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Design of Fractional Order PID Controller for Speed Control of DC Motor

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Abstract- Conventional PID controller is one of the most widely used controllers in industry, but the recent advancement in fractional calculus has introduced applications of fractional order calculus in control theory. One of the prime applications of fractional calculus is fractional order PID controller and it has received a considerable attention in academic studies and in industrial applications. Fractional order PID controller is an advancement of classical integer order PID controller. In many a cases fractional order PID controller has outperformed classical integer order PID controller. This research paper, studies the control aspect of fractional order controller in speed control of DC motor. A comparative study of classical PID controller and fractional order PID controller has been performed.

Index Terms- PID controller, Fractional order PID controller, DC motor

I. Introduction

Fractional order calculus has gained acceptance in last couple of decades. J Liouville made the first major study of fractional calculus in 1832. In 1867, A. K. Grunwald worked on the fractional operations. G. F. B. Riemann developed the theory of fractional integration in 1892. Fractional order mathematical phenomena allow us to describe and model a real object more accurately than the classical "integer" methods. Earlier due to lack of available methods, a fractional order system was used to be approximated as an integer order model. But at the present time, there are many available numerical techniques which are used to approximate the fractional order derivatives and integrals. A typical example of a non-integer (fractional) order system is the voltage-current relation of a semiinfinite lossy transmission line [30].

The past decade has seen an increase in research efforts related to fractional calculus and use of fractional calculus in control system. For a control loop perspective there are four situations like (i) integer order plant with integer order controller, (ii) integer order plant with fractional order controller, (iii) fractional order plant with integer order controller, (iv) fractional order plant with fractional order controller. Fractional order control enhances the dynamic system control performance.

This paper studies the control effect of fractional order PID controller in speed control of DC motor and performs a comparative study of classical PID controller and fractional order PID controller for speed control of DC motor.

II. Fractional Order Calculus: Mathematical Overview

Fractional order calculus is an area where the mathematicians deal with derivatives and integrals from non-integer orders. Gamma function is simply the generalization of the factorial for all real numbers. The definition of the gamma function is given by $\Gamma(x) = \int_0^\infty z^{x-1} e^{-z} dz$ (1)

$$\Gamma(x) = (x-1)!$$

Differintegral operator is denoted by ${}_aD_t^\alpha$. It is the combination of differentiation and integration operation commonly used in fractional calculus. Reimann- Liouville definition for ${}_aD_t^\alpha$ is

$${}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \quad (2)$$

Here α is the fractional order. a and t are the limits.

There are two commonly used definitions for general Differintegral ${}_aD_t^\alpha$.

1. Grunwald - Letnikov
2. Riemann- Liouville

Grunwald – Letnikov definition

$$ {}_aD_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) \qquad (3) $$

$[.]$ is a flooring-operator

Here $$ \binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \qquad (4) $$

Riemann- Liouville definition

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (5) $$

The condition for above equation is $n-1<\alpha<n$. $\Gamma(.)$ is called the gamma function.

Laplace Transform of Differintegral operator

Differintegral operator is denoted by ${}_aD_t^\alpha$ and the Laplace transform of Differintegral operator is represented as

$$ L\left[{}_aD_\alpha^t f(t)\right] = \int_0^\infty e^{-st}\, {}_aD_\alpha^t f(t)\, dt \qquad (6) $$

$$ L\left[{}_aD_\alpha^t f(t)\right] = s^\alpha F(s) - \sum_{m=0}^{n-1} s(-1)^j\, {}_0D_t^{\alpha-m-1} f(t) $$

Here n lies in between $n-1<\alpha\le n$

III. Fractional Order Calculus in Control

One of the primary controllers is PID controller, which is widely used. Fractional controller is denoted by $PI^\lambda D^\mu$ was proposed by Igor Podlubny in 1997 [1], here λ and μ have non-integer values. Figure 1 shows the block diagram of fractional order PID controller.

