

SIX QUARKS' DECAY
MODES, WITH MASS
INCREASE:
A RERUM COGNOSCERE
CAUSES AND
RAMIFICATIONS

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Preface

Comprehensive envelope of expression, abstract problematic structures, and functional topology of Super symmetric QCD corrections to the top quark decay of a heavy charged Higgs boson have been done by some authors like Ricardo A. Jiménez, Joan Solà. The super symmetric QCD corrections to the hadronic width of a heavy charged Higgs boson, basically dominated by the top-quark decay mode $H^+ \rightarrow t\bar{b}$, are evaluated at $O(\alpha_s)$ within the MSSM and compared with the standard QCD corrections. The study of such quantum effects, which turn out to be rather large, is essential to understand the hypothetical super symmetric nature of a heavy charged Higgs boson potentially produced in the near future at the Tevatron and/or at the LHC. It is true that there has been some identical instances of semantic and experimental jugglery in so far as QCD corrections to the top quark decay mode $t \rightarrow \tau \chi^0$: (Djouadi, A.; Hollik, W.; Jünger, C. Physical Review D (Particles and Fields), Volume 54, Issue 9, 1 November 1996, pp.5629-5635) and impassibility of characteristics and ramified resonations thereof is concerned. Imprisoned complementarities', survigorous eurhythmic's of super symmetric theories, the top quark can decay into its scalar partner plus a neutralino, with an appreciable rate. Authors like Djouadi et al. Calculate the $O(\alpha_s)$ QCD corrections to this decay mode in the minimal super symmetric extension of the standard model. These corrections can be either positive or negative and increase logarithmically with the gluino mass. For gluino masses below 1 TeV, they are at most of the order of ten percent and, therefore, well under control.

In connection with the Flavor-changing top quark decay within the minimal super symmetric standard model G. Couture, C. Hamzaoui, and H. König present the results of the gluino and scalar quark contribution to the flavor-changing top quark decay into a charm quark and a photon, gluon, or a Z0 boson within the minimal super symmetric standard model. We include the mixing of the scalar partners of the left- and right-handed top quark. This mixing has several effects, the most important of which is to greatly enhance the cZ decay mode for large values of the soft SUSY-breaking scalar mass m_S and to give rise to GIM-like suppressions in the $c\gamma$ mode for certain combinations of parameters. Sheldon, Elizabeth E. Jenkins studied A massive charged scalar, such as is present in a two Higgs-doublet electroweak theory, can alter the nature of top-quark decay, ensuring that the top quark would not have been detected at hadron colliders. Box diagrams involving charged-scalar exchange can produce the observed level of $B_0 - \bar{B}_0$ mixing, thus relaxing the need for a heavy top quark. Future experiments can readily establish the existence of the charged scalar. Production and decay properties of ultra-heavy quarks (I. BigiStanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA Y. Dokshitzer, V. Khoze P. Zerwas have studied strum and drag of the widths of ultra-heavy quarks that can be decay into W, Z or Higgs bosons are discussed. If the lifetimes become much shorter than the typical strong interaction time scale $\Lambda^{-1} \text{QCD} \sim 10^{-23}$ s, then open-flavor hadrons and quarkonium bound states cannot be formed any more. Consequences for the jet evolution are investigated.

On the other hand, if such quarks can decay only through tiny mixing angles - as it could happen for sequential down-type quarks and for SU (2) singlet quarks in E6 models - then these bound states do form. Production rates for quarkonium in e^+e^- annihilation and in hadronic collisions are estimated and their decay signatures are discussed. Some authors have also studied a sort of rouse sarcoma in quarks with harum sacrum violent reckless dynamics, phosphoric wasp and mysterious muddle. Investigation has been strange behavior sometimes like sang frond like salamander and sometimes saber rattling of quarks have also been efficiently explained. Testing of CP invariance in the weak decay of top quarks is also studied. A prediction of CP violation from the Standard Model is made by an explicit calculation. The effect is very small as expected. We then use a form factor approach to parameterize possible new interactions of CP violation. The differences between the form factor approach and an effective Lagrangian approach are discussed. The sensitivity of CP-odd observables to the form factors is estimated. When it comes to the study of quarks, there is strange pugilistic provocation and verbal hyperbole and the necessity of quick

and skewering eye because of the solipsisti Transformation of Quark Flavors by the Weak Interaction. The decay of hadrons by the weak interaction can be viewed as a process of decay of their constituent quarks. There is a pattern of these quark decays: a quark of charge $+2/3$ (u, c,t) is always transformed to a quark of charge $-1/3$ (d,s,b) and vice versa. This is because the transformation proceeds by the exchange of charged W bosons, which must change the charge by one unit. The general pattern is that the quarks will decay to the most massive quark possible, leading the pattern c nature of the bellicose and blustering particles.

Here we directly tackle the problem of mechanism of interaction of quarks. The modes of quark flavor transformation by the weak interaction can be shown in Feynman diagrams. These diagrams are useful in analyzing decay processes to help keep track of what is happening on the quark level. The most common transformations are delineated and disseminated by the quark transformation. These transformations take place by means of the W vector boson. Epluribus Unum and leit motif of the study is the interaction and decay of quarks. I may mention in unmistakable terms that the Standard Model is the theoretical framework describing all the currently known elementary particles, as well as the Higgs boson. This model contains six flavors of quarks (q), named up (u), down (d), strange (s), charm (c), bottom (b), and top (t). Antiparticles of quarks are called antiquarks, and are denoted by a bar over the symbol for the corresponding quark, such as \bar{u} for an up antiquark. As with antimatter in general, antiquarks have the same mass, mean lifetime, and spin as their respective quarks, but the electric charge and other charges have the opposite sign. It is to be stated in unequivocal terms that the differential and determinate and pronounced study of the concatenated equations have to made with great deal of ardor and fervor to get the further results on the subject matter in question. We intend to undertake these works in future Monograph.

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive.

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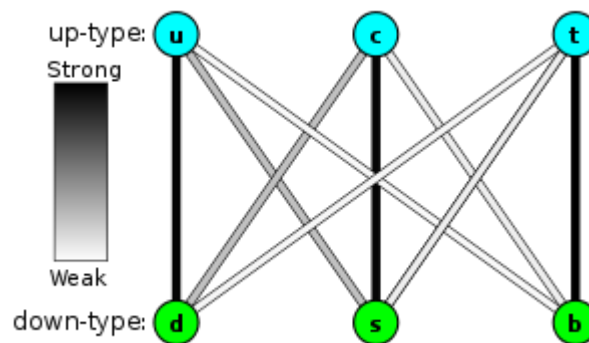
INTRODUCTION:

Flavour (Particle Physics)

In particle physics, flavor or flavor is a quantum number of elementary particles. In quantum Chromodynamics, flavour is a global symmetry. In the electroweak theory, on the other hand, this symmetry is **broken**, and flavour-changing processes exist, such as quark **decay** or neutrino oscillations.

In the Standard Model of particle physics, the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix, quark mixing matrix, sometimes also called KM matrix) is a unitary matrix which **contains** information on the strength of flavour-changing weak decays. Technically, it **specifies** the mismatch of quantum states of quarks when they propagate freely and when they take part in the weak interactions. It is important **in the understanding** of CP violation. This matrix was introduced for three generations of quarks by Makoto Kobayashi and Toshihide Maskawa, adding one generation to the matrix previously introduced by Nicola Cabibbo. This matrix is also an **extension of** the GIM mechanism, which only includes two of the three current families of quarks

The matrix



A pictorial representation of the six quarks' decay modes, with mass increasing from left to right.

In 1963, Nicola Cabibbo introduced the Cabibbo angle (θ_c) to preserve the universality of the weak interaction. Cabibbo was inspired by previous work by Murray Gell-Mann and Maurice Lévy, on the effectively rotated nonstrange and strange vector and axial weak currents, which he references.

In light of current knowledge (quarks were not yet theorized), the Cabibbo angle is related to the relative probability that down and strange quarks decay into up quarks ($|V_{ud}|^2$ and $|V_{us}|^2$ respectively). In particle physics parlance, the object that couples to the up quark via charged-current weak interaction is a superposition of down-type quarks, here denoted by d' . Mathematically this is:

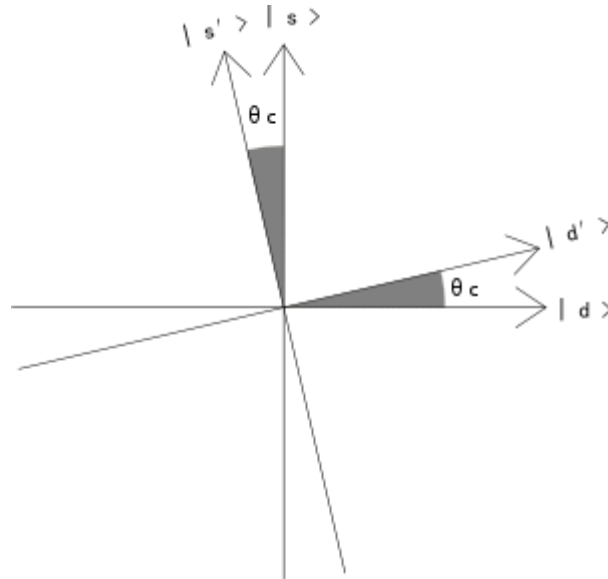
$$d' = V_{ud}d + V_{us}s,$$

Or using the Cabibbo angle:

$$d' = \cos \theta_c d + \sin \theta_c s.$$

Using the currently accepted values for $|V_{ud}|$ and $|V_{us}|$ (see below), the Cabibbo angle can be calculated using

$$\tan \theta_c = \frac{|V_{us}|}{|V_{ud}|} = \frac{0.2257}{0.97419} \rightarrow \theta_c = 13.04^\circ.$$



The Cabibbo angle represents the rotation of the mass Eigen state vector space formed by the mass Eigen states $|d\rangle$, $|s\rangle$ into the weak Eigen state vector space formed by the weak Eigen states $|d'\rangle$, $|s'\rangle$. $\theta_c = 13.04^\circ$.

When the charm quark was discovered in 1974, it was noticed that the down and strange quark could decay into either the up or charm quark, leading to two sets of equations:

$$d' = V_{ud}d + V_{us}s;$$

$$s' = V_{cd}d + V_{cs}s,$$

or using the Cabibbo angle:

$$d' = \cos \theta_c d + \sin \theta_c s;$$

$$s' = -\sin \theta_c d + \cos \theta_c s.$$

This can also be written in matrix notation as:

$$\begin{bmatrix} d' \\ s' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix},$$

MODULE NUMBERED ONE (u&d)

NOTATION :

G_{13} :CATEGORY ONE OF u quark

G_{14} :CATEGORY TWO OF u quark(here we are talking of the Penchance, predilection, proclivities, propensities, idiosyncrasies, characteristics, Parametricization of the systems under investigation in which u quark become a sine qua non form)

G_{15} :CATEGORY THREE OF u (here we are talking of the Penchance, predilection, proclivities, propensities, idiosyncrasies, characteristics, Parametricization of the systems under investigation in which u quark become a sine qua non form)

T_{13} : Category One Of D (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theopries Mentioned)

T_{14} : Category Two Of D (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

T_{15} :Category Three Of D (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

u & s QUARKS

MODULE NUMBERED TWO:

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=

G_{16} : CATEGORY ONE OF u (Here the systemic investigation may be different from the other one that is classified in the foregoing. In the eventuality of the systemic Conformality and congruence being the same we can as well take the classificatory parameters to be equal. This classification is done based on the assumption that the Parametricization of the systems studied are different from the ones which are stated in the foregoing)

G_{17} : CATEGORY TWO OF u (Here the systemic investigation may be different from the other one that is classified in the foregoing. In the eventuality of the systemic Conformality and congruence being the same we can as well take the classificatory parameters to be equal. This classification is done based on the assumption that the Parametricization of the systems studied are different from the ones which are stated in the foregoing)

G_{18} : CATEGORY THREE OF u (Here the systemic investigation may be different from the other one that is classified in the foregoing. In the eventuality of the systemic Conformality and congruence being the same we can as well take the classificatory parameters to be equal. This classification is done based on the assumption that the Parametricization of the systems studied are different from the ones which are stated in the foregoing)

T_{16} :CATEGORY ONE OF s (here we are talking of the penchance,predilection, proclivities, propensities, idiosyncrasies, characteristics, parametricization of the systems under investigation in which u quark become a sine qua non form)

T_{17} : CATEGORY TWO OF s (here we are talking of the Penchance, predilection, proclivities, propensities, idiosyncrasies, characteristics, Parametricization of the systems under investigation in which u quark become a sine qua non form)

T_{18} : CATEGORY THREE OF s (here we are talking of the Penchance, predilection, proclivities, propensities, idiosyncrasies, characteristics, Parametricization of the systems under investigation in which u quark become a sine qua non form)

d & c Quarks

MODULE NUMBERED THREE:

=====

G_{20} : Category One Of D (Like There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “ D ” Quark Forms A Part And Parcel A summum bonum Of The Systems)

G_{21} : Category Two Of D (There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter

Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Marko Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “D” Quark Forms A Part And Parcel A Summum Bonum Of The Systems)

G_{22} : Category Three Of “D” (There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Marko Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “D” Quark Forms A Part And Parcel A Summum Bonum Of The Systems)

T_{20} : Category One Of “C” Quarks (There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Marko Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “D” Quark Forms A Part And Parcel A Summum Bonum Of The Systems)

T_{21} : Category Two Of “C” Quarks (There Are Lot Of Markov Spaces Hilbert Spaces And Quantum Fields And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Marko Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “D” Quark Forms A Part And Parcel A Summum Bonum Of The Systems)

T_{22} : Category Three Of “C” Quarks (There Are Lot Of Markov Spaces Hilbert Spaces And Quantum Fields And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Marko Theory Is Applicable There Are Systems In The Case Of Which Investigations Are Carried On Of Which “D” Quark Forms A Part And Parcel A Summum Bonum Of The Systems)

QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARAIBLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS)

: MODULE NUMBERED FOUR:

G_{24} : Category One Of Quantum Field Theory(Evaluative Parametricization Of Situational Orientations And Essential Cognitive Orientation And Choice Variables Of The System To Which Qft Is Applicable)

G_{25} : Category Two Of Quantum Field Theory

G_{26} : Category Three Of Quantum Field Theory

T_{24} :Category One Of Renormalization Theory

T_{25} :Category Two Of Renormalization Theory(Systemic Instrumental Characterizations And Action Orientations And Functional Imperatives Of Change Manifested Therein)

T_{26} : Category Three Of Quantum Field Theory

Quark “c” and Quark ”b”:

MODULE NUMBERED FIVE:

G_{28} : Category One Of Quark “C”(Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Acolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel)

G_{29} : Category Two Of Quark “C” (Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Accolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel)

G_{30} :Category Three Of Quark “C” (Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Accolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel. We May Note In The Passing That There Might Be Some Quantum Corrections Due To Virtual Electron-Positron Pairs)

T_{28} :Category One Of Quark “B” (Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Acolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel. Mention May Be Made That In Some Cases Linear Corrections In Maxwell’s Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell’s Equations Are The Same)

T_{29} :Category Two Of Quark “B” (Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Acolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel. Mention May Be Made That In Some Cases Linear Corrections In Maxwell’s Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell’s Equations Are The Same)

T_{30} :Category Three Of Quark “B” (Predicational Anteriority, Character Constitution, Primordial Exactitude, Ontological Consonance, Accolytish Representation, Atrophied Asseverations ,Apocryphal Aneurism Of The Systems Under Investigation Of Which Quark “C” Forms Part And Parcel. Mention May Be Made That In Some Cases Linear Corrections In Maxwell’s Equation Applicable To Various Systems With Defined Characteristics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell’s Equations Are The Same)

QUARK “t” and QUARK b”:

MODULE NUMBERED SIX:

G_{32} : Category One Of Quark “T”

G_{33} : Category Two Of Quark “T” (Decisive Regularities With Respect To Cases Wherein Sobolev Inequalities Are In Border Line Transition Might Be Conspicuous By Its Presence)

G_{34} : Category Three Of Quark “T” (Decisive Regularities With Respect To Cases Wherein Sobolev Inequalities Are In Border Line Transition Might Be Conspicuous By Its Prescience)

- T_{32} : Category One Of **Quark B**”:
 T_{33} : Category Two Of **Quark B**”:
 T_{34} : Category Three Of **Quark B**”:

EXPANSION RATE OF QUANTUM FIELD DESCRIBING ELECTRONS AND OTHER FERMIONS AND CREATION AND ANNIHILATION OF OPERATORS ACCORDING TO PAULI'S EXCLUSION PRINCIPLE (AGAIN WE TALK OF THE SYSTEMIC CHARACTERISTICS TO WHICH PAULI'S EXCLUSION PRINCIPLE IS APPLIED, AND THE PARAMETRIZATION AND STRATIFICATION FOLLOWS)

MODULE NUMBERED SEVEN

-
- G_{36} : Category One Of Creation And Annihilation Operators Due To Pauli's Exclusion Principle (Note Pauli's Exclusion Principle Denies The Two States For The Electrons)
 G_{37} : Category Two Of Creation And Annihilation Operators Due To Pauli's Exclusion Principle (Note Pauli's Exclusion Principle Denies The Two States For The Electrons)
 G_{36} : Category Three Of Orthogonal Energy State Of Vacuum (Energy Excitation Of The Vacuum And Concomitant Generation Of Energy Differential-Time Lag Or Instantaneousness might Exist Whereby Accentuation And Attritions Model May Assume Zero Positions)
 G_{38} : category three Of Creation And Annihilation Operators Due To Pauli's Exclusion Principle
 T_{36} : Category One Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions
 T_{37} : Category Two Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions
 T_{38} : Category Three Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions
-

QUARK “t” and QUARK “s”

MODULE NUMBERED EIGHT

- G_{40} : *CATEGORY ONE OF QUARK “t”* (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Quarks, Electrons In The Universe)
 G_{41} : *CATEGORY TWO OF QUARK “T”* (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Quarks, Electrons In The Universe)
 G_{42} : *CATEGORY TWO OF Neutrinos QUARK “T”* (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Quarks, Electrons In The Universe) T_{40} : *CATEGORY ONE QUARK “s”*

- T_{40} : Category Two Of **Quark “S”**
 T_{41} : Category Two Of **Quark “S”**
 T_{42} : *CATEGORY THREE OF QUARK “s”*

$$(a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}:$$

$$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$$

$$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$$

$$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$$

$$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$$

$$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$$

are Accentuation coefficients

$$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$$

$$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$$

$$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}, (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)},$$

$$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$$

$$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$$

$$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$$

are Dissipation coefficients

(u&d)

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
 $-(b''_{13})^{(1)}(G, t) =$ First detritions factor

u&s QUARKS

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 12$$

$$+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor}$$

$$-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor}$$

d & c Quarks

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 18$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor}$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor}$$

QUANTUM FIELD THEORY (AGAIN, PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY (BASED ON CERTAIN VARIABLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLSSIFIABLE ON PARAMETERS))

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 24$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor}$$

Quark “c” and Quark ”b”:

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 30$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

QUARK “t” and QUARK b”:

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 36$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}$$

EXPANSION RATE OF QUANTUM FIELD DESCRIBING ELECTRONS AND OTHER FERMIONS AND CREATION AND ANNIHILATION OF OPERATORS ACCORDING TO PAULI'S EXCLUSION PRINCIPLE (AGAIN WE TALK OF THE SYSTEMIC CHARACTERISTICS TO WHICH PAULI'S EXCLUSION PRINCIPLE IS APPLIED, AND THE PARAMETRICIZATION AND STRATIFICATION FOLLOWS)

Module Numbered Seven:

The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)] G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)] G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)] G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)] T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)] T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)} T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)] T_{38} \quad 42$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor}$$

QUARK “t” and QUARK “s”

Module Numbered Eight

GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)} G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)] G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)} G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)] G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)} G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)] G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)} T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)] T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)} T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)] T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)} T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)] T_{42} \quad 48$$

First Module Concatenation

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[\begin{array}{c} (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37}, t)} \boxed{+(a''_{40})^{(8,8)}(T_{41}, t)} \end{array} \right] G_{13} \quad 49$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[\begin{array}{c} (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37}, t)} \boxed{+(a''_{41})^{(8,8)}(T_{41}, t)} \end{array} \right] G_{14} \quad 50$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[\begin{array}{c} (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37}, t)} \boxed{+(a''_{42})^{(8,8)}(T_{41}, t)} \end{array} \right] G_{15} \quad 51$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7)}(T_{37}, t)}$, $\boxed{+(a''_{36})^{(7,7)}(T_{37}, t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41}, t)}$, $\boxed{+(a''_{41})^{(8,8)}(T_{41}, t)}$, $\boxed{+(a''_{42})^{(8,8)}(T_{41}, t)}$ are eight augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{l} \boxed{(b'_{13})^{(1)} - \boxed{-(b''_{13})^{(1)}(G, t)} - \boxed{-(b''_{16})^{(2,2)}(G_{19}, t)} - \boxed{-(b''_{20})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)} - \boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)} - \boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,7)}(G_{39}, t)} - \boxed{-(b''_{40})^{(8,8)}(G_{43}, t)} \end{array} \right] T_{13} \quad 52$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{l} \boxed{(b'_{14})^{(1)} - \boxed{-(b''_{14})^{(1)}(G, t)} - \boxed{-(b''_{17})^{(2,2)}(G_{19}, t)} - \boxed{-(b''_{21})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)} - \boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)} - \boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,7)}(G_{39}, t)} - \boxed{-(b''_{41})^{(8,8)}(G_{43}, t)} \end{array} \right] T_{14} \quad 53$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{l} \boxed{(b'_{15})^{(1)} - \boxed{-(b''_{15})^{(1)}(G, t)} - \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} - \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)} - \boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)} - \boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39}, t)} - \boxed{-(b''_{42})^{(8,8)}(G_{43}, t)} \end{array} \right] T_{15} \quad 54$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7)}(G_{39}, t)}$, $\boxed{-(b''_{36})^{(7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7)}(G_{39}, t)}$ are seventh detrition coefficients for

category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43}, t)$, $-(b''_{41})^{(8,8)}(G_{43}, t)$, $-(b''_{42})^{(8,8)}(G_{43}, t)$ are eight detrition coefficients for category 1, 2 and 3

