

QUANTUM CHROMODYNAMICS AND QUARK GLUON PLASMA SEA-A ABSTRACTION AND ATTRITION MODEL

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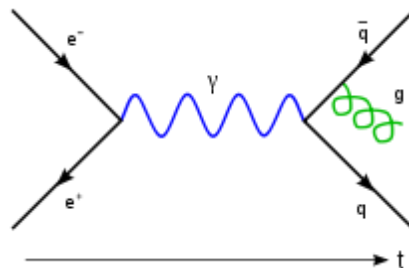
ABSTRACT: A system of quark, gluon, Plasma Sea (QGP) dissipating axiomatic predications and postulates governing QCD (COLOR FIELDS MEDIATED BY EXCHANGED PARTICLES WHICH COULD BE STRATIFIABLE BY AGE) and parallel system of QGP-QCD systems that contribute to the dissipation of the velocity of production of QCD, NAMELY COLOR FIELDS AND CONCOMITANT EXCHANGE PARTICLES-, THEREBY BREAKING THE GOVERNING LAWS OF QCD ITSELF- is investigated. It is shown that the time independence of the contributions portrays another system by itself and constitutes the equilibrium solution of the original time independent system. A system of dominant asymptotic freedom (DAF) dissipating increased energy is annexed to the systems Annexure gives the governing equations of the QGP-QCD-DAF-IES system by the consummation and consolidation of the governing equations which could be solved by the same procedural regularities as has been expatiated herein below.

INDEX TERMS: Quantum Chromodynamics, Quark, Gluon, Plasma Sea

INTRODUCTION:

QUANTUM CHROMODYNAMICS

Standard model of particle physics



In theoretical physics, quantum chromodynamics (QCD) is a theory of **the strong interaction**(color force), a fundamental force describing the **interactions between quarks and gluons** which make up hadrons (such as the proton, neutron or pion). It is the study of the SU (Yang–Mills theory of color-charged fermions (the quarks). QCD is a quantum field theory of a special kind called a non-abelian gauge theory, consisting of a 'color field' mediated by a set of exchange particles (the gluons). The theory is an important part of the Standard Model of particle physics. A huge body of experimental evidence for QCD has been gathered over the years.

QCD enjoys two peculiar properties:

The force between quarks does not diminish as they are separated. Because of this, it would take an **infinite amount of energy to separate two quarks**; they are forever bound into hadrons such as the proton and the neutron. Although analytically unproven, confinement is widely believed to be true because it explains the consistent failure of free quark searches, and it is easy to demonstrate in lattice QCD.

Asymptotic freedom, which means that in very high-energy reactions, quarks and gluons interact very weakly. No known phase-transition line separating these two properties; confinement is dominant in low-energy scales but, as energy increases, asymptotic freedom becomes dominant.

The three kinds of charge in QCD (as opposed to one in quantum electrodynamics or QED) are usually referred to as "color charge" by loose analogy to the three kinds of color (red, green and blue) perceived by humans. With the invention of bubble chambers and spark chambers in the 1950s, experimental particle physics discovered a large and ever-growing number of particles called hadrons. It seemed that such a large number of particles could not all be fundamental. To gain greater insight, the hadrons were sorted into groups having similar properties and masses using the eightfold way, Gell-Mann and George Zweig, correcting an earlier approach of Shoichi Sakata, went on to propose in 1963 that the structure of the groups could be explained by the existence of three flavors of smaller particles inside the hadrons: the quarks.

Perhaps the first remark that quarks should possess an additional quantum number was made by Boris Struminsky in connection with Ω -hyperon composed of three strange quarks with parallel spins (this situation was peculiar, because since quarks are fermions, such **combination is forbidden by the Pauli exclusion principle**):

Three identical quarks cannot form an anti symmetric S-state. In order to realize an anti symmetric orbital S-state, it is necessary for the quark to have an additional quantum number.

A similar mysterious situation was with the Δ^{++} baryon; in the quark model, it is composed of three up quarks with parallel spins. In 1965, Moo-Young Han with Yoichiro Nambu and Oscar W. Greenberg independently resolved the problem by proposing that quarks possess an additional SU(3) gauge degree of freedom, later called color charge. Han and Nambu noted that quarks might interact via an octet of vector gauge bosons: **the gluons**.

Since free quark searches consistently failed to turn up any evidence for the new particles, and because an elementary particle back then was defined as a particle which could be separated and isolated, Gell-Mann often said that quarks were merely convenient mathematical constructs, not real particles. The meaning of this statement was usually clear in context: Implying that the strong interactions could probably not be fully described by quantum field theory.

