Approach of Crashing Critical Activity (CCA) and Stretching Non-critical Activities (SNA) of Time Cost Trade off problems using Critical Path Method

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Abstract: The three important objectives of any project are time, cost and quality. In today scenario delivering projects in least possible time at least cost within maximum available budget has got a critical issue to the manager. Firstly, two different approaches crashing critical activities (CCA) and stretching non critical activities (SNA) are compared to illustrate the gauge of which approach to be used to complete the project within the maximum available budget and within least number of steps. Depending on five construction projects it is observed that the SNA approach is better than CCA approach. Secondly, evaluation of projects with respect to the criteria is generated. Next these evaluations are compared with respect to the criteria. Finally, the interactive technique is used for the selection of most desirable project. A numerical example is presented to illustrate the approaches and selection of desirable project out of the given projects.

Index Terms: Critical path, Crashing activities, Stretching activities, Multi criteria decision problem, Stochastic dominance

I. Introduction

Tradeoff between project duration and total cost are extensively discussed in the project scheduling literature because of its practical significance. It is generally realized that when project duration is compressed, the project requires more labor, increase more productive equipment, demanding procurement and construction management and at the same time the cost will also increase. The main objective of critical path method is to establish a feasible and desirable relationship between the time and cost of the project by decreasing the target time and taking into account the cost of expending as presented by Taha (1) .Several studies conducted on critical path method for time cost trade off , like Anagnostopoulos (2) Ke at al (3) claimed in the real life projects, both trade off between project cost and project completion time and the un certainity of the environment are considerable aspects for decision maker.

The two different approaches are presented for time cost trade off to complete the project in shortest possible duration at smallest cost within the available budget. They are Crashing Critical Activities (CCA), Stretching Non- Critical Activities (SNA).The first approach was developed along with critical path method for planning and controlling large scale projects. Then crashing is selecting the lowest cost slope activities which shorten the critical path, this procedure is repeated until the project has been reduced sufficiently or the cost to shorten the project exceeds the benefits. The second approach will reduce the cost of the project by stretch or increase any non – critical activity by certain amount without extending the project, and the non – critical activities can be stretched until all the slack in the network is used up. Stretching non – critical activities crash all the activities in the project parallel to find the shortest possible project duration by avoiding the intermediate steps.

These approaches have been applied on five different construction projects as mentioned in table-I below. Where the data is collected from various companies, where the companies are specialized in designing and execution of bridges, housing complexes, towers, industrial buildings, plants, dams, etc,.. These companies are willing to construct the new projects, with maximum available budget within the given time. Various heuristically approaches were presented by Siemans (4) and Moselhi and Deb (5). Multicriteria techniques are widely used in project scheduling problems. Nowak (6) proposed a technique based on simulation model, stochastic dominance rules and multi criteria aggregation procedure. In paper (7) a new discrete resource time cost quality constrained trade off problem, renewable and non-renewable resources are considered. And in paper (8) the interactive technique is used for the selection of the most desirable resources allocation.

II. Problem Formulation

Crashing critical activities as discussed earlier finds step – by – step activities to reduce the project completion time. This stepwise reduction of the project duration eventually leads to shortest possible project duration and its associated cost. However, if we want to directly find the shortest possible project duration by avoiding the intermediate steps, a simpler procedure represent with stretch non – critical activities which simultaneously crash all activities at once. This also yields the shortest project duration. However, the

expenses of crashing all activities needs more amount, as it is necessary to crash activity to finish the project in shortest time. So we must stretch non – critical activities to reduce the cost of the project completion.

