

On a class of αg^{**} -closed sets in topological spaces and some mappings

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Abstract- In this paper, we introduced the concepts of αg^{**} -closed sets as well as αg^{**} -open sets and studied some of their basic properties. Further we defined αg^{**} -continuous mappings and investigated their basic properties. In addition we obtained αg^{**} -irresolute mappings and some of their properties are derived.

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I. INTRODUCTION

The concept of generalized closed (g-closed) sets in a topological space was introduced by Levine [8] and concept of $T_{1/2}$ spaces was initiated by Dunham [5] further investigated the properties of $T_{1/2}$ spaces and defined a new closure operator cl^* by using generalized closed sets. We begin with some basic concepts. A subset A of a topological space (X, τ) is called α -open [12] (resp. semi open [9]) if $A \subseteq \text{int}(cl(\text{int}(A)))$ (resp. $A \subseteq cl(\text{int}(A))$). Also A is said to be α -closed (resp. semi closed) if $X - A$ is α -open (resp. semi open). The collection of all α -open (resp. semi open) subsets in (X, τ) is denoted by τ^α (resp. $SO(X)$). The α -closure (resp. semi closure) of a subset A is smallest α -closed (resp. semi closed) set containing A and this is denoted by $\alpha cl(A)$ (resp. $scl(A)$) in the present paper. We recall definitions of some generalized closed sets. A subset A is called $g\alpha$ -closed [10] (resp. αg -closed [10], $g^\# \alpha$ -closed [13]) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an α -open (resp. open, g-open) set in (X, τ) . Moreover G. Sai Sundara Krishnan et.al [16] introduced the concept of g^{**} -open sets and defined a subset A of X is said to be g^{**} -open if and only if there exist an open set U of X such that $U \subseteq A \subseteq cl^{**}(U)$. Also its complement is called g^{**} -closed. A topological space (X, τ) is said to be a g^{**} - $T_{1/2}$ space if every g^{**} -closed set is closed. J.Chitra and D.Saravanakumar [2] introduced the concept of sg^{**} -closed sets and defined a subset A of X is said to be sg^{**} -closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a g^{**} -open set in (X, τ) .

In this paper we introduced the concept of αg^{**} -closed sets, which is analogous to α -generalized closed sets and introduced the notion $\alpha G^{**}C(X)$ which is the set of all αg^{**} -closed sets in a topological space. In addition, we defined the concept of αg^{**} -open sets and studied of its some basic properties. Finally we introduced the concept of αg^{**} -continuous and αg^{**} -irresolute maps in a topological space and investigated relationship between them. Throughout this

paper we denoted cl^* by cl^{**} and we represented the topological spaces (X, τ) , (Y, σ) and (Z, η) as X , Y and Z respectively. Unless otherwise no separation axiom mentioned.

II. αg^{**} -CLOSED SET

In this section we introduce a new class of closed set called αg^{**} -closed set and study further some of their properties.

Definition 2.1. Let A be subset of a topological space X . It is called αg^{**} -closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^{**} -open. The set of all αg^{**} -closed sets denoted by $\alpha G^{**}C(X)$. A subset A of X is called αg^{**} -open if $X - A$ is αg^{**} -closed in X .

Example 2.1. If $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{c, d\}\}$, then αg^{**} -closed sets of X are $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Remark 2.1. The concepts of α -open and g^{**} -open are independent.

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a, c\}$ and $\{b, c\}$ are g^{**} -open sets but not α -open.

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}\}$. Then $\{a, b\}$ and $\{a, c\}$ are α -open sets but not g^{**} -open.

Remark 2.2. The concepts of g-closed sets and αg^{**} -closed sets are independent.

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{a, b, d\}$ is a g-closed set but not αg^{**} -closed. Also $\{c\}$, $\{a, c\}$ and $\{c, d\}$ are αg^{**} -closed sets but not g-closed.

Theorem 2.1. Let A be a subset of a topological space X and if A is a α -closed set in X , then A is αg^{**} -closed.

Proof: Let A be a α -closed set in X and $A \subseteq U$ where U is g^{**} -open. Since A is α -closed, $\alpha cl(A) = A \subseteq U$. Hence A is αg^{**} -closed in X .

The reverse implication does not hold.

Example 2.2. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$. Then $\{b, c\}$ is a αg^{**} -closed set but not α -closed.

