

OPTIMIZATION OF PORTFOLIO AND THE ESTIMATION OF RISKS IN THE RWANDA FOREX EXCHANGE MARKET USING COPULA APPROACH

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Abstract: This research develops a model to forecast portfolio risk such as VaR (Value-at-Risk) and ES (Expected Shortfall) using GARCH-EVT (extreme value theory)-Copula based approach. We first extract the filtered residuals from each return series through an asymmetric GJR-GARCH-ARMA (1,1) model, residuals show the presence of fat tail. Secondary, we then constructs the semi-parametric empirical marginal cumulative distribution function (CDF) of each asset using a Gaussian kernel estimate for the interior and a generalized Pareto distribution (GPD) estimate for the upper and lower tails (our approach focuses on the entire distribution rather than the tail distribution only). Thirdly, we test the best-fitted copula among Gaussian copula, student's t-copula and Gumbel copula, where the student's t copula is found to be best-fitted copula. Finally, the t copula is then fitted to the data and used to induce correlation between the simulated residuals of each market. In order to test the accuracy of this model we backtest the estimated VaR and ES using 263 days that represent 10th percentile, after backtesting VaR and ES, results show that both are significant at critical level of $\alpha=0.05$.

Keywords: Volatility, Leverage effect, GARCH-family models, Extreme value theory, Value at risk, expected shortfall and Copula function.

I. INTRODUCTION

Nowadays due to the acceleration in globalization of markets, the rise in fluctuation of currencies exchange rate has become frequent and consequently affecting volatility in exchange rate of currency. Frequent changes in the foreign exchange market have a high impact on firms' operations and profitability; this does not only affect multinational firms but also firms operating in their home countries.

This rise in volatility of the currency market implies the high risks for investors, but it may provide more opportunities of earning high profits. Many traders engage themselves in highly volatile market looking for bigger profit; it requires attention using good statistical tools and methods, unless this can lead to a serious loss.

Due to unexpected significant fluctuations in these few past years in the currency market, many economists have questioned the existence of a good risk management methodology to solve these financial instabilities, this is why

a huge effort has been invested into developing statistical method to protect financial systems against unpredictable fluctuations and losses.

Embrechts et al. [4] have proven that, leptokurtosis behaviour and volatility clustering of financial data makes inappropriate to apply EVT approach to return series directly.

McNeil [8] Engle demonstrated that by combining GARCH models with extreme value theory (EVT) can be successfully applied in financial market to predict the risk for a financial system McNeil [8]. The EVT refers to the branch of statistics, which deals with the extreme deviations from the mean of a probability distribution. EVT in finance deals with tails of a distribution to evaluate the highest loss of a financial market with certain probability over a certain time horizon, as is applied to event with a very low probability of occurrence. This paper shows how we can model use GARCH based dynamic EVT to model the two risk measures VaR (value at risk) and ES (expected shortfall) for a short term forecasting. The value at risk refers to the amount risked over some period with a fixed probability. since VaR is considered as the measure of tail risk it shows the degree of sensitivity to the

financial market loss; In practice it provides a loss threshold exceeded with some small predefined probability usually 1% and 5%; in other word it shows the maximum loss that can't be exceeded with a certain level of confidence in a given period of time.

Large, small and medium-sized enterprises are influenced by the currency volatility. This shows how important is to analyse, understanding and deal with foreign exchange risk to business owners, policy makers and investors because of the huge influence it can have on their investment. This paper proposes a methods that takes into consideration volatility in returns by focusing on tail behaviour of a portfolio and predict the risk measures (Value at Risk and Expected Shortfall) and loss as well using GARCH-EVT-COPULA approach. It uses daily data of currency exchange rate using Rwandan francs vs US dollars(USD), vs euros(EUR), vs Chinese yuan (CYN) and vs Kenyan shillings (KES) using data from 13th march 2010, up to 21st April 2017.

This research is organized as follow. Part one and part two consists of introduction and review of literature, part three consists of methodology in which we introduce volatility modeling, extreme value theory by using on POT method from GPD and give more detail on Copula decomposition of a general multivariate distribution, its parameters estimation used and how to predict risk measures. In part four we carry out data analysis; by combining GARCH model with EVT then build a Copula-GARCH-EVT to use in estimation of VaR and ES. In part, five we give conclusion and recommendations on findings.

II. RELATED WORK

The ARCH model was first introduced in Engle [5], for capturing time variant variance exhibited by almost all-financial time, series and many economic time series. The generalized version of ARCH model (GARCH model) which gives more parsimonious results than ARCH model was formulated by Bollerslev [2] and Engle, R. & Nelson D. [5]. This paper uses GARCH-family model that allow for capturing asymmetric shocks which is GJR-GARCH (Glosten-Jagannathan-Runkle GARCH) model introduced by Glosten et al. [9].

