Some Theorems on Intuitionistic Multi Fuzzy Subgroups

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Abstract- In this paper, we define the algebric structures of Intuitionistic Multi fuzzy subgroup and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in intuitionistic multi fuzzy subgroups.

Index Terms- Fuzzy set, Fuzzy subgroup, intuitionistic fuzzy set (IFS), Multi fuzzy set (MFS), Intuitionistic fuzzy subgroup (IFSG), Multi fuzzy subgroup (MFSG), Intuitionistic Multi fuzzy Subgroup (IMFSG)

I. Introduction

The idea of Intuitionistic fuzzy set was given by krassimiri, T.Atanassov [1]. S.Sabu and T.V. Ramkrishan [6] Proposed the theory of multi fuzzy sets interms of multi dimensional membership functions. N. Palaniappan, S.Naganathan and K.Arjunan introduced the theory of Intuitionistic fuzzy subgroup is an extension theories of fuzzy subgroups. R.Muthuraj and S.Balamurugan [4] proposed the multi fuzzy group and its level subgroups. In this paper, we define a new algebraic structure of Intuitionistic multi fuzzy subgroups and study some of their related properties.

II. Preliminaries

2.1 : Definition

Let x be any non – empty set. A fuzzy subset α of x is $\alpha : x \to [0,1]$

2.2 : Definition

Let A be a fuzzy set on a group G. Then A is called a fuzzy subgroup of G if for all x,y □ G.

(i) $A(x, y) \geq \min \{ A(x), A(y)\}$
(ii) $A(x^{-1}) = A(x)$

2.3 Definition

Let X be a fixed non – empty set. An Intuitionistic fuzzy set (IFS) A of X is an object of the form
$A=\{(x,\alpha_A(x), \beta_A(x)) : x$ □ $x\}$, where $\alpha_A : x \to [0,1]$ and $\beta_A : x \to [0,1]$
Define the degree of membership and degree of non – membership of the element x □ X respectively and for any x □ X, We have $0 \leq \alpha_A(x) + \beta_A(x) \leq 1$

2.4 Definition

Let x be a non empty set. A multi fuzzy set (IFS) A in x is defined as a set of ordered sequences.
$A = \{ (x, \alpha_1, (x), \alpha_2(x)\ldots.. \alpha_i(x)\ldots.); x$ □ $x\}$, where $\alpha_i : x \to [0,1]$, for all i

2.5 Definition

An IFS $A = \{(x, \alpha_A(x), \beta_A(x))$; x □ G} of a group G is said to be intuitionistic fuzzy subgroup of G (IFSG) if,

(i) $\alpha_A(xy) \geq \min(\alpha_A(x), \alpha_A(y))$ and $\beta_A(xy) \leq \max(\beta_A(x), \beta_A(y))$
(ii) $\alpha_A(x^{-1}) = \alpha_A(x)$ and $\beta_A(x^{-1}) = \beta_A(x)$, for all x,y □ G

2.6 Definition

A multi fuzzy set A of a group G is called a multi – fuzzy subgroup of G (MFSG) if for all x, y □ G

(i) $A(x,y) \geq \min(A(x), A(y))$
(ii) $A(x^{-1}) = A(x)$

2.7 Definition

An IFS $A = \{(x, \alpha_A(x), \beta_A(x))$: x □ G} of a group G is said to be intuitionistic Multi fuzzy subgroup of G(IMFSG) if,

(i) $\alpha_A(xy) \geq \min(\alpha_A(x), \alpha_A(y)$ and $\beta_A(xy) \leq \max(\beta_A(x), \beta_A(y))$
(ii) $\alpha_A(x^{-1}) = \alpha_A(x)$ and $\beta_A(x^{-1}) = \beta_A(x)$

2.8 Definition

Let $A= \{(x, \alpha_A(x), \beta_A(x))$, x □ x} and $B = \{(x, \alpha_B(x), \beta_B(x)$, x □ x} be any two IFS's of X, then

(i) $A \subseteq B$ if and only if $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \geq \beta_B(x)$ for all x □ X
(ii) A = B if and only if $\alpha_A(x) = \alpha_B(x)$ and $\beta_A(x) = \beta_B(x)$ for all x □ X
(iii) $A \cap B = \{(x, (\alpha_A \cap \alpha_B)(x), (\beta_A \cap \beta_B)(x))$; x □ X} where,

