

Comparison of Image Compression using Wavelet for Curvelet Transform & Transmission over Wireless Channel

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Abstract- For image compression, it is very necessary that the selection of transform should reduce the size of the resultant data as compared to the original data set. In this paper, a new lossless image compression method is proposed. Image Compression is a widely addressed research area. Many compression standards have been in place. But still there is a scope for higher compression with quality reconstruction. The introduction of wavelets gave a different dimension to the compression. But there are some limitations of wavelets while handling the line and curve singularities in the image. There are transforms beyond wavelets namely – Curvelet Transforms. This paper aims at the analysis of compression using Curvelet & Wavelet Transform. The Curvelet Transform gives better performance in terms of PSNR. Wavelet performs the least and is also affected by the blocking artifacts. By selecting proper thresholding method, better results for PSNR have been obtained. Again This paper presents an efficient scheme to transmit JPEG coded images over wireless channels. The compressed image is protected against various channel effects of wireless channel using Reed soloman block code and transmitted over wireless channel.

Index Terms- Image compression, Curvelet transform, Wavelet Transform, Wireless channel effect.

I. INTRODUCTION

Image compression means reducing the volume of data for representing an image. The main aim of image compression is to reduce both spatial and spectral redundancy to store or transmit data in a proper manner. After the compression of an image, it is reconstructed at the receiver to reproduce the original image. Various compression techniques are used for this purpose. In lossless image compression, some form of entropy coding is used, while in lossy compression transform coding and predictive coding is used. This paper is connected with lossy image compression using two transforms- Curvelet & Wavelet. Objective Metrics for Compression are:

1. Compression Factor (CF) = (size of input stream) / (size of output stream)

2. Mean squared error (MSE) = $\frac{1}{k} \sum_{i=1}^k (P_i - Q_i)^2$

3. Root mean squared error (RMSE) = $\sqrt{\text{MSE}}$

Where P_i – Original image data, Q_i – Reconstructed image data, k – size of image.

4. PSNR = $20 \log_{10} \{ \max_i(p_i) / \text{RMSE} \}$ [17].

II. WAVELET TRANSFORM

Wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function (t) as a superposition of a set of such wavelets or basic functions. These basic functions or baby wavelets are obtained from a single prototype wavelet called the mother wavelet, by dilation and translation operation. Discrete Wavelet Transform of a finite length signal $s(n)$ having N components, for image is expressed by an $N \times N$ matrix. [1][12]

A. Wavelet Filter Decomposition and Sub- Band Coding

1) Wavelet Filter Decomposition

For the designing of filters, sub-band coding is used. Sub-band coding is a coding strategy that tries to isolate different characteristics of a signal in a way that collects the signal energy into few components. This is referred to as energy compaction. Energy compaction is desirable because it is easier to efficiently encode these components than the signal.

The most commonly used implementation of the discrete wavelet transform (DWT) consists of recursive application of the low-pass/high-pass one-dimensional (1-D) filter bank successively along the horizontal and vertical directions of the image. The low-pass filter provides the smooth approximation coefficients while the high-pass filter is used to extract the detail coefficients at a given resolution.

Both low-pass and high-pass filters are called sub-bands. The number of decompositions performed on original image to obtain sub bands is called sub-band decomposition level. The high pass sub-band represents residual information of the original image, needed for the perfect reconstruction of the original image from the low-pass sub-band while the low pass sub-band represents a down sampled low- resolution version of the original image. It is used for computer and human vision, musical tone generation, FBI finger print compression.

The filtering step is followed by a sub-sampling operation that decreases the resolution from one transformation level to the other. After applying the 2-D filter bank at a given level n , the detail coefficients are output, while the whole filter bank is applied again upon the approximation image until the desired maximum resolution is achieved. Fig.1 shows wavelet filter decomposition. The sub-bands are labeled by using the following symbols [4][8]

1. **LL_n** is the approximation image at resolution (level decomposition) n , resulting from low-pass filtering in the vertical and horizontal directions.
2. **HL_n** represents the vertical details at resolution n , and results from vertical low-pass filtering and horizontal high-pass filtering.
3. **LH_n** represents the horizontal details at resolution n , and results from horizontal low-pass filtering and vertical high-pass filtering.
4. **HH_n** represents the diagonal details at resolution n , and results from high-pass filtering in both directions.