Richard Feynman argued that high energy experiments showed quarks are real particles: he called them partons (since they were parts of hadrons). By particles, Feynman meant objects which travel along paths, elementary particles in a field theory.

The difference between Feynman's and Gell-Mann's approaches reflected a deep split in the theoretical physics community. Feynman thought the quarks have a distribution of position or momentum, like any other particle, and he (correctly) believed that the diffusion of **parton** momentum explained diffractive scattering. Although Gell-Mann believed that certain quark charges could be localized, he was open to the possibility that the **quarks** themselves could not be localized because space and time break down. This was the more radical approach of S-matrix theory. **Non localization of quarks leads to space time break down**

James Bjorken proposed that point like partons would imply certain relations should hold in deep in elastic scattering of electrons and protons, which were spectacularly verified in experiments at SLAC in 1969. This led physicists to abandon the S-matrix approach for the strong interactions.

The discovery of asymptotic freedom in the strong interactions by David Gross, David Politzer and Frank Wilczek allowed physicists to make precise predictions of the results of many high energy experiments using the quantum field theory technique of perturbation theory. Evidence of gluons was discovered in three jet events at PETRA in 1979. These experiments became more and more precise, culminating in the verification of perturbative QCD at the level of a few percent at the LEP in CERN.

The other side of asymptotic freedom is confinement. Since the **force between color charges does not decrease with distance, (decrease in distance does not produce decrease in force between color charge)** it is believed that quarks and gluons can never be liberated from hadrons. This aspect of the theory is verified within lattice QCD computations, but is not mathematically proven.. Two possibilities could be there: either quarks or gluons are gormandized by hadrons or the vice versa may be true. We give a model in one of the forthcoming papers. Other aspects of non-perturbative QCD are the exploration of phases of quark matter, including the quark-gluon plasma.

The relation between the **short-distance particle limit** and the **confining long-distance limit** is one of the topics recently explored using string theory, the modern form of S-matrix theory. It may cause either positive or negative changes in both the factors.

Every field theory of particle physics is based on **certain symmetries of nature whose existence is deduced from observations**. These can be local symmetries, that is the symmetry acts independently at each point in space-time. Each such symmetry is the basis of a gauge theory and requires the introduction of its own gauge bosons, which are symmetries whose operations must be simultaneously applied to all points of space-time.

QCD is a gauge theory of the SU(3) gauge group is obtained by taking the color charge to define a local symmetry. Since the strong interaction does not discriminate between different flavors of quark, QCD has approximate **flavor symmetry**, which is **broken** by the **differing masses of the quarks**.

There are additional global symmetries whose definitions require the notion of chirality, discrimination between left and right-handed. If the spin of a particle has a positive projection on its direction of motion then it is called left-handed; otherwise, it is right-handed. **Chirality** and **handedness** are not the same, but become approximately

equivalent at high energies. Chiral symmetries involve independent transformations of these two types of particle. Vector symmetries (also called diagonal symmetries) mean the same transformation is applied on the two chiralities. Axial symmetries are those in which one transformation is applied on left-handed particles and the inverse on the right-handed.

DUALITY

As mentioned, **asymptotic freedom means that at large energy - this corresponds also to short distances - there is practically no interaction between the particles.** This is in contrast - more precisely one would say dual - to what one is used to, since usually one **connects the absence of interactions with large distances.** However, as already mentioned in the original paper of Franz Wegner a solid state theorist who introduced 1971 simple gauge invariant lattice models, the high-temperature behavior of the original model, e.g. **the strong decay** of correlations at large distances, corresponds to the **low-temperature behavior of the (usually ordered) dual model**, namely the asymptotic **decay** of non-trivial correlations, e.g. short-range deviations from almost perfect arrangements, for short distances. Here, in contrast to Wegner, we have only the dual model, which is that one described in this article.

SYMMETRY GROUPS

The color group SU (3) corresponds to the local symmetry whose gauging **gives rise to QCD.** The electric charge labels a representation of the local symmetry group U (1) which is gauged to give QED: this is an abelian group. If one considers a version of QCD with Nf flavors of massless quarks, then there is a global (chiral) flavor symmetry group $SU(N_f) \times SU(N_f) \times U(1) \times U(1)$. The chiral symmetry is spontaneously broken by the QCD vacuum to the vector (L+R) $SU(N_f)$ with the formation of a chiral condensate. The vector symmetry, U(1) **corresponds** to the baryon number of quarks and is an exact symmetry. **The axial symmetry UA(1)** is exact in the classical theory, but **broken** in the **quantum theory**, an occurrence called an anomaly. Gluon field configurations called instantons are closely related to this anomaly.