Second Module Concatenation

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{l} (a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \quad + (a''_{13})^{(1,1)}(T_{14}, t) \quad + (a''_{20})^{(3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4,4)}(T_{25}, t) \quad + (a''_{28})^{(5,5,5,5,5)}(T_{29}, t) \quad + (a''_{32})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{36})^{(7,7,7)}(T_{37}, t) \quad + (a''_{40})^{(8,8,8)}(T_{41}, t) \end{array} \right] G_{16} \quad 55$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{l} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) \quad + (a''_{14})^{(1,1)}(T_{14}, t) \quad + (a''_{21})^{(3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4,4)}(T_{25}, t) \quad + (a''_{29})^{(5,5,5,5,5)}(T_{29}, t) \quad + (a''_{33})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{37})^{(7,7,7)}(T_{37}, t) \quad + (a''_{41})^{(8,8,8)}(T_{41}, t) \end{array} \right] G_{17} \quad 56$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{l} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) \quad + (a''_{15})^{(1,1)}(T_{14}, t) \quad + (a''_{22})^{(3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4,4)}(T_{25}, t) \quad + (a''_{30})^{(5,5,5,5,5)}(T_{29}, t) \quad + (a''_{34})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a''_{38})^{(7,7,7)}(T_{37}, t) \quad + (a''_{42})^{(8,8,8)}(T_{41}, t) \end{array} \right] G_{18} \quad 57$$

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1)}(T_{14}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37}, t)$, $+(a''_{37})^{(7,7,7)}(T_{37}, t)$, $+(a''_{38})^{(7,7,7)}(T_{37}, t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41}, t)$, $+(a''_{41})^{(8,8,8)}(T_{41}, t)$, $+(a''_{42})^{(8,8,8)}(T_{41}, t)$ are eight augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{l} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) \quad - (b''_{13})^{(1,1)}(G, t) \quad - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4,4)}(G_{27}, t) \quad - (b''_{28})^{(5,5,5,5,5)}(G_{31}, t) \quad - (b''_{32})^{(6,6,6,6,6)}(G_{35}, t) \\ - (b''_{36})^{(7,7,7)}(G_{39}, t) \quad - (b''_{40})^{(8,8,8)}(G_{43}, t) \end{array} \right] T_{16} \quad 58$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{l} (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) \quad - (b''_{14})^{(1,1)}(G, t) \quad - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4,4)}(G_{27}, t) \quad - (b''_{29})^{(5,5,5,5,5)}(G_{31}, t) \quad - (b''_{33})^{(6,6,6,6,6)}(G_{35}, t) \\ - (b''_{37})^{(7,7,7)}(G_{39}, t) \quad - (b''_{41})^{(8,8,8)}(G_{43}, t) \end{array} \right] T_{17} \quad 59$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \quad \boxed{-(b'_{15})^{(1,1)}(G, t)} \quad \boxed{-(b'_{22})^{(3,3,3)}(G_{23}, t)} \\ \boxed{-(b'_{26})^{(4,4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b'_{30})^{(5,5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b'_{34})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{38})^{(7,7,7)}(G_{39}, t)} \quad \boxed{-(b'_{42})^{(8,8,8)}(G_{43}, t)} \end{array} \right] T_{18} \quad 60$$

where $\boxed{-(b'_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b'_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b'_{18})^{(2)}(G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1)}(G, t)}$, $\boxed{-(b'_{14})^{(1,1)}(G, t)}$, $\boxed{-(b'_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{20})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b'_{21})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b'_{22})^{(3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b'_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b'_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b'_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b'_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b'_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b'_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{36})^{(7,7,7)}(G_{39}, t)}$, $\boxed{-(b'_{37})^{(7,7,7)}(G_{39}, t)}$, $\boxed{-(b'_{38})^{(7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)}$, $\boxed{-(b'_{41})^{(8,8,8)}(G_{43}, t)}$, $\boxed{-(b'_{42})^{(8,8,8)}(G_{43}, t)}$ are eight detrition coefficients for category 1,2 and 3

Third Module Concatenation

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21}, t)} \quad \boxed{+(a'_{16})^{(2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{13})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a'_{24})^{(4,4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a'_{28})^{(5,5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a'_{32})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{40})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{20} \quad 61$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} \quad \boxed{+(a'_{17})^{(2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{14})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a'_{25})^{(4,4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a'_{29})^{(5,5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a'_{33})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{37})^{(7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{41})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{21} \quad 62$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} \quad \boxed{+(a'_{18})^{(2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{15})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a'_{26})^{(4,4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a'_{30})^{(5,5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a'_{34})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{38})^{(7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{42})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{22} \quad 63$$

$\boxed{+(a'_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a'_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a'_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a'_{16})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a'_{17})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a'_{18})^{(2,2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a'_{13})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a'_{14})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a'_{15})^{(1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37}, t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41}, t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41}, t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41}, t)}$ are eight augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{l} \boxed{(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23}, t)}\boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)}\boxed{-(b'_{13})^{(1,1,1)}(G, t)}} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{20} \quad 64$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{l} \boxed{(b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23}, t)}\boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)}\boxed{-(b'_{14})^{(1,1,1)}(G, t)}} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39}, t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{21} \quad 65$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{l} \boxed{(b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23}, t)}\boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)}\boxed{-(b'_{15})^{(1,1,1)}(G, t)}} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39}, t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{22} \quad 66$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43}, t)}$ are eight detrition coefficients for category 1, 2 and 3

Fourth Module Concatenation

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[\begin{array}{l} (a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) + (a''_{28})^{(5,5)}(T_{29}, t) + (a''_{32})^{(6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1)}(T_{14}, t) + (a''_{16})^{(2,2,2,2)}(T_{17}, t) + (a''_{20})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{36})^{(7,7,7,7)}(T_{37}, t) + (a''_{40})^{(8,8,8,8)}(T_{41}, t) \end{array} \right] G_{24} \quad 67$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[\begin{array}{l} (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t) + (a''_{29})^{(5,5)}(T_{29}, t) + (a''_{33})^{(6,6)}(T_{33}, t) \\ + (a''_{14})^{(1,1,1,1)}(T_{14}, t) + (a''_{17})^{(2,2,2,2)}(T_{17}, t) + (a''_{21})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{37})^{(7,7,7,7)}(T_{37}, t) + (a''_{41})^{(8,8,8,8)}(T_{41}, t) \end{array} \right] G_{25} \quad 68$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[\begin{array}{l} (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t) + (a''_{30})^{(5,5)}(T_{29}, t) + (a''_{34})^{(6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1)}(T_{14}, t) + (a''_{18})^{(2,2,2,2)}(T_{17}, t) + (a''_{22})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{38})^{(7,7,7,7)}(T_{37}, t) + (a''_{42})^{(8,8,8,8)}(T_{41}, t) \end{array} \right] G_{26} \quad 69$$

$$(a''_{24})^{(4)}(T_{25}, t), (a''_{25})^{(4)}(T_{25}, t), (a''_{26})^{(4)}(T_{25}, t)$$

are first augmentation coefficients category 1, 2 3

$$+(a''_{28})^{(5,5)}(T_{29}, t), +(a''_{29})^{(5,5)}(T_{29}, t), +(a''_{30})^{(5,5)}(T_{29}, t)$$

are second augmentation coefficient for category 1, 2 and 3

$$+(a''_{32})^{(6,6)}(T_{33}, t), +(a''_{33})^{(6,6)}(T_{33}, t), +(a''_{34})^{(6,6)}(T_{33}, t)$$

are third augmentation coefficient for category 1, 2 and 3

$$+(a''_{13})^{(1,1,1,1)}(T_{14}, t), +(a''_{14})^{(1,1,1,1)}(T_{14}, t), +(a''_{15})^{(1,1,1,1)}(T_{14}, t)$$

are fourth augmentation coefficients for category 1, 2 and 3

$$+(a''_{16})^{(2,2,2,2)}(T_{17}, t), +(a''_{17})^{(2,2,2,2)}(T_{17}, t), +(a''_{18})^{(2,2,2,2)}(T_{17}, t)$$

are fifth augmentation coefficients for category 1, 2 and 3

$$+(a''_{20})^{(3,3,3,3)}(T_{21}, t), +(a''_{21})^{(3,3,3,3)}(T_{21}, t), +(a''_{22})^{(3,3,3,3)}(T_{21}, t)$$

are sixth augmentation coefficients for category 1, 2 and 3

$$+(a''_{36})^{(7,7,7,7)}(T_{37}, t), +(a''_{37})^{(7,7,7,7)}(T_{37}, t), +(a''_{38})^{(7,7,7,7)}(T_{37}, t)$$

are seventh augmentation coefficients for category 1, 2 and 3

$$+(a''_{40})^{(8,8,8,8)}(T_{41}, t), +(a''_{41})^{(8,8,8,8)}(T_{41}, t), +(a''_{42})^{(8,8,8,8)}(T_{41}, t)$$

are eighth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[\begin{array}{l} (b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}, t) - (b''_{28})^{(5,5)}(G_{31}, t) - (b''_{32})^{(6,6)}(G_{35}, t) \\ - (b''_{13})^{(1,1,1,1)}(G, t) - (b''_{16})^{(2,2,2,2)}(G_{19}, t) - (b''_{20})^{(3,3,3,3)}(G_{23}, t) \\ - (b''_{36})^{(7,7,7,7)}(G_{39}, t) - (b''_{40})^{(8,8,8,8)}(G_{43}, t) \end{array} \right] T_{24} \quad 70$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[\begin{array}{l} (b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}, t) - (b''_{29})^{(5,5)}(G_{31}, t) - (b''_{33})^{(6,6)}(G_{35}, t) \\ - (b''_{14})^{(1,1,1,1)}(G, t) - (b''_{17})^{(2,2,2,2)}(G_{19}, t) - (b''_{21})^{(3,3,3,3)}(G_{23}, t) \\ - (b''_{37})^{(7,7,7,7)}(G_{39}, t) - (b''_{41})^{(8,8,8,8)}(G_{43}, t) \end{array} \right] T_{25} \quad 71$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[\begin{array}{l} (b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}, t) - (b''_{30})^{(5,5)}(G_{31}, t) - (b''_{34})^{(6,6)}(G_{35}, t) \\ - (b''_{15})^{(1,1,1,1)}(G, t) - (b''_{18})^{(2,2,2,2)}(G_{19}, t) - (b''_{22})^{(3,3,3,3)}(G_{23}, t) \\ - (b''_{38})^{(7,7,7,7)}(G_{39}, t) - (b''_{42})^{(8,8,8,8)}(G_{43}, t) \end{array} \right] T_{26} \quad 72$$

Where $-(b''_{24})^{(4)}(G_{27}, t), -(b''_{25})^{(4)}(G_{27}, t), -(b''_{26})^{(4)}(G_{27}, t)$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43}, t)}$ are eighth detrition coefficients for category 1, 2 and 3

Fifth Module Concatenation

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{l} \boxed{(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{24})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{32})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{13})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{16})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{36})^{(7,7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{40})^{(8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{28} \quad 73$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{l} \boxed{(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{25})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{33})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{14})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{17})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{21})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{37})^{(7,7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{41})^{(8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{29} \quad 74$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{l} \boxed{(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{26})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{34})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{15})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{18})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{22})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{38})^{(7,7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{42})^{(8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{30} \quad 75$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$

are first augmentation coefficients for category 1, 2 and 3

And $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$

are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$

are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a'_{13})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a'_{14})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a'_{15})^{(1,1,1,1,1)}(T_{14}, t)}$ are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a'_{16})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a'_{17})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a'_{18})^{(2,2,2,2,2)}(T_{17}, t)}$ are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a'_{20})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a'_{21})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a'_{22})^{(3,3,3,3,3)}(T_{21}, t)}$ are sixth augmentation coefficients for category 1, 2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t)}$ are eighth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{l} \boxed{(b'_{28})^{(5)} - \boxed{(b''_{28})^{(5)}(G_{31}, t)} - \boxed{(b''_{24})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)} \quad \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)} \quad \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{28} \quad 76$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{l} \boxed{(b'_{29})^{(5)} - \boxed{(b''_{29})^{(5)}(G_{31}, t)} - \boxed{(b''_{25})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)} \quad \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)} \quad \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{29} \quad 77$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{l} \boxed{(b'_{30})^{(5)} - \boxed{(b''_{30})^{(5)}(G_{31}, t)} - \boxed{(b''_{26})^{(4,4)}(G_{27}, t)} - \boxed{(b''_{34})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)} \quad \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)} \quad \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{30} \quad 78$$

where $\boxed{-(b''_{28})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4)}(G_{27}, t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t)}$ are eighth detrition coefficients for category 1,2, and 3

Sixth Module Concatenation

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{l} \boxed{(a'_{32})^{(6)} + \boxed{(a''_{32})^{(6)}(T_{33}, t)} + \boxed{(a''_{28})^{(5,5,5)}(T_{29}, t)} + \boxed{(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{32} \quad 79$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{l} \boxed{(a'_{33})^{(6)} + \boxed{(a''_{33})^{(6)}(T_{33}, t)} + \boxed{(a''_{29})^{(5,5,5)}(T_{29}, t)} + \boxed{(a''_{25})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{33} \quad 80$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[\begin{array}{l} (a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t) + (a''_{30})^{(5,5,5)}(T_{29}, t) + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t) + (a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{34} \quad 81$$

$(a'_{32})^{(6)}(T_{33}, t), (a'_{33})^{(6)}(T_{33}, t), (a'_{34})^{(6)}(T_{33}, t)$ are first augmentation coefficients for category 1, 2 and 3

$(a''_{28})^{(5,5,5)}(T_{29}, t), (a''_{29})^{(5,5,5)}(T_{29}, t), (a''_{30})^{(5,5,5)}(T_{29}, t)$ are second augmentation coefficients for category 1, 2 and 3

$(a''_{24})^{(4,4,4)}(T_{25}, t), (a''_{25})^{(4,4,4)}(T_{25}, t), (a''_{26})^{(4,4,4)}(T_{25}, t)$ are third augmentation coefficients for category 1, 2 and 3

$(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t), (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t), (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$ - are fourth augmentation coefficients

$(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t), (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t), (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$ - fifth augmentation coefficients

$(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t), (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t), (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$ sixth augmentation coefficients

$(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t), (a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t), (a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t)$ seventh augmentation coefficients

$(a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t), (a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t), (a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t)$ Eighth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[\begin{array}{l} (b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}, t) - (b''_{28})^{(5,5,5)}(G_{31}, t) - (b''_{24})^{(4,4,4)}(G_{27}, t) \\ - (b''_{13})^{(1,1,1,1,1,1)}(G, t) - (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t) - (b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t) \end{array} \right] T_{32} \quad 82$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[\begin{array}{l} (b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}, t) - (b''_{29})^{(5,5,5)}(G_{31}, t) - (b''_{25})^{(4,4,4)}(G_{27}, t) \\ - (b''_{14})^{(1,1,1,1,1,1)}(G, t) - (b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t) - (b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t) \end{array} \right] T_{33} \quad 83$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[\begin{array}{l} (b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}, t) - (b''_{30})^{(5,5,5)}(G_{31}, t) - (b''_{26})^{(4,4,4)}(G_{27}, t) \\ - (b''_{15})^{(1,1,1,1,1,1)}(G, t) - (b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t) - (b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t) - (b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t) \end{array} \right] T_{34} \quad 84$$

$-(b''_{32})^{(6)}(G_{35}, t), -(b''_{33})^{(6)}(G_{35}, t), -(b''_{34})^{(6)}(G_{35}, t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5)}(G_{31}, t), -(b''_{29})^{(5,5,5)}(G_{31}, t), -(b''_{30})^{(5,5,5)}(G_{31}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4)}(G_{27}, t), -(b''_{25})^{(4,4,4)}(G_{27}, t), -(b''_{26})^{(4,4,4)}(G_{27}, t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G, t), -(b''_{14})^{(1,1,1,1,1,1)}(G, t), -(b''_{15})^{(1,1,1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)$, $-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)$ are eighth detrition coefficients for category 1, 2, and 3

Seventh Module Concatenation

$$\frac{dG_{36}}{dt} \tag{85}$$

$$= (a_{36})^{(7)} G_{37} - \left[\begin{array}{c} (a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) + (a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4,4,4)}(T_{25}, t) + (a''_{28})^{(5,5,5,5,5,5)}(T_{29}, t) + (a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{13}$$

$$\frac{dG_{37}}{dt} \tag{86}$$

$$= (a_{37})^{(7)} G_{36} - \left[\begin{array}{c} (a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t) + (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t) + (a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t) + (a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{14}$$

$$\frac{dG_{38}}{dt} \tag{87}$$

$$= (a_{38})^{(7)} G_{37} - \left[\begin{array}{c} (a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t) + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t) + (a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t) + (a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) + (a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{15}$$

Where $(a''_{36})^{(7)}(T_{37}, t)$, $(a''_{37})^{(7)}(T_{37}, t)$, $(a''_{38})^{(7)}(T_{37}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$ are seventh augmentation coefficient for category 1, 2 and 3

$$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41}, t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41}, t)}, \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41}, t)}$$

are eighth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \tag{88}$$

$$(b_{36})^{(7)}T_{37} - \left[\begin{array}{cccc} \boxed{(b'_{36})^{(7)}} & \boxed{-(b''_{36})^{(7)}(G_{39}, t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)} & \\ & \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)} & \end{array} \right] T_{13}$$

$$\frac{dT_{37}}{dt} = \tag{89}$$

$$(b_{37})^{(7)}T_{36} - \left[\begin{array}{cccc} \boxed{(b'_{37})^{(7)}} & \boxed{-(b''_{37})^{(7)}(G_{39}, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)} & \\ & \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)} & \end{array} \right] T_{14}$$

$$\frac{dT_{38}}{dt} = \tag{90}$$

$$(b_{38})^{(7)}T_{37} - \left[\begin{array}{cccc} \boxed{(b'_{38})^{(7)}} & \boxed{-(b''_{38})^{(7)}(G_{39}, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)} & \\ & \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)} & \end{array} \right] T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G, t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G, t)}, \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G, t)}$$

are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43}, t)}$ are eighth detrition coefficients for category 1, 2 and 3

Eighth Module Concatenation

$$\frac{dG_{40}}{dt} \tag{91}$$

$$= (a_{40})^{(8)}G_{41} - \left[\begin{array}{l} (a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) \quad + (a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17}, t) \quad + (a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25}, t) \quad + (a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29}, t) \quad + (a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14}, t) \quad + (a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{13}$$

$$\frac{dG_{41}}{dt} \tag{92}$$

$$= (a_{41})^{(8)}G_{40} - \left[\begin{array}{l} (a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t) \quad + (a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17}, t) \quad + (a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25}, t) \quad + (a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29}, t) \quad + (a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14}, t) \quad + (a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{14}$$

$$\frac{dG_{42}}{dt} \tag{93}$$

$$= (a_{42})^{(8)}G_{41} - \left[\begin{array}{l} (a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t) \quad + (a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17}, t) \quad + (a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25}, t) \quad + (a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29}, t) \quad + (a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14}, t) \quad + (a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{15}$$

Where $(a'_{40})^{(8)}(T_{41}, t)$, $(a'_{41})^{(8)}(T_{41}, t)$, $(a'_{42})^{(8)}(T_{41}, t)$ are first augmentation coefficients for category 1, 2 and 3

$(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17}, t)$, $(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17}, t)$, $(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3

$(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21}, t)$, $(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21}, t)$, $(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25}, t)$, $(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25}, t)$, $(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29}, t)$, $(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29}, t)$, $(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33}, t)$, $(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33}, t)$, $(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14}, t)$, $(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14}, t)$, $(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14}, t)$ are seventh augmentation coefficient for 1,2,3

$(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37}, t)$, $(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37}, t)$, $(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37}, t)$ are eighth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = \tag{94}$$

$$(b_{40})^{(8)}T_{41} - \left[\begin{array}{l} (b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}, t) \quad - (b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19}, t) \quad - (b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27}, t) \quad - (b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31}, t) \quad - (b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{13})^{(1,1,1,1,1,1,1,1)}(G, t) \quad - (b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39}, t) \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$

95

$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43}, t)} & \boxed{-(b'_{17})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)} & \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$

96

$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43}, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)} & \end{array} \right] T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39}, t)}$ are eighth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(A) \quad (a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i, j = 13, 14, 15$$

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(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$(C) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \\ \lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

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Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$ are positive constants and $i = 13, 14, 15$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(1)}(T'_{14}, t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions

$(a_i'')^{(1)}(T'_{14}, t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

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(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

101

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, i, j = 16, 17, 18$

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$(H) \lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}(G_{19}, t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

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Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}, t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} \|(G_{19}) - (G_{19})'\| e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}, t)$ and $(a_i'')^{(2)}(T_{17}, t) \cdot (T_{17}, t)$ and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$$(1) \quad (\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, \text{ are positive constants} \quad 109$$

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(j) \quad (a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$ are positive constants and $i = 20, 21, 22$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}, t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} \|G_{23} - G_{23}'\| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}, t)$ and $(a_i'')^{(3)}(T_{21}, t) \cdot (T_{21}, t)$ and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24, 25, 26$ 117

