds and gradually get less prominent with higher motor.

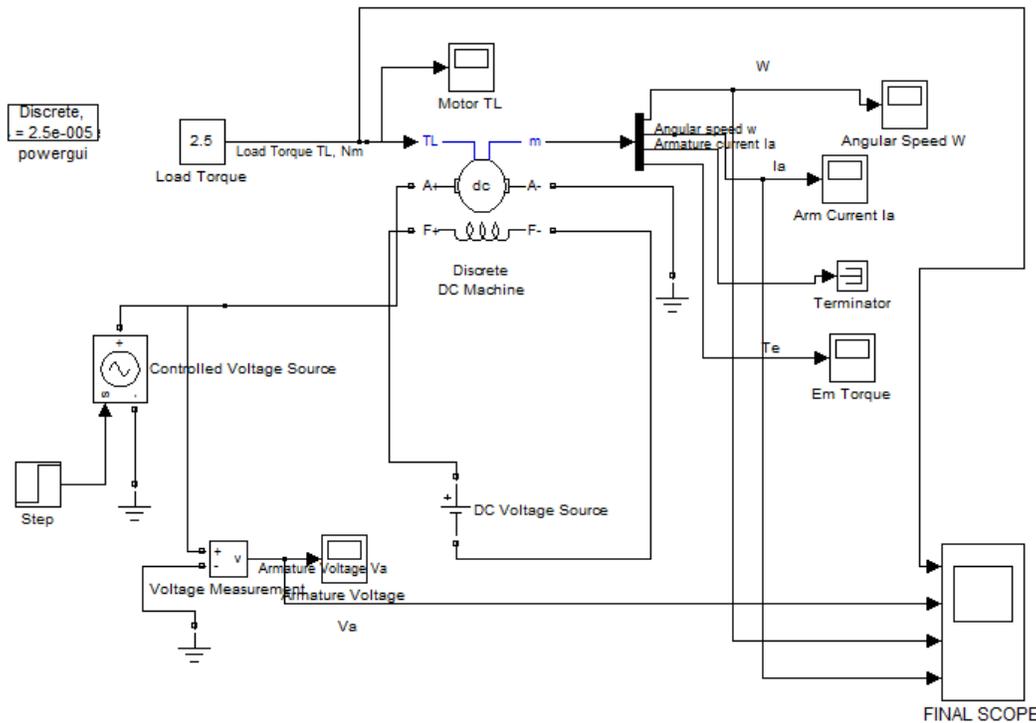
Theoretically in DC motor speed control using armature voltage variation, the relationship between angular speed  $\omega$  and armature voltage  $V_a$  is given by a linear equation:

$\omega = V_a / K\Phi$ , as  $V_a = E_b$  as drop across armature is small as compared to back emf  $E_b$ .

Therefore, if the field flux is held constant, motor speed should vary linearly with  $V_a$ . However, in actual practice the motor speed shall be dependant of friction torques opposing the electromechanical torque developed by the motor. The non – linear behaviour of DC motor, especially at low speeds with changes in load torque can be studied using modelling and simulation as depicted in next section.

**III. SIMULATION OF DC MOTOR BEHAVIOUR**

In order to test the behaviour of a DC motor connected to mechanical load of a machine with step change in load and / or step change on armature voltage, an innovative Simulink model of the DC motor without the conventional PI speed controller (i.e. in open loop) is developed as shown below.



**SIMULINK MODEL FOR DC MOTOR WITHOUT CONTROLLER (MOTOR 5HP, 240V, 1750RPM, Field: 150V DC)**

In the model the DC motor field excitation is provided with constant DC source and the armature voltage is provided by selecting the controlled voltage source and a DC mode is selected for the same with a step signal as input. The mechanical system is modelled to provide load torque to the DC motor model. The dynamic response of the DC motor (speed and armature current) is captured using scope with step change of armature voltage for different loads. Also the simulation results are tabulated and conclusions are drawn from the results.