Both the approaches aims to reduce the project duration at the smallest total cost, and these approaches identifie critical and non - critical activities. The normal time and cost as well as crash time and cost must be found for all activities in order to find the cost slope for the two approaches. These two approaches finds the expected project completion time within maximum available budget. Crashing critical activities is the expression of crashing the critical path activities to reduce the completion time of project duration gradually, while stretching non – critical activities crash all activities simultaneously to yield the shortest possible duration of project completion and stretching the non- critical activities to reduce the total cost. Crashing critical activity starts to crash the critical activity that has the least cost slope to reduce the project duration at least cost. While stretching non – critical activity depends on the stretching non-critical activity that has greatest cost slope to obtain the greatest saving. Crashing critical activity can obtain the desired project completion time at least cost and also obtain the shortest possible duration to complete the project at least cost within the maximum available budget, while stretching non – critical activity obtains only the shortest duration at least cost within the maximum available budget. Stretching non – critical activity requires less computation as compared to crashing critical activity because crashing critical activity will be by virtue of crashing one time unit from critical activity that has smallest cost slope.

The procedure proposed for the desirable project consists of three steps. First, evaluations of projects with respect to the criteria are generated. Next, these evaluations are compared with respect to the criteria. Finally, the interactive technique is used for the selection of the most desirable resource allocation. The steps required to perform the analysis are , the approach uses the evaluations of projects with respect to criteria. One of the most important point is identifying appropriate probability distribution for the input data. Usually, it requires analyzing the data and fitting these data to distributions. Sometimes, however, such data are not available and an appropriate distribution has to be selected according to the decision maker's or expert's judgment. Once the project evaluations are obtained, the relations between the projects with respect to the criteria can be analyzed. Two methods are usually used for comparing uncertain outcomes: mean risk analysis and stochastic dominance. The former is based on two criteria: one measuring expected outcome another representing variability of outcomes. In the stochastic dominance approach random variables are compared by point wise comparison of their distribution functions. In this paper both techniques are used. While the stochastic dominance is employed for constructing rankings of projects with respect to each criterion, mean risk technique is used when a final solution is chosen.

III. Notations

x_{ijk} = 1 if activity (i,j) is executed in mode k
        = 0 otherwise

m= number of modes

t_{ijk} = duration of activity (i,j) in mode k

r_{ijk} = the amount of renewable resource r

P_r = price of the renewable resource r

c_{ijk} = cost of the activity $(i,j) = t_{ijk} \times r_{ijk} \times P_r + the\ cost\ of\ non\ renewable\ resources$

q_{ijk} = quality of activity (i,j) in mode k

M_T = maximum time to complete the project

B_{max} = maximum available budget

R is the project activities

T is the shortest possible duration to complete the project at least cost within the maximum available budget

F is the desired project completion time at least cost

N_T is the normal time for an activity

C_T is the crash time for an activity

N_c is the normal cost for an activity

C_c is the crash cost for an activity

U_i is the cost slope for an activity

CP is the critical path

TC_N is the total cost to complete the project in normal conditions

NC_j is the normal cost for non – critical activity j, where j = *1,2,…*

TC_a is the total cost complete the project by crashing all activities (crash conditions)

TE_c is the extra cost that added to all crash critical activities

D_{rs} is the maximum reduction in duration for critical activity S

U_s is the cost slope for the critical activity S

TC_c is the total cost to complete the project using obtained critical activity

TSC_n is the total saving cost obtained by stretching critical activity

D_{rx} is the maximum stretching in duration for non- critical activity x

l_{ij} = lower bound for quality of activity *(i,j)*

In normal conditions the critical path of project network is given by

$$CP = \sum_{q=1}^{L} N_{Tq}$$

Also the total cost in normal conditions is given by

$$TC_N = \sum_{j=1}^{n} N_{Cj}$$

The total cost of the project after crashing

$$TCa = \sum_{i=1}^{n} C_{Ci}$$

Critical path of the project after crashing

$$CPc = \sum_{q=1}^{L} CTq$$

The total cost after crashing the critical activity

$$TC_C = TC_N + TC_E$$

The total extra cost that adding to crash critical activities

$$TE_c = \sum_{s=1}^{Y} D_{rs} U_s$$

Total saving cost obtained by stretching non-critical activities

$$TSC_n = \sum_{xs=1}^{z} D_{rx} U_x$$

Total cost of the project within the given time by stretching non- critical activity

$$TCS_n = TC_a - TSC_n$$

Three mathematical models using 0-1 integer linear programming are

(i) $Max\ z_1 = \left[\prod_{(i,j) \in A} \left[\sum_{k=1}^{m} q_{ijk} x_{ijk} \right] \right]^{\frac{1}{|A|}}$,