(L) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

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(M) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$
 $\lim_{G \rightarrow \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$ are positive constants and $i = 24, 25, 26$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(4)}(T_{25}, t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} |(G_{27}) - (G_{27})'| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}, t)$ and $(a_i'')^{(4)}(T_{25}, t) \cdot (T_{25}, t)$ and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\widehat{M}_{24})^{(4)} = 4$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\widehat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\widehat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\widehat{P}_{24})^{(4)}, (\widehat{Q}_{24})^{(4)}$: 121

(N) There exists two constants $(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\widehat{k}_{24})^{(4)}, (\widehat{A}_{24})^{(4)}$ and $(\widehat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\widehat{B}_{24})^{(4)} + (\widehat{Q}_{24})^{(4)} (\widehat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, i, j = 28, 29, 30$ 122

(O) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\widehat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\widehat{B}_{28})^{(5)}$$

123

(P) $\lim_{T \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$
 $\lim_{G \rightarrow \infty} (b_i'')^{(5)}((G_{31}), t) = (r_i)^{(5)}$

Definition of $(\widehat{A}_{28})^{(5)}, (\widehat{B}_{28})^{(5)}$:

Where $(\widehat{A}_{28})^{(5)}, (\widehat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\widehat{k}_{28})^{(5)} |T_{29}' - T_{29}| e^{-(\widehat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\widehat{k}_{28})^{(5)} |(G_{31}') - (G_{31})| e^{-(\widehat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\widehat{k}_{28})^{(5)}, (\widehat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{28})^{(5)} = 5$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to

terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants
 $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with
 $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants
 $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, i, j = 32, 33, 34$ 127

(Q) The functions $(a''_i)^{(6)}, (b''_i)^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b''_i)^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b'_i)^{(6)} \leq (\hat{B}_{32})^{(6)}$$

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(R) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(6)}(T_{33}, t) = (p_i)^{(6)}$
 $\lim_{G \rightarrow \infty} (b''_i)^{(6)}((G_{35}), t) = (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$ are positive constants and $i = 32, 33, 34$

They satisfy Lipschitz condition:

$$|(a''_i)^{(6)}(T'_{33}, t) - (a''_i)^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T'_{33} - T_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b''_i)^{(6)}((G_{35})', t) - (b''_i)^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35})' - (G_{35})| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(6)}(T'_{33}, t)$ and $(a''_i)^{(6)}(T_{33}, t)$. (T'_{33}, t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a''_i)^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a''_i)^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants
 $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and $(\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$(a_i)^{(7)}, (a'_i)^{(7)}, (a''_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (b''_i)^{(7)} > 0$,
 $i, j = 36, 37, 38$ 131

The functions $(a''_i)^{(7)}, (b''_i)^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a''_i)^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b''_i)^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b'_i)^{(7)} \leq (\hat{B}_{36})^{(7)}$$

$\lim_{T_2 \rightarrow \infty} (a''_i)^{(7)}(T_{37}, t) = (p_i)^{(7)}$
 $\lim_{G \rightarrow \infty} (b''_i)^{(7)}(G_{39}, t) = (r_i)^{(7)}$ 132

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$ are positive constants
 and $i = 36, 37, 38$

They satisfy Lipschitz condition: 133

$$|(a''_i)^{(7)}(T'_{37}, t) - (a''_i)^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T'_{37}| e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b''_i)^{(7)}((G_{39})', t) - (b''_i)^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} |(G_{39})' - (G_{39})| e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(7)}(T'_{37}, t)$ and $(a''_i)^{(7)}(T_{37}, t)$. (T'_{37}, t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a''_i)^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 7$ then the function $(a''_i)^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

$(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}$ and $(\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a'_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36, 37, 38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(a_i)^{(7)} + (a'_i)^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(b_i)^{(7)} + (b'_i)^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

A. $(a_i)^{(8)}, (a'_i)^{(8)}, (a''_i)^{(8)}, (b_i)^{(8)}, (b'_i)^{(8)}, (b''_i)^{(8)} > 0, \quad i, j = 40, 41, 42$ 136

B. The functions $(a''_i)^{(8)}, (b''_i)^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a''_i)^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \quad 138$$

$$(b''_i)^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b'_i)^{(8)} \leq (\hat{B}_{40})^{(8)} \quad 139$$

C. $\lim_{T_2 \rightarrow \infty} (a''_i)^{(8)}(T_{41}, t) = (p_i)^{(8)} \quad 140$

$$\lim_{G \rightarrow \infty} (b''_i)^{(8)}((G_{43}), t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}$ are positive constants and $i = 40, 41, 42$

They satisfy Lipschitz condition:

$$|(a''_i)^{(8)}(T'_{41}, t) - (a''_i)^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T'_{41}| e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b''_i)^{(8)}((G_{43})', t) - (b''_i)^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} |(G_{43}) - (G_{43})'| e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(8)}(T'_{41}, t)$ and $(a''_i)^{(8)}(T_{41}, t)$. (T'_{41}, t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a''_i)^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 8$ then the function $(a''_i)^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

D. $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}} , \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

E. There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a'_i)^{(8)}, (b_i)^{(8)}, (b'_i)^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42$, Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a'_i)^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b'_i)^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13} \right)^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right] G_{13}(s_{(13)}) ds_{(13)}$$

$$\begin{aligned} \bar{G}_{14}(t) &= G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \\ \bar{G}_{15}(t) &= G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \\ \bar{T}_{13}(t) &= T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \\ \bar{T}_{14}(t) &= T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \\ \bar{T}_{15}(t) &= T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \end{aligned}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

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Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By

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$$\begin{aligned} \bar{G}_{16}(t) &= G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)} \\ \bar{G}_{17}(t) &= G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \\ \bar{G}_{18}(t) &= G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \\ \bar{T}_{16}(t) &= T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \\ \bar{T}_{17}(t) &= T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \\ \bar{T}_{18}(t) &= T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \end{aligned}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

By

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$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20} \right)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}$$

By

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$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24} \right)^{(4)} (T_{25}(s_{(24)}), s_{(24)}) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}$$

By

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$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

By

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$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}$$

By

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$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{40})^{(8)}, T_i^0 \leq (\hat{Q}_{40})^{(8)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t}$$

By

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$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$

$$\left(1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$
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From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0)e^{-(\tilde{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\tilde{M}_{20})^{(3)}} \left[((\hat{P}_{20})^{(3)} + G_{21}^0)e^{-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself. Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)}s_{(24)}} \right) \right] ds_{(24)} =$$

$$(1 + (a_{24})^{(4)}t)G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\tilde{M}_{24})^{(4)}} \left(e^{(\tilde{M}_{24})^{(4)}t} - 1 \right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0)e^{-(\tilde{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\tilde{M}_{24})^{(4)}} \left[((\hat{P}_{24})^{(4)} + G_{25}^0)e^{-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}} + (\hat{P}_{24})^{(4)} \right]$$

(G_t^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)}s_{(28)}} \right) \right] ds_{(28)} =$$

$$(1 + (a_{28})^{(5)}t)G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\tilde{M}_{28})^{(5)}} \left(e^{(\tilde{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0)e^{-(\tilde{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\tilde{M}_{28})^{(5)}} \left[((\hat{P}_{28})^{(5)} + G_{29}^0)e^{-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}} + (\hat{P}_{28})^{(5)} \right]$$

(G_t^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)}s_{(32)}} \right) \right] ds_{(32)} =$$

$$(1 + (a_{32})^{(6)}t)G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left(e^{(\tilde{M}_{32})^{(6)}t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0)e^{-(\tilde{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left[((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}} + (\hat{P}_{32})^{(6)} \right]$$

(G_t^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} = \\ (1 + (a_{36})^{(7)} t) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 179

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \\ (1 + (a_{40})^{(8)} t) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{-\left(\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d \left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t} \}$$

Indeed if we denote

Definition of \tilde{G}, \tilde{T} : $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{-(\bar{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} | (a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)}) | e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)} \end{aligned}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\bar{M}_{13})^{(1)}t} \leq \frac{1}{(\bar{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)} \right) d \left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \tag{186}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a'_{13})^{(1)}$ and $(b'_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ and $(\hat{Q}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$, $i = 13, 14, 15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \} ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

Definition of $(\bar{M}_{13})^{(1)}_1, (\bar{M}_{13})^{(1)}_2$ and $(\bar{M}_{13})^{(1)}_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < (\bar{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq ((\bar{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\bar{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\bar{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b'_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded.}$$

The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b'_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\widehat{P}_{16})^{(2)}$ and $(\widehat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[(\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d \left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{16})^{(2)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{16})^{(2)} t} \right\}$$

Indeed if we denote 195

$$\widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$$

It results 196

$$\begin{aligned}
 &|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\
 &\int_0^t \{(a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} + \\
 &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} + \\
 &G_{16}^{(2)} |(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)})| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}
 \end{aligned}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\begin{aligned}
 &|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\widehat{M}_{16})^{(2)}t} \leq \\
 &\frac{1}{(\widehat{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{K}_{16})^{(2)}) d((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})
 \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as 199
 not conformal with the reality, however we have put this hypothesis, in order that we can postulate
 condition necessary to prove the uniqueness of the solution bounded by
 $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it
 suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$, $i = 16, 17, 18$ depend only on T_{17} and respectively on
 (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1$, $((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)})_1 \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)} G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)} ((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is 202
 analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 10: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$ then $T_{17} \rightarrow \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_2} \text{ By taking now } \varepsilon_2 \text{ sufficiently small one sees that } T_{17} \text{ is unbounded.} \quad 205$$

The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b_{18}'')^{(2)}((G_{19})(t), t) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 206

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[(\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \}$$

Indeed if we denote 212

$$\text{Definition of } \widetilde{G}_{23}, \widetilde{T}_{23} : (\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$$

It results 213

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} + \end{aligned}$$

$$G_{20}^{(2)} |(a_{20}'')^{(3)}(T_{21}^{(1)}, s_{(20)}) - (a_{20}'')^{(3)}(T_{21}^{(2)}, s_{(20)})| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{20})^{(3)}t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}} ((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}) d((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}) \quad 214$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ and $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ respectively of \mathbb{R}_+ . 215

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i')^{(3)} - (a_i'')^{(3)})(T_{21}(s_{(20)}), s_{(20)}) ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(3)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{20})^{(3)})_1$, $((\widehat{M}_{20})^{(3)})_2$ and $((\widehat{M}_{20})^{(3)})_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$$G_{20} < (\widehat{M}_{20})^{(3)} \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a_{21}')^{(3)} G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)} ((\widehat{M}_{20})^{(3)})_1 / (a_{21}')^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)} ((\widehat{M}_{20})^{(3)})_2 / (a_{22}')^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(3)}((G_{23})(t), t)) = (b_{21}')^{(3)}$ then $T_{21} \rightarrow \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21}')^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = \log \frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.

The same property holds for T_{22} if $\lim_{t \rightarrow \infty} (b_{22}'')^{(3)}((G_{23})(t), t) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[(\hat{P}_{24})^{(4)} + ((\hat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[((\hat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)} \right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying 224
Equations into itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right), \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)} t} \right\}$$

Indeed if we denote

$$\underline{\text{Definition of}} (\overline{(G_{27})}, \overline{(T_{27})}) : (\overline{(G_{27})}, \overline{(T_{27})}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_{24}^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\bar{M}_{24})^{(4)} s_{(24)}} e^{(\bar{M}_{24})^{(4)} s_{(24)}} ds_{(24)} + \\ &\int_0^t \{(a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)} s_{(24)}} e^{-(\bar{M}_{24})^{(4)} s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)} s_{(24)}} e^{(\bar{M}_{24})^{(4)} s_{(24)}} + \\ &G_{24}^{(2)} |(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)})| e^{-(\bar{M}_{24})^{(4)} s_{(24)}} e^{(\bar{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)} \end{aligned}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\begin{aligned} & |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\widehat{M}_{24})^{(4)}t} \leq \hspace{15em} 226 \\ & \frac{1}{(\widehat{M}_{24})^{(4)}} ((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}) d \left(((G_{27})^{(1)}, (T_{27})^{(1)}); (G_{27})^{(2)}, (T_{27})^{(2)} \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of \mathbb{R}_+ . 227

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a'_i)^{(4)}$ and $(b'_i)^{(4)}$, $i = 24, 25, 26$ depend only on T_{25} and respectively on (G_{27}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 228

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{24})^{(4)})_1, ((\widehat{M}_{24})^{(4)})_2$ and $((\widehat{M}_{24})^{(4)})_3$: 229

Remark 18: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$$G_{24} < (\widehat{M}_{24})^{(4)} \text{ it follows } \frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)}G_{25} \text{ and by integrating}$$

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)}((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way, one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$ then $T_{25} \rightarrow \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.

The same property holds for T_{26} if $\lim_{t \rightarrow \infty} (b_{26}'')^{(4)}((G_{27})(t), t) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[(\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[((\hat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d \left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \}$$

Indeed if we denote

Definition of $(\widetilde{G}_{31}), (\widetilde{T}_{31})$: $(\widetilde{G}_{31}), (\widetilde{T}_{31}) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\begin{aligned} |\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t ((a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} |(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)})| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)} \end{aligned}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\begin{aligned} & |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widehat{M}_{28})^{(5)}t} \leq \hspace{15em} 237 \\ & \frac{1}{(\widehat{M}_{28})^{(5)}} ((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}) d \left(((G_{31})^{(1)}, (T_{31})^{(1)}); (G_{31})^{(2)}, (T_{31})^{(2)} \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ respectively of \mathbb{R}_+ . 238

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$, $i = 28, 29, 30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{28})^{(5)})_1$, $((\widehat{M}_{28})^{(5)})_2$ and $((\widehat{M}_{28})^{(5)})_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$$G_{28} < (\widehat{M}_{28})^{(5)} \text{ it follows } \frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)}G_{29} \text{ and by integrating}$$

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)}((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$ then $T_{29} \rightarrow \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b'_i)^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded.}$$

The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[(\hat{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d \left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}, \widetilde{T_{35}})$: $(\widetilde{G_{35}}, \widetilde{T_{35}}) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$|\widetilde{G}_{32}^{(1)} - \widetilde{G}_{32}^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} +$$

$$\int_0^t \{ (a_{32}')^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$(a_{32}'')^{(6)}(T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} +$$

$$G_{32}^{(2)} | (a_{32}'')^{(6)}(T_{33}^{(1)}, s_{(32)}) - (a_{32}'')^{(6)}(T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} & |(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\widehat{M}_{32})^{(6)}t} \leq \hspace{15em} 248 \\ & \frac{1}{(\widehat{M}_{32})^{(6)}} ((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}) d\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)}\right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ and $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of \mathbb{R}_+ . 249

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a'_i)^{(6)}$ and $(b'_i)^{(6)}$, $i = 32, 33, 34$ depend only on T_{33} and respectively on (G_{35}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 250

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(6)} - (a''_i)^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(6)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{32})^{(6)})_1$, $((\widehat{M}_{32})^{(6)})_2$ and $((\widehat{M}_{32})^{(6)})_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$$G_{32} < (\widehat{M}_{32})^{(6)} \text{ it follows } \frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a'_{33})^{(6)}G_{33} \text{ and by integrating}$$

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a'_{33})^{(6)}$$

In the same way, one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a'_{34})^{(6)}$$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(6)}((G_{35})(t), t)) = (b'_{33})^{(6)}$ then $T_{33} \rightarrow \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$$

If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_6}$$

By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.

The same property holds for T_{34} if $\lim_{t \rightarrow \infty} (b''_{34})^{(6)}((G_{35})(t), t(t), t) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose $(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[(\widehat{P}_{36})^{(7)} + ((\widehat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{36})^{(7)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[((\widehat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{36})^{(7)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{36})^{(7)} \right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{36})^{(7)}t} \right\}$$

Indeed if we denote

Definition of $(\widehat{G}_{39}), (\widehat{T}_{39}) : (\widehat{G}_{39}), (\widehat{T}_{39}) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$

It results

$$\begin{aligned} |\widetilde{G}_{36}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{36})^{(7)} |G_{37}^{(1)} - G_{37}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} ds_{(36)} + \\ &\int_0^t ((a'_{36})^{(7)}) |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} + \\ &(a''_{36})^{(7)} (T_{37}^{(1)}, s_{(36)}) |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} + \\ &G_{36}^{(2)} |(a''_{36})^{(7)} (T_{37}^{(1)}, s_{(36)}) - (a''_{36})^{(7)} (T_{37}^{(2)}, s_{(36)})| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} ds_{(36)} \end{aligned}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\begin{aligned} & |(G_{39})^{(1)} - (G_{39})^{(2)}| e^{-(\widehat{M}_{36})^{(7)}t} \leq \hspace{15em} 259 \\ & \frac{1}{(\widehat{M}_{36})^{(7)}} ((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}) d \left(((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)}) \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ and $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)}t}$ respectively of \mathbb{R}_+ . 260

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}$, $i = 36, 37, 38$ depend only on T_{37} and respectively on (G_{39}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(7)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{36})^{(7)})_1$, $((\widehat{M}_{36})^{(7)})_2$ and $((\widehat{M}_{36})^{(7)})_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} and G_{38} . indeed if

$$G_{36} < (\widehat{M}_{36})^{(7)} \text{ it follows } \frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a'_{37})^{(7)} G_{37} \text{ and by integrating}$$

$$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)} ((\widehat{M}_{36})^{(7)})_1 / (a'_{37})^{(7)}$$

In the same way, one can obtain

$$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)} ((\widehat{M}_{36})^{(7)})_2 / (a'_{38})^{(7)}$$

If G_{37} or G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} is bounded, from below, the same property holds for G_{37} and G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(7)} ((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \rightarrow \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b'_i)^{(7)} ((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7} \right) (1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_7 t} = \frac{1}{2} \text{ it results}$$

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_7} \text{ By taking now } \varepsilon_7 \text{ sufficiently small one sees that } T_{37} \text{ is unbounded.}$$

The same property holds for T_{38} if $\lim_{t \rightarrow \infty} (b_{38})^{(7)}((G_{39})(t), t) = (b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\bar{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\bar{M}_{40})^{(8)}} < 1$ and to choose 266

$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\bar{M}_{40})^{(8)}} \left[(\hat{P}_{40})^{(8)} + ((\hat{P}_{40})^{(8)} + G_j^0) e^{-\left(\frac{(\hat{P}_{40})^{(8)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\bar{M}_{40})^{(8)}} \left[((\hat{Q}_{40})^{(8)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{40})^{(8)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{40})^{(8)} \right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d \left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t} \} \quad 269$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}, \widetilde{T_{43}}) : ((\widetilde{G_{43}}, \widetilde{T_{43}})) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\begin{aligned} |\tilde{G}_{40}^{(1)} - \tilde{G}_{40}^{(2)}| &\leq \int_0^t (a_{40})^{(8)} |G_{41}^{(1)} - G_{41}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} ds_{(40)} + \\ &\int_0^t \{ (a'_{40})^{(8)} |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{-(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &(a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &G_{40}^{(2)} | (a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) - (a''_{40})^{(8)} (T_{41}^{(2)}, s_{(40)}) | e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} \} ds_{(40)} \end{aligned}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\begin{aligned} |(G_{43})^{(1)} - (G_{43})^{(2)}| e^{-(\bar{M}_{40})^{(8)}t} &\leq \\ \frac{1}{(\bar{M}_{40})^{(8)}} &((a_{40})^{(8)} + (a'_{40})^{(8)} + (\bar{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}) d \left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) \end{aligned} \quad 273$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)}t}$ and $(\widehat{Q}_{40})^{(8)} e^{(\widehat{M}_{40})^{(8)}t}$ respectively of \mathbb{R}_+ . 274

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(8)}$ and $(b''_i)^{(8)}$, $i = 40, 41, 42$ depend only on T_{41} and respectively on (G_{43}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(8)} - (a''_i)^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(8)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{40})^{(8)})_1, ((\widehat{M}_{40})^{(8)})_2$ and $((\widehat{M}_{40})^{(8)})_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} and G_{42} . indeed if

$G_{40} < ((\widehat{M}_{40})^{(8)})_1$ it follows $\frac{dG_{41}}{dt} \leq ((\widehat{M}_{40})^{(8)})_1 - (a'_{41})^{(8)}G_{41}$ and by integrating

$$G_{41} \leq ((\widehat{M}_{40})^{(8)})_2 = G_{41}^0 + 2(a_{41})^{(8)}((\widehat{M}_{40})^{(8)})_1 / (a'_{41})^{(8)}$$

In the same way, one can obtain

$$G_{42} \leq ((\widehat{M}_{40})^{(8)})_3 = G_{42}^0 + 2(a_{42})^{(8)}((\widehat{M}_{40})^{(8)})_2 / (a'_{42})^{(8)}$$

If G_{41} or G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} is bounded, from below, the same property holds for G_{41} and G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(8)}((G_{43})(t), t)) = (b'_{41})^{(8)}$ then $T_{41} \rightarrow \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b''_i)^{(8)}((G_{43})(t), t) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8} \right) (1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_8 t} = \frac{1}{2} \text{ it results}$$

$$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_8} \text{ By taking now } \varepsilon_8 \text{ sufficiently small one sees that } T_{41} \text{ is unbounded.}$$

The same property holds for T_{42} if $\lim_{t \rightarrow \infty} (b''_{42})^{(8)}((G_{43})(t), t(t), t) = (b'_{42})^{(8)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 82

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

- (a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
 $-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$
 $-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

- (b) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations
 $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

- By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations
 $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 283

- (c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
 $(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, if $(v_0)^{(1)} < (v_1)^{(1)}$
 $(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$, if $(v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{C_{13}^0}{C_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, if $(\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

- $(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, if $(u_0)^{(1)} < (u_1)^{(1)}$
 $(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, if $(u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}$, if $(\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G_{13}^0 e^{((s_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(s_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