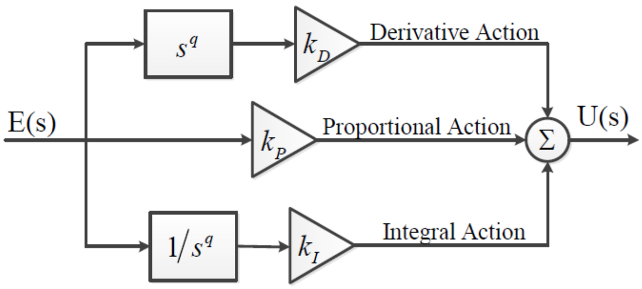


Figure 1: Fractional PID Controller

The transfer function for conventional PID controller is

$$ G_{PID}(s) = \frac{u(s)}{e(s)} = K_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \qquad (7) $$

Transfer function for fractional order PID controller is

$$ G_{FOPID}(s) = \frac{u(s)}{e(s)} = K_c\left(1 + \frac{1}{\tau_i s^\lambda} + \tau_d s^\mu\right) \qquad (8) $$

Where λ and μ are an arbitrary real numbers, K_p is amplification (gain), T_i is integration constant and T_d is differentiation constant. Taking λ=1 and μ=1, a classical PID controller is obtained. For further practical digital realization, the derivative part has to be complemented by first order filter. The filter is used to remove high frequency noise.

$$ G_{FOPID}(s) = K_c\left(1 + \frac{1}{\tau_i s^\lambda} + \frac{\tau_d s^\mu}{\frac{\tau_d}{N} s + 1}\right) \qquad (9) $$

The $PI^\lambda D^\mu$ controller is more flexible and gives an opportunity to better adjust the dynamics of control system. Its compact and simple but the analog realization of fractional order system is very difficult.

Intuitively, the FOPID has more degree of freedom than the conventional PID. It can be expected that the FOPID can provide better performance with proper choice of controller parameters. However, with more parameters to be tuned, the associated optimization problem will be more difficult to deal with. It is motivated to develop a systematic procedure for the FOPID optimization to achieve a certain performance.

Different fractional controllers are summarized below.

CRONE Controller (Commande Robuste d'Ordre Non Entier)

CRONE has a French acronym which means fractional order robust control.

TID Controller (Tilted Proportional Integral Controller)

TID controller has the same structure as classical PID controller, but the proportional gain is replaced with a function $s^{-\alpha}$

Fractional Lead-Lag Controller

Fractional lead-lag controller is an extension of classical lead-lag controller.

Related Works

Fractional order PID controllers are used in many control applications. This section summarizes some of the contribution in this field.

Schlegel Milos et.al [5], performed a comparison between classical controller and fractional controller and summarized that the fractional order controller outperforms the classical controller. D Xue et.al [6], implemented fractional order PID control in DC motor. Chuang Zhao et.al [9], Ramiro S Barbosa et.al [12] and Ying Luo et.al [17], implemented fractional order controller in position servo mechanism. Varsha Bhambhani et.al [11] implemented fractional order controller in water level control. Hyo-Sung Ahn et.al [15] implemented fractional order integral derivative controller for temperature profile control. Concepcion A. Monje et.al [16], and Fabrizio Padula et.al [26], devised methods for tuning and auto tuning of fractional order controller for industry level control. Venu Kishore Kadiyala et.al [21] implemented fractional order controller for aerofin control.

Jun-Yi Cao et.al [2, 4] implemented genetic algorithm and particle swarm optimization methods for optimization of fractional order controller. Majid Zamani et.al [7] implemented particle swarm optimization for robust performance of fractional order controller. Deepyaman Maiti et.al [8], implemented particle swarm optimization for design of fractional order controller. Li Meng et.al [18] implemented multi objective genetic algorithm optimization for fractional order PID controller. Arijit Biswas et.al [20] implemented improved differential evolution techniques for design of fractional PID controller. Ammar A Aldair et.al [23] implemented evolutionary algorithm based fractional order controller for a full vehicle nonlinear activation suspension system.