**TABULATION OF RESULTS OF SIMULATION WITH CONSTANT  $V_a (= 50 V)$**

Sr. No	Load Torque in N-m	Maximum $\omega$ in rad/sec	Time required to reach max speed	Armature Current Ia Amps in steady state
1.	1.0	32	1.25 sec	1.6
2.	1.5	25	1.0 sec	2
3.	2.0	20	1.0 sec	2.25

4.	2.5	15	1.0 sec	3.0
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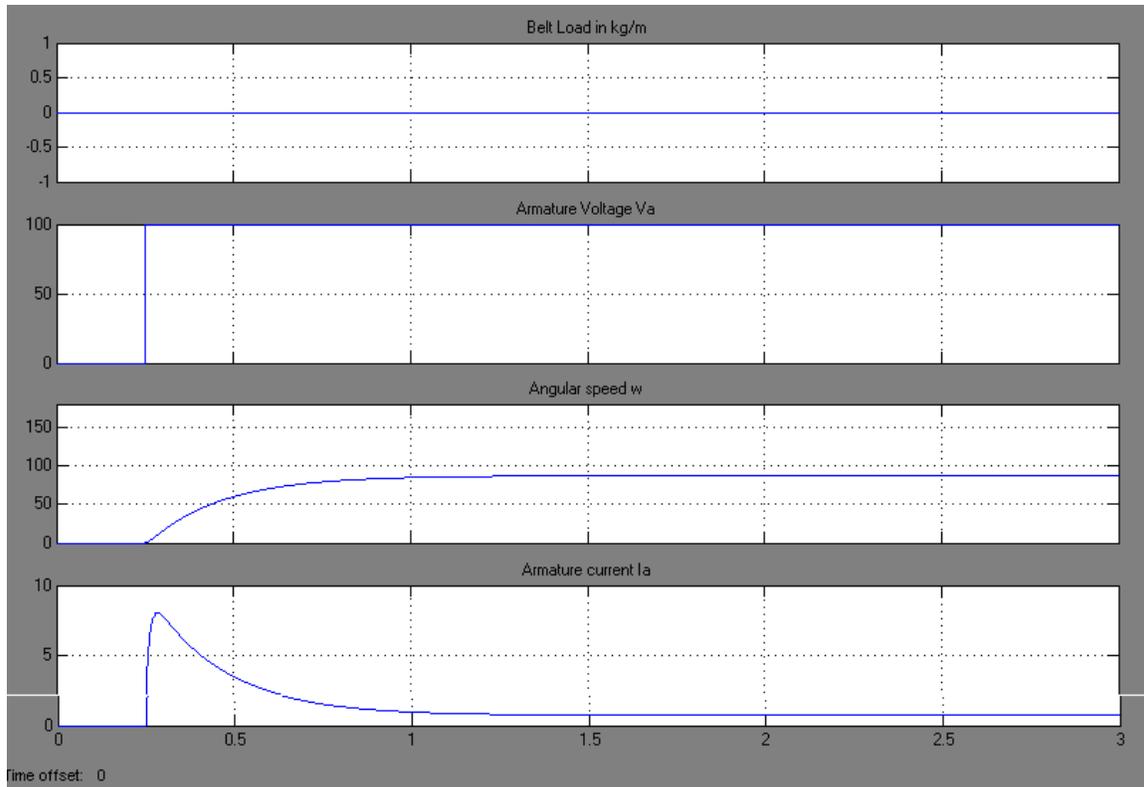
1.	25	0		2.2
2.	50	15	1.25 sec	3.0
3.	100	60	1.10 sec	3.0
4.	150	110	1.00 sec	3.2
5.	200	175	0.75 sec	3.5