(d) $\sigma_1^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
 $-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a'_{16})^{(2)}(T_{17}, t) + (a'_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b'_{16})^{(2)}((G_{19}), t) - (b'_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$ 294

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots 296

(e) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 301

$$\text{and } (b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \quad 302$$

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \quad 305$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 306$$

$$\text{and } \boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)} \quad 307$$

and analogously 308

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } \boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 309$$

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)} \quad 317$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

(b) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations

$$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the roots of the equations

$$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

and
$$(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously 322

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } (u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t} \quad 324$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}, t) - (b''_{25})^{(4)}((G_{27}, t) \leq -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

(e) By $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$$(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the

roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

$$\text{and } (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

$$\text{and } (u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

Then the solution of global equations satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq \left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t} \right)$$

$$T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b'_{24})^{(4)}$$

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$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions of equation

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$:

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(h) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations

$$(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

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By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the

roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

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$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } (u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)} \text{ where } (u_1)^{(5)}, (\bar{u}_1)^{(5)}$$

Then the solution of global equations satisfies the inequalities

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$$G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(s_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \right) \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t} \quad 344$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

(k) By $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations

$$(a_{33})^{(6)} (v^{(6)})^2 + (\sigma_1)^{(6)} v^{(6)} - (a_{32})^{(6)} = 0$$

$$\text{and } (b_{33})^{(6)} (u^{(6)})^2 + (\tau_1)^{(6)} u^{(6)} - (b_{32})^{(6)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the

roots of the equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

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$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities

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$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t} \quad 355$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)} ((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)} ((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$$

$$\begin{aligned}(S_2)^{(6)} &= (a_{34})^{(6)} - (p_{34})^{(6)} \\ (R_1)^{(6)} &= (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)} \\ (R_2)^{(6)} &= (b'_{34})^{(6)} - (r_{34})^{(6)}\end{aligned}$$

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Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$\begin{aligned}- (\sigma_2)^{(7)} &\leq - (a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq - (\sigma_1)^{(7)} \\ - (\tau_2)^{(7)} &\leq - (b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq - (\tau_1)^{(7)}\end{aligned}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$:

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By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations

$$\begin{aligned}(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} &= 0 \\ \text{and } (b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} &= 0 \text{ and}\end{aligned}$$

Definition of $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$:

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By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$ and $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$\begin{aligned}(m_2)^{(7)} &= (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)} \\ (m_2)^{(7)} &= (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ if } (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}, \\ \text{and } (v_0)^{(7)} &= \frac{a_{36}^0}{a_{37}^0}\end{aligned}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously

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$$\begin{aligned}(\mu_2)^{(7)} &= (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)} \\ (\mu_2)^{(7)} &= (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}, \\ \text{and } (u_0)^{(7)} &= \frac{T_{36}^0}{T_{37}^0}\end{aligned}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \right) \left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((S_1)^{(7)} - (a'_{38})^{(7)})} \left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t} \quad 366$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)} ((R_1)^{(7)} - (b'_{38})^{(7)})} \left[e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)} ((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

(m) $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \leq -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}((G_{43}), t) - (b''_{41})^{(8)}((G_{43}), t) \leq -(\tau_1)^{(8)}$$

Definition of $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$: 372

(n) By $(v_1)^{(8)} > 0, (v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}, (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0, (\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

(o) If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}, \text{ if } (v_0)^{(8)} < (v_1)^{(8)}$$

$$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}, \text{ if } (v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)},$$

$$\text{and } (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}$$

$$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}, \text{ if } (\bar{v}_1)^{(8)} < (v_0)^{(8)}$$

and analogously

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$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \text{ if } (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \text{ if } (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

$$\text{and } (u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \text{ if } (\bar{u}_1)^{(8)} < (u_0)^{(8)} \text{ where } (u_1)^{(8)}, (\bar{u}_1)^{(8)}$$

Then the solution of global equations satisfies the inequalities

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$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)} ((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})} \left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t} \right] + G_{42}^0 e^{-(S_2)^{(8)}t} \right) \leq G_{42}(t) \leq \quad 377$$

$$\frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)} ((S_1)^{(8)} - (a'_{42})^{(8)})} \left[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t} \right] + G_{42}^0 e^{-(a'_{42})^{(8)}t}$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})} \left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t} \right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \leq T_{42}(t) \leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})} \left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t} \right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

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Proof: From global equations we obtain

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$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $v^{(1)} = \frac{G_{13}}{G_{14}}$

It follows

$$- \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(a) For $0 < \frac{G_{13}^0}{G_{14}^0} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}} , \quad (C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

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$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} , \quad (\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

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$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{c})^{(1)} (\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)} (\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}] t}}{1 + (\bar{c})^{(1)} e^{[-(a_{14})^{(1)} (\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}] t}} \leq (\bar{v}_1)^{(1)}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$, we obtain

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$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{c})^{(1)} (\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)} (\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}] t}}{1 + (\bar{c})^{(1)} e^{[-(a_{14})^{(1)} (\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}] t}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)} G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain

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$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)} (T_{17}, t) \right) - (a'_{17})^{(2)} (T_{17}, t) v^{(2)} - (a_{17})^{(2)} v^{(2)}$$

Definition of $v^{(2)}$:-

$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

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It follows

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$$- \left((a_{17})^{(2)} (v^{(2)})^2 + (\sigma_2)^{(2)} v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)} (v^{(2)})^2 + (\sigma_1)^{(2)} v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

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Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

(d) For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get

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$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

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(e) If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case,

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$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(f) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

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$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:-

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$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)} , \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

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Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)} , \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

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If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

Proof: From global equations we obtain

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$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a''_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:-

$$v^{(3)} = \frac{G_{20}}{G_{21}}$$

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It follows

$$- \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

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From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}} , \quad (C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get

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$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} , \quad (\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case,

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$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}} \leq v^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (\bar{v}_1)^{(3)}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

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$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

Proof : From global equations we obtain

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$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

(d) For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get

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$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case,

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$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)} (\nu_2)^{(4)} e^{[-(a_{25})^{(4)} ((\nu_1)^{(4)} - (\nu_2)^{(4)}) t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)} ((\nu_1)^{(4)} - (\nu_2)^{(4)}) t]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)} (\bar{\nu}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}) t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}) t]}} \leq (\bar{\nu}_1)^{(4)}$$

(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

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$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)} (\bar{\nu}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}) t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}) t]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines $(\nu_0)^{(4)}$ for the special case.**

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}$, then $(\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_1)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of $(u_0)^{(4)}$.**

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Proof: From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$- \left((a_{29})^{(5)} (\nu^{(5)})^2 + (\sigma_2)^{(5)} \nu^{(5)} - (a_{28})^{(5)} \right) \leq \frac{d\nu^{(5)}}{dt} \leq - \left((a_{29})^{(5)} (\nu^{(5)})^2 + (\sigma_1)^{(5)} \nu^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(g) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get

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$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

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$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (\bar{v}_1)^{(5)}$$

(i) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

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$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines** $(v_0)^{(5)}$ **for the special case .**

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, and definition of $(u_0)^{(5)}$.

Proof: From global equations we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:-
$$v^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

It follows

$$- \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

(j) For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}} , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

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$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case,

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$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} \leq (\bar{v}_1)^{(6)}$$

(l) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}$, then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines $(v_0)^{(6)}$ for the special case .**

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of $(u_0)^{(6)}$.**

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Proof: From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) \right) - \frac{(a''_{37})^{(7)}(T_{37}, t)v^{(7)} - (a_{37})^{(7)}v^{(7)}}{(a_{37})^{(7)}} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $v^{(7)} = \frac{G_{36}}{G_{37}}$

It follows

$$-\left((a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq -\left((a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$:-

$$\text{For } 0 < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$$

$$v^{(7)}(t) \geq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}, \quad (C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}$$

it follows $(v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}$

In the same manner , we get

$$v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}, \quad (\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}$$

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From which we deduce $(v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \leq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]t}} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (C)^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}} \leq (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \leq (\bar{v}_1)^{(7)} \leq \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(v_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (C)^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}} \leq (v_0)^{(7)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}''^{(7)}) = (a_{37}''^{(7)})$, then $(\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(v_1)^{(7)} = (\bar{v}_1)^{(7)}$ if in addition $(v_0)^{(7)} = (v_1)^{(7)}$ then $v^{(7)}(t) = (v_0)^{(7)}$ and as a consequence $G_{36}(t) = (v_0)^{(7)}G_{37}(t)$ **this also defines $(v_0)^{(7)}$ for the special case .**

Analogously if $(b_{36}''^{(7)}) = (b_{37}''^{(7)})$, then $(\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$ if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(v_1)^{(7)}$ and $(\bar{v}_1)^{(7)}$, **and definition of $(u_0)^{(7)}$.**

Proof : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) \right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

Definition of $v^{(8)}$:- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(8)}, (v_0)^{(8)}$:-

$$(m) \quad \text{For } 0 < \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (v_1)^{(8)} < (\bar{v}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(v_1)^{(8)} + (C)^{(8)}(v_2)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_0)^{(8)}]t}}{1 + (C)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_0)^{(8)}]t}}, \quad \boxed{(C)^{(8)} = \frac{(v_1)^{(8)} - (v_0)^{(8)}}{(v_0)^{(8)} - (v_2)^{(8)}}$$

it follows $(v_0)^{(8)} \leq v^{(8)}(t) \leq (v_1)^{(8)}$

In the same manner , we get

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$$v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)} + (\bar{C})^{(8)}(\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}{1 + (\bar{C})^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}, \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{v}_1)^{(8)} - (v_0)^{(8)}}{(v_0)^{(8)} - (\bar{v}_2)^{(8)}}$$

From which we deduce $(v_0)^{(8)} \leq v^{(8)}(t) \leq (\bar{v}_8)^{(8)}$

$$(n) \quad \text{If } 0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)} \text{ we find like in the previous case,}$$

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$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)} + (C)^{(8)}(v_2)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_2)^{(8)}]t}}{1 + (C)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_2)^{(8)}]t}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(8)} + (\bar{C})^{(8)}(\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}{1 + (\bar{C})^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}} \leq (\bar{v}_1)^{(8)}$$

$$(o) \quad \text{If } 0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}, \text{ we obtain}$$

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$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)} + (\bar{C})^{(8)}(\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}{1 + (\bar{C})^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}$, then $(\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines $(v_0)^{(8)}$ for the special case .**

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}$, then $(\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of $(u_0)^{(8)}$.**

We can prove the following

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Theorem : If $(a'_i)^{(1)}$ and $(b'_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a'_i)^{(2)}$ and $(b'_i)^{(2)}$ are independent on t , and the conditions with the notations

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 ,$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

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with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a'_i)^{(3)}$ and $(b'_i)^{(3)}$ are independent on t , and the conditions with the notations

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$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0 ,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following

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Theorem 3: If $(a'_i)^{(4)}$ and $(b'_i)^{(4)}$ are independent on t , and the conditions with the notations

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0 ,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with $(p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0 ,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with $(p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a_i'')^{(7)}$ and $(b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$$

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0 ,$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$$

with $(p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a_i'')^{(8)}$ and $(b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$$

$$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$$

$$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0 ,$$

$$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{40})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$$

with $(p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 442$$

has a unique positive solution, which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 448$$

has a unique positive solution, which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 454$$

has a unique positive solution, which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})]T_{42} = 0 \quad 484$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a'_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a'_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a'_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a'_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) +$$

$$(a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof:

490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof:

491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof:

492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Definition and uniqueness of T_{14}^* :-

493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :-

494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 495$$

Definition and uniqueness of T_{21}^* :-

496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]} \quad 497$$

Definition and uniqueness of T_{25}^* :-

497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :-

500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value, we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :-

501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value, we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^*)]}$$

(b) By the same argument, the equations admit solutions G_{13}, G_{14} if

502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(c) By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

(a) By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(b) By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(c) By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(d) By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

(e) By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - [(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - [(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{41}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0, T_{14}^*$ given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} , T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0, T_{17}^*$ given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0, T_{21}^*$ given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} , T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]}$$
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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} , T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]}$$
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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} , G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^*)]} , T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^*)]}$$
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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^*)]} , G_{38}^* = \frac{(a_{38})^{(7)}G_{37}^*}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)}T_{37}^*}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^*)]} , T_{38}^* = \frac{(b_{38})^{(7)}T_{37}^*}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)}G_{41}^*}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^*)]} , G_{42}^* = \frac{(a_{42})^{(8)}G_{41}^*}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)}T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} , \quad T_{42}^* = \frac{(b_{42})^{(8)}T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}(s_{(13)(j)})T_{13}^*\mathbb{G}_j \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}(s_{(14)(j)})T_{14}^*\mathbb{G}_j \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}(s_{(15)(j)})T_{15}^*\mathbb{G}_j \quad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \quad 535$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \quad 536$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*G_j \quad 537$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j \quad 538$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(3)}$ and $(b''_i)^{(3)}$ belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a'_{21})^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \quad \frac{\partial (b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \quad 541$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21} \quad 542$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \quad 543$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^*G_j \quad 544$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^*G_j \quad 545$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^*G_j \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(4)}$ and $(b''_i)^{(4)}$ belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

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$$\frac{\partial(a'_{25})^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}, \quad \frac{\partial(b'_i)^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^*T_{25} \quad 549$$

$$\frac{dG_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^*T_{25} \quad 550$$

$$\frac{dG_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^*T_{25} \quad 551$$

$$\frac{dT_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^*G_j \quad 552$$

$$\frac{dT_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^*G_j \quad 553$$

$$\frac{dT_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^*G_j \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(5)}$ and $(b'_i)^{(5)}$ belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :- 556

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a'_{29})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial(b'_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^*T_{29} \quad 557$$

$$\frac{dG_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^*T_{29} \quad 558$$

$$\frac{dG_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^*T_{29} \quad 559$$

$$\frac{dT_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*G_j \quad 560$$

$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*G_j \quad 561$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*G_j \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \quad \frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(32)(j)})T_{32}^*\mathbb{G}_j \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}(s_{(33)(j)})T_{33}^*\mathbb{G}_j \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(34)(j)})T_{34}^*\mathbb{G}_j \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ and $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \quad \frac{\partial(b_i'')^{(7)}}{\partial G_j}((G_{39})^{**}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -((b'_{36})^{(7)} - (r_{36})^{(7)})\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}(s_{(36)(j)})T_{36}^*\mathbb{G}_j \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -((b'_{37})^{(7)} - (r_{37})^{(7)})\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38} (s_{(37)(j)} T_{37}^* \mathbb{G}_j) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -((b'_{38})^{(7)} - (r_{38})^{(7)})\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38} (s_{(38)(j)} T_{38}^* \mathbb{G}_j) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(8)}$ and $(b'_i)^{(8)}$ belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a'_{41})^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \quad \frac{\partial (b'_{41})^{(8)}}{\partial G_j} ((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -((a'_{40})^{(8)} + (p_{40})^{(8)})\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^* \mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -((a'_{41})^{(8)} + (p_{41})^{(8)})\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^* \mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -((a'_{42})^{(8)} + (p_{42})^{(8)})\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^* \mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -((b'_{40})^{(8)} - (r_{40})^{(8)})\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42} (s_{(40)(j)} T_{40}^* \mathbb{G}_j) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -((b'_{41})^{(8)} - (r_{41})^{(8)})\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42} (s_{(41)(j)} T_{41}^* \mathbb{G}_j) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -((b'_{42})^{(8)} - (r_{42})^{(8)})\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42} (s_{(42)(j)} T_{42}^* \mathbb{G}_j) \quad 586$$

The characteristic equation of this system is 587

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}) \{ (\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)} \} \\ & \left[\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)} \right) (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)} G_{13}^* \right] \\ & \left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^* \\ & + \left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)} \right) (q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)} (q_{14})^{(1)} G_{14}^* \\ & \left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^* \\ & \left((\lambda)^{(1)} \right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} \end{aligned}$$

$$\begin{aligned} & \left((\lambda^{(1)})^2 + (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)} \right) (\lambda^{(1)}) \\ & + \left((\lambda^{(1)})^2 + (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda^{(1)}) (q_{15})^{(1)} G_{15} \\ & + \left((\lambda^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \left((a_{15})^{(1)} (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (a_{15})^{(1)} (q_{13})^{(1)} G_{13}^* \right) \right. \\ & \left. \left((\lambda^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^* \right) \right\} = 0 \end{aligned}$$

+

$$\begin{aligned} & \left((\lambda^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}) \left\{ (\lambda^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \right. \right. \\ & \left. \left[\left((\lambda^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \right] \right. \\ & \left. \left((\lambda^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}) s_{(17),(17)} T_{17}^* + (b_{17})^{(2)} s_{(16),(17)} T_{17}^* \right) \right. \\ & \left. + \left((\lambda^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}) (q_{16})^{(2)} G_{16}^* + (a_{16})^{(2)} (q_{17})^{(2)} G_{17}^* \right) \right. \\ & \left. \left((\lambda^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}) s_{(17),(16)} T_{17}^* + (b_{17})^{(2)} s_{(16),(16)} T_{16}^* \right) \right. \\ & \left. \left((\lambda^{(2)})^2 + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda^{(2)}) \right. \\ & \left. \left((\lambda^{(2)})^2 + (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} \right) (\lambda^{(2)}) \right. \\ & \left. + \left((\lambda^{(2)})^2 + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda^{(2)}) (q_{18})^{(2)} G_{18} \right. \\ & \left. + \left((\lambda^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) \left((a_{18})^{(2)} (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \right. \right. \\ & \left. \left. \left((\lambda^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}) s_{(17),(18)} T_{17}^* + (b_{17})^{(2)} s_{(16),(18)} T_{16}^* \right) \right\} = 0 \end{aligned}$$

+

$$\begin{aligned} & \left((\lambda^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}) \left\{ (\lambda^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \right. \right. \\ & \left. \left[\left((\lambda^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \right] \right. \\ & \left. \left((\lambda^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \right) \right. \\ & \left. + \left((\lambda^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}) (q_{20})^{(3)} G_{20}^* + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^* \right) \right. \\ & \left. \left((\lambda^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \right) \right. \\ & \left. \left((\lambda^{(3)})^2 + (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda^{(3)}) \right. \\ & \left. \left((\lambda^{(3)})^2 + (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda^{(3)}) \right. \end{aligned}$$

$$\begin{aligned}
 &+ \left(((\lambda)^{(3)})^2 + (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22} \\
 &+ \left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left((a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \\
 &\left. \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right) \right\} = 0
 \end{aligned}$$

+

$$\begin{aligned}
 &((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)}) \{ (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \} \\
 &\left[\left(((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right) \right] \\
 &\left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \right) \\
 &+ \left(((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)}) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \right) \\
 &\quad \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \right) \\
 &\left(((\lambda)^{(4)})^2 + (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \\
 &\quad \left(((\lambda)^{(4)})^2 + (b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \\
 &+ \left(((\lambda)^{(4)})^2 + (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26} \\
 &+ \left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) \left((a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right) \\
 &\left. \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}) s_{(25),(26)} T_{25}^* + (b_{25})^{(4)} s_{(24),(26)} T_{24}^* \right) \right\} = 0
 \end{aligned}$$

+

$$\begin{aligned}
 &((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}) \{ (\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)} \} \\
 &\left[\left(((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \right] \\
 &\left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) s_{(29),(29)} T_{29}^* + (b_{29})^{(5)} s_{(28),(29)} T_{29}^* \right) \\
 &+ \left(((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)}) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \right) \\
 &\quad \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) s_{(29),(28)} T_{29}^* + (b_{29})^{(5)} s_{(28),(28)} T_{28}^* \right) \\
 &\left(((\lambda)^{(5)})^2 + (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} \\
 &\quad \left(((\lambda)^{(5)})^2 + (b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} \right) (\lambda)^{(5)}
 \end{aligned}$$

$$\begin{aligned}
 &+ \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} (q_{30})^{(5)} G_{30} \\
 &+ \left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) \left((a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \\
 &\left. \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(30)} T_{29}^* + (b_{29})^{(5)} s_{(28),(30)} T_{28}^* \right\} = 0
 \end{aligned}$$

+

$$\begin{aligned}
 & \left((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)} \right) \left\{ (\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)} \right\} \\
 & \left[\left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right] \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(33)} T_{33}^* + (b_{33})^{(6)} s_{(32),(33)} T_{33}^* \\
 & + \left((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)} \right) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \\
 & \quad \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(32)} T_{33}^* + (b_{33})^{(6)} s_{(32),(32)} T_{32}^* \\
 & \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & \quad \left((\lambda)^{(6)} \right)^2 + \left((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & + \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\
 & + \left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) \left((a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left. \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(34)} T_{33}^* + (b_{33})^{(6)} s_{(32),(34)} T_{32}^* \right\} = 0
 \end{aligned}$$

+

$$\begin{aligned}
 & \left((\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)} \right) \left\{ (\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)} \right\} \\
 & \left[\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)} \right) (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (q_{36})^{(7)} G_{36}^* \right] \\
 & \left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(37)} T_{37}^* + (b_{37})^{(7)} s_{(36),(37)} T_{37}^* \\
 & + \left((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)} \right) (q_{36})^{(7)} G_{36}^* + (a_{36})^{(7)} (q_{37})^{(7)} G_{37}^* \\
 & \quad \left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(36)} T_{37}^* + (b_{37})^{(7)} s_{(36),(36)} T_{36}^* \\
 & \left((\lambda)^{(7)} \right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} \\
 & \quad \left((\lambda)^{(7)} \right)^2 + \left((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)} \right) (\lambda)^{(7)}
 \end{aligned}$$

$$\begin{aligned}
 &+ \left((\lambda^{(7)})^2 + (a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda^{(7)}) (q_{38})^{(7)} G_{38} \\
 &+ \left((\lambda^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}) \left((a_{38})^{(7)} (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (a_{38})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \right. \\
 &\left. \left((\lambda^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) S_{(37),(38)} T_{37}^* + (b_{37})^{(7)} S_{(36),(38)} T_{36}^* \right) \right\} = 0
 \end{aligned}$$