IV. Speed Control of DC Motor Using Fractional Order PID Controller

In this section the authors investigate the control performance of fractional order PID controller by designing a fractional order PID controller to control the speed of an armature controlled DC motor. Figure 2 shows the schematic diagram of armature controlled DC motor.

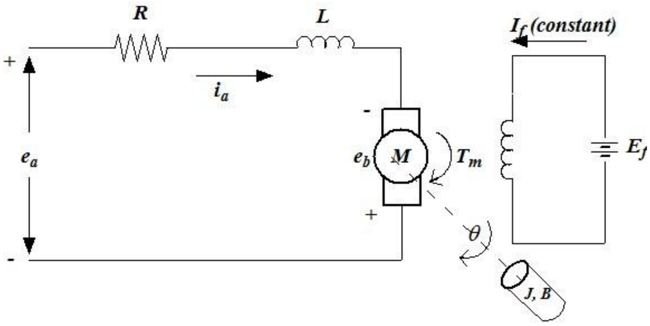


Figure 2: Schematic diagram of armature controlled DC motor

Notations

R = Armature Resistance (Ω).
L = Inductance of armature winding (H).
i_a = Armature current (A).
i_f = Field current (A).
e_a = applied armature voltage (V)
e_b = back emf (V)
T_m = torque developed by motor (Nm)
θ = angular displacement of motor shaft (rad).
ω = angular speed of motor shaft (rad/sec.)
J = equivalent moment of inertia of motor and load referred to motor shaft (kg-m^2)
B = equivalent friction coefficient of motor and load referred to motor shaft (Nm*s/rad)

Mathematical Modeling

In servo applications, the DC motors are generally used in the linear range of magnetization curve. Therefore, the air gap flux ϕ is proportional of field current, i.e. $\phi = K_f i_f$

Where K_f is constant.

The torque T_m developed by the motor is proportional to the product of armature current and air gap flux, i.e.

$$T_m = K_1 K_f i_f i_a \quad (10)$$

Here K_1 is constant.

In armature controlled dc motor, the field current is kept constant, so $T_m = K_T i_a$ (11)

Here K_T is known as the motor torque constant.

The motor back EMF being proportional to speed is given as

$$e_b = K_b \frac{d\theta}{dt} \quad (12)$$

Here K_b is the back EMF constant.

The differential equation of the armature circuit is

$$L\frac{di_a}{dt} + Ri_a + e_b - e_a = 0 \quad (13)$$

The torque equation is $J\dfrac{d^2\theta}{dt^2} + B\dfrac{d\theta}{dt} = T_m = K_T i_a$ (14)

Applying Laplace transform

$$e_b(s) = K_b s\theta(s) \quad (15)$$

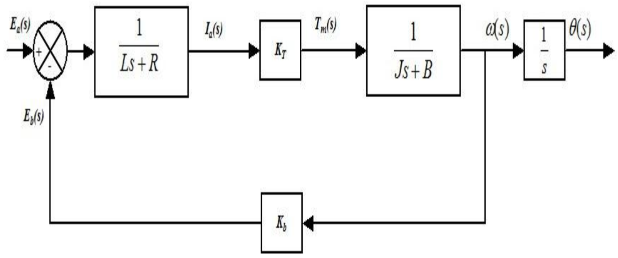
$$(Ls+R)I_a(s) = e_a(s) - e_b(s) \quad (16)$$

$$(Js^2+Bs)\theta(s) = T_m(s) = K_T i_a(s) \quad (17)$$

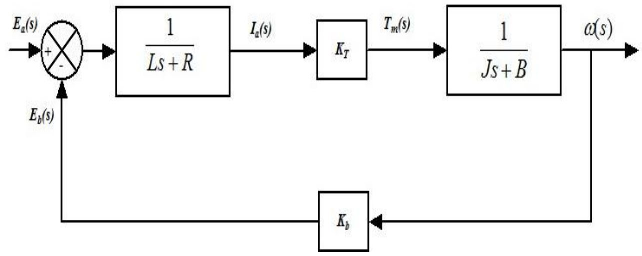
So the final transfer function will be

$$\frac{\theta(s)}{E_a(s)} = \frac{K_T}{s\left[(R+Ls)(Js+B)+K_T K_b\right]} \quad (18)$$

$$G(s) = \frac{\omega(s)}{E_a(s)} = \frac{K_T}{\left[(R+Ls)(Js+B)+K_T K_b\right]} \quad (19)$$