$(\alpha_A \cap \alpha_B)(x) = \text{Min}\{ \alpha_A(x), \alpha_A(x)\}$ and
$(\beta_A \cap \beta_B)(x) = \text{Max}\{ \beta_A(x), \beta_B(x)\}$

(iv) $AUB =\{(x (\alpha_A U \alpha_B)(x), (\beta_A U \beta_B)(x))$; x □ X}

Where,
$(\alpha_A U \alpha_B)(x) = \text{Max}(\alpha_A(x), \alpha_B(x))$
$(\beta_A U \beta_B)(x) = \text{Min}(\beta_A(x), \beta_B(x))$

III. PROPERTIES OF INTUITIONISTIC MULTI FUZZY SUBGROUPS

3.1 Theorem

Let A be a IMFSG of a group G and e is the identity element of G. Then

(i) $\alpha_A(x) \leq \alpha_A(e)$ and $\beta_A(x) \geq \beta_A(e)$, for all $x \square G$

(ii) The subset $H = \{ x \square G / \alpha_A(x) = \alpha_A(e)$ and $\beta_A(x) = \beta_A(e)\}$ is a group of G.

Proof:

(i) Let $x \square G$

$\alpha_A(x) = \min\{\alpha_A(x), \alpha_A(x)\}$

$= \min\{\alpha_A(x), \alpha_A(x^{-1})\} \leq \alpha_A(x x^{-1}) = \alpha_A(e)$

$\alpha_A(x) \leq \alpha_A(e)$

Hence, $\alpha_A(x) \leq \alpha_A(e)$

and

$\beta_A(x) = \max\{\beta_A(x), \beta_A(x)\}$

$= \max\{\beta_A(x), \beta_A(x^{-1})\} \geq \beta_A(x x^{-1}) = \beta_A(e)$

$\beta_A(x) \geq \beta_A(e)$

(ii) Let $H = \{ x \square G / \alpha_A(x) = \alpha_A(e)$ and $\beta_A(x) = \beta_A(e)\}$

Clearly, H is non – empty set as $e \square H$

Let $x,y \square H$

Then $\alpha_A(x) = \alpha_A(y) = \alpha_A(e)$

$\alpha_A(xy^{-1}) \geq \min(\alpha_A(x), \alpha_A(y^{-1}))$

$= \min(\alpha_A(x), \alpha_A(y))$

$= \min(\alpha_A(e), \alpha_A(e))$

$= \alpha_A(e)$

Therefore, $\alpha_A(xy^{-1}) \geq \alpha_A(e)$

and Obviously

$\alpha_A(e) \geq \min\{\alpha_A(e), \alpha_A(e)\}$

$= \min\{\alpha_A(x), \alpha_A(y)\} = \min\{(\alpha_A(x), \alpha_A(y^{-1})\}$

$\alpha_A(e) \geq \{\alpha_A(xy^{-1})\}$

Therefore, $\alpha_A(e) \geq \alpha_A(xy^{-1})$

$\alpha_A(xy^{-1}) = \alpha_A(e)$

Let $x,y \square H$, Then $\beta_A(x) = \beta_A(y) = \beta_A(e)$

$\beta_A(xy^{-1}) \leq \max\{\beta_A(x), \beta_A(y^{-1})\}$

$= \max\{\beta_A(x), \beta_A(y)\}$

$= \max\{\beta_A(e), \beta_A(e)\}$

$= \beta_A(e)$

Therefore, $\beta_A(xy^{-1}) \leq \beta_A(e)$

and $\beta_A(e) \leq \max\{\beta_A(e), \beta_A(e)\}$

$= \max\{\beta_A(x), \beta_A(y)\}$

$= \max\{\beta_A(x), \beta_A(y^{-1})\}$

$= \beta_A(xy^{-1})$

Therefore, $\beta_A(e) \leq \beta_A(xy^{-1})$

Hence, $\beta_A(xy^{-1}) = \beta_A(e)$

Clearly, H is a subgroup of G

3.2 Theorem : A is a IFSG of G if and only if A^C is a IMFSG of G

Proof:

