

# A Systematic Study of “Estimation of Ionospheric Delay Errors in GPS”

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**Abstract-** The precision of the GPS navigation solution is affected by several types of error factors, in which the GPS signal delay by the ionosphere is the greatest after the omission of selective availability. This delay can be approximated by using one of the Ionospheric error correction models i.e. Klobuchar Algorithm, which estimate ionospheric time delay up to 50% or more, on a Root Mean Square (RMS) basis which is crucial to give the appropriate user position for single frequency GPS receivers. By using the Klobuchar algorithm, ionospheric range delay, the ionospheric time delay and Total Electron Content of ionosphere are estimated. In this paper the estimation of different ionospheric delay errors are presented after a systematic study of different parameters involved in this estimation.

**Index Terms-** GPS, Ionospheric Error Correction, Total Electron Content,

## I. INTRODUCTION

The Ionosphere is the zone of the terrestrial atmosphere that extends itself from about 60 kilometres until more than 2000 kilometres in high. As it names says it contains partially ionized medium, as a result of the X and UV rays of solar radiation and the incidence of charged particles. The propagation speed of the electromagnetic signals in the ionosphere depends on its electronic density which is typically driven by 2 main processes during the day. Sun radiation causes ionisation of neutral atoms producing free electrons and ions. During the night; the recombination process prevails, where free electrons are recombined with ions to produce neutral particles, which leads to a reduction in the electron density medium where the angular frequency ‘w’ and the wave number ‘k’ are most proportional is a dispersive media, that is, the wave propagation speed and hence the R.I depends on the frequency. This is the case with the Ionosphere where W and K are related,

$$W^2 = c^2k^2 + w_p^2(1) \text{ [Crawford, 1968]}$$

where ‘c’ is the velocity of light /signal in the vacuum.

$$\begin{aligned} W_p &= 2\pi f_p \\ f_p &= 8.98\sqrt{N_e} \quad (2) \end{aligned}$$

where  $N_e$  = electron density in  $e^-/m^3$

Equation (1) is named as the relation of dispersion of Ionosphere and signal with  $w > w_p$  will cross through the plasma [Davies, 1989]. The electron density in the Ionosphere changes with the height having a max of  $N_e \approx 10^{11} - 10^{12} e^-/m^3$ . According to equation (2), electromagnetic signal with  $f > f_p \approx 10^6$  Hz will be able to cross the ionosphere. This is the case of GNSS signals whose frequencies are at order of  $10^9$  Hz. Radio

frequency signals whose frequency under  $f_p$  will be reflected in the Ionosphere.

From equation (1),  $w = 2\pi f$

From the definition of phase and group velocity,

$$V_p = \frac{w}{k}, \quad V_g = \frac{dw}{dk} \quad (3)$$

$$V_p = \frac{c}{\sqrt{1 - (\frac{f_p}{f})^2}} \quad (4)$$

$$\text{Hence, } n_p = \text{ and } n_g = \frac{c}{v_g} \quad (5)$$

The phase refractive index of the Ionosphere can be approximated as:

$$\begin{aligned} n_p &= \frac{c}{v_p} = \sqrt{1 - (\frac{f_p}{f})^2} \\ &= 1 - \frac{1}{2} (\frac{f_p}{f})^2 \\ &= 1 - (\frac{40.3}{f^2}) N_e \quad (6) \end{aligned}$$

Given that for each point  $f_p^2 = 80.6N$  Hz<sup>2</sup> is valid (N is density of electrons in  $\frac{e^-}{m^3}$ )

At the frequency of GNSS signals, the equation (6) accounts for more than 99.9% of the refractory, that is, less than 0.1% error, it can be assumed

$$n_p = 1 - (\frac{40.3}{f^2}) N_e \quad (7)$$

Differentiating equation 1, with respect to ‘k’ and taking into account 3, 5 and the approximation

$$(1 - \mathcal{E}^2)^{-1/2} = 1 + \frac{1}{2}\mathcal{E}^2, \text{ yields the group R.I,}$$

$$n_g = 1 + (\frac{40.3}{f^2}) N_e \quad (8)$$

Hence, phase measurements suffer advance when crossing the ionosphere i.e a negative delay, and the group/code measurements suffer a positive delay.  $n_p, n_g$  are called phase and code ionospheric refraction and the integral is defined as the slant TEC (STEC).