Subject to constraints

$$\sum_{k=1}^{m} x_{ijk} = 1$$

$$\sum_{u \in N_J} u y_{ju} - \sum_{w \in Ni} w y_{iw} \geq \sum_{k=1}^{m} t_{ijk} x_{ijk}$$

$$\sum_{u \in Nn} u y_{nu} \leq M_T$$

$$\sum_{(i,j) \in A} \sum_{k=1}^{m} c_{ijk} x_{ijk} \leq B_{\max} ,$$

$$\sum_{k=1}^{m} q_{ijk} x_{ijk} \geq l_{ij}$$

where (i,j) is an activity , $x_{ijk} = 0$ or 1 for each activity(i,j) executed in each mode k and $y_{iu} = 0$ or 1 for each event i and for each $u \in N_i$,

(ii) $Min\ Z_2 = \sum_{k=1}^{m} c_{ijk}\ x_{ijk}$

Subject to

$$\sum_{k=1}^{m} x_{ijk} = 1$$

$$\sum_{u \in N_J} u y_{ju} - \sum_{w \in Ni} w y_{iw} \geq \sum_{k=1}^{m} t_{ijk} x_{ijk}$$

$$\sum_{u \in Nn} u y_{nu} \leq M_T$$

$$\sum_{k=1}^{m} q_{ijk} x_{ijk} \geq l_{ij}$$

Where (i,j) is an activity , $x_{ijk} = 0$ or 1 for each activity(i,j) executed in each mode k and

$y_{iu} = 0$ or 1 for each event i and for each $u \in N_i$,

(iii) *Min z_3* = $\sum_{u \in N_n} u y_{nu}$

$$\sum_{k=1}^{m} x_{ijk} = 1$$

$$\sum_{u \in N_J} u y_{ju} - \sum_{w \in Ni} w y_{iw} \geq \sum_{k=1}^{m} t_{ijk} x_{ijk}$$

$$\sum_{(i,j) \in A} \sum_{k=1}^{m} c_{ijk} x_{ijk} \leq B_{\max} ,$$

$$\sum_{k=1}^{m} q_{ijk} x_{ijk} \geq l_{ij}$$

where (i,j) is an activity , $x_{ijk} = 0$ or 1 for each activity(i,j) executed in each

mode k and $y_{iu} = 0$ or 1 for each event i and for each $u \in N_i$.

IV. Solution Procedure

Firstly, applying both the approaches to the construction project by drawing the project network, computing the cost slope for each activity and crashing all the activities simultaneously. Once again, draw the project network after crashing all the activities and determine the critical path, critical activities and non critical path, now compute the new total cost after crashing all activities in the project. Start with those non critical activities that yield the greatest savings those with the greatest cost slope. The non critical activities can be stretched up to their normal time until all the slack in the different non critical paths network is used up and then the saving cost of stretching all non critical activities are found and finally cost of project completion within maximum available budget is computed. And two approaches are compared for minimum number of steps to complete the project. Secondly, stochastic dominance and multi criteria decisions are used to select the best desirable project.