(a)



(b)

Figure 3(a) (b): Block diagram of armature controlled DC Motor

After applying the values of DC motor parameters as mentioned in appendix A, final transfer function can be represent as:

$$G(s) = \frac{0.0924}{8.49\times10^{-7}s^2 + 0.00585s + 0.01729} \quad (20)$$

Eq(20) represents the transfer function of DC motor. The primary objective is to control the speed of DC motor using fractional order PID controller. Section V shows the results of speed control using conventional and fractional PID controller.

V. Results And Discussions

This section shows the results of speed control of DC motor using conventional PID controller and fractional order PID controller. Ziegler-Nicholas tuning method is used to tune the conventional PID controller. Proportional Gain K_p, Derivative Gain K_d and Integral Gain K_i of conventional PID controller are 0.05, 0.0525 and 0.98 respectively. Unit step response of DC motor for different values of K_p, K_d, K_i are shown in figure 4.

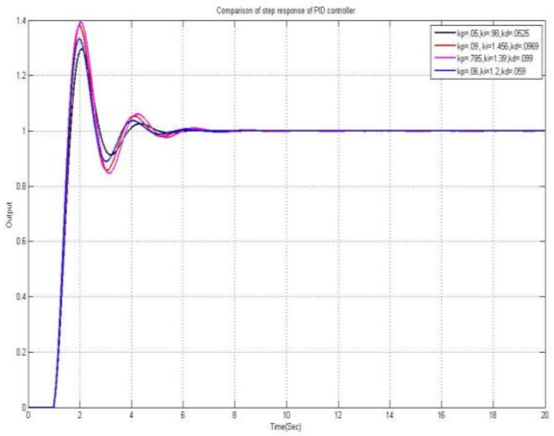


Figure 4: Unit step response for different values of K_p, K_d, K_i

To evaluate the performance of the unit step response different steady state and transient state parameters are taken in to consideration. The parameters are peak overshoot, peak time and settling time. For different values of proportional gain, derivative gain and integral gain the values of the parameter are shown in table 1.

Table 1: Performance parameters for different values of K_p, K_d, K_i

Kp	Ki	Kd	%Overshoot	Peak Time	Settling Time
0.05	0.98	0.0525	29.5939	1.9937	5.5025
0.09	1.456	0.0929	37.9562	2.0736	6.5158
0.785	1.39	0.099	39.3546	2.0241	6.7980
0.06	1.2	0.059	33.3067	1.9835	5.7706

The unit step response gives an overshoot of 29.5% which is undesirable. To minimize the overshoot, fractional order PID controller can be used in place of conventional PID controller. In fractional order PID controller the order of the integral and derivative are in fractions. This paper evaluates the performance of the controller with different combinations of μ and λ, and tries to find the best combination of μ and λ in a heuristic method. Different combinations of μ and λ are shown below.

i. λ=1 and μ<1
ii. λ<1 and μ=1
iii. λ<1 and μ<1
iv. λ=1 and μ>1
v. λ>1 and μ=1
vi. λ>1 and μ>1
vii. λ<1 and μ>1
viii. λ>1 and μ<1

(i) With λ=1 and varying values of μ<1

Figure 5 shows the unit step response of speed control of DC motor with λ=1 and varying values of μ<1