$$n_p = 1 - 40.3 \cdot \frac{N}{f^2} \quad (9)$$

$$n_g = 1 + 40.3 \cdot \frac{N}{f^2} \quad (10)$$

The electromagnetic distance measured between the satellite and the receiver can be written as:

$$S = \int_{\text{satellite}}^{\text{Receiver}} nds. \quad (11)$$

Substituting equation (9) in (11) then,

$$\begin{aligned} S &= \rho - 40.3 \cdot \frac{1}{f^2} \int_{\text{Satellite}}^{\text{Receiver}} Nds \\ &= \rho - 40.3 \cdot \frac{TEC}{f^2}. \quad (12) \end{aligned}$$

Where TEC is Total Electron Content, i.e. integrated electron density along the signal path given in the TEC units (1 TEC =  $10^{16} \frac{1}{m^2}$ ),  $\rho$  is Right distance. The equivalent equation for modulated signal is given as:

$$S = \rho + 40.3 \cdot \frac{TEC}{f^2}. \quad (13)$$

Equation (12) and (13) shows that signal during the passage through the ionosphere, phase of the carrier wave will accelerate (12), the distance  $S$  is shorter than the actual distance  $\rho$  and the modulated signal will be delayed (13), the distance  $S$  is longer than the actual distance  $\rho$ .

The true distance from satellite to the receiver is given as  $\rho$ , the remaining part of the equations (12) and (13) represents the error caused by signal propagation through the ionosphere, known as ionospheric signal delay.

$$d_{ion} = 40.3 \cdot \frac{TEC}{f^2}. \quad (14)$$

The ionospheric refraction depends on the geographical location of the  $R_x$ , the hour of day and the solar activity.

## II. IONOSPHERE EFFECTS ON ELECTROMAGNETIC WAVE PROPAGATION

When radio waves, such as those emitted from GPS satellites pass through the ionized path, there are two effects: the trajectory of the beam is bent and the signal comes to a destination with a delay [3]. The free electrons in the ionosphere are the wrong doing for this phenomenon, due to the effect called refraction. Refraction of beam is defined by Snell-law. However, the behaviour of waves in the ionosphere cannot be described with this relatively simple equation only. To adequately describe the behaviour of radio waves passing through the ionosphere, it must be borne in mind that the ionosphere is only partially ionized, spherically stratified plasma with a broad spectrum of unevenly spaced irregularities, which extends along the uneven magnetic field, which is distorted in itself due to the disorder that arises as a

result of the occurrence of solar winds. The signal beamed from satellites must pass through the ionosphere on their way to earth. Free electrons, as the most massive particles in the ionosphere affect the propagation of the signal, changing their speed, direction and shape of the signal path (figure 1). Positioning error that occurs due to this effect is called the ionospheric delay. Sun radiation causes ionisation of neutral atoms producing free electrons and ions. During the night; there combination process prevails, where free electrons are recombined with ions to produce neutral particles, which leads to a reduction in the electron density.

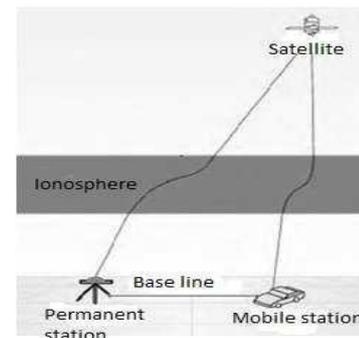


Figure 1. Appearance of signal path while passing through the ionosphere

The parameter that most affects the propagation of GPS signal is called total electron content or abbreviated TEC. Knowing the parameters of TEC, estimation of errors and calculation of corrections can be made.

## III. ESTIMATION OF RANGE EQUATION

The Range delay from the incoming GPS signal is estimated as [2]

$$p = \rho + c(dt - dT) + d_{ion} + d_{tro} + \epsilon$$

where

“ $p$ ” is measured pseudo range

“ $\rho$ ” is geometric or true range

“ $c$ ” represents speed of light

“ $dt$ ” and “ $dT$ ” are offsets of satellite and receiver clocks.

“ $d_{ion}$ ”, “ $d_{tro}$ ” are the delays due to ionosphere and troposphere.

“ $\epsilon$ ” represent effects of multipath and receiver measurement noise.

Here the delay due to  $d_{tro}, c(dt - dT), \epsilon$  are negligible

when compared to  $d_{ion}$ , so neglecting those three errors, the Range equation can be reduced and is given by

$$p = \rho + d_{ion}$$

The Range delay due to ionosphere  $d_{ion}$  is estimated using **Klobuchar Algorithm**.