V. Numerical Example

Five different construction projects have been selected for illustration of above

Table I : The five different construction projects

Project No.	Project Type	Project Name	Start date of the Project	Expected Completion time
1	House Construction	SSR Constructions	01/01/2011	30 weeks
2	Plant Construction	VGR Constructions	20/01/2011	27 weeks
3	House Construction	Reddy Constructions	10/02/2011	29 weeks
4	Plant Construction	NNR Group	01/01/2011	80 weeks
5	Plant construction	SiddiVinayaka constructions	11/03/2011	70 weeks

The Project-I is considered to illustrate the above said data (Housing construction-SSR Constructions) is summarized in Table-II in which there are 11 activities with maximum available budget ₹. 78,500,with shortest possible duration of 17 weeks to complete the project at least cost within maximum available budget and desired project completion time is 18 weeks

Table II: Data obtained from Project I

SNo	Activity	Predecessor	Normal Time	Crash Time	Normal Cost	Crash Cost	Max Reduction in time	Cost Slope
1	P_1	---	3	2	5000	7000	1	2000
2	P_2	P_1	4	2	4000	5000	2	500
3	P_3	P_2	4	4	7000	7000	0	---
4	P_4	P_2	3	1	3000	5000	2	1000
5	P_5	P_2	5	2	6000	10500	3	1500
6	P_6	P_5, P_3	4	3	8000	10000	1	2000
7	P_7	P_4	3	1	4000	5500	2	750
8	P_8	P_7	6	4	6000	9000	2	1500
9	P_9	P_6	7	4	5000	8000	3	1000
10	P_{10}	P_8, P_9	4	2	6000	7500	2	750
11	P_{11}	P_3, P_5	9	7	3000	4000	2	500
Total Cost					57,000	78,000		

Project –I network diagram in Fig -1, which is considered for explaining the mechanism of Crashing the Critical Activity (CCA) and Stretching the Non-critical Activity (SNA)
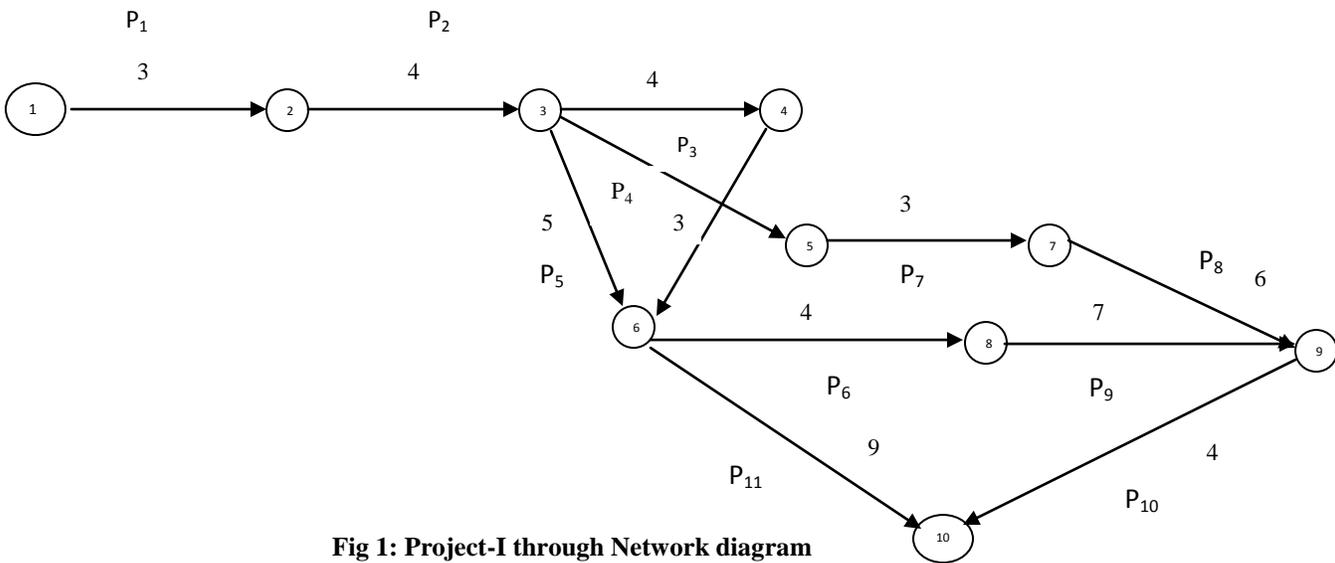