+

$$\begin{aligned}
 & \left((\lambda^{(8)} + (b'_{42})^{(8)} - (r_{42})^{(8)}) \left\{ (\lambda^{(8)} + (a'_{42})^{(8)} + (p_{42})^{(8)}) \right. \right. \\
 & \left. \left[\left((\lambda^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}) (q_{41})^{(8)} G_{41}^* + (a_{41})^{(8)} (q_{40})^{(8)} G_{40}^* \right) \right] \right. \\
 & \left. \left((\lambda^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}) S_{(41),(41)} T_{41}^* + (b_{41})^{(8)} S_{(40),(41)} T_{41}^* \right) \right. \\
 & \left. + \left((\lambda^{(8)} + (a'_{41})^{(8)} + (p_{41})^{(8)}) (q_{40})^{(8)} G_{40}^* + (a_{40})^{(8)} (q_{41})^{(8)} G_{41}^* \right) \right. \\
 & \left. \left((\lambda^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}) S_{(41),(40)} T_{41}^* + (b_{41})^{(8)} S_{(40),(40)} T_{40}^* \right) \right. \\
 & \left. \left((\lambda^{(8)})^2 + (a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)} \right) (\lambda^{(8)}) \right. \\
 & \left. \left((\lambda^{(8)})^2 + (b'_{40})^{(8)} + (b'_{41})^{(8)} - (r_{40})^{(8)} + (r_{41})^{(8)} \right) (\lambda^{(8)}) \right. \\
 & \left. + \left((\lambda^{(8)})^2 + (a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)} \right) (\lambda^{(8)}) (q_{42})^{(8)} G_{42} \right. \\
 & \left. + \left((\lambda^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}) \left((a_{42})^{(8)} (q_{41})^{(8)} G_{41}^* + (a_{41})^{(8)} (a_{42})^{(8)} (q_{40})^{(8)} G_{40}^* \right) \right. \right. \\
 & \left. \left. \left((\lambda^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}) S_{(41),(42)} T_{41}^* + (b_{41})^{(8)} S_{(40),(42)} T_{40}^* \right) \right\} \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

QUANTUM GRAVITY HOLOGRAPHIC RENORMALIZATION GROUP ETAL.,A MODEL AB UNO DISCE OMNES

PHYSICAL PARTICLE STATES (PROTON,NEUTRON AND PION) AND INVARIANT SU(3),THE PHYSICAL PARAMETER STATES

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF PHYSICAL PARTICLE STATES

G_{14} : CATEGORY TWO OF PHYSICAL PARTICLE STATES

G_{15} : CATEGORY THREE OF PHYSICAL PARTICLE STATES

T_{13} : CATEGORY ONE OF INVARIANT SU(3), THE PHYSICAL PARAMETER STATES

T_{14} : CATEGORY TWO OF INVARIANT SU(3) THE PJYSICAL PARAMETER STATES

T_{15} :CATEGORY THREE OF INVARIANT SU(3),THE PHYSICAL PARAMETER STATES

Quantum gravity, field theory and signatures of noncommutative space-time Richard J. Szabo

Field theories on quantized space times with emphasis on what such field theories divinizing about the problem of quantizing gravity, leads to the question extent noncommutative gauge theories **may be regarded** as gauge theories of gravity, with consummative and concomitant UV/IR mixing and its **relations to** renormalization, to gravitational dynamics, and to deformed dispersion relations in models of quantum spacetime of interest in string theory and in doubly special relativity, spacetime noncommutativity are direct ramification of the study.

MODULE NUMBERED TWO:

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G_{16} : CATEGORY ONE OF Field theories on quantized spacetimes with emphasis on what such field theories divinizing about the problem of quantizing gravity, leads to the question extent noncommutative gauge theories **may be regarded** as gauge theories of gravity, with consummative and concomitant UV/IR mixing

G_{17} : CATEGORY TWO OF Field theories on quantized spacetimes with emphasis on what such field theories divinizing about the problem of quantizing gravity, leads to the question extent noncommutative gauge theories **may be regarded** as gauge theories of gravity, with consummative and concomitant UV/IR mixing

G_{18} : CATEGORY THREE OF Field theories on quantized space times with emphasis on what such field theories divinizing about the problem of quantizing gravity, leads to the question extent noncommutative gauge theories **may be regarded** as gauge theories of gravity, with consummative and concomitant UV/IR mixing

T_{16} :CATEGORY ONE OF renormalization, to gravitational dynamics, and to deformed dispersion relations in models of quantum space-time of interest in string theory and in doubly special relativity, space-time noncommutativity

T_{17} : CATEGORY TWO OF renormalization, to gravitational dynamics, and to deformed dispersion relations in models of quantum space-time of interest in string theory and in doubly special relativity, space-time noncommutativity

T_{18} : CATEGORY THREE OF renormalization, to gravitational dynamics, and to deformed dispersion relations in models of quantum space-time of interest in string theory and in doubly special relativity, space-time noncommutativity

On the holographic renormalization group Jan de Boer^{1,2}, Erik Verlinde³ and Herman Verlinde^{3,4}

Direct correspondence between the classical evolution equations of 5-d supergravity and the renormalization group (RG) equations of the dual 4-d large-N gauge theory, using standard Hamilton-Jacobi theory, first order flow equations for the classical supergravity action S that take the usual form of the Callan-Symanzik equations including the corrections due to the conformal anomaly giving accentuatory and corroboratory **evidence for the** identification of S with the quantum effective action of the gauge theory. (By implication interesting new relations between the beta-functions and the counter terms that affect the 4-d cosmological and Newton constant).

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF Direct correspondence between the classical evolution equations of 5-d supergravity and the renormalization group (RG) equations of the dual 4-d large-N gauge theory, using standard Hamilton-Jacobi theory, first order flow equations for the classical supergravity action S that take the usual form of the Callan-Symanzik equations including the corrections due to the conformal anomaly

G_{21} :CATEGORY TWO OF Direct correspondence between the classical evolution equations of 5-d supergravity and the renormalization group (RG) equations of the dual 4-d large-N gauge theory, using standard Hamilton-Jacobi theory, first order flow equations for the classical supergravity action S that take the usual form of the Callan-Symanzik equations including the corrections due to the conformal anomaly

G_{22} : CATEGORY THREE OF Direct correspondence between the classical evolution equations of 5-d supergravity and the renormalization group (RG) equations of the dual 4-d large-N gauge theory, using standard Hamilton-Jacobi theory, first order flow equations for the classical supergravity action S that take the usual form of the Callan-Symanzik equations including the corrections due to the conformal anomaly

T_{20} : CATEGORY ONE OF identification of S with the quantum effective action of the gauge theory. (By implication interesting new relations between the beta-functions and the counterterms that affect the 4-d cosmological and Newton constant).

T_{21} : CATEGORY TWO OF identification of S with the quantum effective action of the gauge theory. (By implication interesting new relations between the beta-functions and the counterterms that affect the 4-d cosmological and Newton constant).

T_{22} : CATEGORY THREE OF identification of S with the quantum effective action of the gauge theory. (By implication interesting new relations between the beta-functions and the counterterms that affect the 4-d cosmological and Newton constant).

Exact superconformal and Yangian symmetry of scattering amplitudes: Till Bargheer, Niklas Beisert and Florian Loebbert

Authors review recent progress in the understanding of symmetries for scattering amplitudes in $\mathcal{N} = 4$ superconformal Yang–Mills theory. It is summarized how the superficial breaking of superconformal symmetry by collinear anomalies and the renormalization process **can be cured** at tree and loop level by **correcting** the representation of the superconformal group on amplitudes. (Moreover, authors comment on the Yangian symmetry of scattering amplitudes and how **it inherits** these correction terms from the ordinary Lie algebra symmetry. Invariants under this algebra and their **relation** to the Graßmannian generating function

: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OF symmetries for scattering amplitudes in $\mathcal{N} = 4$ superconformal Yang–Mills theory. It is summarized how the superficial breaking of superconformal symmetry by collinear anomalies and the renormalization process **can be cured** at tree and loop level

G_{25} : CATEGORY TWO OF symmetries for scattering amplitudes in $\mathcal{N} = 4$ superconformal Yang–Mills theory. It is summarized how the superficial breaking of superconformal symmetry by

collinear anomalies and the renormalization process **can be cured** at tree and loop level

G_{26} : CATEGORY THREE OF symmetries for scattering amplitudes in $\mathcal{N} = 4$ superconformal Yang–Mills theory. It is summarized how the superficial breaking of superconformal symmetry by collinear anomalies and the renormalization process **can be cured** at tree and loop level

T_{24} :CATEGORY ONE OF **correcting** the representation of the superconformal group on amplitudes. (Moreover, authors comment on the Yangian symmetry of scattering amplitudes and how it **inherits** these correction terms from the ordinary Lie algebra symmetry. Invariants under this algebra and their **relation** to the Graßmannian generating function

T_{25} :CATEGORY TWO OF **correcting** the representation of the superconformal group on amplitudes. (Moreover, authors comment on the Yangian symmetry of scattering amplitudes and how it **inherits** these correction terms from the ordinary Lie algebra symmetry. Invariants under this algebra and their **relation** to the Graßmannian generating function

T_{26} : CATEGORY THREE OF **correcting** the representation of the superconformal group on amplitudes. (Moreover, authors comment on the Yangian symmetry of scattering amplitudes and how it **inherits** these correction terms from the ordinary Lie algebra symmetry. Invariants under this algebra and their **relation** to the Graßmannian generating function

The picture of the Bianchi I model via gauge fixing in Loop Quantum Gravity: F. Cianfrani¹, A. Marchini and G. Montani

The implications of the SU (2) gauge fixing associated with the choice of invariant triads in Loop Quantum Cosmology for a Bianchi I model, in particular, via the analysis of Dirac brackets, it is outlined how the holonomy-flux algebra **coincides with** the one of Loop Quantum Gravity if paths are parallel to fiducial vectors only. In this way the quantization procedure for the Bianchi I model is **performed by** applying the techniques developed in Loop Quantum Gravity but **restricting** the admissible paths. Furthermore, the local character **retained by** the reduced variables provides a relic diffeomorphisms constraint, whose imposition **implies** homogeneity on a quantum level. **The resulting** picture for the fundamental spatial manifold is that of a cubical knot with attached SU (2) irreducible representations. The discretization of geometric operators is outlined and a new perspective for the super-Hamiltonian regularization in Loop Quantum Cosmology is proposed

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF The implications of the SU (2) gauge fixing associated with the choice of invariant triads in Loop Quantum Cosmology for a Bianchi I model, in particular, via the analysis of Dirac brackets, it is outlined how the holonomy-flux algebra **coincides with** the one of Loop Quantum Gravity if paths are parallel to fiducial vectors only. In this way the quantization procedure for the Bianchi I model is **performed by** applying the techniques developed in Loop Quantum Gravity but **restricting** the admissible paths. Furthermore, the local character **retained by** the reduced variables provides a relic diffeomorphisms constraint, whose imposition **implies** homogeneity on a quantum level

G_{29} : CATEGORY TWO OF The implications of the SU (2) gauge fixing associated with the choice of invariant triads in Loop Quantum Cosmology for a Bianchi I model, in particular, via the analysis of Dirac brackets, it is outlined how the holonomy-flux algebra **coincides with** the one of Loop Quantum Gravity if paths are parallel to fiducial vectors only. In this way the quantization procedure for the Bianchi I model is **performed by** applying the techniques developed in Loop Quantum Gravity but **restricting** the admissible paths. Furthermore, the local character **retained by** the reduced variables provides a relic diffeomorphisms constraint, whose imposition **implies** homogeneity on a quantum level

G_{30} :CATEGORY THREE OF The implications of the SU (2) gauge fixing associated with the choice of invariant triads in Loop Quantum Cosmology for a Bianchi I model, in particular, via the analysis of Dirac brackets, it is outlined how the holonomy-flux algebra **coincides with** the one of Loop Quantum Gravity if paths are parallel to fiducial vectors only. In this way the quantization procedure for the Bianchi I model is **performed by** applying the techniques developed in Loop Quantum Gravity but **restricting** the admissible paths. Furthermore, the local character **retained by** the reduced variables provides a relic diffeomorphisms constraint, whose imposition **implies** homogeneity on a quantum level

T_{28} : CATEGORY ONE OF picture for the fundamental spatial manifold is that of a cubical knot with attached SU (2) irreducible representations. The discretization of geometric operators is outlined and a new perspective for the super-Hamiltonian regularization in Loop Quantum Cosmology

T_{29} : CATEGORY TWO OF picture for the fundamental spatial manifold is that of a cubical knot with attached SU (2) irreducible representations. The discretization of geometric operators is outlined and a new perspective for the super-Hamiltonian regularization in Loop Quantum Cosmology

T_{30} :CATEGORY THREE OF picture for the fundamental spatial manifold is that of a cubical knot with attached SU (2) irreducible representations. The discretization of geometric operators is outlined and a new perspective for the super-Hamiltonian regularization in Loop Quantum Cosmology(NOTE THAT WE ARE TALKING OF THE PARAMETRICIZATIONS AND CHARACTERSTICS OF THE SYSTEMS WHICH ARE UBNDER INVESTIGATION)

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Zero-Lag Synchronization in Spatiotemporal Chaotic Systems with Long Range Delay Couplings: Li Jian-Ping, Yu Lian-Chun, Yu Mei-Chen and Chen Yong

Under the appellation of “synchronization of spatiotemporal chaos patterns between two delay-coupled excitable layers”, it is found that zero-lag synchronization (ZLS) can be **achieved by** dynamical relay via a third mediating layer. Based on simulations with large parameter ranges, time delay and coupling strength **influences on** transition time is investigated. ZLS with a stronger coupling strength and shorter time delay appears to have a shorter transition time, which phenomenon has **possible implications** in network communication

MODULE NUMBERED SIX:

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G_{32} : CATEGORY ONE OF synchronization of spatiotemporal chaos patterns between two delay-coupled excitable layers”, it is found that zero-lag synchronization (ZLS) can be **achieved by** dynamical relay via a third mediating layer. Based on simulations with large parameter ranges, time delay and coupling strength

G_{33} : CATEGORY TWO OF synchronization of spatiotemporal chaos patterns between two delay-coupled excitable layers”, it is found that zero-lag synchronization (ZLS) can be **achieved by** dynamical relay via a third mediating layer. Based on simulations with large parameter ranges, time delay and coupling strength

G_{34} : CATEGORY THREE OF synchronization of spatiotemporal chaos patterns between two delay-coupled excitable layers”, it is found that zero-lag synchronization (ZLS) can be **achieved by** dynamical relay via a third mediating layer. Based on simulations with large parameter ranges, time delay and coupling strength

T_{32} : CATEGORY ONE OF **influences on** transition time is investigated. ZLS with a stronger coupling

strength and shorter time delay appears to have a shorter transition time, which phenomenon has **possible implications** in network communication

T_{33} : CATEGORY TWO OF **influences on** transition time is investigated. ZLS with a stronger coupling strength and shorter time delay appears to have a shorter transition time, which phenomenon has **possible implications** in network communication

T_{34} : CATEGORY THREE OF **influences on** transition time is investigated. ZLS with a stronger coupling strength and shorter time delay appears to have a shorter transition time, which phenomenon has **possible implications** in network communication

De Moivre's formula: are Sands' H-functions the same as Chao's? P Nishikawa

It has often been said that there is no clear extension of the Courant-Snyder theory to coupled dynamics. In particular, one never sees an extension of the symplectic de Moivre type formula beyond one degree of freedom. In the process of analysing the existence and meaning of such a formula, author **discovers** quite accidentally that Sands' formalism, if expressed in terms of Ripken-like lattice functions germane to the full three dimensional oscillator, **gives the exact same result as** the more accurate Chaos theory . This semi-serious paper displays this elegant connection. It is based in the lattice functions of Ripken as extended by Forest. A review Forest's derivation from the point of view de Moivre's formula extended to three degrees of freedom is also undertaken

MODULE NUMBERED SEVEN

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G_{36} : CATEGORY ONE OF Courant-Snyder theory to coupled dynamics. In particular, one never sees an extension of the symplectic de Moivre type formula beyond one degree of freedom. In the process of analysing the existence and meaning of such a formula, author **discovers** quite accidentally that Sands' formalism, if expressed in terms of Ripken-like lattice functions germane to the full three dimensional oscillator

G_{37} : CATEGORY TWO OF Courant-Snyder theory to coupled dynamics. In particular, one never sees an extension of the symplectic de Moivre type formula beyond one degree of freedom. In the process of analysing the existence and meaning of such a formula, author **discovers** quite accidentally that Sands' formalism, if expressed in terms of Ripken-like lattice functions germane to the full three dimensional oscillator

G_{38} : CATEGORY THREE OF Courant-Snyder theory to coupled dynamics. In particular, one never sees an extension of the symplectic de Moivre type formula beyond one degree of freedom. In the process of analysing the existence and meaning of such a formula, author **discovers** quite accidentally that Sands' formalism, if expressed in terms of Ripken-like lattice functions germane to the full three dimensional oscillator (ENERGY EXCITATION OF THE VACUUM AND CONCOMITANT GENERATION OF ENERGY DIFFERENTIAL-TIME LAG OR INSTANTANEOUSNESSMIGHT EXISTS WHEREBY ACCENTUATION AND ATTRITIONS MODEL MAY ASSUME ZERO POSITIONS)

T_{36} : CATEGORY ONE OF Chaos theory . (This semi-serious paper displays this elegant connection. It is based in the lattice functions of Ripken as extended by Forest. A review Forest's derivation from the point of view de Moivre's formula extended to three degrees of freedom is also undertaken)

T_{37} : CATEGORY TWO OF Chaos theory . (This semi-serious paper displays this elegant connection.

It is based in the lattice functions of Ripken as extended by Forest. A review Forest's derivation from the point of view de Moivre's formula extended to three degrees of freedom is also undertaken)

T₃₈ : CATEGORY THREE OF Chaos theory . (This semi-serious paper displays this elegant connection. It is based in the lattice functions of Ripken as extended by Forest. A review Forest's derivation from the point of view de Moivre's formula extended to three degrees of freedom is also undertaken)

$$(a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)} (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}:$$

$$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$$

$$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$$

$$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$$

$$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$$

are Accentuation coefficients

$$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$$

$$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$$

$$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$$

$$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$$

$$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$$

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 7$$

$$\begin{aligned} \frac{dG_{17}}{dt} &= (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} & 8 \\ \frac{dG_{18}}{dt} &= (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} & 9 \\ \frac{dT_{16}}{dt} &= (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} & 10 \\ \frac{dT_{17}}{dt} &= (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} & 11 \\ \frac{dT_{18}}{dt} &= (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} & 12 \\ &+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \\ &-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor} \end{aligned}$$

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\begin{aligned} \frac{dG_{20}}{dt} &= (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} & 13 \\ \frac{dG_{21}}{dt} &= (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} & 14 \\ \frac{dG_{22}}{dt} &= (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} & 15 \\ \frac{dT_{20}}{dt} &= (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} & 16 \\ \frac{dT_{21}}{dt} &= (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} & 17 \\ \frac{dT_{22}}{dt} &= (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} & 18 \\ &+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor} \\ &-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor} \end{aligned}$$

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\begin{aligned} \frac{dG_{24}}{dt} &= (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} & 19 \\ \frac{dG_{25}}{dt} &= (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} & 20 \\ \frac{dG_{26}}{dt} &= (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} & 21 \\ \frac{dT_{24}}{dt} &= (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} & 22 \\ \frac{dT_{25}}{dt} &= (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} & 23 \\ \frac{dT_{26}}{dt} &= (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} & 24 \\ &+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor} \\ &-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor} \end{aligned}$$

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 30$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 36$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}$$

Module Numbered Seven:

The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)]T_{38} \quad 42$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor}$$

First Module Concatenation

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37}, t)} \end{array} \right] G_{13} \tag{49}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{c} (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37}, t)} \end{array} \right] G_{14} \tag{50}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37}, t)} \end{array} \right] G_{15} \tag{51}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7)}(T_{37}, t)}$ are seventh augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{c} (b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,7)}(G_{39}, t)} \end{array} \right] T_{13} \tag{52}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{c} (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,7)}(G_{39}, t)} \end{array} \right] T_{14} \tag{53}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{c} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \quad \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} \quad \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)} \quad \boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)} \quad \boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39}, t)} \end{array} \right] T_{15} \quad 54$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7)}(G_{39}, t)}$, $\boxed{-(b''_{36})^{(7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2 and 3