Theorem 2.2. Let A be a subset of a topological space X and if A is a g^{**} -closed set in X , then A is αg -closed.

Proof: Let A be a αg -closed set in X and $A \subseteq U$ where U is open. Since every open set is g^{**} -open and A is αg^{**} -closed, $\alpha cl(A) \subseteq U$. Hence A is αg -closed in X .

The reverse implication does not hold.

Example 2.3. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$. Then $\{a, c, d\}$ is a αg -closed set but not αg^{**} -closed.

Remark 2.3. The concepts of $g\alpha$ -closed and αg^{**} -closed are independent.

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}\}$. Then $\{a, b\}$ and $\{a, c\}$ are αg^{**} -closed sets but not αg -closed.

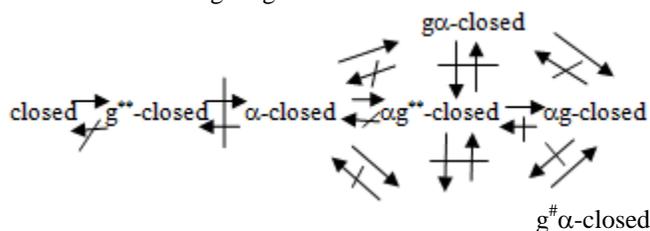
Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$. Then $\{b, c\}$ and $\{b, d\}$ are αg -closed sets but not αg^{**} -closed.

Remark 2.4. The concepts of $g^{\#}\alpha$ -closed and αg^{**} -closed are independent.

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$. Then $\{b, c\}$ and $\{a, b, c\}$ are αg^{**} -closed sets but not $g^{\#}\alpha$ -closed.

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{a, b, d\}$ is a $g^{\#}\alpha$ -closed set but not αg^{**} -closed.

Remark 2.5. From the Theorems 2.1, 2.2 and Remarks 2.3, 2.4, we have the following diagram



where $A \rightarrow B$ represents A imply B , $A \dashrightarrow B$ represents A does not imply B .

Theorem 2.3. If A is a αg^{**} -closed set in X , then it is sg^{**} -closed.

Proof: Let A be a αg^{**} -closed set such that $A \subseteq U$, where U is g^{**} -open. By Definition 2.1, we have $\alpha cl(A) \subseteq U$. Since $\alpha cl(A)$ is a semi closed set and $scl(A)$ is the least semi closed set containing A , we have $scl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is sg^{**} -closed.

The reverse implication does not hold.

Example 2.4. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\{a\}, \{b\}, \{a, c\}$ are sg^{**} -closed sets in X but not αg^{**} -closed.

Theorem 2.4. Let A be a αg -closed set in a topological space X . Then A is αg^{**} -closed if X is $g^{**}-T_{1/2}$.

Proof: Let A be a αg -closed set in X and $A \subseteq U$ where U is g^{**} -open. Since X is a $g^{**}-T_{1/2}$ space and A is αg -closed, every g^{**} -open set is open and hence $\alpha cl(A) \subseteq U$. Therefore A is αg^{**} -closed in X .

Corollary 2.1. If X is $g^{**}-T_{1/2}$, then every $g\alpha$ -closed set is αg^{**} -closed.

Proof: Follows from the above Theorem 2.4.

Corollary 2.2. If X is $g^{**}-T_{1/2}$, then every $g^{\#}\alpha$ -closed set is αg^{**} -closed.

Proof: Follows from the above Theorem 2.4.

Theorem 2.5. Let $\{A_i : i \in J\}$ be the collection of αg^{**} -closed sets in a topological space X . Then $\cup_{i \in J} A_i$ is also a αg^{**} -closed set in X .

Proof. Let $\cup_{i \in J} A_i \subseteq U$ where U is g^{**} -open. Then $A_i \subseteq U$ for each $i \in J$. Since A_i is αg^{**} -closed for each $i \in J$, we have $\alpha cl(A_i) \subseteq U$ for each $i \in J$. This implies that $\alpha cl(\cup_{i \in J} A_i) = \cup_{i \in J} \alpha cl(A_i) \subseteq U$. Hence $\cup_{i \in J} A_i$ is αg^{**} -closed in X .

Remark 2.6. (i) Intersection of two αg^{**} -closed sets need not be αg^{**} -closed.

Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$. Then αg^{**} -closed sets of X are $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

If $A = \{a, b\}$ and $B = \{a, c\}$, then the sets A and B are αg^{**} -closed but $A \cap B = \{a\}$ is not αg^{**} -closed.