Fig 1: Project-I through Network diagram

Table III: CCA approach for Project – I

Crashed Activity	Normal Project duration	Cost Slope	Total Cost with addition of extra cost
P_2	22	500	57,500
P_2	21	500	58,000
P_{10}	20	750	58,750
P_{10}	19	750	59,500
P_9	19	1000	60,500
P_9	19	1000	61,500
P_9	19	1000	62,500
P_5	19	1500	64,000
P_1	18	2000	66,000
P_6	18	2000	68,000
P_7	17	750	68,750

The total extra cost that added to crashed critical activities

$$\text{TE}_c = \sum_{s=1}^{Y} D_{rs} U_s$$

i.e., $TE_c = ₹. 11,750$ and the steps to complete the project within the shortest possible duration with maximum available budget by

CCA approach is 11.

Table IV : SNA approach for Project-I

Stretched Non-critical Activity	Normal Project duration	Cost Slope	Total Cost with subtraction of saving cost
P_5	15	1,500	75,500
P_8	15	1,500	72,500
P_4	15	1,000	70,500
P_7	15	750	69,750
P_{11}	17	500	68,750

Total saving cost obtained by stretching non-

critical activities

$$TSC_n = \sum_{xs=1}^{z} D_{rx} U_x$$

i.e., $TSC_n = ₹. 9,750$ and the steps to complete the project within the shortest possible duration with maximum available budget by

SNA approach is 5.

We generate the set of efficient projects by comparing evaluations of projects employing stochastic dominance rules as stated above. Firstly, evaluations of projects with respect to the criteria are generated. Next, these evaluations are compared with respect to the criteria. Finally, the interactive technique is used for the selection of the most desirable resource allocation. The steps required to perform the analysis are described below.

The efficient set consist of five projects, namely

Z = {proj(1), proj(2), proj(3), proj(4), proj(5)},data related to the five projects are given in the following table

TableV : Time, cost & Quality of 5 construction Projects

Projects	Time	Cost	Quality
Proj(1)	27	57,000	0.95
Proj(2)	63	1,86,500	0.91
Proj(3)	35	1,58,500	0.88
Proj(4)	77	51,20,000	0.92
Proj(5)	98	3,45,000	0.89

Criterion :

X_1 : Time will not exceed 90 days

X_2 : Cost will not exceed Rs. 2,00,000

X_3 : Quality will not be less than 0.94

The set of projects satisfying criterion X_1 is

Z_1 = { proj(1), proj(2), proj(3), proj(4) }

The best project satisfying the criterion from set 1 is proj(1)

The set of projects satisfying criterion X_2 is

Z_2 = { proj(1), proj(2), proj(3) }

The best project satisfying the criterion from set 2 is proj(1)

The set of projects satisfying criterion X_3 is

Z_3 = { proj(2), proj(3), proj(4) }

The best project satisfying the criterion from set 3 is proj(4)

If the decision maker needs to select a project from the above set of projects, then he would select project (1) as the final by giving preference to quality and cost constraint

Table VI: Data presented to decision maker

Projects	Time	Cost	Quality
Proj(1)	27	57,000	0.95
Proj(4)	77	51,20,000	0.92

We conclude by saying proj (1) is the best project. That is SSR Construction projects are chosen compared to Reddy Constructions projects.

VI. Conclusion

The time cost trades off problems are explained through CCA and SNA in this paper. Based on five projects and above calculations shows that the minimum number of steps required to complete the project within the maximum available budget is SNA which could complete the project in least number of steps than CCA. Therefore, we can conclude that SNA is a faster and easier to complete the project within the maximum available budget. And by using interactive technique we conclude that project-I is the best desirable project.

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