Direct Product of B-Algebra Using Some Cycles of Aunu Permutation Pattern; Application in Graph Theory

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Abstract

In this paper we provide another method of constructions of direct product of B-algebra using a class of AUNU Permutation Pattern, were we used cycles of AUNU permutation pattern for the construction of direct product of B-algebra which yield some result, the pattern was also applied in graph theory.

Keywords*: AUNU Groups, AUNU Patterns, B-Algebra, direct product of B-algebra, Graph.*

1. Introduction

Algebra is a very wide area of study and it has so many classes such as BF and B –Algebra, in this research we limited our work on B-Algebra.

B-algebra is non-empty set X with a binary operation "$*$" and a constant 0 that satisfied the following axioms.

i. $x * 0 = x$
ii. $x * x = 0$
iii. $(x * y) * z = x * (z * (0 * y))$

$\forall x, y$ and z in X (Negger and Kim, 2002)

Another proof of the relationship of B-algebra with Group using the observation that the zero adjoint mapping is subjective, moreover a condition for an algebra defined on real numbers to a B-algebra using analytical method was found and in addition certain other facts about commutative B-algebra was reported by Allen *et al*, (2003). The direct product of B-Algebra was investigated by Angeline and Endam (2016), in which they introduced some properties. The detailed application of this algebra can be found in Park and Kim, (2001); Cho and Kim, (2001) Angeline *et al* (2016); introduced two canonical mapping of the direct product of B-algebra and obtained some of their properties.

2. Some Basic Defination

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In order to make reader a friendly reading and provide good understanding of the method and results of the paper, some basic notations are briefly discussed in the section following.

2.1. B-**algebra-:** A B-algebra is a non empty set x with constant 0 and binary operation "*" satisfying the following axioms.

B1 $x * x = 0$

B2 $x * 0 = x$

B3 $(x * y) * z = x * \left(z * (0 * y)\right)$

For all x, y and z in X

2.2. Sub Algebra: - Let $(X,* ,0)$ be a B-algebra. A non empty subset N of X is said to be sub algebra if $x * y \in X \; \forall \; x, y \in N$

2.3. Direct product of B-algebra: - let $A = (A,*, 0_A,)$ and $B = (B,*, 0_B)$ be two B-algebra, define the direct product of A and B to be the structure $A \times B = (A \times B * (0_A \times 0_B))$ where $A \times B$ is the set $\{(a, b): a \in A, b \in B\}$ whose binary operation * is given by$(a_1, b_1) * (a_2, b_2) = a_1 * a_2, b_1 b_2$.

2.4. Aunu Numbers: There are two types of Aunu Numbers; The (123)-avoiding class obtained from a recursion relation (Ibrahim and Audu 2005) as follows:

$$N(A_n(123)) = \frac{P_n - 1}{2}$$

Give rise to: 2, 3, 5, 6, 8, 9, 11, 14,

Corresponding to the length of 5, 7,11,13,17, 19,

The sequence in (2) is called the Aunu numbers correspond to the (123)-avoiding class of permutation.

Where $N(A_n(123))$ is the number of the class of numbers expressed as permutations, that avoid (123) patterns while P_n is the n^{th} prime number n ≥ 5.

On the other hand the (132)-avoiding class of Aunu permutation patterns is obtained from a relation (Ibrahim 2006, Ibrahim 2004) as follows:

$$N\left(A_n(132)\right) = n + (m - 1), m \le n.$$

Give rise to: 5,7,9,11,13,…,

Corresponding to the length of 3, 4, 5, 6, 7,

The sequence in (5) is called the Aunu numbers corresponding to the (132)-avoiding class of permutation.

3. Construction of Direct Product of B-Algebra Using Aunu Permutation Pattern of (123)-Avoiding Pattern

Let $A = (A,*,0_A) and\ B = (B,*,0_B)$ be B-algebra. Defined the direct product of A and B to be the structure $A \times B = \big(A \times B(0_A;0_B)\big)$, were

$A \times B$ Is the set $\{(a,b): a \in A\ and\ b \in B\}$ whose binary operation* is given by $(a_1,b_1)*(a_2,b_2) = (a_1 * a_2, b_1 * b_2)$.note that the binary operation is componentwise. Thus, the properties (I),(II) and (III) of $A \times B$ from those of A and B. Angeline, J. V & Endam, C. E (2016). Direct Product of B-Algebra.

Hence the construction easily follows.

(123)-avoiding Pattern

Proposition.

Let Ω= {1,4,2,0,3} ⊂ Z_5 be a first cycle of B-algebra of order$p = 5$, were $p \geq 5$ under binary operation "×" then Ω× Ω is a direct product of B-Algebra under ×.

Proof

For a Ω × Ω to be a direct product of B-Algebra

Then, regarding {1,4,2,0,3} as elements to construct a table of direct product of B-Algebra

Table 1: Operation table of Z_5 of direct product (123)

×	**1**	**4**	**2**	**0**	**3**
1	1	4	2	0	3
4	4	1	3	0	2
2	2	3	4	0	1
0	0	0	0	0	0
3	3	2	1	0	4

Theorem

If $\Omega_1$and Ω_2 are two different AUNU numbers that are B-algebra then the direct product of two B-algebra is also a B-algebra.