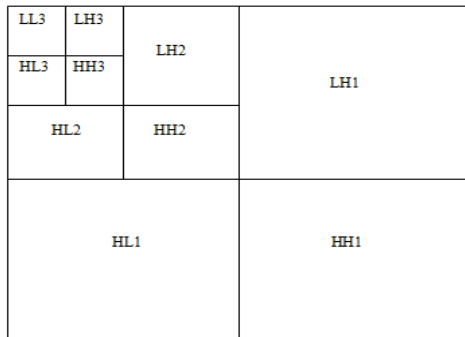


Fig.1 Wavelet Filter Decomposition

III. CURVELET TRANSFORM (CT)

A new multi-resolution transform was developed by Candés and Donoho [13] in 1999 known as curvelet transform as a result of motivation to take away the drawbacks associated with wavelet transform. The transform that is a two-dimensional anisotropic extension of wavelet, originally designed to represent edges and other singularities along curves much more efficiently than traditional wavelet transforms. Although curvelets is an extension of wavelets but there exists a correspondence between curvelet and wavelet subbands. The general rule that represents correspondence between curvelet subband (Cs) and wavelet subband (Ws) is [11]

$$C_s \leftrightarrow W_s \in \{2 * C_s, 2 * C_s + 1\}$$

Contrary to wavelets' isotropic principle where length and width of support frame is of equal size, in curvelet transform the width and length are related by the relation $width \approx length^2$ that is known as parabolic or anisotropic scaling [14]. Moreover, frame elements in curvelets indexed by scale, location and orientation parameters in contrast to wavelets where elements have only scale and location parameters.

The basic flaw that wavelet transform exhibits, is its inability to represent edge discontinuities along curves. Less number of coefficients is required in compression process but several wavelet coefficients are used to reconstruct edges properly along the curves.

This is due to the reason that in a map of large wavelet coefficients, edges repeat at scale after scale. There was a need of a transform that handle two dimensional singularities along the curves sparsely. This led to the birth of new multi-resolution curvelet transform. Curvelet basis elements possess wavelet

basis function qualities but these also oriented at a variety of directions and so represent edge discontinuities and other singularities well than wavelet transform [7].

Curvelet transform is a special member of the multiscale geometric transforms [15, 16, 17]. It is a transform with multiscale pyramid with many directions at each length and scale. Curvelets will be superior over wavelets in following cases:

- i) Optimally sparse representation of objects with edges.
 - ii) Optimal image reconstruction in severely ill-posed problems
 - iii) Optimal sparse representation of wave propagators
- Curvelets are initially introduced by Candés and Donoho [16].

Suppose we have a function f which has a discontinuity across a curve, and which is smooth otherwise, and consider approximating f from the best m -terms in the Fourier expansion.

The squared error of such an m -term expansion obeys:

$$\|f - f_F^{\sim}\|^2 \propto 1/\sqrt{m}, \quad m \rightarrow +\infty$$

In a wavelet expansion, we have

$$\|f - f_W^{\sim}\|^2 \propto 1/m, \quad m \rightarrow +\infty$$

$f_W^{\sim}$ is the approximation from m best Wavelet coefficients) In a curvelet expansion (Donoho and Candés, 2000), we have

$$\|f - f_C^{\sim}\|^2 \propto m^{-2}(\log m)^2, \quad m \rightarrow +\infty,$$

$f_C^{\sim}$ is the approximation from m best Curvelet coefficients.) This shows that the mean squared error will be reduced in curvelets.

A fast and accurate discrete curvelet transform operating on digital data is required to use curvelet transform in various applications. This Fast Discrete Curvelet Transform (FDCT) is described in [15]. Fig.2 indicates the Curvelet tiling.

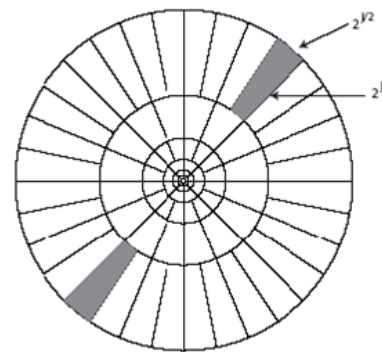


Fig. 2 Curvelet Tiling of the Frequency Plane. The shaded area represents such a generic wedge.

Ψ_j is used as a "mother" curvelet and all other curvelets at scale 2^{-j} are derived by rotations and translations of Ψ_j .