The use of extreme value theory has become popular in finance, after its publication in some papers such as Embrechts et al [4], Bensalah [19] and Brodin and Klüppelberg [3]; results showed that extreme value theory methods fit the tails of heavy-tailed financial time series better than more conventional distributional approaches and that it was the best approach in estimating the tail of a loss distribution. In 1990s due to the currency crisis, stock market turbulence and credit default many research studies such as Gilli and Këllezzi [8] and Bensalah [19], showed the potential of EVT approach in

finance and illustrated EVT using block maxima method(BMM) and peak over threshold (POT) in modelling VaR, ES and return level. Result showed that POT was considered more efficient in modelling limited data and not depending on the requirement for large data set as BMM because it exploits better the information in sampling.

This research will apply Copula to model the inter-dependence between innovations of the return series, which can characterize various dependence structures comprehensively and effectively.

Skylar [17] first proposes Copula to measure the non-linear inter-dependence between variables. After that, Schweitzer and Wolff, Genest and Mackay, Joe. H., Nelsen and so on further develop and improve Copula and it has become an important approach to construct multivariable joint distribution and capture dependence structure between variables. The use of copula started with Embrechts et al. [4] who first introduced Copula to financial research; Cherubini [11] systematically summarizes the applications of Copula in finance; Jondeau and Rockinger [10] proposed Copula-GARCH model and apply it to extract the dependence structure between stock markets. In China, Zhang [18] first introduces Copula; Wei and Zhang [20] apply Copula-GARCH model to extract the dependence structure between Shanghai Stock Market and Shenzhen Stock Market; Wu and Chen [18] apply Copula-GARCH model to analyse portfolio risk in Chinese Stock Market; however, most empirical researches about Copula have focused on capturing bivariate inter-dependence structure. The reason is that the complexity of multivariate Copula increases sharply along with its dimensions and it is inclined to ignore the influence of its dimensions and the differences of tail dependence between variables.

III. METHODOLOGY

III.1 Modeling volatility using GARCH-family models

This paper uses the GJR-GARCH model combined with ARMA model to model volatility.

Glosten-Jagannathan-Runkle GARCH(1,1) it allows for asymmetry effects in volatility modelling which is used to handle leverage effects. Assuming that a portfolio consists of n assets, the return series for asset i is $\{y_{i,t}, t=1,2,\dots\}$. Then GJR-GARCH (1, 1) can be written as:

$$y_{i,t} = \mu_i + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} = \sigma_{i,t} Z_{i,t}, \varphi Z_{i,t} \sim i.i.d$$

$$\alpha_{i,t}^2 = w_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i h_{i,t-1} \varepsilon_{i,t-1}^2 \quad \text{Eq. (1)}$$

Where α_i and β_j are respectively ARCH and GARCH terms for return series and W_i is a constant; $\sigma_{i,t}^2$ and $\sigma_{i,t-1}^2$ are respectively the fitted conditional variance from the model and its previous value. $\varepsilon_{i,t}^2$ Are the squared residuals terms of the model.

$h_{i,t-1}$ Is an indicator function for return series i that takes the value of 1 if $\varepsilon_{i,t-1} \leq 0$ and 0 otherwise; γ the asymmetric parameter.

Typically, $Z_{i,t}$ follows a fat-tailed distribution.

In the empirical studies, we assume $Z_{i,t}$ follows Student t distribution.

III.2 Extreme risk modelling

The extreme value theory has two results: the use the block maxima model (BMM) which fit the generalized extreme value distribution and the peak over threshold (POT) which fit the Generalized Pareto Distribution (GPD). The advantage of POT over BMM model is that in PTO all data which exceed the fixed threshold are used while in BMM they use only the maximum data in a block is selected.

Pickands-Balkema theorem:

Given a large class of distribution function F_u tend to fit the GPD for an increasing threshold u . Then:
 $F_u(y) \approx G_{\xi,\sigma}(y), \quad u \rightarrow \infty.$ **Eq. (2)**

Where $G_{\xi,\sigma}$ is the Generalized Pareto Distribution (GPD) which is summarized and given by the equation below.

$$G_{\xi,\sigma}(y) = \begin{cases} \left(1 - \frac{\xi}{\sigma} y\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}} & \text{if } \xi = 0 \end{cases} \quad \text{Eq.(3)}$$

Where ξ is the shape parameter and σ is the scale parameter for the GPD (Generalized Pareto Distribution).For

$$y \in [0, (x_F - u)] \text{ if } \xi \geq 0 \text{ and } y \in \left[0, -\frac{\sigma}{\xi}\right] \text{ if } \xi < 0.$$