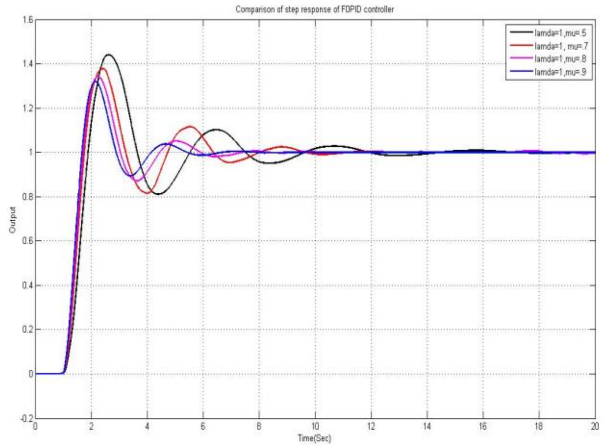


Figure 5: Unit Step Response of DC motor Control using FPID for Different Values of μ< 1

The transient and steady state parameters of unit step response with different combinations of λ and μ are shown in table 2.

Table 2: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
1	.5	44.0002	2.6109	20	.6693	2.072	13.2
1	.7	37.9076	2.4184	20	.4894	1.504	7.794
1	.8	34.2127	2.2361	19.2436	.3864	.8966	2.351
1	.9	31.9307	2.1435	6.4481	.3397	.7420	1.53

It can be seen from the above table 1 that with the increase in the value of μ, control parameters are improved.

(ii) With varying values of λ<1 and μ=1

Figure 6 shows the unit step response of speed control of DC motor with λ< 1

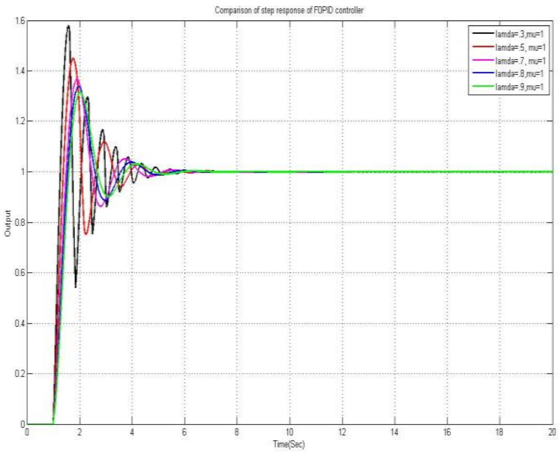


Figure 6: Unit Step Response of DC motor Control using FPID for different Values of λ <1

The transient and steady state parameters of unit step response with different combinations of λ and μ are shown in table 3.

Table 3: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
.3	1	57.8344	1.578	6.0182	.2274	.61	1.235
.5	1	44.9034	1.7552	5.7181	.2361	.5899	1.133
.7	1	36.6742	1.9012	5.7881	.2569	.6036	1.147
.8	1	33.7712	1.9640	5.3927	.2707	.6131	1.154
.9	1	31.4517	2.0222	5.6193	.2859	.6247	1.166

It can be seen from table 3 that with the increase in the value of λ, control parameters are almost remained constant.

(iii) With varying values of λ<1 and μ<1

Figure 7 shows the unit step response of speed control of DC motor with λ< 1 and μ<1

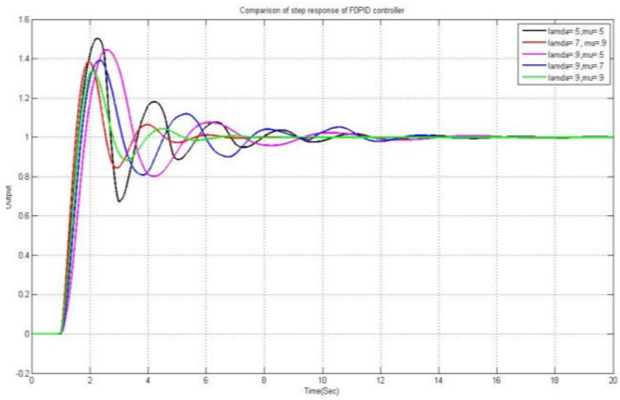


Figure 7: Unit Step Response of DC motor Control using FOPID for Different Values of λ<1and μ<1

The transient and steady state parameters of unit step response with different combinations of λ and μ are shown in table 4.