* Rotation angles $\theta_l = 2\pi \cdot 2^{-[j/2]} \cdot l$, with $l = 0, 1, \dots$ such that $0 < \theta_l < 2\pi$, 2^{-j} indicates scale. (the spacing between consecutive angles is scale dependent),

* Sequence of translation parameters

$$k = (k_1, k_2) \in \mathbb{Z}^2$$

and the Curvelets are defined (as a function of $x = (x_1, x_2)$) at scale 2^{-j} , orientation angle θ_l and position.

$$x_k^{(j,l)} = R_{\theta_l}^{-1} (k_1 \cdot 2^{-j}, k_2 \cdot 2^{-j/2}) \text{ by}$$

$$\Psi_{j,l,k}(x) = \Psi_j(R_{\theta_l}(x - x_k^{(j,l)}))$$

where $R\theta$ is the rotation by θ radians.

A curvelet coefficient is the inner product between an element $f \in L^2(\mathbb{R}^2)$ and a curvelet $\Psi_{j,l,k}$,

$$c(j, l, k) = \int_{\mathbb{R}^2} f(x) \overline{\Psi_{j,l,k}(x)} dx$$

(where $\mathbb{R}$ denotes the real line.)

This theory is explained in detail in [13]. Curvelet transform obeys an anisotropy scaling relation,

$$length \approx 2^{-j/2}, width = 2^{-j} \Rightarrow width \approx length^2$$

This is also called as a Curve scaling law [15]

Fast digital curvelet transforms can be implemented via two methods i) using Unequispaced FFTs ii) using Wrapping. This paper makes use of the method using unequispaced FFTs [13].

IV. EXPERIMENTATION USING CURVELET TRANSFORM

The experimentation is based on three transforms namely, Curvelet, Wavelet and Ridgelet Transform. The images are selected from the set of standard images. The image is compressed with 'sym4' filter. The transform is invertible, non-redundant and computed via fast algorithms. Further, this construction experimentation is done on many images. But the results of only five images are given here. Similar trend is observed in other images too.

Results for Compression metrics, the Root Mean Square Error (RMSE) and Peak Signal-to-noise Ratio (PSNR) are tabulated. Specific thresholding method (hard thresholding) is used in each of the transform. Details are indicated below in each of the cases. MATLAB computation platform is used with necessary tool boxes.

i) Curvelet Transform:

Number of scales used is **three** in both the transforms. Flowchart for the method is given in Figure. 3. v indicates threshold value.

Since the effect of transform on compression is the emphasis of this paper, the stages of entropy coding and decoding is not discussed. Other methods are expressed using algorithms.

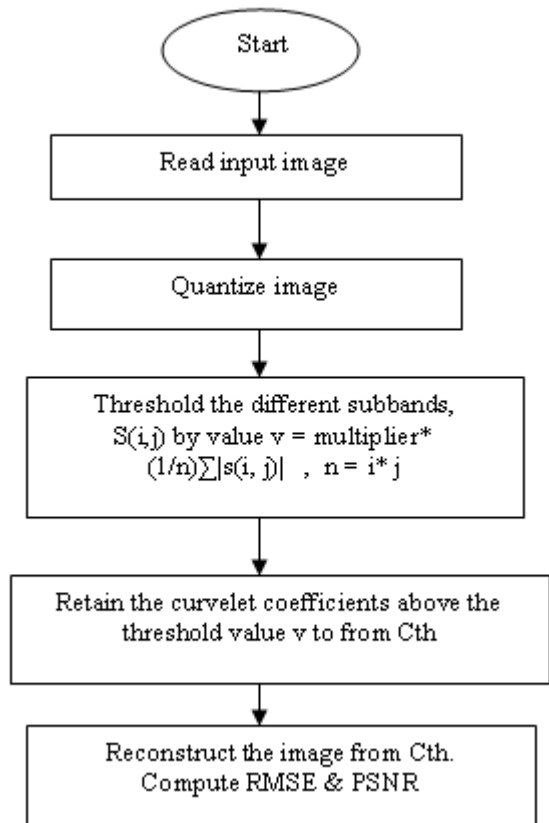


Figure 3: Flowchart for method using Curvelet

V. WIRELESS TRANSMISSION CHANNELS AND THEIR EFFECTS

Nowadays, wireless communications have captured a great attention. Due to multipath effects of wireless channel the received signal is equal to the sum of attenuated, delayed, and phase-shifted replicas of the transmitted signal. The transmission of JPEG images over noisy channels can lead to severe consequences on decoded visual information. The reason for such high error sensitivity is the Variable Length Coding schemes. Thus designing a system for image transmission over wireless channel remains a major issue. A number of solutions have already been designed for image transmission over wireless channels. Optimized product codes consisting of Turbo codes and Reed- Solomon code have been implemented for JPEG transmission. RS code is tested over an AWGN and Rayleigh fading channel [9] [10]. RS codes provide highly reliable performance in the mobile environment.