VaR and Expected Shortfall in GPD

Here the VaR and ES are the functions of estimated parameter of the GPD. If F is an extreme distribution with the right endpoint X_F , we can assume that for some threshold u

Then the $F_u(x) = G_{\xi,\sigma}(x)$ where $0 \leq x < x_F - u$ and $\xi \in \mathbb{R}$ and $\sigma > 0$. If $x \geq u$, then:

$$\begin{aligned} \bar{F}(x) &= P(X > u)P(X > x \mid X > u) \\ &= \bar{F}(u)P(X - u > x - u \mid X > u) \\ &= \bar{F}(u)\bar{F}(x - u) \\ &= \bar{F}(u)\left(1 + \xi \frac{x - u}{\sigma}\right) \end{aligned} \quad \text{Eq. (4)}$$

Given $\bar{F}(u)$, $\bar{F}(x)$ is the formula for tail probabilities, its inverse gives the highest quantile of the distribution which represent the value at risk VaR and is given by:

$$VaR_\alpha = q_\alpha(F) = u + \frac{\sigma}{\xi} \left(\left(\frac{1 - \alpha}{\bar{F}(u)} \right)^{-\xi} - 1 \right) \quad \text{Eq. (5)}$$

For $\xi < 1$, the expected shortfall ES is given by:

$$ES_\alpha = \frac{1}{1 - \alpha} \int_\alpha^1 q_x(F) dx = \frac{VaR_\alpha}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \quad \text{Eq. (6)}$$

If n is the total observation and N_u the number of observations above the threshold u . If replace F_u with $G_{\xi,\sigma}(x)$ and $F(u)$ by $(n - N_u)/n$, the tail distribution is estimated by:

$$\hat{F}(x) = 1 - \frac{N_u}{n} \left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x - u)\right)^{-1/\hat{\xi}} \quad \text{Eq. (7)}$$

The inverse of the estimator for the tail distribution with a probability p gives the estimator of parameters $\hat{\sigma}$ and $\hat{\xi}$ which are used in estimating VaR and ES, by replacing them respectively in equation (2.22) and (2.23). And the ES with the probability p is given by:

$$ES_p = \frac{VaR_p}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \quad \text{Eq. (8)}$$

III.3. Modeling the dependence structure

Definition1. A d -dimensional copula is a function that links or married univariate marginal distribution.

A d -dimensional copula is a multivariate cumulative distribution function $C : [0, 1]^d \rightarrow [0, 1]$, whose margins have the uniform distribution on the interval $[0, 1]$.

The following theorem is a very significant result in the copula theory.

Skyklar's theorem. Let F denote a d -dimensional distribution functions with marginal distribution functions

$$F_{X_1}, \dots, F_{X_d}. \text{ Then, there exists a copula } C, \text{ such that}$$

$$F(x_1, \dots, x_d) = C(F_{X_1}(x_1), \dots, F_{X_d}(x_d)) \text{ for any } (x_1, \dots, x_d) \in R^d.$$

In addition, we have that, if F_{X_1}, \dots, F_{X_d} are continuous, then the copula C is a unique one.

Conversely, if C is a copula and F_{X_1}, \dots, F_{X_d} are distribution functions, then the function F , defined by (3.19), is the joint distribution function with marginal distribution functions F_{X_1}, \dots, F_{X_d} .

Below, we present the four families of copulas used in our paper, namely: normal copula, student t-copula, Plackett copula and Clayton copula.

III.3.1 The bivariate normal copula

The bivariate normal copula is the function of the form:

$$C(u_1, u_2; \rho) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{r^2 - 2\rho rs + s^2}{2(1-\rho^2)}\right\} dr ds \quad \text{Eq. (9)}$$

Where ρ is the linear correlation coefficient between the two random variables and Φ^{-1} stands for the inverse of the univariate standard normal distribution function.

III.3.2. The bivariate Student t-copula

The bivariate student t-copula is given by the following function:

$$C(u_1, u_2; \rho, \nu) = \int_{-\infty}^{t_v^{-1}(u_1)} \int_{-\infty}^{t_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{1 + \frac{r^2 - 2\rho rs + s^2}{\nu(1-\rho^2)}\right\}^{-\frac{(\nu+2)}{2}} dr ds$$

Eq. (10)

Where ρ is the linear correlation coefficient between the two random variables and t_v^{-1} denotes the inverse of the univariate Student- t distribution function with ν degrees of freedom.