Theorem 2.6. If a set A is αg^{**} -closed in X , then $\alpha cl(A) - A$ contains no non-empty g^{**} -closed set.

Proof: Let F be a g^{**} -closed set such that $F \subseteq \alpha cl(A) - A$. Since $X - F$ is g^{**} -open and $A \subseteq X - F$, from the definition of αg^{**} -closed set it follows that $\alpha cl(A) \subseteq X - F$. Thus $F \subseteq X - \alpha cl(A)$. This implies that $F \subseteq \alpha cl(A) \cap (X - \alpha cl(A)) = \emptyset$.

Corollary 2.3. Let A be a αg^{**} -closed set in X . Then A is α -closed if and only if $\alpha cl(A) - A$ is g^{**} -closed.

Proof: Necessity. Let A be α -closed in X . Then $\alpha cl(A) \subseteq A$. This implies that $\alpha cl(A) - A = \emptyset$. Therefore $\alpha cl(A) - A$ is g^{**} -closed.

Sufficiency, Suppose $\alpha cl(A) - A$ is g^{**} -closed. Then by Theorem 2.6, $\alpha cl(A) - A$ contains does not contain any non-empty g^{**} -closed set and hence $\alpha cl(A) - A = \emptyset$. This implies that $\alpha cl(A) = A$. Therefore A is α -closed in X .

Theorem 2.7. Suppose that X is an α -space (i.e., $\tau = \tau^\alpha$). A αg^{**} -closed set A is α -closed in X if and only if $\alpha cl(A) - A$ is α -closed in X .

Proof: Necessity. If A is α -closed in X , then $\alpha cl(A) \subseteq A$. This implies that $\alpha cl(A) - A = \emptyset$, we have $\alpha cl(A) - A$ is α -closed in X .

Sufficiency, Suppose that A is αg^{**} -closed and $\alpha cl(A) - A$ is α -closed. It follows from the assumption that $\tau = \tau^\alpha$. Then $\alpha cl(A) - A$ is g^{**} -closed in X . By Theorem 2.6, we have $\alpha cl(A) - A = \emptyset$ this implies that $\alpha cl(A) = A$. Hence A is α -closed in X .

Theorem 2.8. Let A be a αg^{**} -closed set in X . If A is g^{**} -open, then $\alpha cl(A) - A = \emptyset$.

Proof: Let A be g^{**} -open in X . Since A is αg^{**} -closed, $\alpha cl(A) \subseteq A$. This implies that $\alpha cl(A) - A = \emptyset$.

Theorem 2.9. Every subset is αg^{**} -closed in X if and only if every g^{**} -open set is α -closed.

Proof: Necessity. Let A be g^{**} -open in X . Then by hypothesis A is αg^{**} -closed in X . By Theorem 2.8 $\alpha cl(A) - A = \emptyset$. Hence A is α -closed.

Sufficiency. Let A be a subset of X and U a g^{**} -open set such that $A \subseteq U$. Then by hypothesis, U is α -closed. This implies that $\alpha cl(A) \subseteq \alpha cl(U) = U$. Hence A is αg^{**} -closed.

Theorem 2.10. If A is a αg^{**} -closed set in X and $A \subseteq B \subseteq \alpha cl(A)$, then B is αg^{**} -closed in X .

Proof: Let $B \subseteq U$ where U is g^{**} -open. Since A is αg^{**} -closed in X and $A \subseteq U$, it follows that $\alpha cl(A) \subseteq U$. By hypothesis, $B \subseteq \alpha cl(A)$ and hence $\alpha cl(B) \subseteq \alpha cl(A)$. Consequently, $\alpha cl(B) \subseteq U$ and B becomes αg^{**} -closed in X .

Theorem 2.11. For each $x \in X$, $\{x\}$ is g^{**} -closed or its complement $X - \{x\}$ is αg^{**} -closed in a space X .

Proof: Suppose that $\{x\}$ is not g^{**} -closed in X . Since $X - \{x\}$ is not g^{**} -open, the space X itself is only g^{**} -open set containing $X - \{x\}$. Therefore, $\alpha cl(X - \{x\}) \subseteq X$ holds and so $X - \{x\}$ is αg^{**} -closed.

III. αg^{**} -OPEN SET

In this section, we introduce a new class of open set called αg^{**} -open set and study some of their properties.