**TABULATION OF RESULTS OF SIMULATION WITH CONSTANT LOAD TORQUE (TL= 2.5N-m)**

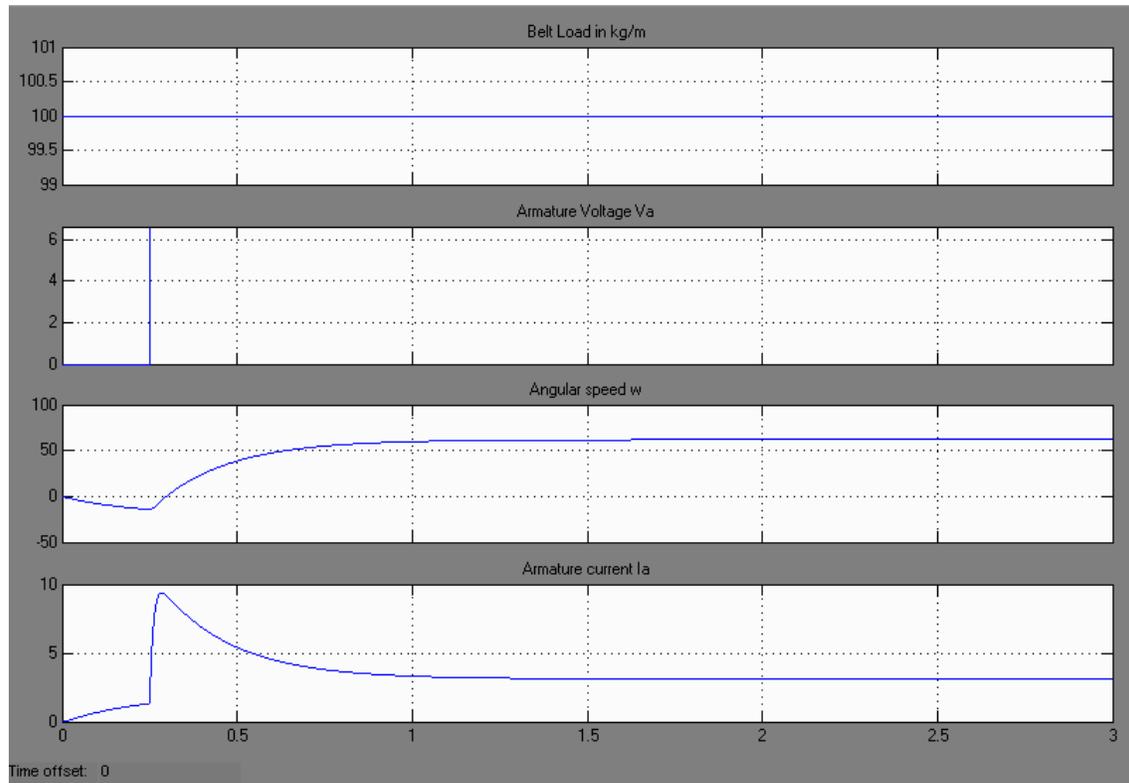
Sr. No	Armature Voltage Va Volts	Maximum $\omega$ in rad/sec	Time required to reach max speed	Armature Current Ia Amps in steady state
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Table above gives the clear picture of the Non linear response of DC motor captured using the MATLAB Simulink Model proposed above.