III.3.3. The bivariate Plackett copula

The bivariate Plackett copula is given by:

$$C(u_1, u_2; \theta) = \frac{1}{2(\theta-1)} \left[1 + (\theta-1)(u_1 + u_2) - \left([1 + (\theta-1)(u_1 + u_2)]^2 - 4u_1u_2\theta(\theta-1) \right)^{1/2} \right]$$

$$C(u_1, u_2; \theta) = u_1u_2, \text{ For, } \theta = 0 \quad \text{Eq. (11)}$$

Where θ stands for the given parameter value.

III.3.4. The bivariate Clayton copula

The following function is called the bivariate Clayton (or Cook Johnson) copula:

$$C(u_1, u_2; \theta) = \max\left\{u_1^{-\theta} + u_2^{-\theta} - 1, 0\right\}^{-1/\theta} \quad \text{Eq. (12)}$$

III.4 Estimation of aggregated loss and VaR

Finally after the marginal and dependence structure modeling separately, Value at Risk (VaR) can be estimated based on the selected best GJR-GARCH Copula model. The steps are described as follows:

Step1: we use Monte Carlo simulation, where we generate a large $N=565$ from the fitted copula.

Step2: Quantile-transform to standardized t distributions. Given that the simulated numbers come from copulas which are defined on the interval $[0,1]$ uniform distribution, we are required to transform the numbers into the original scales of returns. Which is $X_{1,t}, X_{2,t}, X_{3,t}$ and $X_{4,t}$ that represents the simulated returns of each corresponding margins.

Step3: Use these multivariate dependent t innovations to sample from the time series Calculate the portfolio return which is given by:

$$X_t = w_1X_{1,t} + w_2X_{2,t} + w_3X_{3,t} + w_4X_{4,t} \quad \text{Eq. (13)}$$

Where $w_i = 1/4$ stand for weight of each simulated returns.

Step4: the aggregated loss function is given by:

$$L_{t,N} = w_1X_{1,t}(e^{R_{1,N}^{-1}}) + w_2X_{2,t}(e^{R_{2,N}^{-1}}) + w_3X_{3,t}(e^{R_{3,N}^{-1}}) + w_4X_{4,t}(e^{R_{4,N}^{-1}})$$

Eq. (14)

Step5: Finally, we calculate the value at risk of a portfolio. The one-day horizon VaR at time t .

The main objective of this paper is to predict the aggregated loss and the value at risk in the Rwandan stock exchange market. The analysis of this is done in the following steps:

- (1) Filtered Residuals from each stock closing values using asymmetric GARCH
- (2) Kernel Gaussian to construct sample marginal cumulative distribution function (CDF)
- (3) Generalized Pareto distribution (GPD) to estimate the lower and upper tails.

- (4) Student's t copula is fitted to the data and induced correlation between simulated residuals of each asset using Monte Carlo Simulation.
- (5) Calculated the Value at Risk (VaR) and expected shortfall (ES) of the forex exchange portfolio over the horizon of one day and 10 days.

Mean	0.003	0.001	0.000	0.006
std	0.012	0.017	0.010	0.011
kurtosis	15.95	62.77	24.76	17.44
skewness	-0.4601	-0.1123	-0.3541	-0.1851
minimum	-0.1310	-0.1341	-0.1306	-0.1309
maximum	0.1310	0.1307	0.1312	0.1317
Jarque-bera test	433050 (0.00)	361600 (0.00)	396710 (2.2e-16)	304460 (2.2e-16)

IV. DATA ANALYSIS AND DISCUSSION

In this paper, I analysis the time-varying dependence structures of four representative currency in the Rwanda forex exchange asset by modelling each asset residuals extracted from GJR-GARCH combined with ARMA model, then the research apply GPD model to residuals, finally the t-copula is fitted on data to extract the dependence structure between forex exchange stock . Second, I calculate risk measures such as VaR of the portfolio from fitted and time-varying t-copula. Lastly, we predict the VaR and the loss in the market.

This study used negative log-returns extracted from a GJR-GARCH model which reveals the existence of volatility clustering as shown in Figure1.

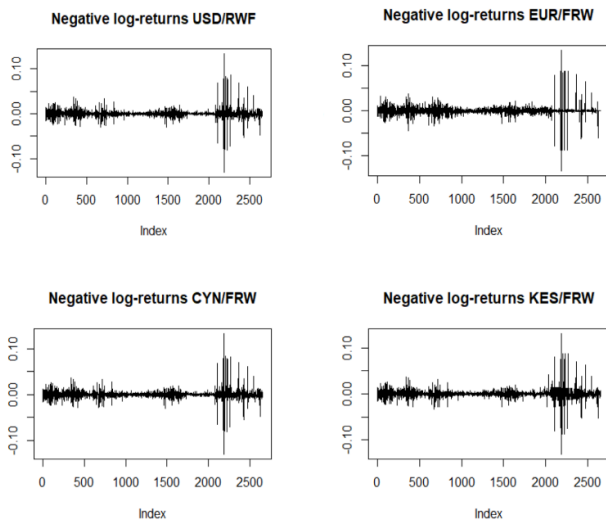


Fig 1. Negative log-returns for the portfolio

This paper choose the four indexes of the Rwandan forex market using Rwanda francs(RWF), Euros(EUR), Chinese yuan(KES) and Kenyan shillings(KES). To analyse the loss in this market we use negative log-returns. Table 1, shows summary statistics of log returns of each index.