Definition 3.1. A subset A of a topological space is called semi g^{**} -open (briefly αg^{**} -open) if and only if $X - A$ is αg^{**} -closed. The set of all αg^{**} -open set denoted by $\alpha G^{**}O(X)$.

Theorem 3.1. Let $\{A_i : i \in J\}$ be the collection of αg^{**} -open sets in a topological space X . Then $\bigcap_{i \in J} A_i$ is also a αg^{**} -open set in X .

Proof. Since A_i is αg^{**} -open in X for each $i \in J$. This implies that $X - A_i$ is αg^{**} -closed in X for each $i \in J$. Then by Theorem 2.5, we have $X - \bigcap_{i \in J} A_i = \bigcup_{i \in J} (X - A_i)$ is also αg^{**} -closed in X . Hence $\bigcap_{i \in J} A_i$ is αg^{**} -open in X .

Remark 3.1. Union of two αg^{**} -open sets need not be αg^{**} -open.

Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$. Then αg^{**} -open sets of X are $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$

If $A = \{b\}$ and $B = \{c\}$. Then the sets A and B are αg^{**} -open but $A \cup B = \{b, c\}$ is not αg^{**} -open.

Theorem 3.2. A set A is αg^{**} -open in X if and only if $F \subseteq \alpha int(A)$ whenever F is g^{**} -closed and $F \subseteq A$.

Proof: Necessity, Let A be αg^{**} -open and suppose $F \subseteq A$ where F is g^{**} -closed. By Definition 3.1, $X - A$ is αg^{**} -closed. Also $X - A$ is contained in the g^{**} -open set $X - F$. This implies that $\alpha cl(X - A) = X - \alpha int(A)$. Hence $X - \alpha int(A) \subseteq X - F$. Therefore $F \subseteq \alpha int(A)$.

Sufficiency. If F is a g^{**} -closed set with $F \subseteq \alpha int(A)$ whenever $F \subseteq A$, it follows that $X - A \subseteq X - F$ and $X - \alpha int(A) \subseteq X - F$. Thus $\alpha cl(X - A) \subseteq X - F$. Hence $X - A$ is αg^{**} -closed and A becomes αg^{**} -open. This proves the theorem.

Theorem 3.3. If $\alpha int(A) \subseteq B \subseteq A$ and A is αg^{**} -open in X , then B is αg^{**} -open.

Proof: By hypothesis, $X - A \subseteq X - B \subseteq X - \alpha int(A)$. Thus $X - A \subseteq X - B \subseteq X - (X - \alpha cl(X - A)) = \alpha cl(X - A)$. Now $X - A$ is αg^{**} -closed and hence by Theorem 2.10, $X - B$ is αg^{**} -closed. Hence B is αg^{**} -open in X .

IV. αg^{**} -CONTINUOUS MAP and αg^{**} -IRRESOLUTE MAP

In this section we introduce the concepts of αg^{**} -continuous map and αg^{**} -irresolute map and study relationship between some other mappings.

Definition 4.1. A map $f : X \rightarrow Y$ is called αg^{**} -continuous if $f^{-1}(V)$ is αg^{**} -closed in X for every closed set V of Y .

Definition 4.2. A map $f : X \rightarrow Y$ is called αg^{**} -irresolute if $f^{-1}(V)$ is αg^{**} -closed in X for every αg^{**} -closed set V of Y .

Theorem 4.1. Let $f : X \rightarrow Y$ be αg^{**} -continuous. Then f is αg^{**} -irresolute but not conversely.

Proof: Let V be a closed set in Y . Then $f^{-1}(V)$ is αg^{**} -closed in X since f is αg^{**} -continuous. By Theorem 2.2, every αg^{**} -closed set is αg -closed. Therefore $f^{-1}(V)$ is αg -closed in X . Hence f is αg -continuous. The converse need not be true as seen from the following example.

Example 4.1. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$, $Y = \{1, 2, 3, 4\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = 3, f(b) = 2, f(c) = 1, f(d) = 4$. Then f is αg -continuous. $f^{-1}(\{2, 3, 4\}) = \{a, b, d\}$ is not αg^{**} -closed in X for the closed set $\{2, 3, 4\}$ of Y . So f is not αg^{**} -continuous.

Theorem 4.2. Let $f : X \rightarrow Y$ be a mapping from a topological space X into a topological space Y . Then the following statements are equivalent.