**SCOPES FOR SIMULATION OF DC MOTOR RESPONSE IN OPEN LOOP**



SCOPE FOR DC MOTOR WITH NO LOAD & Va = 100V DC



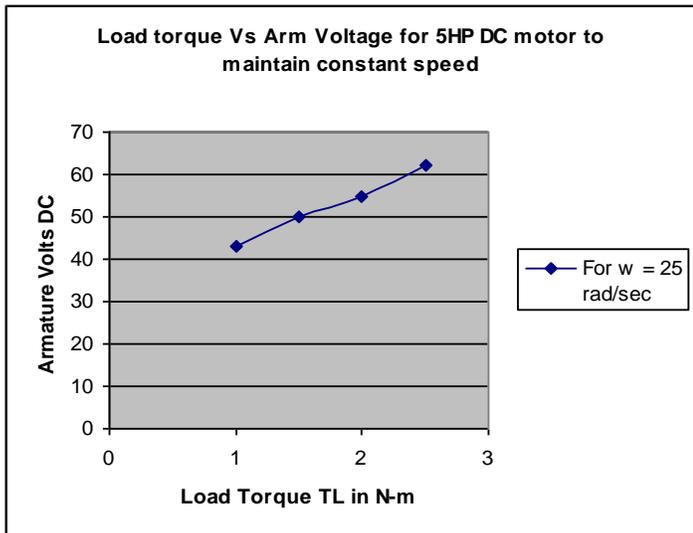
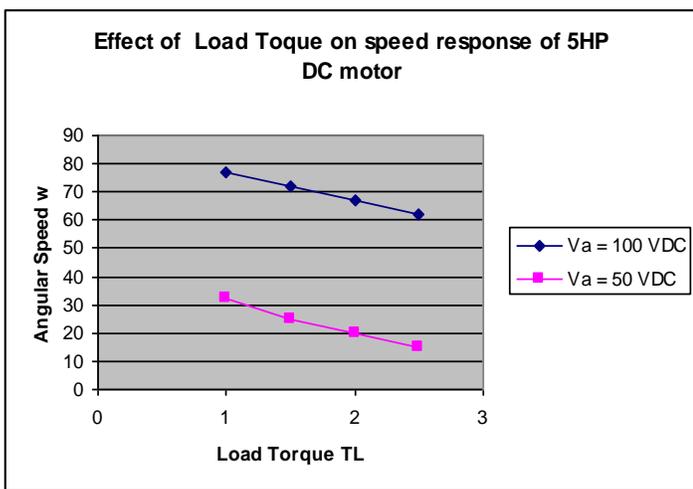
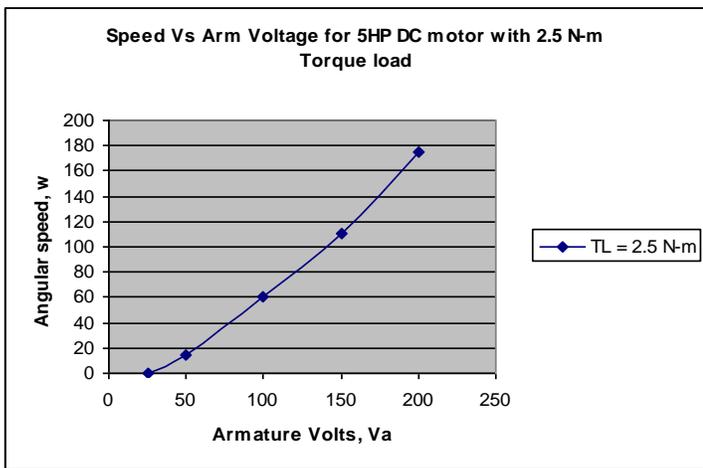
SCOPE WITH STEP CHANGE OF LOAD &  $V_a = 100V$  DC

#### IV. CONCLUSIONS AND ANALYSIS OF RESULTS

Following observations and conclusions can be drawn from the results of the simulations:

1. The response of motor speed and armature voltage is non-linear for 1.5-2 sec after the step change due to moment of inertia and friction effects (Viscous Friction Coeff  $B_m = 0.002953$  N.m.sec and Columb friction torque  $T_f: 0.5761$  N.m)
2. The maximum motor speed drops in response to the increase in load torque and also the dynamic response gets sluggish even for the same  $V_a$ .

For further analysis, the data captured from the simulation are plotted as X-Y chart to see the co-relation between the variable of DC Motor as shown below.



From the chart 1, it is evident that the response of Armature voltage and speed is not linear for constant field voltage and load torque, and the non-linearity is more pronounced at the lower speeds, as the friction forces are more prominent in that region.

Chart 2 depicts that the motor speeds drops significantly as the load torque changes and also varies with the change in armature voltage. In order to compensate for the drop in the speed due to change in load torque, the armature voltage needs to be gradually increased to bring the motor back to desired speed.

In order to overcome the Non-linearity of DC motor response the conventional approach is to implement a linear controller with P+I strategy where the integral action is used to incrementally adjust the armature voltage to bring the motor speed to desired level and remove any steady state error as depicted in chart 3.

However, a fixed PI controller tuning at low load may not be suitable for higher load and vice versa in slow speed applications with variable load torque such as in case of belt weigh feeders. In view of these challenges, the performance of PI controller alone is sub-optimal. It is recommended to overcome the shortcomings of conventional PI controller by implementing the adaptive PI controller or Fuzzy logic based PI controller where the gain of PI controller gets adjusted continuously based on dynamic response of the DC motor.

The results of the present study are meant to constitute a starting point for ongoing studies on identification of non linearity in DC motor based electromechanical systems and adaptive and Fuzzy control applications for nonlinear systems.

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