The table below shows the sumary statistics of 4 indexes.

Table 1: Summary statistics of log-returns

Variables	EUR	USD	CYN	KES
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In Table1, reasults shows that the kurtosis of each index is larger than 3 and the skewness is less than 0, wich means that all the series have fat tails and are leptokurtosis. Returns are negatively skewed and marginals are not distributed normally. This allows the use of the GPD on volatility residuals.

4.1 Modelling volatility log returns

TABLE 2. Estimates of the ARMA(1,1) with GJR-GARCH(1,1) for log return series with standardized t residuals.

Table 2: estimated parameter from ARMA and GJR-GARCH model.

series	USD	EURO	CYN	KES
μ	0.0011	0.0031	0.0057	-0.0046
ar_1	0.5819	0.35042	0.3432	0.3130
ma_1	-0.8858	-0.4549	-0.5667	-0.4622
c_o	0.00004	0.00003	0.0001	0.0004
α	0.04452	0.43969	0.1533	0.1765
β	0.94370	0.55775	0.8451	0.8546
γ	0.02155	0.00311	0.0011	-0.0646
δ	1.0021	2.89075	3.0868	2.7843

After ARCH effects are tested on squared residuals to check whether the time series model exhibit autocorrelation, the results show that all p-values of the Ljung-Box and ARCH LM tests are greater than 0.05, which indicates that the heteroscedasticity and autocorrelation of each series are removed. There is no autocorrelation in the series. Squared residuals from GJR-GARCH model.

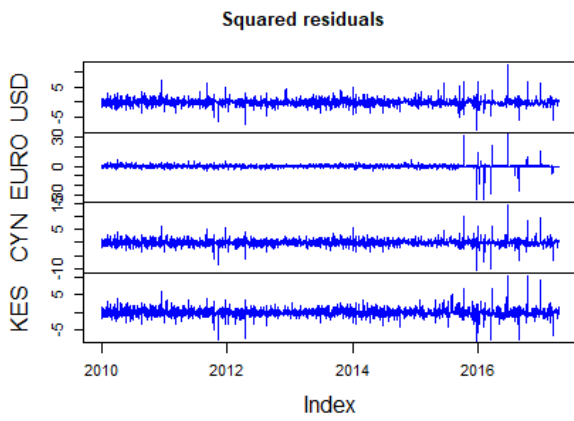


Fig 2. Residuals extracted from the fitted GJR-GARCH model

The qqplot are shown of squared residuals are shown in the Figure 3 below.

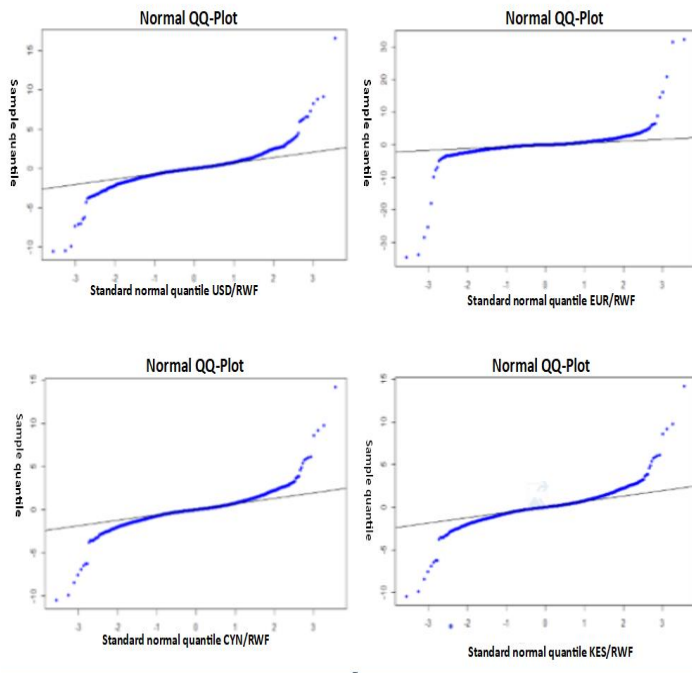


Fig 2. Q-Q plot for residuals

The squared residual in figure 3 show sign of normality but shows the presence of fat tails in the portfolio. In the next stage this paper use PTO method to model the tail behaviour.