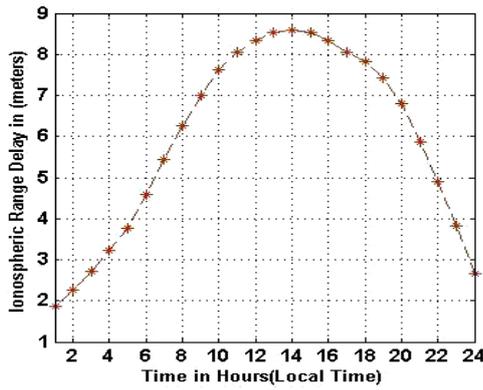


Figure 2. Model graph for Ionospheric Range delay for a complete day

#### IV. ESTIMATION OF ELEVATION ANGLE AND AZIMUTH ANGLE

According to the Klobuchar model, the ionospheric layer is assumed at 350 km above earth surface and the satellite at 20,200km above earth surface the LOS between satellite and the GPS ground receiver is intersected at a point called Ionospheric Pierce Point (IPP) on ionosphere layer. It is necessary to calculate the elevation angle and azimuth angle for the estimation of Slant TEC in Ionosphere [4].

The Elevation Angle and Azimuth Angle can be calculated using the equations given below:

$$E = \arctan \left[ \frac{\cos(G) \cos(L) - 0.1512}{\sqrt{1 - \cos^2(G) \cos^2(L)}} \right]$$

where  $G = S - N$

$$A = 180 + \arctan \left[ \frac{\tan(G)}{\sin(L)} \right]$$

'E' is the Elevation Angle of antenna in degrees

'L' is the Site Latitude in degrees

'N' is the Site Longitude in degrees

'A' is the Azimuth Angle of antenna in degrees

'S' is the Satellite Longitude in degrees.

#### V. IONOSPHERIC TIME DELAY ESTIMATION USING KLOBUCHAR ALGORITHM

1. The Earth-centered angle is calculated using the Elevation angle of the satellites with respect to the ground station GPS receivers. [4]

$$\Psi = \frac{0.0137}{(E+0.11)} - 0.022 \quad (\text{semicircles}) \quad (1)$$

"Ψ" is the Earth-centered angle units in semicircles

"E" is the Elevation Angle (convert degrees in to semicircles)

2. Then Compute the sub-ionospheric latitude value using azimuth angle, earth-centered angle and the geodetic latitude

$$\phi_I = \phi_U + \psi \cos A \quad (\text{semicircles}) \quad (2)$$

where  $\phi_U = 17^\circ$  (degrees).

which is the geodetic latitude value of NGRI (Ground station GPS receiver).

If  $\phi_I \geq 0.416$ , then  $\phi_I = 0.416$ .

If  $\phi_I \leq -0.416$ , then  $\phi_I = -0.416$ .

" $\phi_I$ " is the Sub-Ionospheric Latitude units in semicircles.

" $\phi_U$ " is the Geodetic Latitude (convert degrees in to Semicircles).

"A" is the Azimuth Angle (convert degrees in to semicircles).

3. Compute the sub-ionospheric longitude value using Geodetic longitude, Geodetic latitude, Azimuth angle, Earth-centered angle.

$$\lambda_I = \lambda_U \left[ \frac{\sin(A)}{\cos(\phi_I \times 3.14)} \right] \quad (3)$$

where  $\lambda_U = 78^\circ$  (degrees).

This is the geodetic Longitude value of NGRI (Ground station GPS receiver).

' $\lambda_I$ ' is the sub-Ionospheric Longitude units in semicircles.

' $\lambda_U$ ' is the Geodetic Longitude (convert degrees in to semicircles).

4. Then Find the geomagnetic latitude of the sub-ionospheric location looking toward each GPS satellite. It is shown below[1]

$$\phi_m = \phi_I + 0.064 \cos[(\lambda_I - 1.617) \times 3.14] \quad (\text{semicircles}) \quad (4)$$

' $\phi_m$ ' is the geomagnetic latitude units in semicircles

5. Find the local time, at the sub-ionospheric point and here we have to use the GPS time value in seconds.[1]

$$t = 4.32 \times 10^4 \lambda_I + Time_{GPS} (\text{seconds}) \quad (5)$$

' $Time_{GPS}$ ' is the GPS time value in seconds.

't' is the local time in seconds.

If  $t > 86400$  use  $t = t - 86400$

If  $t < 86400$  use  $t = t + 86400$

6. Compute the slant factor[1]

$$SF = 1 + 16(0.53 - E)^3 \quad (6)$$

'SF' is the Slant factor.