Proof

Suppose $\Omega_i$be a family of the product of a finite B-algebra then $\Omega_i, \{i = 1,2,3, \dots n\}$ for $i = 1,2, \dots n$ then the direct product of B-algebra of Ω_i is defined by $\prod_{i=1}^{n} \Omega_i$ where the π is the direct product given by $\Omega_1 \times \Omega_2 \times \Omega_3 \times \dots \times \Omega_n$ that is for any

$(a_1\, a_2\, a_3 \dots a_n) \in \Omega_1$ and $(b_1, b_2, b_3, \dots b_n) \in \Omega_2$ then $\prod_{i=1}^{2} \Omega_i = (a_1\, a_2\, a_3 \dots a_n) \times (b_1, b_2, b_3, \dots b_n) \in \Omega_2)$ were $\times$ is the binary operation on $\prod_{i=1}^{n} \Omega_i$

$(a_1 \times b_1)$,$(a_2 \times b_2)$,…$(a_n \times b_n)$

$$= (a_1\, a_2\, a_3 \dots a_n) \times (b_1, b_2, b_3, \dots b_n)$$

$$= \Omega_1 \times \Omega_2$$

Then the results follow

4. Application to Graph Theory

Example 1. Suppose a function g is defined on B-algebraΩ, that is $\Omega = \{1,4,2,0,3\}$ by $g(x) = 3x + 2\ \forall x \in \Omega$. then the following pairs of point were obtained from the function $V(G) = \{(1,0), (4,4), (2,3), (0,2), (3,1)\}$

Adjency matrix

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

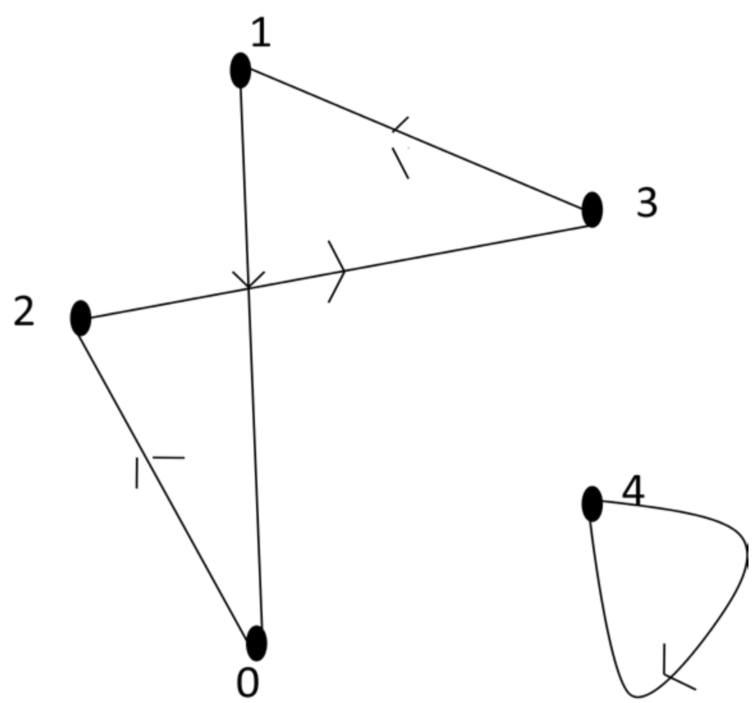

Fig 1

Example 2. Suppose a function g is defined on B-algebraΩ, that is $\Omega = \{1,4,2,0,3\}$ by $g(x) = 3x - 2\ \forall x \in \Omega$. then the following pairs of point were obtained from the function $V(G) = \{(1,1), (4,0), (2,4), (0,3), (3,2)\}$

Adjency matrix

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

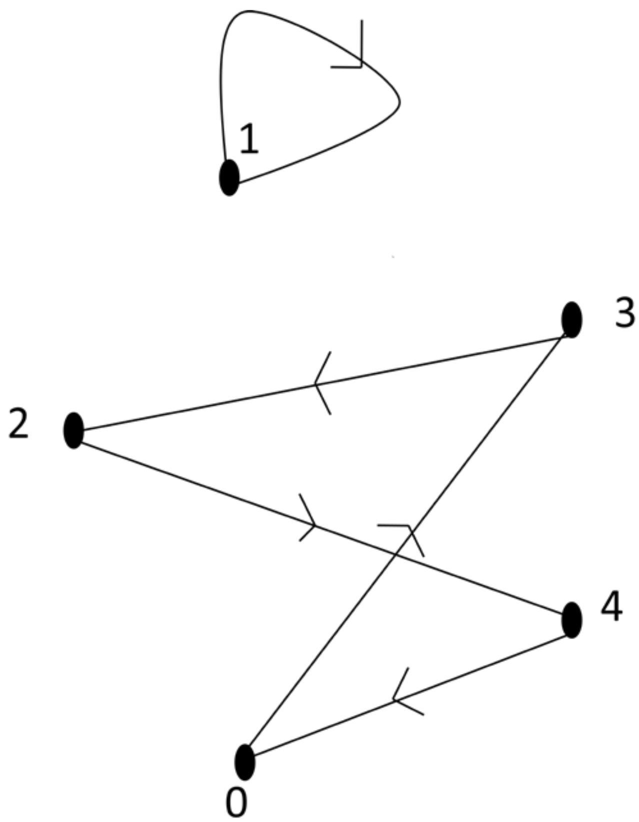

Fig 2

Conclusion

We have used (132)-avoiding pattern of AUNU Permutation Pattern to construct a direct product of B-algebra. This has been achieved by considering the direct product on sequence of the (132)-avoiding pattern of AUNU Permutation Pattern which gives a Square table see operation table; the product of two B-Algebra will give another B-Algebra.

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