4.2 Modelling tail behaviour

In order to estimate the marginal distribution of each series of standardized residuals we first analyse the empirical result of the GJR-GARCH and the GPD model. In this study, we use POT method to model the tail distribution of the residuals. The fitted scale and shape parameter are shown in table3 below.

Table 3: estimated parameter from GPD, and risk measures of each market.

	USD	EUR0	CYN	KES
Scale-parameter	0.242	0.592	0.3643	0.3526
Shape parameter	0.8337	0.6962	0.6541	0.6063
Threshold At 95%	1.6546	1.65468	1.6546	1.65468
ES _{99%}	3.3674	4.675731	5.9339	4.8467
VaR _{99%}	1.4285	3.640507	4.0274	3.9456

We test the fitness of GPD through excess distribution, results reveals that the standardized residuals fit well our series. The graph shows that approximately 10% of the standardized residuals used, the fitted distribution closely follows the exceedances data, so the GPD model seems to be a good choice

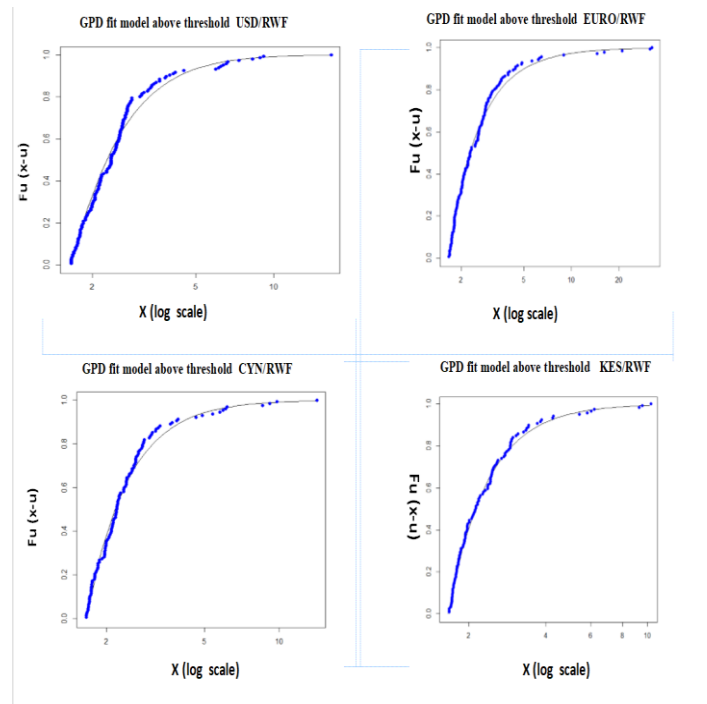


Fig 3. GPD fit model above threshold (excess distribution)