Table 4: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
.5	.5	50.459	2.2573	20	.4779	1.421	6.702
.5	.7	47.667	2.0182	19.972	.3849	1.445	8.026
.5	.9	45.904	1.8137	6.9300	.2656	.678	1.431
.7	.5	46.446	2.4501	20	.5575	1.639	8.389
.7	.9	38.335	1.7679	6.4501	.2106	.6006	1.421
.9	.5	44.169	2.5774	20	.6057	1.704	8.775
.9	.7	38.974	2.3607	20	.4783	1.52	8.147
.9	.9	38.595	2.0397	7.1159	.3223	.7257	1.501

It can be seen from the above table 4 that from all the different combinations of λ and μ, control parameters for the values of λ=0.7 and μ=0.9 are less than other values of λ and μ.

(iv) With λ=1 and varying values of μ>1

Figure 8 shows the unit step response of speed control of DC motor with λ< 1

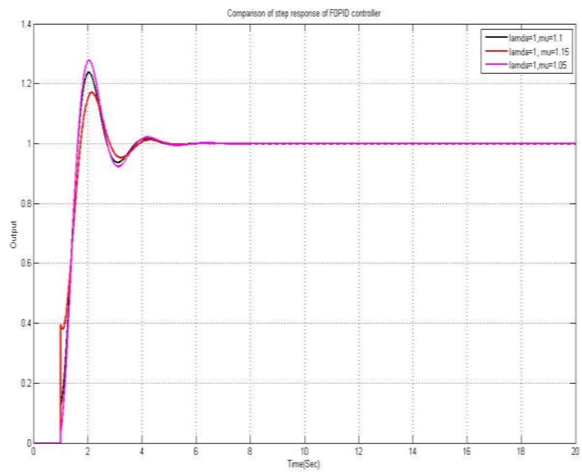


Figure 8: Unit Step Response of DC motor Control using FPID for Different Values of μ>1

The transient and steady state parameters of unit step response with different combinations of λ and μ are shown in table 5.

Table 5: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
1	1.05	27.8089	2.0473	5.5645	.2777	.5891	.9141
1	1.1	23.7612	2.0413	4.6560	.232	.5226	.8725
1	1.15	17.1389	2.1465	4.7784	.1523	.4428	1.052

It can be seen that from the above table 5 that with the increase in the value of μ, control parameters are reduce upto the value of λ =1 and μ=1.15.after these response will be more slow and more oscillatory.

(iv) With varying values of λ>1 and μ=1

Figure 9 shows the unit step response of speed control of DC motor with λ> 1 and μ=1

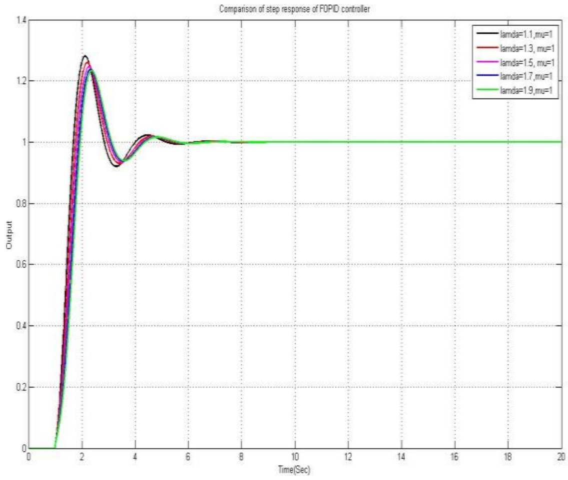


Figure 9: Unit Step Response of DC motor using FOPID for Different Values of λ >1

Table 6: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
1.1	1	28.1038	2.213	5.8632	.3186	.6525	1.199
1.2	1	26.9088	2.1637	5.906	.3354	.668	1.22
1.3	1	25.9522	2.2026	5.8922	.3521	.684	1.243
1.4	1	25.1895	2.2377	5.2537	.3686	.7003	1.267
1.5	1	24.5857	2.2695	5.2965	.3847	.7167	1.293
1.6	1	24.1127	2.2978	5.3295	.4005	.733	1.319
1.7	1	23.7982	2.3228	5.3536	.4158	.7491	1.345
1.8	1	23.476	2.3457	5.3705	.4307	.965	1.373
1.9	1	23.2754	2.3669	5.3817	.445	.7805	1.4

It can be seen that from the above table 6 with the increase in the value of λ, peak overshoot reduces but system will be slow.