Second Module Concatenation

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \quad \boxed{+(a''_{13})^{(1,1)}(T_{14}, t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)} \end{array} \right] G_{16} \quad 55$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \quad \boxed{+(a''_{14})^{(1,1)}(T_{14}, t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)} \end{array} \right] G_{17} \quad 56$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \quad \boxed{+(a''_{15})^{(1,1)}(T_{14}, t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)} \quad \boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)} \quad \boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)} \end{array} \right] G_{18} \quad 57$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1)}(T_{14}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)}$ are fifth augmentation

coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$ are sixth augmentation

coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)}$ are seventh augmentation

coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} \boxed{(b'_{16})^{(2)} - \boxed{(b''_{16})^{(2)}(G_{19}, t)} - \boxed{(b''_{13})^{(1,1)}(G, t)} - \boxed{(b''_{20})^{(3,3,3)}(G_{23}, t)} & & \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} - \boxed{(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} - \boxed{(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} & & \\ & \boxed{-(b''_{36})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{16} \quad 58$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} \boxed{(b'_{17})^{(2)} - \boxed{(b''_{17})^{(2)}(G_{19}, t)} - \boxed{(b''_{14})^{(1,1)}(G, t)} - \boxed{(b''_{21})^{(3,3,3)}(G_{23}, t)} & & \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} - \boxed{(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} - \boxed{(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} & & \\ & \boxed{-(b''_{37})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{17} \quad 59$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} \boxed{(b'_{18})^{(2)} - \boxed{(b''_{18})^{(2)}(G_{19}, t)} - \boxed{(b''_{15})^{(1,1)}(G, t)} - \boxed{(b''_{22})^{(3,3,3)}(G_{23}, t)} & & \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)} - \boxed{(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)} - \boxed{(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)} & & \\ & \boxed{-(b''_{38})^{(7,7,7)}(G_{39}, t)} & \end{array} \right] T_{18} \quad 60$$

where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1,2 and 3

Third Module Concatenation

$$\frac{dG_{20}}{dt} =$$

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$$(a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} \boxed{(a'_{20})^{(3)} + \boxed{(a''_{20})^{(3)}(T_{21}, t)} + \boxed{(a''_{16})^{(2,2,2)}(T_{17}, t)} + \boxed{(a''_{13})^{(1,1,1)}(T_{14}, t)} & & \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)} + \boxed{(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)} + \boxed{(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} & & \\ & \boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)} & \end{array} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[\begin{array}{l} (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t) + (a'_{17})^{(2,2,2)}(T_{17}, t) + (a'_{14})^{(1,1,1)}(T_{14}, t) \\ + (a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t) + (a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t) + (a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{37})^{(7,7,7,7)}(T_{37}, t) \end{array} \right] G_{21} \quad 62$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[\begin{array}{l} (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t) + (a'_{18})^{(2,2,2)}(T_{17}, t) + (a'_{15})^{(1,1,1)}(T_{14}, t) \\ + (a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t) + (a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t) + (a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t) \\ + (a''_{38})^{(7,7,7,7)}(T_{37}, t) \end{array} \right] G_{22} \quad 63$$

$(a''_{20})^{(3)}(T_{21}, t)$, $(a''_{21})^{(3)}(T_{21}, t)$, $(a''_{22})^{(3)}(T_{21}, t)$ are first augmentation coefficients for category 1, 2 and 3

$(a''_{16})^{(2,2,2)}(T_{17}, t)$, $(a''_{17})^{(2,2,2)}(T_{17}, t)$, $(a''_{18})^{(2,2,2)}(T_{17}, t)$ are second augmentation coefficients for category 1, 2 and 3

$(a''_{13})^{(1,1,1)}(T_{14}, t)$, $(a''_{14})^{(1,1,1)}(T_{14}, t)$, $(a''_{15})^{(1,1,1)}(T_{14}, t)$ are third augmentation coefficients for category 1, 2 and 3

$(a''_{24})^{(4,4,4,4,4,4)}(T_{25}, t)$, $(a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t)$, $(a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficients for category 1, 2 and 3

$(a''_{28})^{(5,5,5,5,5,5)}(T_{29}, t)$, $(a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t)$, $(a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficients for category 1, 2 and 3

$(a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t)$, $(a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t)$, $(a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficients for category 1, 2 and 3

$(a''_{36})^{(7,7,7,7)}(T_{37}, t)$, $(a''_{37})^{(7,7,7,7)}(T_{37}, t)$, $(a''_{38})^{(7,7,7,7)}(T_{37}, t)$ are seventh augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[\begin{array}{l} (b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b'_{16})^{(2,2,2)}(G_{19}, t) - (b'_{13})^{(1,1,1)}(G, t) \\ - (b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t) - (b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t) - (b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{36})^{(7,7,7,7)}(G_{39}, t) \end{array} \right] T_{20} \quad 64$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[\begin{array}{l} (b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t) - (b'_{17})^{(2,2,2)}(G_{19}, t) - (b'_{14})^{(1,1,1)}(G, t) \\ - (b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t) - (b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t) - (b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{37})^{(7,7,7,7)}(G_{39}, t) \end{array} \right] T_{21} \quad 65$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[\begin{array}{l} (b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t) - (b'_{18})^{(2,2,2)}(G_{19}, t) - (b'_{15})^{(1,1,1)}(G, t) \\ - (b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t) - (b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t) - (b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t) \\ - (b''_{38})^{(7,7,7,7)}(G_{39}, t) \end{array} \right] T_{22} \quad 66$$

$(b''_{20})^{(3)}(G_{23}, t)$, $(b''_{21})^{(3)}(G_{23}, t)$, $(b''_{22})^{(3)}(G_{23}, t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2)}(G_{19}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1)}(G, t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7)}(G_{39}, t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39}, t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39}, t)$ are seventh detrition coefficients for category 1, 2 and 3

Fourth Module Concatenation

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{c} (a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) + (a''_{28})^{(5,5)}(T_{29}, t) + (a''_{32})^{(6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1)}(T_{14}, t) + (a''_{16})^{(2,2,2,2)}(T_{17}, t) + (a''_{20})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{36})^{(7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{24} \quad 67$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{c} (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t) + (a''_{29})^{(5,5)}(T_{29}, t) + (a''_{33})^{(6,6)}(T_{33}, t) \\ + (a''_{14})^{(1,1,1,1)}(T_{14}, t) + (a''_{17})^{(2,2,2,2)}(T_{17}, t) + (a''_{21})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{37})^{(7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{25} \quad 68$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{c} (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t) + (a''_{30})^{(5,5)}(T_{29}, t) + (a''_{34})^{(6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1)}(T_{14}, t) + (a''_{18})^{(2,2,2,2)}(T_{17}, t) + (a''_{22})^{(3,3,3,3)}(T_{21}, t) \\ + (a''_{38})^{(7,7,7,7,7)}(T_{37}, t) \end{array} \right] G_{26} \quad 69$$

$(a''_{24})^{(4)}(T_{25}, t)$, $(a''_{25})^{(4)}(T_{25}, t)$, $(a''_{26})^{(4)}(T_{25}, t)$ are first augmentation coefficients category 1, 2 3

$+(a''_{28})^{(5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5)}(T_{29}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6)}(T_{33}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14}, t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17}, t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21}, t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37}, t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37}, t)$ are seventh augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{c} (b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27}, t)} \quad \boxed{-(b''_{28})^{(5,5)}(G_{31}, t)} \quad \boxed{-(b''_{32})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)} \quad \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{24} \quad 70$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{c} (b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27}, t)} \quad \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)} \quad \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)} \quad \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{25} \quad 71$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{c} (b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27}, t)} \quad \boxed{-(b''_{30})^{(5,5)}(G_{31}, t)} \quad \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)} \quad \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{26} \quad 72$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2 and 3

Fifth Module Concatenation

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29}, t)} \quad \boxed{+(a''_{24})^{(4,4)}(T_{25}, t)} \quad \boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{28} \quad 73$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29}, t)} \quad \boxed{+(a''_{25})^{(4,4)}(T_{25}, t)} \quad \boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{29} \quad 74$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29}, t)} \quad \boxed{+(a''_{26})^{(4,4)}(T_{25}, t)} \quad \boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37}, t)} \end{array} \right] G_{30} \quad 75$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$

are first augmentation coefficients for category 1, 2 and 3

And $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$

are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$

are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$ are fifth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t)}$ are seventh augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{l} \boxed{(b'_{28})^{(5)}} \boxed{-(b''_{28})^{(5)}(G_{31}, t)} \boxed{-(b''_{24})^{(4,4)}(G_{27}, t)} \boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{28} \quad 76$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{l} \boxed{(b'_{29})^{(5)}} \boxed{-(b''_{29})^{(5)}(G_{31}, t)} \boxed{-(b''_{25})^{(4,4)}(G_{27}, t)} \boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{29} \quad 77$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{l} \boxed{(b'_{30})^{(5)}} \boxed{-(b''_{30})^{(5)}(G_{31}, t)} \boxed{-(b''_{26})^{(4,4)}(G_{27}, t)} \boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)} \end{array} \right] T_{30} \quad 78$$

where $\boxed{-(b''_{28})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31}, t)}$

are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4)}(G_{27}, t)}$ are second detrition coefficients

for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35}, t)}$ are third detrition coefficients

for category 1,2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)}$ are fourth detrition coefficients

for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)}$ are fifth detrition

coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)}$ are sixth detrition

coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)}$ are seventh detrition

coefficients for category 1,2, and 3

Sixth Module Concatenation

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \tag{79}$$

$$- \left[\begin{array}{ccc} \boxed{(a'_{32})^{(6)}} & \boxed{+(a''_{32})^{(6)}(T_{33}, t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)} & \\ & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37}, t)} & & \end{array} \right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} \tag{80}$$

$$- \left[\begin{array}{ccc} \boxed{(a'_{33})^{(6)}} & \boxed{+(a''_{33})^{(6)}(T_{33}, t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{25})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)} & \\ & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37}, t)} & & \end{array} \right] G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} \tag{81}$$

$$- \left[\begin{array}{ccc} \boxed{(a'_{34})^{(6)}} & \boxed{+(a''_{34})^{(6)}(T_{33}, t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{26})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)} & \\ & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37}, t)} & & \end{array} \right] G_{34}$$

$\boxed{+(a'_{32})^{(6)}(T_{33}, t)}$, $\boxed{+(a'_{33})^{(6)}(T_{33}, t)}$, $\boxed{+(a'_{34})^{(6)}(T_{33}, t)}$ are first augmentation coefficients

for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29}, t)}$

are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4)}(T_{25}, t)}$

are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)}$ - are fourth

augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t)}$ seventh augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} \boxed{(b'_{32})^{(6)}} & \boxed{-(b''_{32})^{(6)}(G_{35}, t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)} & \\ & \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \end{array} \right] T_{32} \quad 82$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} \boxed{(b'_{33})^{(6)}} & \boxed{-(b''_{33})^{(6)}(G_{35}, t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)} & \\ & \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \end{array} \right] T_{33} \quad 83$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} \boxed{(b'_{34})^{(6)}} & \boxed{-(b''_{34})^{(6)}(G_{35}, t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)} & \\ & \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \end{array} \right] T_{34} \quad 84$$

$\boxed{-(b''_{32})^{(6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)}$

are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)}$

are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)}$ are seventh detrition coefficients for category 1, 2, and 3

Seventh Module Concatenation

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} \tag{85}$$

$$- \left[\begin{array}{ccc} (a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) & + (a''_{16})^{(2,2,2,2,2,2,2)}(T_{17}, t) & + (a''_{20})^{(3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6,6,6,6)}(T_{33}, t) \\ & + (a''_{13})^{(1,1,1,1,1,1,1)}(T_{14}, t) & \end{array} \right] G_{13}$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} \tag{86}$$

$$- \left[\begin{array}{ccc} (a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t) & + (a''_{17})^{(2,2,2,2,2,2,2)}(T_{17}, t) & + (a''_{21})^{(3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6,6,6,6)}(T_{33}, t) \\ & + (a''_{13})^{(1,1,1,1,1,1,1)}(T_{14}, t) & \end{array} \right] G_{14}$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} \tag{87}$$

$$- \left[\begin{array}{ccc} (a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t) & + (a''_{18})^{(2,2,2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6,6,6,6)}(T_{33}, t) \\ & + (a''_{15})^{(1,1,1,1,1,1,1)}(T_{14}, t) & \end{array} \right] G_{15}$$

Where $(a'_{36})^{(7)}(T_{37}, t)$, $(a'_{37})^{(7)}(T_{37}, t)$, $(a'_{38})^{(7)}(T_{37}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+ (a''_{16})^{(2,2,2,2,2,2,2)}(T_{17}, t)$, $+ (a''_{17})^{(2,2,2,2,2,2,2)}(T_{17}, t)$, $+ (a''_{18})^{(2,2,2,2,2,2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+ (a''_{20})^{(3,3,3,3,3,3,3)}(T_{21}, t)$, $+ (a''_{21})^{(3,3,3,3,3,3,3)}(T_{21}, t)$, $+ (a''_{22})^{(3,3,3,3,3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+ (a''_{24})^{(4,4,4,4,4,4,4)}(T_{25}, t)$, $+ (a''_{25})^{(4,4,4,4,4,4,4)}(T_{25}, t)$, $+ (a''_{26})^{(4,4,4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+ (a''_{28})^{(5,5,5,5,5,5,5)}(T_{29}, t)$, $+ (a''_{29})^{(5,5,5,5,5,5,5)}(T_{29}, t)$, $+ (a''_{30})^{(5,5,5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+ (a''_{32})^{(6,6,6,6,6,6,6)}(T_{33}, t)$, $+ (a''_{33})^{(6,6,6,6,6,6,6)}(T_{33}, t)$, $+ (a''_{34})^{(6,6,6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+ (a''_{13})^{(1,1,1,1,1,1,1)}(T_{14}, t)$, $+ (a''_{13})^{(1,1,1,1,1,1,1)}(T_{14}, t)$, $+ (a''_{15})^{(1,1,1,1,1,1,1)}(T_{14}, t)$ are seventh augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{36}}{dt} = \tag{88}$$

$$(b_{36})^{(7)}T_{37} - \left[\begin{array}{ccc} (b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}, t) & - (b''_{16})^{(2,2,2,2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6,6,6,6)}(G_{35}, t) \\ & - (b''_{13})^{(1,1,1,1,1,1,1)}(G, t) & \end{array} \right] T_{13}$$

$$\frac{dT_{37}}{dt} = \tag{89}$$

$$(b_{37})^{(7)}T_{36} - \left[\begin{array}{cccc} (b'_{37})^{(7)} \boxed{-(b''_{37})^{(7)}(G_{39}, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23}, t)} & \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35}, t)} & \\ & \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G, t)} & & \end{array} \right] T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[\begin{array}{cccc} (b'_{38})^{(7)} \boxed{-(b''_{38})^{(7)}(G_{39}, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23}, t)} & \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35}, t)} & \\ & \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G, t)} & & \end{array} \right] T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39}, t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39}, t)}$ are first detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3
 $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G, t)}$ are seventh detrition coefficients for category 1, 2 and 3

Where we suppose

$$(X) \quad (a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i, j = 13,14,15 \quad 97$$

(Y) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$(Z) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \quad 98$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{K}_{13})^{(1)}|T_{14} - T'_{14}|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T'_{14}, t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

(AA) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

(BB) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

(CC) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, i, j = 16, 17, 18$

(DD) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \tag{102}$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \tag{103}$$

(EE) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$ 104

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}(G_{19}, t) = (r_i)^{(2)} \tag{105}$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T'_{17}, t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T'_{17} - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \tag{107}$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} \|(G_{19}) - (G_{19})'\| e^{-(\hat{M}_{16})^{(2)}t} \tag{108}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T'_{17}, t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T'_{17}, t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if

$(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$$(FF) \quad (\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, \text{ are positive constants} \quad 109$$

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(GG) \quad (a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$ are positive constants and $i = 20, 21, 22$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21}' - T_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} |G_{23}' - G_{23}| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

(HH) $(\widehat{M}_{20})^{(3)}, (\widehat{k}_{20})^{(3)}$, are positive constants
 $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$

There exists two constants $(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ which together with $(\widehat{M}_{20})^{(3)}, (\widehat{k}_{20})^{(3)}, (\widehat{A}_{20})^{(3)}$ and $(\widehat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a'_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the inequalities 116

$$\frac{1}{(\widehat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a'_i)^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\widehat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b'_i)^{(3)} + (\widehat{B}_{20})^{(3)} + (\widehat{Q}_{20})^{(3)} (\widehat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i, j = 24, 25, 26 \quad 117$$

(II) The functions $(a''_i)^{(4)}, (b''_i)^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$$(a''_i)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\widehat{A}_{24})^{(4)}$$

$$(b''_i)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b'_i)^{(4)} \leq (\widehat{B}_{24})^{(4)}$$

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$$(JJ) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(4)}(T_{25}, t) = (p_i)^{(4)} \\ \lim_{G \rightarrow \infty} (b''_i)^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\widehat{A}_{24})^{(4)}, (\widehat{B}_{24})^{(4)}$:

Where $(\widehat{A}_{24})^{(4)}, (\widehat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$ are positive constants and $i = 24, 25, 26$

They satisfy Lipschitz condition:

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$$|(a''_i)^{(4)}(T'_{25}, t) - (a''_i)^{(4)}(T_{25}, t)| \leq (\widehat{k}_{24})^{(4)} |T'_{25} - T_{25}| e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b''_i)^{(4)}((G_{27})', t) - (b''_i)^{(4)}((G_{27}), t)| < (\widehat{k}_{24})^{(4)} |(G_{27})' - (G_{27})| e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(4)}(T'_{25}, t)$ and $(a''_i)^{(4)}(T_{25}, t) \cdot (T'_{25}, t)$ and (T_{25}, t) are points belonging to the interval $[(\widehat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}]$. It is to be noted that $(a''_i)^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 4$ then the function $(a''_i)^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\widehat{k}_{24})^{(4)}$:

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$(\widehat{M}_{24})^{(4)}, (\widehat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

(KK) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$ and $(\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a'_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a'_i)^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b'_i)^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a'_i)^{(5)}, (a''_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (b''_i)^{(5)} > 0, i, j = 28, 29, 30$ 122

(LL) The functions $(a''_i)^{(5)}, (b''_i)^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a''_i)^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b''_i)^{(5)}(G_{31}, t) \leq (r_i)^{(5)} \leq (b'_i)^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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(MM) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(5)}(T_{29}, t) = (p_i)^{(5)}$
 $\lim_{G \rightarrow \infty} (b''_i)^{(5)}(G_{31}, t) = (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition: 124

$$|(a''_i)^{(5)}(T'_{29}, t) - (a''_i)^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29} - T'_{29}| e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b''_i)^{(5)}((G_{31})', t) - (b''_i)^{(5)}(G_{31}, t)| < (\hat{k}_{28})^{(5)} |(G_{31}) - (G_{31})'| e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(5)}(T'_{29}, t)$ and $(a''_i)^{(5)}(T_{29}, t)$. (T'_{29}, t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a''_i)^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a''_i)^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, i, j = 32, 33, 34$ 127

(NN) The functions $(a'_i)^{(6)}, (b'_i)^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b''_i)^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b'_i)^{(6)} \leq (\hat{B}_{32})^{(6)}$$

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$$(OO) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(6)}(T_{33}, t) = (p_i)^{(6)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$ are positive constants and $i = 32, 33, 34$

They satisfy Lipschitz condition:

$$|(a''_i)^{(6)}(T'_{33}, t) - (a''_i)^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T'_{33} - T_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b''_i)^{(6)}((G_{35})', t) - (b''_i)^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35})' - (G_{35})| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(6)}(T'_{33}, t)$ and $(a''_i)^{(6)}(T_{33}, t) \cdot (T'_{33}, t)$ and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a''_i)^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a''_i)^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and $(\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(a_i)^{(7)}, (a'_i)^{(7)}, (a''_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (b''_i)^{(7)} > 0, \quad 131$$

$$i, j = 36,37,38$$

The functions $(a''_i)^{(7)}, (b''_i)^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a''_i)^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b''_i)^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b'_i)^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(7)}(T_{37}, t) = (p_i)^{(7)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(7)}(G_{39}, t) = (r_i)^{(7)}$$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$ are positive constants and $i = 36,37,38$

They satisfy Lipschitz condition:

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$$|(a''_i)^{(7)}(T'_{37}, t) - (a''_i)^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T'_{37}| e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b''_i)^{(7)}((G_{39})', t) - (b''_i)^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} |(G_{39})' - (G_{39})| e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(7)}(T'_{37}, t)$ and $(a''_i)^{(7)}(T_{37}, t) \cdot (T'_{37}, t)$ and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a''_i)^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 7$ then the function $(a''_i)^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$:

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$(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$:

135

There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}$ and $(\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a'_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(a_i)^{(7)} + (a'_i)^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(b_i)^{(7)} + (b'_i)^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

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Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By

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$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

By

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$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By

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$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By

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$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}$$

By

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$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}$$

By

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$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + (a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that 167

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

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$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \left(1 + (a_{16})^{(2)} t \right) G_{17}^0 +$$

$$\frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

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From which it follows that

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$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that

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$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself. Indeed it is obvious that 173

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}} \right) \right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t \right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1 \right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0 \right) e^{-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t \right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}} + (\hat{P}_{28})^{(5)} \right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself. Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t \right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0)e^{-(\tilde{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left[((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)}s_{(36)}} \right) \right] ds_{(36)} = \\ \left(1 + (a_{36})^{(7)}t \right) G_{37}^0 + \frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)}t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0)e^{-(\tilde{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0)e^{-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}} + (\hat{P}_{36})^{(7)} \right]$$

(G_i^0) is as defined in the statement of theorem 7

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 179

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}} \right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_j)^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\tilde{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\tilde{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote

Definition of \tilde{G}, \tilde{T} : $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} |(a'_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a'_{13})^{(1)} (T_{14}^{(2)}, s_{(13)})| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)} \end{aligned}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} &\leq \\ \frac{1}{(\widehat{M}_{13})^{(1)}} &\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)} \right) d \left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \end{aligned} \quad 186$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ and $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13, 14, 15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \quad \text{for } t > 0 \end{aligned}$$

Definition of $((\widehat{M}_{13})^{(1)})_1, ((\widehat{M}_{13})^{(1)})_2$ and $((\widehat{M}_{13})^{(1)})_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$G_{13} < ((\widehat{M}_{13})^{(1)})_1$ it follows $\frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13}, G_{15} and G_{13}, G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \rightarrow \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d \left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote 195

$$\text{Definition of } \widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$$

It results 196

$$|\widetilde{G}_{16}^{(1)} - \widetilde{G}_{16}^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} +$$

$$\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} |(a_{16}'')^{(2)}(T_{17}^{(1)}, s_{(16)}) - (a_{16}'')^{(2)}(T_{17}^{(2)}, s_{(16)})| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\frac{|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\widehat{M}_{16})^{(2)}t} \leq \frac{1}{(\widehat{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)}) d((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ . 199