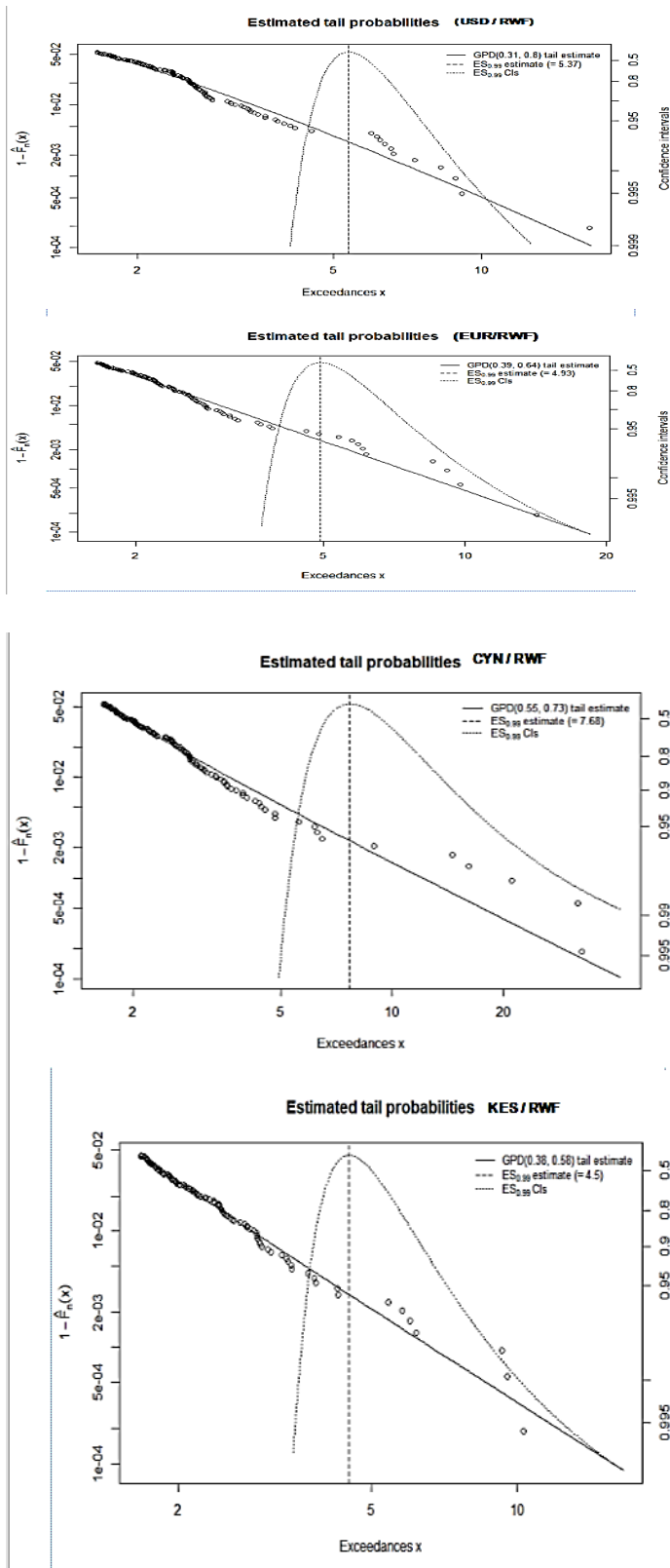


Fig 4. Estimated tail probabilities of the 4 stocks

4.3 Dependence Structure Modeling

Finally, I model the margins with copulas. Since the margins of the copulas are considered to be in uniform distribution, the

transformation of the marginal standardized residuals into uniform distribution should be done before modeling.

In this section, we consider only three types of copula functions (Gaussian copula, t-copula and Gumbel copula) and we choose the well-fitted copula. Then by using the estimation method, the dependence models can be estimated. The results are summarized in Table 4.

Table 4: estimated parameter from the 3 copula

Copula type	parameter	AIC	GoF(p-value)
Gaussian	$\rho_{Gauss} = 0.87332$	-892.1	0.06874
t-student	$\rho_t = 0.92300$	-2129.1	0.00048
Gumbel	$\rho_{Gum} = 1.807$	-1301.2	0.00238

Table4. Shows that the well-fitted type of copula model is t-student copula. Because it has the smallest AIC compared to other models. The goodness of fit shows that t-copula is significant at $\alpha = 0.005$.

The degrees-of-freedom parameter from the fitted t-copula is found to be $\nu = 2.69$. The estimated correlation matrix from a fitted t-copula is summarized in Table 5 below.

	USD	EURO	CYN	KES
USD	1.00000	0.507851	0.948680	0.80848
EUR	0.50785	1.000000	0.521364	0.50511
CYN	0.94868	0.521364	1.000000	0.77696
KES	0.80848	0.505109	0.776959	1.00000

The matrices of pairwise Kendall's tau and upper tail-dependence coefficients below shows there is no much concordance for for bivariate random vectors.

$$\begin{pmatrix} 1.000 & & & & \\ 0.339 & 1.000 & & & \\ 0.795 & 0.349 & 1.000 & & \\ 0.599 & 0.337 & 0.566 & 1.000 & \end{pmatrix}$$

4.5 estimation of risk measures and loss

4.5.1 Estimation of VaR and ES using t-copula

Finally, given the simulated returns of each index, we form a 1/4 equally weighted index of our portfolio and calculate the VaR and ES at 95% and 99% confidence levels, over the 10 days risk horizon.

Table 5: VaR and ES from the fitted t-copula

VaR & ES	VaR _{99%}	VaR _{95%}	ES _{99%}	ES _{95%}
	4.72%	1.23%	10.1%	6.97%
Backtesting (p-value)	0.08	0.030	0.098	0.01

After backtesting VaR and ES, results shows that both are significant at critical level of $\alpha=0.05$.

4.5.2 Estimation of loss in portfolio

In this study we simulated paths from the full model using the fitted t copula predict the aggregated loss using confidence level of 99% (non-parametrically).

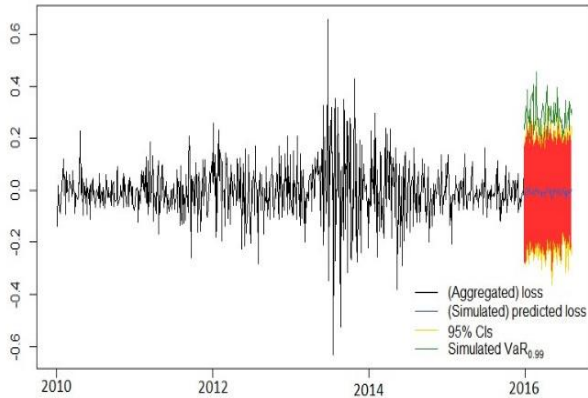


Fig 5. Comparison between estimated loss, predicted loss and predicted VaR

In figure 5 the simulated predicted loss and aggregated loss shows that they fit the model. The maximum predicted loss is found to be 15.97% while the minimum predicted loss is 4.39%.