There are two different types of SU(3) symmetry: there is the symmetry that acts on the different colors of quarks, and this is an exact gauge symmetry **mediated** by the gluons, and there is also a flavor symmetry which rotates different flavors of quarks to each other, or flavor SU(3). Flavor SU (3) is an approximate symmetry of the vacuum of QCD, and is not a fundamental symmetry at all. It is an accidental consequence of the small mass of the three lightest quarks.

In the QCD vacuum there are vacuum condensates of all the quarks whose mass is less than the QCD scale. This includes the up and down quarks, and to a lesser extent the strange quark, but not any of the others. The vacuum is symmetric under SU(2) isospin rotations of up and down, and to a lesser extent under rotations of up, down and strange, or full flavor group SU(3), and the observed particles make isospin and SU(3) multiplets.

The approximate flavor symmetries do have associated gauge bosons, observed particles like the rho and the omega, but these particles are nothing like the gluons and they are not massless. They are emergent gauge bosons in an approximate string description of QCD.

LAGRANGIAN

The dynamics of the quarks and gluons are **controlled** by the quantum chromo dynamics Lagrangian. **The gauge invariant QCD Lagrangian is**

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} , \end{aligned}$$

where $\psi_i(x)$ is the quark field, a dynamical function of space-time, in the fundamental representation of the SU(3) gauge group, indexed by i, j, \dots ; $G_\mu^a(x)$ are the gluon fields, also a dynamical function of space-time, in the adjoint representation of the SU(3) gauge group, indexed by a, b, The γ_μ are Dirac matrices connecting the spinor representation to the vector representation of the Lorentz group; and T_{ij}^a are the generators connecting the fundamental, anti fundamental and ad joint representations of the SU(3) gauge group. The Gell-Mann matrices provide one such representation for the generators.

The symbol $G_{\mu\nu}^a$ represents the gauge invariant gluonic field strength tensor, analogous to the electromagnetic field strength tensor, $F_{\mu\nu}$, where f_{abc} are the structure constants of SU(3). Note that the rules to move-up or pull-down the a, b, or c indexes are trivial, (+.....+), so that $f_{abc} = f_{bca} = f_{cab}$ whereas for the μ or ν indexes one has the non-trivial relativistic rules, corresponding e.g. to the signature (+---). Furthermore, for mathematicians, according to this formula the gluon color field can be represented by a SU(3)-Lie algebra-valued "curvature"-2-form $\mathbf{G} = d\tilde{\mathbf{G}} - g \tilde{\mathbf{G}} \wedge \tilde{\mathbf{G}}$, where $\tilde{\mathbf{G}}$ is a "vector potential"-1-form corresponding to G and \wedge is the (anti symmetric) "wedge product" of this algebra, producing the "structure constants" f_{abc} . The Cartan-derivative of the field form (i.e. essentially the divergence of the field) would be zero in the absence of the "gluon terms", i.e. those $\sim g$, which represent the non-abelian character of the SU(3).

The constants m and g **control** the quark mass and coupling constants of the theory, subject to renormalization in the full quantum theory.

An important theoretical notion concerning the final term of the above Lagrangian is the Wilson loop variable. This loop variable plays a most-important role in discretized forms of the QCD (see lattice QCD), and more generally, it distinguishes confined and deconfined states of a gauge theory.

FIELDS

Quarks are massive spin-1/2 fermions which **carry** color charge whose gauging is the **content** of QCD. Quarks are represented by Dirac fields in the fundamental representation 3 of the gauge group SU(3). They also carry electric charge (either -1/3 or 2/3) and participate in weak interactions as part of weak isospin doublets. They carry global quantum numbers including the baryon number, which is 1/3 for each quark, hypercharge and one of the flavor quantum numbers.

Gluons are spin-1 bosons which also carry color charges, since they lie in the adjoint representation 8 of SU(3). They have no electric charge, do not participate in the weak interactions, and have no flavor. They lie in the singlet representation 1 of all these symmetry groups.

Every quark has its own antiquark. The charge of each antiquark is exactly the opposite of the corresponding quark. Relationship deduction between quark and antiquark is of rich repository and receptacle and is of indepth interest.

DYNAMICS

According to the rules of quantum field theory, and the associated Feynman diagrams, the above theory gives rise to three basic interactions: a quark may **emit (or absorb)** a gluon, a gluon may **emit (or absorb) a** gluon, and two gluons may directly **interact**. This contrasts with QED, in which only the first kind of interaction occurs, since photons have no charge. Diagrams involving Faddeev–Popov ghosts must be considered too (except in the unitarity gauge).