VI. RESULTS

The RMSE and PSNR for both the transforms is tabulated. The PSNR values using curvelets are better. The PSNR in wavelet case is the least. Table 1 gives the comparison of compression performance. Compression Factor (CF) is calculated as the ratio of number of Input pixels to number of retained coefficients. The CF itself is called as Compression Ratio in the Figures.

VII. CONCLUSION

In this work ,we have presented a comparison of image compression using two transform i.e wavelet & curvelet transform. Curvelet Transform gives the best performance for PSNR . But the subjective visual inspection shows that the Curvelet is the best for Compression out of two transforms. This can be seen from the images given below in Figure 4. Even though the most significant subband is retained in case of wavelet, its performance is poor and has annoying blocking artifacts when the numbers of retained coefficients is low (higher compression factor).

This shows that the Curvelet Transforms are more suitable for the image data to represent the singularities over geometric structures in the image, than the Wavelet counterpart. Curvelet is designed to age data to represent handle the singularities on curves . Wavelets are effective for point singularities.

Transmission over wireless channel remains a major issue. Many techniques have been proposed for the transmission of images over wireless networks.

Table 1: Compression Performance

	Transform	Compression	PSNR
		Factor (CF)	(dB)
1. F2 - lili.jpg, Size - 512 x 512			
	Curvelet	7.49	20.93
	Wavelet	7.49	16
	Curvelet	5.24	24.48
	Wavelet	5.24	16.37
	Curvelet	4.03	22.64
	Wavelet	4.03	16.7
2. F1 - cameraman.tif, Size - 256 x 256			
	Curvelet	8.19	15.3
	Wavelet	8.19	13.3
	Curvelet	6.55	18
	Wavelet	6.55	15.7
	Curvelet	5.04	21.8
	Wavelet	5.04	16.1

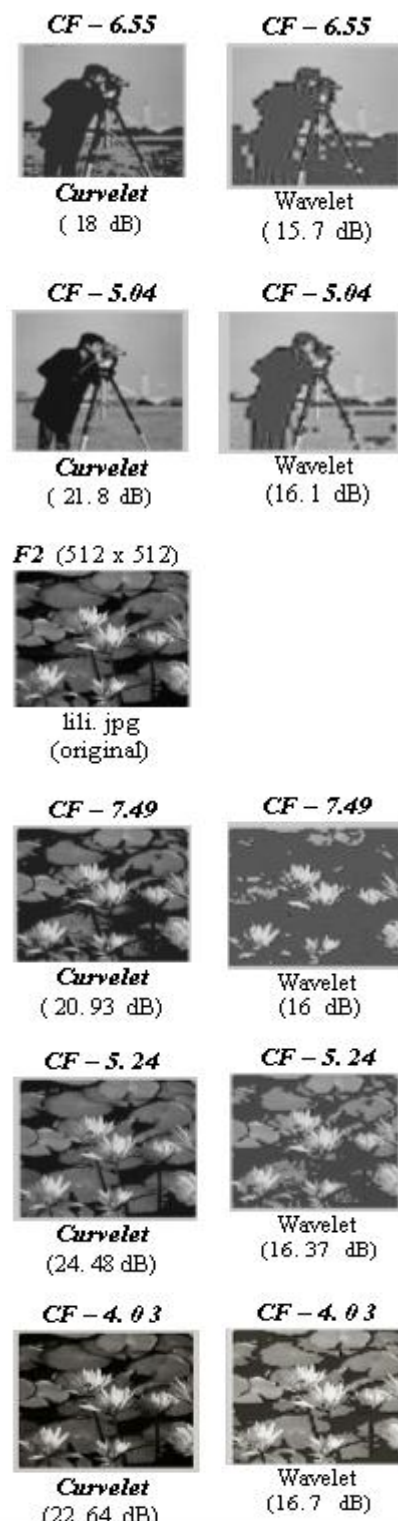
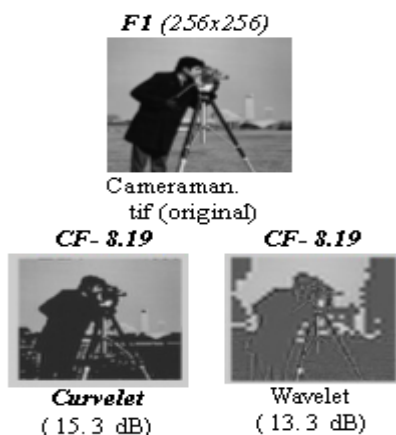


Figure 4: A Few Result Images with their Reconstruction for different Compression Factors CF – Compression factor

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