- (i) f is αg^{**} -continuous;
- (ii) the inverse image of each open set in Y is αg^{**} -open in X .

Proof: Straight forward from the Definition 3.1.

Definition 4.3. A space X is called $\alpha g^{**}\text{-}T_{1/2}$ space if every αg^{**} -closed set is g^{**} -closed.

Theorem 4.3. Let $f : X \rightarrow Y$ be a map. If f is αg^{**} -continuous, then f is g^{**} -irresolute where X is $\alpha g^{**}\text{-}T_{1/2}$ and Y is $g^{**}\text{-}T_{1/2}$.

Proof: Let V be g^{**} -closed in Y . Since Y is $g^{**}\text{-}T_{1/2}$, we have V is closed in Y . Then $f^{-1}(V)$ is αg^{**} -closed in X , since f is αg^{**} -continuous. But X is $\alpha g^{**}\text{-}T_{1/2}$ and so $f^{-1}(V)$ is g^{**} -closed. Hence f is g^{**} -irresolute.

Theorem 4.4. Let $f : X \rightarrow Y$ be a map. If f is g^{**} -continuous, then f is αg^{**} -irresolute where X is $g^{**}\text{-}T_{1/2}$ and Y is $\alpha g^{**}\text{-}T_{1/2}$.

Proof: Let V be g^{**} -closed in Y . Since Y is $\alpha g^{**}\text{-}T_{1/2}$, we have V is g^{**} -closed in Y . Then $f^{-1}(V)$ is g^{**} -closed in X , since f is g^{**} -continuous. But X is $g^{**}\text{-}T_{1/2}$ and so $f^{-1}(V)$ is αg^{**} -closed. Hence f is αg^{**} -irresolute.

Note that if $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are both αg^{**} -continuous, then the composition $g \circ f : X \rightarrow Z$ is not αg^{**} -continuous mapping.

Example 4.2. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ and $\eta = \{\emptyset, Z, \{a\}, \{a, b\}, \{a, c\}\}$. Define $f : X \rightarrow Y$ by $f(a) = c, f(b) = a$ and $f(c) = b$ and define $g : Y \rightarrow Z$ by $g(a) = a, g(b) = c, g(c) = b$. Then f and g are αg^{**} -continuous mappings. The set $\{b\}$ is closed in Z . $(g \circ f)^{-1}(\{b\}) = f^{-1}(g^{-1}(\{b\})) = f^{-1}(\{c\}) = \{a\}$ which is not αg^{**} -closed in X . Hence $g \circ f$ is not αg^{**} -continuous.

Theorem 4.5. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two mappings.

- (i) If f is αg^{**} -irresolute and g is αg^{**} -continuous, then the composition $g \circ f$ is αg^{**} -continuous;

- (ii) If f and g are αg^{**} -irresolute, then the composition $g \circ f$ is αg^{**} -irresolute;

- (iii) Let a space Y be $\alpha g^{**}\text{-}T_{1/2}$. If $f : X \rightarrow Y$ is g^{**} -irresolute $g : Y \rightarrow Z$ is g^{**} -continuous, then the composition $g \circ f : X \rightarrow Z$ is g^{**} -continuous.

Proof: (i) Let V be closed in Z . Then αg^{**} -continuity of g implies that $g^{-1}(V)$ is αg^{**} -closed in Y . Since f is αg^{**} -irresolute, it follows that $f^{-1}(g^{-1}(V))$ is αg^{**} -closed in X . Hence $g \circ f$ is αg^{**} -continuous.

(ii) Let V be αg^{**} -closed in Z . Then αg^{**} -irresolute of g implies that $g^{-1}(V)$ is αg^{**} -closed in Y . Since f is αg^{**} -irresolute, it follows that $f^{-1}(g^{-1}(V))$ is αg^{**} -closed in X . Hence $g \circ f$ is αg^{**} -irresolute.

(iii) Let V be closed in Z . Then αg^{**} -continuity of g implies that $g^{-1}(V)$ is αg^{**} -closed in Y . Thus $g^{-1}(V)$ is g^{**} -closed in Y since Y is $\alpha g^{**}\text{-}T_{1/2}$. Therefore $f^{-1}(g^{-1}(V))$ is g^{**} -closed in X , because f is g^{**} -irresolute. Hence $g \circ f$ is g^{**} -continuous.

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