(v) With varying values of λ>1 and μ>1

Figure 10 shows the unit step response of speed control of DC motor with λ> 1 and μ>1

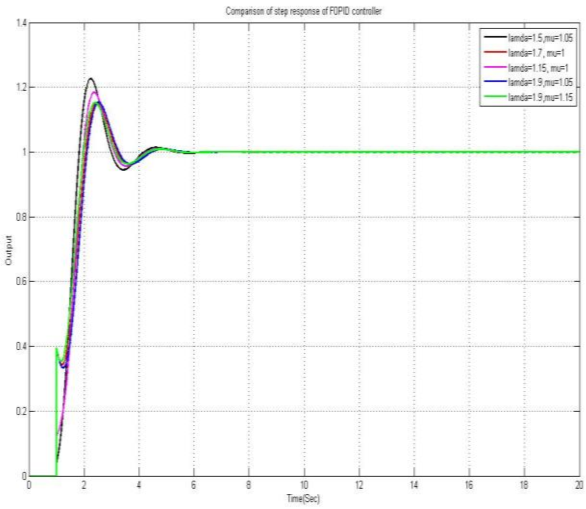


Figure 10: Unit Step Response of DC motor using FPID for Different Values of λ >1 and μ>1

Table 7: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
1.5	1.05	22.6561	2.2488	5.1095	.3588	.669	1.166
1.5	1.1	19.3614	2.2628	5.0276	.3143	.6149	1.054
1.5	1.15	15.2327	2.4100	5.1690	.2349	.5083	1.091
1.7	1.05	21.7979	2.3060	5.1665	.3895	.7018	1.222
1.7	1.1	18.7298	2.3235	5.0846	.3453	.652	1.116
1.7	1.15	15.2073	2.1774	5.0176	.266	.6141	1.033
1.9	1.05	21.3166	2.3520	5.1959	.4183	.7342	1.275
1.9	1.1	18.4303	2.3738	5.1230	.3744	.6873	1.178
1.9	1.15	15.3788	2.5293	5.3213	.2952	.657	1.262

It can be seen from the above table 7 that all the different combinations of λ and μ, control parameters for the values of λ=1.7 and μ=1.15 are less than other values of λ and μ.

(vii) With varying values of λ<1 and μ>1

Figure 11 shows the unit step response of speed control of DC motor with λ<1 and μ>1

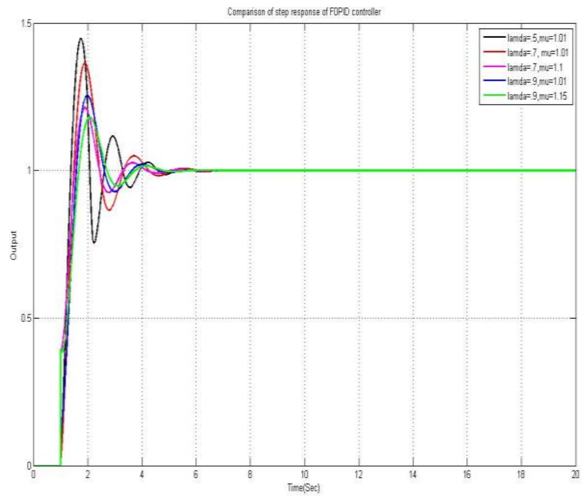


Figure 11: Unit Step Response of DC motor using FOPID for different Values of λ<1 and μ>1