(i) Suppose A is a IFSG of G

Then for all $x,y \square G$

$\alpha_A(xy) \geq \min\{\alpha_A(x), \alpha_A(y)\}$

$1-\alpha_A(xy) \geq 1-\min\{1-\alpha_A(x), 1-\alpha_A(y)\}$

$\beta_A(xy) \leq \max\{\beta_A(x), \beta_A(y)\}$

and

$\beta_A(xy) \leq \max\{\beta_A(x), \beta_A(y)\}$

$1-\beta_A(xy) \leq 1-\max\{1-\beta_A(x), 1-\beta_A(y)\}$

$\alpha_A(xy) \geq \min\{\alpha_A(x), \alpha_A(y)\}$

(ii) $\alpha_A(x) = \alpha_A(x^{-1})$, for all $x \square G$

$1-\alpha_A(x) = 1-\alpha_A(x^{-1})$

$\beta_A(x) = \beta_A(x^{-1})$

and

$\beta_A(x) = \beta_A(x^{-1})$ for all $x \square G$

$1-\beta_A(x) = 1-\beta_A(x^{-1})$

$\beta_A(x) = \beta_A(x^{-1})$

Hence, A^c is a IMFSG of G

3.3 Theorem

Let A be any IMFSG of G with indentity 'e'.Then (i) $\alpha_A(xy^{-1}) = \alpha_A(e) \Rightarrow \alpha_A(x) = \alpha_A(y)$ and $\beta_A(xy^{-1}) = \beta_A(e) \Rightarrow \beta_A(x) = \beta_A(y)$ for all x,y in G

Proof

(i) Given A is a IMFSG of G

$\alpha_A(xy^{-1}) = \alpha_A(e)$ for all $x,y \square G$

$\alpha_A(x) = \alpha_A(xe) = \alpha_A(x(y^{-1}y))$

$= \alpha_A(xy^{-1}y) = \alpha_A(x y^{-1}) y\}$

$\geq \min(\alpha_A(xy^{-1}), \alpha_A(y)\}$

$= \min\{\alpha_A(e), \alpha_A(y)) = \alpha_A(y)$

Therefore, $\alpha_A(x) \geq \alpha_A(y)$

Now $\alpha_A(y) = \alpha_A(y(x^{-1}x)) = \alpha_A(yx^{-1},x)$

$\alpha_A(y) \geq \min(\alpha_A(yx^{-1}), \alpha_A(x))$

$= \min(\alpha_A(e), \alpha_A(x)) = \alpha_A(x)$

$\alpha_A(y) \geq \alpha_A(x)$

Therefore, $\alpha_A(x) = \alpha_A(y)$

(ii) $\beta_A(xy^{-1}) = \beta_A(e)$ for all $x,y \square G$

$\beta_A(x) = \beta_A(xy\, y^{-1}) = \beta_A(xy^{-1} y)$

$\leq \max\{\beta_A(xy^{-1}), \beta_A(y)\}$

$= \max\{\beta_A(e), \beta_A(y)\} = \beta_A(y)$

$\beta_A(x) \leq \beta_A(y)$

$\beta_A(y) = \beta_A(y x^{-1} x)$

$\leq \max\{\beta_A(yx^{-1} x)$

$= \max\{\beta_A(e), \beta_A(x)\} = \beta_A(x)$

$\beta_A(y) \leq \beta_A(x)$

Therefore $\beta_A(x) = \beta_A(y)$

3.4 Theorem

A is a IMFSG of G $\Leftrightarrow \alpha_A(xy^{-1}) \geq \min\{\alpha_A(x), \alpha_A(y)\}$ & $\beta_A(xy^{-1}) \leq \max\{\beta_A(x), \beta_A(y)$

Proof :

Let A be a IMFSG of G for all x,y in G

(i) $\alpha_A(xy^{-1}) \geq \min\{\alpha_A(x), \alpha_A(y^{-1})\}$

$= \min\{\alpha_A(x), \alpha_A(y)\}$

Therefore, $\alpha_A(xy^{-1}) \geq \min\{\alpha_A(x), \alpha_A(y)\}$

(ii) $\beta_A(xy^{-1}) \leq \max\{\beta_A(x), \beta_A(y^{-1})\}$

$= \max\{\beta_A(x), \beta_A(y)\}$

Therefore, $\beta_A(xy^{-1}) \leq \max\{\beta_A(x), \beta_A(y)\}$

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