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$, $i = 16, 17, 18$ depend only on T_{17} and respectively on (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})) ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(2)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1$, $((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$$G_{16} < (\widehat{M}_{16})^{(2)} \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a_{17}'')^{(2)} G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a_{17}'')^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)} ((\widehat{M}_{16})^{(2)})_2 / (a_{18}'')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}'')^{(2)}$ then $T_{17} \rightarrow \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17}'')^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$$

If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is 205

unbounded. The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b_{18}'')^{(2)}((G_{19})(t), t) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

206

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[(\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{20})^{(3)}$$
208

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)}$$
209

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d \left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \right\}$$

Indeed if we denote 212

Definition of $\widetilde{G}_{23}, \widetilde{T}_{23}$: $(\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$|\widetilde{G}_{20}^{(1)} - \widetilde{G}_{20}^{(2)}| \leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} +$$

$$\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}(T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)} | (a''_{20})^{(3)}(T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)}(T_{21}^{(2)}, s_{(20)}) | e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{20})^{(3)}t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d \left(((G_{23})^{(1)}, (T_{23})^{(1)}); (G_{23})^{(2)}, (T_{23})^{(2)} \right) \quad 214$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ and $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ respectively of \mathbb{R}_+ . 215

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(3)} - (a''_i)^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(3)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{20})^{(3)})_1$, $((\widehat{M}_{20})^{(3)})_2$ and $((\widehat{M}_{20})^{(3)})_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$ then $T_{21} \rightarrow \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = \log \frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t \rightarrow \infty} (b''_{22})^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[(\hat{P}_{24})^{(4)} + ((\hat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[((\hat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)} \right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying 224
Equations into itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right), \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}, \widetilde{T_{27}})$: $(\widetilde{G_{27}}, \widetilde{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

$$\text{It } |\tilde{G}_{24}^{(1)} - \tilde{G}_{24}^{(2)}| \leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{-(\bar{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)} |(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)})| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows results

$$|(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\bar{M}_{24})^{(4)}t} \leq \quad 226$$

$$\frac{1}{(\bar{M}_{24})^{(4)}} \left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{K}_{24})^{(4)} \right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right); (G_{27})^{(2)}, (T_{27})^{(2)}\right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of \mathbb{R}_+ . 227

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ and $(b''_i)^{(4)}$, $i = 24, 25, 26$ depend only on T_{25} and respectively on (G_{27}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 228

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{24})^{(4)})_1$, $((\widehat{M}_{24})^{(4)})_2$ and $((\widehat{M}_{24})^{(4)})_3$: 229

Remark 18: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$G_{24} < ((\widehat{M}_{24})^{(4)})_1$ it follows $\frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)}((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way, one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$ then $T_{25} \rightarrow \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $\lim_{t \rightarrow \infty} (b''_{26})^{(4)}((G_{27})(t), t) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[(\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[((\hat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d \left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G}_{31}), (\widetilde{T}_{31})$: $(\widetilde{G}_{31}), (\widetilde{T}_{31}) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\begin{aligned} |\tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)} \end{aligned}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\begin{aligned} & |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widehat{M}_{28})^{(5)}t} \leq \hspace{15em} 237 \\ & \frac{1}{(\widehat{M}_{28})^{(5)}} \left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)} \right) d \left(((G_{31})^{(1)}, (T_{31})^{(1)}); (G_{31})^{(2)}, (T_{31})^{(2)} \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ respectively of \mathbb{R}_+ . 238

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$, $i = 28, 29, 30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 239

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{28})^{(5)})_1$, $((\widehat{M}_{28})^{(5)})_2$ and $((\widehat{M}_{28})^{(5)})_3$: 240

Remark 23: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$$G_{28} < (\widehat{M}_{28})^{(5)} \text{ it follows } \frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)}G_{29} \text{ and by integrating}$$

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)}((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$ then $T_{29} \rightarrow \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b''_i)^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b''_{30})^{(5)}((G_{31})(t), t) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[(\hat{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d \left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}, \widetilde{T_{35}})$: $(\widetilde{G_{35}}, \widetilde{T_{35}}) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$\begin{aligned} & |\tilde{G}_{32}^{(1)} - \tilde{G}_{32}^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \\ & \int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} + \\ & (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} + \\ & G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)} \end{aligned}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} & |(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\widehat{M}_{32})^{(6)}t} \leq \qquad \qquad \qquad 248 \\ & \frac{1}{(\widehat{M}_{32})^{(6)}} ((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}) d \left(((G_{35})^{(1)}, (T_{35})^{(1)}); (G_{35})^{(2)}, (T_{35})^{(2)} \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ and $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ respectively of \mathbb{R}_+ . 249

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32, 33, 34$ depend only on T_{33} and respectively on (G_{35}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 250

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(6)} - (a''_i)^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(6)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{32})^{(6)})_1, ((\widehat{M}_{32})^{(6)})_2$ and $((\widehat{M}_{32})^{(6)})_3$: 251

Remark 28: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$G_{32} < ((\widehat{M}_{32})^{(6)})_1$ it follows $\frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a'_{33})^{(6)}$$

In the same way, one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a'_{34})^{(6)}$$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(6)}((G_{35})(t), t)) = (b'_{33})^{(6)}$ then $T_{33} \rightarrow \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}$$

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $\lim_{t \rightarrow \infty} (b_{34}'')^{(6)}((G_{35})(t), t(t), t) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose $(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[(\widehat{P}_{36})^{(7)} + ((\widehat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{36})^{(7)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[((\widehat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{36})^{(7)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{36})^{(7)} \right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{36})^{(7)}t} \right\}$$

Indeed if we denote

$$\underline{\text{Definition of}} (\widehat{G}_{39}), (\widehat{T}_{39}) : ((\widehat{G}_{39}), (\widehat{T}_{39})) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$$

It results

$$\begin{aligned} |\widehat{G}_{36}^{(1)} - \widehat{G}_{36}^{(2)}| &\leq \int_0^t (a_{36})^{(7)} |G_{37}^{(1)} - G_{37}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} ds_{(36)} + \\ &\int_0^t \{ (a'_{36})^{(7)} |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} + \\ &(a''_{36})^{(7)} (T_{37}^{(1)}, s_{(36)}) |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} + \\ &G_{36}^{(2)} |(a''_{36})^{(7)} (T_{37}^{(1)}, s_{(36)}) - (a''_{36})^{(7)} (T_{37}^{(2)}, s_{(36)})| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} e^{(\widehat{M}_{36})^{(7)}s_{(36)}} \} ds_{(36)} \end{aligned}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)} - (G_{39})^{(2)}| e^{-(\widehat{M}_{36})^{(7)}t} \leq \quad 259$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{K}_{36})^{(7)} \right) d \left(((G_{39})^{(1)}, (T_{39})^{(1)}); (G_{39})^{(2)}, (T_{39})^{(2)} \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ and $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of \mathbb{R}_+ . 260

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}$, $i = 36, 37, 38$ depend only on T_{37} and respectively on (G_{39}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)}\} (T_{37}(s_{(36)}), s_{(36)}) ds_{(36)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(7)} t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{36})^{(7)})_1$, $((\widehat{M}_{36})^{(7)})_2$ and $((\widehat{M}_{36})^{(7)})_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} and G_{38} . indeed if

$$G_{36} < (\widehat{M}_{36})^{(7)} \text{ it follows } \frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a'_{37})^{(7)} G_{37} \text{ and by integrating}$$

$$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)} ((\widehat{M}_{36})^{(7)})_1 / (a'_{37})^{(7)}$$

In the same way, one can obtain

$$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)} ((\widehat{M}_{36})^{(7)})_2 / (a'_{38})^{(7)}$$

If G_{37} or G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} is bounded, from below, the same property holds for G_{37} and G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(7)} ((G_{39})(t), t)) = (b'_{37})^{(7)}$ then $T_{37} \rightarrow \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b'_i)^{(7)} ((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)} (m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)} (m)^{(7)}}{\varepsilon_7} \right) (1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_7 t} = \frac{1}{2} \text{ it results}$$

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $\lim_{t \rightarrow \infty} (b''_{38})^{(7)}((G_{39})(t), t) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Behavior of the solutions of equation

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Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(p) $\sigma_1^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
 $-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$
 $-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

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(q) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

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By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:-

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(r) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
 $(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, if $(v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$, if $(v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{c_{13}^0}{c_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, if $(\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously

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$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, if $(u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$, if $(u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}$, if $(\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities

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$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq \right. \tag{286}$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t} \right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \tag{287}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \tag{288}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq \tag{289}$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)} ((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

(s) $\sigma_1^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying 293

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \tag{294}$$

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots 296

(t) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

(u) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}$, **if** $(v_0)^{(2)} < (v_1)^{(2)}$ 305

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}$, **if** $(v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}$, 306

and $(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}$, **if** $(\bar{v}_1)^{(2)} < (v_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, **if** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, **if** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, **if** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \quad 312$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions

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Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(v) $\sigma_1^{(3)}, \sigma_2^{(3)}, \tau_1^{(3)}, \tau_2^{(3)}$ four constants satisfying
 $-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$
 $-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

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(w) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$ and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

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(x) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
 $(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \mathbf{if} (v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \mathbf{if} (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$

and $(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \mathbf{if} (\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously

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$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \mathbf{if} (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \mathbf{if} (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \mathbf{if} (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq \quad 324$$

$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(y) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

(z) By $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$$(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

(aa) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

$$\text{and } (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

$$\text{and } (u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

Then the solution of global equations satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq \left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t} \right)$$

$$T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

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$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions of equation

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(bb) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$:

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(cc) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations

$$(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

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By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the

roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(dd) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

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$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } (u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)} \text{ where } (u_1)^{(5)}, (\bar{u}_1)^{(5)}$$

Then the solution of global equations satisfies the inequalities

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$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t} \right) \quad 344$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t} \quad 348$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(ee) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

(ff) By $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations

$$(a_{33})^{(6)} (v^{(6)})^2 + (\sigma_1)^{(6)} v^{(6)} - (a_{32})^{(6)} = 0$$

$$\text{and } (b_{33})^{(6)} (u^{(6)})^2 + (\tau_1)^{(6)} u^{(6)} - (b_{32})^{(6)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the

roots of the equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

(gg) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

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$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities

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$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t} \right) \quad 355$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)} ((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)} ((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$$

$$\begin{aligned}(S_2)^{(6)} &= (a_{34})^{(6)} - (p_{34})^{(6)} \\ (R_1)^{(6)} &= (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)} \\ (R_2)^{(6)} &= (b'_{34})^{(6)} - (r_{34})^{(6)}\end{aligned}$$

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Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$\begin{aligned}- (\sigma_2)^{(7)} &\leq - (a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq - (\sigma_1)^{(7)} \\ - (\tau_2)^{(7)} &\leq - (b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq - (\tau_1)^{(7)}\end{aligned}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$:

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By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations

$$\begin{aligned}(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} &= 0 \\ \text{and } (b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} &= 0 \text{ and}\end{aligned}$$

Definition of $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$:

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By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$ and $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$\begin{aligned}(m_2)^{(7)} &= (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)} \\ (m_2)^{(7)} &= (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ if } (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}, \\ \text{and } (v_0)^{(7)} &= \frac{a_{36}^0}{a_{37}^0}\end{aligned}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously

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$$\begin{aligned}(\mu_2)^{(7)} &= (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)} \\ (\mu_2)^{(7)} &= (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}, \\ \text{and } (u_0)^{(7)} &= \frac{T_{36}^0}{T_{37}^0}\end{aligned}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((S_1)^{(7)} - (a'_{38})^{(7)})} \left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t} \quad 366$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)} ((R_1)^{(7)} - (b'_{38})^{(7)})} \left[e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)} ((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Proof: From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$- \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(p) For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t]}} , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

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$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(q) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

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$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t]}} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (\bar{v}_1)^{(1)}$$

(r) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

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$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t]}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)} , \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)} , \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof: From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- 388

$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows 389

$$- \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

(s) For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

(t) If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)}e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(u) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)}e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

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Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

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If $(a''_{16})^{(2)} = (a''_{17})^{(2)}$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

Proof : From global equations we obtain

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$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a''_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:-

$$\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$$

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It follows

$$- \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

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From which one obtains

$$(v) \text{ For } 0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}}{1 + (C)^{(3)}e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}} , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get

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$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)}e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

$$(w) \text{ If } 0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)} \text{ we find like in the previous case,}$$

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$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (\bar{c})^{(3)} (\nu_2)^{(3)} e^{[-(a_{21})^{(3)} ((\nu_1)^{(3)} - (\nu_2)^{(3)}) t]}}{1 + (\bar{c})^{(3)} e^{[-(a_{21})^{(3)} ((\nu_1)^{(3)} - (\nu_2)^{(3)}) t]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{c})^{(3)} (\bar{\nu}_2)^{(3)} e^{[-(a_{21})^{(3)} ((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}) t]}}{1 + (\bar{c})^{(3)} e^{[-(a_{21})^{(3)} ((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}) t]}} \leq (\bar{\nu}_1)^{(3)}$$

(x) If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

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$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{c})^{(3)} (\bar{\nu}_2)^{(3)} e^{[-(a_{21})^{(3)} ((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}) t]}}{1 + (\bar{c})^{(3)} e^{[-(a_{21})^{(3)} ((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}) t]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof: From global equations we obtain

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$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left((a_{25})^{(4)} (\nu^{(4)})^2 + (\sigma_2)^{(4)} \nu^{(4)} - (a_{24})^{(4)} \right) \leq \frac{d\nu^{(4)}}{dt} \leq - \left((a_{25})^{(4)} (\nu^{(4)})^2 + (\sigma_4)^{(4)} \nu^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(y) For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get

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$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(z) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case,

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$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (\bar{v}_1)^{(4)}$$

(aa) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

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$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)} , \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)} , \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$ **this also defines $(v_0)^{(4)}$ for the special case .**

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}$, then $(\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important

consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, and definition of $(u_0)^{(4)}$.

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Proof: From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

Definition of $v^{(5)}$:-
$$v^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

It follows

$$- \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(bb) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}, \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get

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$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}, \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(cc) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

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$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (\bar{v}_1)^{(5)}$$

(dd) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

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$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines $(v_0)^{(5)}$ for the special case .**

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of $(u_0)^{(5)}$.**

Proof : From global equations we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$- \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

(ee) For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}} , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

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$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(ff) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (\bar{C})^{(6)} (v_2)^{(6)} e^{[-(a_{33})^{(6)} ((v_1)^{(6)} - (v_2)^{(6)}) t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} ((v_1)^{(6)} - (v_2)^{(6)}) t]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} ((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}) t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} ((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}) t]}} \leq (\bar{v}_1)^{(6)}$$

(gg) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} ((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}) t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)} ((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}) t]}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{32}'')^{(6)} = (a_{33}'')^{(6)}$, then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)} G_{33}(t)$ **this also defines $(v_0)^{(6)}$ for the special case.**

Analogously if $(b_{32}'')^{(6)} = (b_{33}'')^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of $(u_0)^{(6)}$.**

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Proof: From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)} (T_{37}, t) \right) - (a''_{37})^{(7)} (T_{37}, t) v^{(7)} - (a_{37})^{(7)} v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$- \left((a_{37})^{(7)} (v^{(7)})^2 + (\sigma_2)^{(7)} v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq - \left((a_{37})^{(7)} (v^{(7)})^2 + (\sigma_1)^{(7)} v^{(7)} - (a_{36})^{(7)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$:-

$$\text{For } 0 < \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$$

$$v^{(7)}(t) \geq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_0)^{(7)})t]}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_0)^{(7)})t]}} , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}$$

it follows $(v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}$

In the same manner , we get

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$$v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}} , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}$$

From which we deduce $(v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$ we find like in the previous case,

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$$(v_1)^{(7)} \leq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_2)^{(7)})t]}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_2)^{(7)})t]}} \leq v^{(7)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}} \leq (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \leq (\bar{v}_1)^{(7)} \leq \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain

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$$(v_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}} \leq (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

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$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}''^{(7)}) = (a_{37}''^{(7)})$, then $(\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(v_1)^{(7)} = (\bar{v}_1)^{(7)}$ if in addition $(v_0)^{(7)} =$

$(v_1)^{(7)}$ then $v^{(7)}(t) = (v_0)^{(7)}$ and as a consequence $G_{36}(t) = (v_0)^{(7)}G_{37}(t)$ **this also defines $(v_0)^{(7)}$ for the special case.**

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}$, then $(\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$ if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(v_1)^{(7)}$ and $(\bar{v}_1)^{(7)}$, **and definition of $(u_0)^{(7)}$.**

We can prove the following

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Theorem : If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied, then the system

Theorem : If $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ are independent on t , and the conditions with the notations

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0,$$

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$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

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with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied, then the system

Theorem : If $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ are independent on t , and the conditions with the notations

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$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied, then the system

We can prove the following

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Theorem 3: If $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with $(p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a'_i)^{(5)}$ and $(b'_i)^{(5)}$ are independent on t , and the conditions with the notations 433

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0 ,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a'_i)^{(6)}$ and $(b'_i)^{(6)}$ are independent on t , and the conditions with the notations 434

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with $(p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem 3: If $(a'_i)^{(7)}$ and $(b'_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$$

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0 ,$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$$

with $(p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0 \quad 478$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(b) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Definition and uniqueness of T_{14}^* :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 495$$

Definition and uniqueness of T_{21}^* :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value, we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^*)]}$$

(h) By the same argument, the equations admit solutions G_{13}, G_{14} if 502
 $\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$$[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(i) By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

(j) By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(k) By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(l) By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that

there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(m) By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

(n) By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0, T_{14}^*$ given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} , T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0, T_{17}^*$ given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0, T_{21}^*$ given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution

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G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} , T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]}$$

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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution

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G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} , T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]}$$

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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution

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G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} , G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^*)]} , T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^*)]}$$

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Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution

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G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^*)]} , G_{38}^* = \frac{(a_{38})^{(7)}G_{37}^*}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)}T_{37}^*}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^*)]} , T_{38}^* = \frac{(b_{38})^{(7)}T_{37}^*}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^*T_{14} \quad 525$$

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^*T_{14} \quad 526$$

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^*T_{14} \quad 527$$

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(13)(j)})T_{13}^*G_j \quad 528$$

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15}(s_{(14)(j)})T_{14}^*G_j \quad 529$$

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(15)(j)})T_{15}^*G_j \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \quad 534$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \quad 535$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \quad 536$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*G_j \quad 537$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j \quad 538$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(3)}$ and $(b'_i)^{(3)}$ belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_i)^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \quad \frac{\partial(b''_i)^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \quad 541$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21} \quad 542$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \quad 543$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^*G_j \quad 544$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^*G_j \quad 545$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^*G_j \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(4)}$ and $(b''_i)^{(4)}$ belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :- 548

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_i)^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \quad \frac{\partial(b''_i)^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^*T_{25} \quad 549$$

$$\frac{dG_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^*T_{25} \quad 550$$

$$\frac{dG_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^*T_{25} \quad 551$$

$$\frac{dT_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^*G_j \quad 552$$

$$\frac{dT_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^*G_j \quad 553$$

$$\frac{dT_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^*G_j \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(5)}$ and $(b'_i)^{(5)}$ belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :- 556

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a'_{29})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial (b'_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{dG_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^*T_{29} \quad 557$$

$$\frac{dG_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^*T_{29} \quad 558$$

$$\frac{dG_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^*T_{29} \quad 559$$

$$\frac{dT_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*G_j \quad 560$$

$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*G_j \quad 561$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*G_j \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(6)}$ and $(b'_i)^{(6)}$ belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :- 564

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}, \quad \frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(32)(j)})T_{32}^*\mathbb{G}_j \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}(s_{(33)(j)})T_{33}^*\mathbb{G}_j \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(34)(j)})T_{34}^*\mathbb{G}_j \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ and $(b_i'')^{(7)}$ belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)}, \quad \frac{\partial(b_i'')^{(7)}}{\partial G_j}((G_{39})^{**}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -((b'_{36})^{(7)} - (r_{36})^{(7)})\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}(s_{(36)(j)})T_{36}^*\mathbb{G}_j \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -((b'_{37})^{(7)} - (r_{37})^{(7)})\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}(s_{(37)(j)})T_{37}^*\mathbb{G}_j \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -((b'_{38})^{(7)} - (r_{38})^{(7)})\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}(s_{(38)(j)})T_{38}^*\mathbb{G}_j \quad 579$$