Table 8: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
.5	1.01	43.5083	1.7294	5.4501	.2147	.5439	1.012
.5	1.1	38.8578	1.6955	4.8685	.1687	.4679	.8576
.5	1.15	27.1098	1.6918	4.9308	.08584	.3453	.7442
.7	1.01	36.4376	1.8956	5.7249	.2532	.5946	1.12
.7	1.1	30.6041	1.8496	4.7835	.188	.4819	.866
.7	1.15	21.3701	1.8986	4.8358	.1074	.3751	.7721
.9	1.01	31.1797	2.0156	5.5637	.282	.6155	1.139
.9	1.1	25.5206	1.9830	5.1964	.2162	.5062	.8874
.9	1.15	18.1406	2.0724	4.6213	.1363	.4186	.836

It can be seen from the above table 8 that all the different combinations of λ and μ,peak overshoot will be less for the combination for λ=.9 and μ=1.15.

(viii) With varying values of λ>1 and μ<1

Figure 12 shows the unit step response of speed control of DC motor with λ> 1 and μ<1

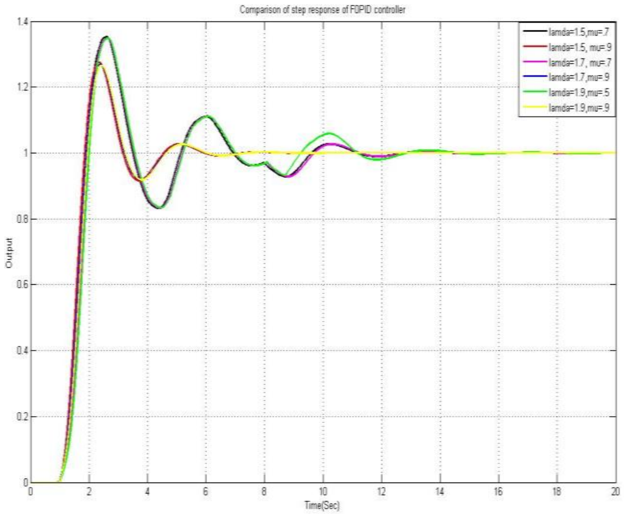


Figure 12: Unit Step Response of DC motor using FOPID for different Values of λ >1 and μ<1

Table 9: Comparison of Parameters for Different Combinations of λ and μ

λ	μ	Mp	Tp	Ts	ISE	IAE	ITAE
1.5	.7	35.220	2.5910	20	.6223	1.937	12.87
1.5	.9	26.488	2.3340	6.920	.4267	.824	1.652
1.7	.7	34.9529	2.6269	20	.6187	1.522	6.55
1.7	.9	26.8613	2.3838	6.9712	.459	.8567	1.705
1.9	.5	42.0534	2.7607	20	.8181	1.989	11.18
1.9	.7	34.8113	2.6542	20	.6517	1.666	8.283
1.9	.9	26.5052	2.4195	6.9815	.4893	.8878	1.761

It can be seen from the above table 9 that from all the different combinations of λ and μ, control parameters for the values of λ=1.5 and μ=0.9 are less than other values of λ and μ.

From the above graphs and tables for different combinations of integral order and derivative order, for λ=1.7and μ=1.15, all the parameters are minimum. Hence combination of λ=1.7 and μ=1.15 is taken for fractional PID controller.

VI. Conclusion

This paper studies the use of fractional calculus in control system and controller design. The authors showed the use of fractional order PID controller to control the speed of armature controlled DC motor and showed the variations in unit step response if the derivative and integer order of the fractional PID controller is varied. To obtain the optimal values of integer order and derivative order, different optimization techniques can be implemented.

APPENDIX A

The parameters of separately excited DC motor

Rated Power (P)	5 Hp
Rated Armature Voltage	240 V
Armature Resistance R_a	2.518 Ω
Armature Inductance L_a	0.028 H
Field Resistance R_f	281.3 Ω
Field Inductance L_f	156 H
Back EMF constant K_b	0.0924
Motor constant K_t	0.0924
Friction coefficient of motor B	0.0005 Nm*s/rad
Moment of inertia of motor J	0.003 kg-m^2
Rated Speed	1750 RPM
Rated Field Voltage	300 V

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