Predicted loss(simulated)	t-copula
Maximum Loss	15.97%
Maximum Gain	4.39%

5. Conclusion

This study estimate the value at risk and the expected shortfall for the 4 foreign exchange market USD/RWF, EURO/RWF, CYN/RWF and KES/RWF in two ways. The first was to estimate VaR and ES using individual currency (by modeling residuals tail from GJR-GARCH model the tail using GPD(Generalized Pareto Distribution) and the second was to estimate VaR and ES using copula function, where the copula shows that it works better than individual estimation. For copula function, we worked with 3 type of copula and the best copula to use was found to be the t-copula.

References

[1] Akgiray, V. and G. G. Booth. "The Stable-law Model of Stock Returns" *Journal of Business & Economic Statistics*, 1988, Vol. 6, No. 1, pp. 51–57,

1988.

[2] T.BOLLESLEV," Generalized autoregressive conditional heteroscedasticity ", *journal of econometrics*, 31, pp.307-327, 1986.

[3] Brodin, E., and Klüppelberg, C., "Extreme Value Theory in Finance", *Encyclopedia of Quantitative Risk Assessment*, 2006.

[4] Embrechts, P., Klüppelberg, C. & Mikosch, T. Modeling Extremal events for insurance and finance, *Springer*, Berlin, 1997.

[5] Engle, R. & Nelson D. B." ARCH models". In *Robert F. Engle and Daniel McFadden (Eds.) Handbook of Econometrics*, (pp. 2959-3038), 1994.

[6] Fernandez, C. and Steel, M.F.J. "On Bayesian modeling of fat tails and skewness", *journal of the American statistical Association*, 93, No 441pgs.359-371, 1998.

[7] Gencay, R. and Selcuk, F. "Extreme value theory and Value-at-risk: relative performance in Emerging markets," *International Journal of Forecasting*, 20:2, pp. 287-303, 2004.

[8] Gilli, M., & Këllezi, E. "An application of Extreme value Theory for measuring financial risk". *Computational Economics*, 27 (2), 207-228, 2006.

[9] Glostten, L., R. Jagannathan, and D. Runkle "On the Relation between Expected Return on Stocks", *Journal of Finance*, 48, 1779-1801, 1993.

[10] Jondeau, Eric and Rockinger, Micheal. "The copula-GARCH model of conditional dependencies: An international stock market application." *Journal of International Money and Finance*, 25, no.5 pp. 827-853, 2006.

[11] U.Cherubini, E.Luciano," Value-at-risk trade-off and capital allocation with copulas", *Economic Notes*30, pp.235–256, 2001.

[12] McNeil, A., & Frey, R. "Estimation of Tail Related Risk Measure for Heteroscedastic Financial Time Series", *An Extreme Value Approach. Journal of Empirical Finance* 7, pp.271- 300, 2000.

[13] McNeil, Alexandre J. Frey, Rudiger, and Embrechts, Paul. "Quantitative risk Management: Concepts, Techniques and Tools", Princeton New Jersey: Princeton University Press, 2005.

- [14] Nelsen, R.B. *an Introduction to Copulas, Lectures Notes In Statistics*, 139, Springer Verlag, New York, 1999.
- [15] Nurrahmat, Bachrudin, "Estimation of Value at Risk in Currency Exchange Rate Portfolio Using Asymmetric GJR-GARCH-Copula". *American Institute of Physics: AIP Publishing*. 978-0-7354, 2017.
- [16] Pickands III, James. "Statistical inference Using extreme Order statistics," *Annals of Statistics*, Volume 3, (1975): pp. 119-131, 1975.
- [17] Skylar, A. "Fonctions de Repartition à n Dimension et Leurs Marges," *Publ.Inst. Statist.* Paris, 8, pp. 229-231, 1959.
- [18] X. Zhang, "Copula and financial risk analysis". *Statistic Research*.4, pp. 48-51, 2002.
- [19] Y. Bensallah, *asset allocation using extreme Value theory*, bank of Canada, Ottawa Ontario Canada K10G9, 2000.
- [20] Y. Wei, S. Zhang, Y. Guo, "Research on degree And patterns of Dependence in financial markets". *Journal of System Engineering*. 19 pp.355-362, 2004.