The characteristic equation of this system is

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$$\begin{aligned}
 & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\
 & \left[((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\
 & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(14)}T_{14}^* + (b_{14})^{(1)}S_{(13),(14)}T_{14}^* \right) \\
 & + \left(((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\
 & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(13)}T_{14}^* + (b_{14})^{(1)}S_{(13),(13)}T_{13}^* \right) \\
 & \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & \left(((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\
 & + \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\
 & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^* \right) \\
 & \left. \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(15)}T_{14}^* + (b_{14})^{(1)}S_{(13),(15)}T_{13}^* \right) \right\} = 0 \\
 & + \\
 & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\
 & \left[((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(17)}T_{17}^* + (b_{17})^{(2)}S_{(16),(17)}T_{17}^* \right) \\
 & + \left(((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(16)}T_{17}^* + (b_{17})^{(2)}S_{(16),(16)}T_{16}^* \right) \\
 & \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & \left(((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & + \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) \left((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \\
 & \left. \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(18)}T_{17}^* + (b_{17})^{(2)}S_{(16),(18)}T_{16}^* \right) \right\} = 0 \\
 & +
 \end{aligned}$$

$$\begin{aligned}
 & ((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)})\{((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \\
 & \left[((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right] \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})S_{(21),(21)}T_{21}^* + (b_{21})^{(3)}S_{(20),(21)}T_{21}^* \right) \\
 & + \left(((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^* \right) \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})S_{(21),(20)}T_{21}^* + (b_{21})^{(3)}S_{(20),(20)}T_{20}^* \right) \\
 & \left(((\lambda)^{(3)})^2 + ((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) (\lambda)^{(3)} \right) \\
 & \left(((\lambda)^{(3)})^2 + ((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}) (\lambda)^{(3)} \right) \\
 & + \left(((\lambda)^{(3)})^2 + ((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) (\lambda)^{(3)} \right) (q_{22})^{(3)}G_{22} \\
 & + ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) ((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^*) \\
 & \left. \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})S_{(21),(22)}T_{21}^* + (b_{21})^{(3)}S_{(20),(22)}T_{20}^* \right) \right\} = 0 \\
 & + \\
 & ((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)})\{((\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)}) \\
 & \left[((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)})(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(q_{24})^{(4)}G_{24}^* \right] \\
 & \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})S_{(25),(25)}T_{25}^* + (b_{25})^{(4)}S_{(24),(25)}T_{25}^* \right) \\
 & + \left(((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)})(q_{24})^{(4)}G_{24}^* + (a_{24})^{(4)}(q_{25})^{(4)}G_{25}^* \right) \\
 & \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})S_{(25),(24)}T_{25}^* + (b_{25})^{(4)}S_{(24),(24)}T_{24}^* \right) \\
 & \left(((\lambda)^{(4)})^2 + ((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}) (\lambda)^{(4)} \right) \\
 & \left(((\lambda)^{(4)})^2 + ((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}) (\lambda)^{(4)} \right) \\
 & + \left(((\lambda)^{(4)})^2 + ((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}) (\lambda)^{(4)} \right) (q_{26})^{(4)}G_{26} \\
 & + ((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) ((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^*) \\
 & \left. \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})S_{(25),(26)}T_{25}^* + (b_{25})^{(4)}S_{(24),(26)}T_{24}^* \right) \right\} = 0 \\
 & + \\
 & ((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)})\{((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)})
 \end{aligned}$$

$$\begin{aligned}
 & \left[\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right] \\
 & \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(29)} T_{29}^* + (b_{29})^{(5)} s_{(28),(29)} T_{29}^* \\
 & + \left((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)} \right) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \\
 & \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(28)} T_{29}^* + (b_{29})^{(5)} s_{(28),(28)} T_{28}^* \\
 & \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} \\
 & \left((\lambda)^{(5)} \right)^2 + \left((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} \right) (\lambda)^{(5)} \\
 & + \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} (q_{30})^{(5)} G_{30} \\
 & + \left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) \left((a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \\
 & \left. \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(30)} T_{29}^* + (b_{29})^{(5)} s_{(28),(30)} T_{28}^* \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)} \right) \left\{ (\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)} \right\} \\
 & \left[\left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right] \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(33)} T_{33}^* + (b_{33})^{(6)} s_{(32),(33)} T_{33}^* \\
 & + \left((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)} \right) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(32)} T_{33}^* + (b_{33})^{(6)} s_{(32),(32)} T_{32}^* \\
 & \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & \left((\lambda)^{(6)} \right)^2 + \left((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & + \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\
 & + \left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) \left((a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left. \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(34)} T_{33}^* + (b_{33})^{(6)} s_{(32),(34)} T_{32}^* \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)} \right) \left\{ (\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \left[\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)} \right) (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (q_{36})^{(7)} G_{36}^* \right] \\
 & \left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(37)} T_{37}^* + (b_{37})^{(7)} s_{(36),(37)} T_{37}^* \\
 & + \left((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)} \right) (q_{36})^{(7)} G_{36}^* + (a_{36})^{(7)} (q_{37})^{(7)} G_{37}^* \\
 & \left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(36)} T_{37}^* + (b_{37})^{(7)} s_{(36),(36)} T_{36}^* \\
 & \left((\lambda)^{(7)} \right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} \\
 & \left((\lambda)^{(7)} \right)^2 + \left((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)} \right) (\lambda)^{(7)} \\
 & + \left((\lambda)^{(7)} \right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} (q_{38})^{(7)} G_{38} \\
 & + \left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)} \right) \left((a_{38})^{(7)} (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (a_{38})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \\
 & \left. \left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(38)} T_{37}^* + (b_{37})^{(7)} s_{(36),(38)} T_{36}^* \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

References

- [1] "Quark (subatomic particle)". Encyclopædia Britannica. Retrieved 2008-06-29.
- [2] R. Nave. "Confinement of Quarks". HyperPhysics. Georgia State University, Department of Physics and Astronomy. Retrieved 2008-06-29.
- [3] R. Nave. "Bag Model of Quark Confinement". HyperPhysics. Georgia State University, Department of Physics and Astronomy. Retrieved 2008-06-29.
- [4] a b R. Nave. "Quarks". HyperPhysics. Georgia State University, Department of Physics and Astronomy. Retrieved 2008-06-29. a b c d B. Carithers, P. Grannis (1995). "Discovery of the Top Quark" (PDF). *Beam Line (SLAC)* 25 (3): 4–16. Retrieved 2008-09-23.
- [5] a b E.D. Bloom et al. (1969). "High-Energy Inelastic e–p scattering at 6° and 10°". *Physical Review Letters* 23 (16): 930–934. Bibcode 1969PhRvL...23..930B.doi:10.1103/PhysRevLett.23.930.
- [6] a b M. Breidenbach et al. (1969). "Observed Behavior of Highly Inelastic Electron–Proton Scattering". *Physical Review Letters* 23 (16): 935–939. Bibcode 1969PhRvL...23...935B. doi:10.1103/PhysRevLett.23.935.
- [7] C. Amsler et al. (Particle Data Group) (2008). "Higgs Bosons: Theory and Searches". *Physics Letters B* 667 (1): 1–1340. Bibcode 2008PhLB..667....1P.doi:10.1016/j.physletb.2008.07.018.
- [8] S.S.M. Wong (1998). *Introductory Nuclear Physics* (2nd ed.). Wiley Interscience. p. 30. ISBN 0-471-23973-9.
- [9] K.A. Peacock (2008). *The Quantum Revolution*. Greenwood Publishing Group. p. 125. ISBN 0-313-33448-X.
- [10] B. Povh, C. Scholz, K. Rith, F. Zetsche (2008). *Particles and Nuclei*. Springer. p. 98. ISBN 3-540-79367-4.
- [11] Section 6.1. in P.C.W. Davies (1979). *The Forces of Nature*. Cambridge University Press. ISBN 0-521-22523-X.
- [12] a b c M. Munowitz (2005). *Knowing*. Oxford University Press. p. 35. ISBN 0-19-516737-6.
- [13] a b W.-M. Yao et al. (Particle Data Group) (2006). "Review of Particle Physics: Pentaquark Update". *Journal of Physics G* 33 (1): 1–1232. ArXiv: astro-ph/0601168. Bibcode 2006JPhG...33....1Y. doi:10.1088/0954-3899/33/1/001.
- [14] C. Amsler et al. (Particle Data Group) (2008). "Review of Particle Physics: Pentaquarks". *Physics Letters B* 667 (1): 1–40. Bibcode 2008PhLB..667....1P.doi:10.1016/j.physletb.2008.07.018.
- [15] C. Amsler et al. (Particle Data Group) (2008). "Review of Particle Physics: New Charmonium-Like States". *Physics Letters B* 667 (1): 1–1340. Bibcode 2008PhLB..667....1P. doi:10.1016/j.physletb.2008.07.018.
- [16] E.V. Shuryak (2004). *The QCD Vacuum, Hadrons and Superdense Matter*. World Scientific. p. 59. ISBN 981-238-574-6.
- [17] C. Amsler et al. (Particle Data Group) (2008). "Review of Particle Physics: b' (4th Generation) Quarks, Searches for". *Physics Letters B* 667 (1): 1–1340. Bibcode 2008PhLB...667....1P. doi:10.1016/j.physletb.2008.07.018.
- [18] C. Amsler et al. (Particle Data Group) (2008). "Review of Particle Physics: t' (4th Generation) Quarks, Searches for". *Physics Letters B* 667 (1): 1–1340. Bibcode 2008PhLB...667....1P. doi:10.1016/j.physletb.2008.07.018.
- [19] D. Decamp (1989). "Determination of the number of light neutrino species". *Physics Letters B* 231 (4): 519. Bibcode 1989PhLB..231...519D. doi: 10.1016/0370-2693(89)90704-1.
A. Fisher (1991). "Searching for the Beginning of Time: Cosmic Connection". *Popular Science* 238 (4): 70.
- [20] J.D. Barrow (1997) [1994]. "The Singularity and Other Problems". *The Origin of the Universe* (Reprint

- Ed.). Basic Books. ISBN 978-0-465-05314-8.
- [21] D.H. Perkins (2003). Particle Astrophysics. Oxford University Press. p. 4. ISBN 0-19-850952-9.
- [22] a b M. Gell-Mann (1964). "A Schematic Model of Baryons and Mesons". *Physics Letters* 8 (3): 214–215. Bibcode 1964PhL.....8...214G. Doi: 10.1016/S0031-9163(64)92001-3.
- [23] a b G. Zweig (1964). "An SU (3) Model for Strong Interaction Symmetry and its Breaking". CERN Report No.8182/TH.401.
- [24] a b G. Zweig (1964). "An SU (3) Model for Strong Interaction Symmetry and its Breaking: II". CERN Report No.8419/TH.412.
- [25] M. Gell-Mann (2000) [1964]. "The Eightfold Way: A theory of strong interaction symmetry". In M. Gell-Mann, Y. Ne'emann. *The Eightfold Way*. Westview Press. p. 11. ISBN 0-7382-0299-1.
- [26] Original: M. Gell-Mann (1961). "The Eightfold Way: A theory of strong interaction symmetry". Synchrotron Laboratory Report CTSL-20 (California Institute of Technology).
- [27] Y. Ne'emann (2000) [1964]. "Derivation of strong interactions from gauge invariance". In M. Gell-Mann, Y. Ne'emann. *The Eightfold Way*. Westview Press. ISBN 0-7382-0299-1.
Original Y. Ne'emann (1961). "Derivation of strong interactions from gauge invariance". *nuclear* 26 (2): 222. Bibcode 1961NucPh..26...222N. doi: 10.1016/0029-5582(61)90134-1.
- [28] Companion to the History of Modern Science. Taylor & Francis. 1996. p. 673. ISBN 0-415-14578-3.
- [29] A. Pickering (1984). *Constructing Quarks*. University of Chicago Press. pp. 114–125. ISBN 0-226-66799-5.
- [30] B.J. Bjorken, S.L. Glashow (1964). "Elementary Particles and SU (4)". *Physics Letters* 11 (3): 255–257. Bibcode 1964PhL.....11...255B. Doi: 10.1016/0031-9163(64)90433-0.
- [31] J.I. Friedman. "The Road to the Nobel Prize". Hue University. Retrieved 2008-09-29.
- [32] R.P. Feynman (1969). "Very High-Energy Collisions of Hadrons". *Physical Review Letters* 23 (24): 1415–1417. Bibcode 1969PhRvL..23.1415F. doi:10.1103/PhysRevLett.23.1415.
- [33] S. Kretzer et al. (2004). "CTEQ6 Parton Distributions with Heavy Quark Mass Effects". *Physical Review D* 69 (11): 114005. ArXiv: hep-ph/0307022. Bibcode 2004PhRvD..69k4005K. doi:10.1103/PhysRevD.69.114005.
- [34] a b D.J. Griffiths (1987). *Introduction to Elementary Particles*. John Wiley & Sons. p. 42. ISBN 0-471-60386-4.
- [35] M.E. Peskin, D.V. Schroeder (1995). *An introduction to quantum field theory*. Addison–Wesley. p. 556. ISBN 0-201-50397-2.
- [36] V.V. Ezhela (1996). *Particle physics*. Springer. p. 2. ISBN 1-56396-642-5.
- [37] S.L. Glashow, J. Iliopoulos, L. Maiani (1970). "Weak Interactions with Lepton–Hadron Symmetry". *Physical Review D* 2 (7): 1285–1292. Bibcode 1970PhRvD...2.1285G. doi:10.1103/PhysRevD.2.1285.
- [38] D.J. Griffiths (1987). *Introduction to Elementary Particles*. John Wiley & Sons. p. 44. ISBN 0-471-60386-4.
- [39] M. Kobayashi, T. Maskawa (1973). "CP-Violation in the Renormalizable Theory of Weak Interaction". *Progress of Theoretical Physics* 49 (2): 652–657. Bibcode 1973PTPh..49...652K. doi:10.1143/PTP.49.652.
- [40] a b H. Harari (1975). "A new quark model for hadrons". *Physics Letters B* 57B (3): 265. Bibcode 1975PhLB...57...265H. doi: 10.1016/0370-2693(75)90072-6.
- [41] a b K.W. Staley (2004). *The Evidence for the Top Quark*. Cambridge University Press. pp. 31–33. ISBN 978-0-521-82710-2.
- [42] S.W. Herb et al. (1997). "Observation of a Dimuon Resonance at 9.5 GeV in 400-GeV Proton-Nucleus Collisions". *Physical Review Letters* 39 (5): 252. Bibcode 1977PhRvL..39...252H. doi:10.1103/PhysRevLett.39.252.
- [43] M. Bartusiak (1994). *A Positron named Priscilla*. National Academies Press. p. 245. ISBN 0-309-04893-1.

- [45] F. Abe et al. (CDF Collaboration) (1995). "Observation of Top Quark Production in ppCollisions with the Collider Detector at Fermilab". *Physical Review Letters* 74 (14): 2626–2631. Bibcode 1995PhRvL..74.2626A. doi:10.1103/PhysRevLett.74.2626.PMID 10057978.
- [46] S. Abachi et al. (DØ Collaboration) (1995). "Search for High Mass Top Quark Production in pp Collisions at $\sqrt{s} = 1.8$ TeV". *Physical Review Letters* 74 (13): 2422–2426. Bibcode 1995PhRvL..74.2422A. doi:10.1103/PhysRevLett.74.2422.
- [47] K.W. Staley (2004). *The Evidence for the Top Quark*. Cambridge University Press. p. 144. ISBN 0-521-82710-8.
- [48] "New Precision Measurement of Top Quark Mass". Brookhaven National Laboratory News. Retrieved 2008-09-24.
- [49] J. Joyce (1982) [1939]. *Finnegans Wake*. Penguin Books. p. 383. ISBN 0-14-006286-6. LCCN 59354.
- [50] M. Gell-Mann (1995). *The Quark and the Jaguar: Adventures in the Simple and the Complex*. Henry Holt and Co. p. 180. ISBN 978-0-8050-7253-2.
- [51] J. Gleick (1992). *Genius: Richard Feynman and modern physics*. Little Brown and Company. p. 390. ISBN 0-316-90316-7.
- [52] J.J. Sakurai (1994). S.F Tuan, ed. *Modern Quantum Mechanics (Revised ed.)*. Addison–Wesley. p. 376. ISBN 0-201-53929-2.
- [53] a b D.H. Perkins (2000). *Introduction to high energy physics*. Cambridge University Press. p. 8. ISBN 0-521-62196-8.
- [54] M. Riordan (1987). *The Hunting of the Quark: A True Story of Modern Physics*. Simon & Schuster. p. 210. ISBN 978-0-671-50466-3.
- [55] F. Close (2006). *The New Cosmic Onion*. CRC Press. p. 133. ISBN 1-58488-798-2.
- [56] G. Fraser (2006). *The New Physics for the Twenty-First Century*. Cambridge University Press. p. 91. ISBN 0-521-81600-9.
- [57] "The Standard Model of Particle Physics". BBC. 2002. Retrieved 2009-04-19.
- [58] F. Close (2006). *The New Cosmic Onion*. CRC Press. pp. 80–90. ISBN 1-58488-798-2.
- [59] D. Lincoln (2004). *Understanding the Universe*. World Scientific. p. 116. ISBN 981-238-705-6.
- [60] "Weak Interactions". Virtual Visitor Center. Stanford Linear Accelerator Center. 2008. Retrieved 2008-09-28.
- [61] K. Nakamura et al. (2010). "Review of Particles Physics: The CKM Quark-Mixing Matrix". *J. Phys. G* 37 (75021): 150.
- [62] Z. Maki, M. Nakagawa, S. Sakata (1962). "Remarks on the Unified Model of Elementary Particles". *Progress of Theoretical Physics* 28 (5): 870. Bibcode1962PThPh...28...870M. doi:10.1143/PTP.28.870.
- [63] B.C. Chauhan, M. Picariello, J. Pulido, E. Torrente-Lujan (2007). "Quark–lepton complementarity, neutrino and standard model data predict θ_{PMNS}
 $13 = 9+1$
 $13 = 9+1$ ". *European Physical Journal C* 50 (3): 573–578. ArXiv:hep-ph/0605032. Bibcode2007EPJC...50..573C. Doi:10.1140/epjc/s10052-007-0212-z.
- [64] R. Nave. "The Color Force". HyperPhysics. Georgia State University, Department of Physics and Astronomy. Retrieved 2009-04-26.
- [65] B.A. Schumm (2004). *Deep Down Things*. Johns Hopkins University Press. pp. 131–132. ISBN 0-8018-7971-X. OCLC 55229065.
- [66] a b Part III of M.E. Peskin, D.V. Schroeder (1995). *An Introduction to Quantum Field Theory*. Addison–Wesley. ISBN 0-201-50397-2.
- [67] V. Icke (1995). *The force of symmetry*. Cambridge University Press. p. 216. ISBN 0-521-45591-X.
- [68] M.Y. Han (2004). *A story of light*. World Scientific. p. 78. ISBN 981-256-034-3.
- [69] C. Sutton. "Quantum chromodynamics (physics)". *Encyclopædia Britannica Online*. Retrieved 2009-05-12.

- [71] A. Watson (2004). *The Quantum Quark*. Cambridge University Press. pp. 285–286. ISBN 0-521-82907-0.
- [72] a b c K. Nakamura et al. (Particle Data Group), JP G 37, 075021 (2010) and 2011 partial update for the 2012 edition (URL: <http://pdg.lbl.gov>)
- [73] W. Weise, A.M. Green (1984). *Quarks and Nuclei*. World Scientific. pp. 65–66. ISBN 9971-966-61-1.
- [74] D. McMahon (2008). *Quantum Field Theory Demystified*. McGraw–Hill. p. 17. ISBN 0-07-154382-1.
- [75] S.G. Roth (2007). *Precision electroweak physics at electron–positron colliders*. Springer. p. VI. ISBN 3-540-35164-7.
- [76] R.P. Feynman (1985). *QED: The Strange Theory of Light and Matter* (1st ed.). Princeton University Press. pp. 136–137. ISBN 0-691-08388-6.
- [77] a b M. Veltman (2003). *Facts and Mysteries in Elementary Particle Physics*. World Scientific. pp. 45–47. ISBN 981-238-149-X.
- [78] F. Wilczek, B. Devine (2006). *Fantastic Realities*. World Scientific. p. 85. ISBN 981-256-649-X.
- [79] F. Wilczek, B. Devine (2006). *Fantastic Realities*. World Scientific. pp. 400ff. ISBN 981-256-649-X.
- [80] T. Yulsman (2002). *Origin*. CRC Press. p. 55. ISBN 0-7503-0765-X.
- [81] F. Garberson (2008). "Top Quark Mass and Cross Section Results from the Tevatron". arXiv:0808.0273 [hep-ex].
- [82] J. Steinberger (2005). *Learning about Particles*. Springer. p. 130. ISBN 3-540-21329-5.
- [83] C.-Y. Wong (1994). *Introduction to High-energy Heavy-ion Collisions*. World Scientific. p. 149. ISBN 981-02-0263-6.
- [84] S.B. Rüester, V. Werth, M. Buballa, I.A. Shovkovy, D.H. Rischke (2005). "The phase diagram of neutral quark matter: Self-consistent treatment of quark masses". *Physical Review D* 72 (3): 034003. ArXiv:hep-ph/0503184. Bibcode 2005PhRvD..72c4004R.doi:10.1103/PhysRevD.72.034004.
- [85] M.G. Alford, K. Rajagopal, T. Schaefer, A. Schmitt (2008). "Color superconductivity in dense quark matter". *Reviews of Modern Physics* 80 (4): 1455–1515. ArXiv:0709.4635. Bibcode 2008RvMP..80.1455A.doi:10.1103/RevModPhys.80.1455.
- [86] S. Mrowczynski (1998). "Quark–Gluon Plasma". *Acta Physica Polonica B* 29: 3711. ArXiv:nucl-th/9905005. Bibcode 1998AcPPB..29.3711M.
- [87] Z. Fodor, S.D. Katz (2004). "Critical point of QCD at finite T and μ , lattice results for physical quark masses". *Journal of High Energy Physics* 2004 (4): 50. ArXiv: hep-lat/0402006. Bibcode 2004JHEP..04...050F. Doi:10.1088/1126-6708/2004/04/050.
- [88] U. Heinz, M. Jacob (2000). "Evidence for a New State of Matter: An Assessment of the Results from the CERN Lead Beam Programme". ArXiv: nucl [nucl-th].
- [89] "RHIC Scientists Serve Up "Perfect" Liquid". Brookhaven National Laboratory News. 2005. Retrieved 2009-05-22.
- [90] T. Yulsman (2002). *Origins: The Quest for Our Cosmic Roots*. CRC Press. p. 75. ISBN 0-7503-0765-X.
- [91] A. Sedrakian, J.W. Clark, M.G. Alford (2007). *Pairing in fermionic systems*. World Scientific. pp. 2–3. ISBN 981