7. Period of the model is[1]

$$PER = \sum_{n=0}^3 \beta_n \phi_m^n \quad (7)$$

Expanded form of equation (7) is shown in equation (8)

$$PER = \beta_0 \times \phi_m^1 + \beta_1 \times \phi_m^1 + \beta_2 \times \phi_m^2 + \beta_3 \phi_m^3$$

$$PER = \beta_0 + \beta_1 \phi_m^1 + \beta_2 \phi_m^2 + \beta_3 \phi_m^3 \quad (8)$$

if  $PER < 72000$  then  $PER = 72000$

"PER" is the period of the model.

" $\beta^n$ " is the Klobuchar coefficient.

8. Phase of the model is [1]

$$x = \frac{2\pi(-50400)}{PER} \quad (9)$$

'x' is the phase of the model which is (max at

14hours=50400sec) local time.

9. Amplitude of the model is[1]  

$$AMP = \sum_{n=0}^3 \alpha_n \phi_m^n \quad (10)$$

$$AMP = \alpha_0 \times \phi_m^0 + \alpha_1 \times \phi_m^1 + \alpha_2 \times \phi_m^2 + \alpha_3 \times \phi_m^3$$

$$AMP = \alpha_0 + \alpha_1 \phi_m^1 + \alpha_2 \phi_m^2 + \alpha_3 \phi_m^3 \quad (11)$$

If  $AMP < 0$  then  $AMP = 0$ .

“AMP” is the amplitude of the amplitude.

10. If  $x > 1.57$  then use [1]  

$$T_{iono} = SF \times (5 \times 10^{-9}) \quad (12)$$

Otherwise use

$$T_{iono} = SF \times \left[ (5 \times 10^{-9}) + AMP \times \left( 1 - \frac{x^2}{2} + \frac{x^4}{4} \right) \right] \quad (13)$$

‘ $T_{iono}$ ’ is the ionospheric time delay.

11. The range delay can be calculated using ‘ $T_{iono}$ ’ value[1]

Where  $C = 3 \times 10^8$  m/ sec

$$Rd_{iono} = T_{iono} \times C \quad (14)$$

‘ $Rd_{iono}$ ’ is the ionospheric range delay in meters.

‘ $C$ ’ is the velocity of light.

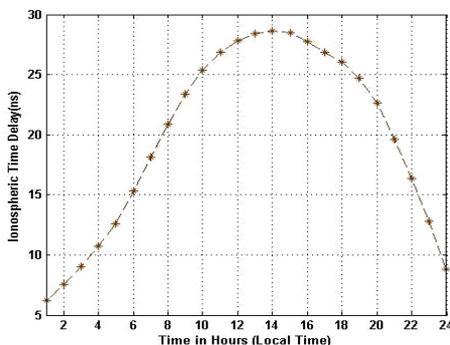


Figure 3. Model graph for Ionospheric Time delay for a complete day

12. Using,  $Rd_{iono}$  the TEC can be estimated[1]

$$Rd_{iono} = \left[ \frac{40.3}{f_1^2} \right] \cdot TEC \quad (15)$$

$$TEC = \frac{Rd_{iono} \times f_1^2}{40.3} \quad (16)$$

' $f_1$ ' Represents the  $L_1$  frequency = 1575.42 MHz  
 TEC Values are often represented in terms of Total Electron Content Units (TECU).

Where  $1TECU = 10^{16} \text{ el} / \text{m}^2$

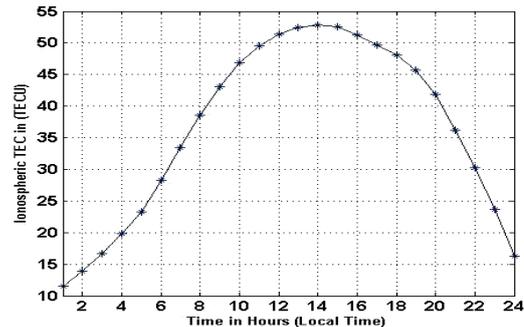


Figure 4. Model graph for Ionospheric TEC for a complete day

## VI. CONCLUSION

In this paper, by estimating the azimuth angle and elevation angle of satellites with respect to the GPS receiver antenna location and by using Klobuchar algorithm Ionospheric time delay is estimated and then by using Ionospheric time delay value range delay and TEC of ionosphere are calculated for any day. Estimation of Ionospheric delay is important as it causes a delay in ranging measurements which in turn cause an error in navigation solution. This work can be extended further by eliminating the ionospheric range delay from the ranging measurements, which it is possible to obtain more precise navigation solution for GPS users.

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