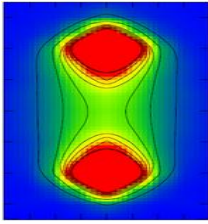
Area law and confinement

Detailed computations with the above-mentioned Lagrangian show that the effective potential between a quark and its anti-quark in a meson contains a term $\propto T$, which represents some kind of "stiffness" of **the interaction** between the **particle and its anti-particle at large distances**, similar to the entropic elasticity of a rubber band. This leads to confinement of the quarks to the interior of hadrons, i.e. mesons and nucleons, with typical radii R_c , corresponding to former "Bag models" of the hadrons. The order of magnitude of the "bag radius" is 1 fm (=10⁻¹⁵ m). Moreover, the above-mentioned **stiffness is quantitatively related to the so-called "area law"** behaviour of the expectation value of the Wilson loop product PW of the ordered coupling constants around a closed loop W; i.e. $\langle P_W \rangle$ is **proportional** to the area enclosed by the loop. For this behaviour the non-abelian behaviour of the gauge group is essential.

Perturbative QCD

This approach is based on asymptotic freedom, which allows perturbation theory to be used accurately in experiments performed at very high energies. Although limited in scope, this approach has resulted in the most precise tests of QCD to date.

Lattice QCD



A quark and an antiquark (red color) **are glued together** (green color) to form a meson (result of a lattice QCD simulation by M. Cardoso et al[(PLEASE SEE REFERENCE)

Among non-perturbative approaches to QCD, the most well established one is **lattice QCD**. This approach uses a discrete set of space-time points (called the lattice) to reduce the analytically intractable path integrals of the continuum theory to a very difficult numerical computation which is then carried out on super computers like the QCDOC which was constructed for precisely this purpose. While it is a slow and resource-intensive approach, it has wide applicability, giving insight into parts of the theory inaccessible by other means, in particular into the explicit forces acting between quarks and anti quarks in a meson. However, the numerical sign problem makes it difficult to use lattice methods to study QCD at high density and low temperature (e.g. nuclear matter or the interior of neutron stars).

1/N EXPANSION

One known approximation scheme, the 1/N expansion, starts from the premise that the number of colors is infinite, and makes a series of corrections to account for the fact that it is not. Until now it has been the source of qualitative insight rather than a method for quantitative predictions. Modern variants include the AdS /CFT approach.

EFFECTIVE THEORIES

. In the best of cases, these may then be obtained as systematic expansions in some parameter of the QCD Lagrangian. One such effective field theory is chiral perturbation theory or ChiPT, which is the QCD effective theory at low energies. More precisely, it is a low energy expansion based on the spontaneous **chiral symmetry breaking** of QCD, which is an exact symmetry when quark masses are equal to zero, but for the u,d and s quark, which have small mass, it is still a good approximate symmetry. Depending on the number of quarks which are treated as light, one uses either SU(2) ChiPT or SU(3) ChiPT . Other effective theories are heavy quark effective theory (which expands around heavy quark mass near infinity), and soft-collinear effective theory (which expands around large ratios of energy scales). In addition to effective theories, models like the Nambu-Jona-Lasinio model and the chiral model are often used when discussing general features.

NAMBU-JONA-LASINIO MODEL

In one of his recent works, Kei-Ichi Kondo derived as a low-energy limit of QCD, a theory linked to the Nambu-Jona-Lasinio model since it is basically a particular non-local version of the Polyakov-Nambu-Jona-Lasinio model. The later being in its local version, nothing but the Nambu-Jona-Lasinio model in which one has included the Polyakov loop effect, in order to describe a 'certain confinement'.

The Nambu-Jona-Lasinio model in itself is, among many other things, used because it is a 'relatively simple' model of **chiral symmetry breaking**, phenomenon present up to certain conditions (Chiral limit i.e. massless fermions) in QCD itself.

EXPERIMENTAL TESTS

The notion of **quark flavors** was prompted by the necessity of explaining the properties of **hadrons** during the development of the quark model. The notion of color was necessitated by the puzzle of the Δ^{++} . Good quantitative tests of perturbative QCD are the running of the QCD coupling as deduced from many observations scaling violation in polarized and un polarized deep inelastic scattering vector boson production at colliders (this includes the Drell-Yan process) jet cross sections in colliders event shape observables at the LEP heavy-quark production in colliders .Quantitative tests of non-perturbative QCD are fewer, because the predictions are harder to make. The best is probably the running of the QCD coupling as probed through lattice computations of heavy-quarkonium spectra. There is a recent claim about the mass of the heavy

meson Bc [4]. Other non-perturbative tests are currently at the level of 5% at best. Continuing work on masses and form factors of hadrons and their weak matrix elements are promising candidates for future quantitative tests. The whole subject of quark matter and the quark-gluon plasma is a non-perturbative test bed for QCD which still remains to be properly exploited.

CROSS-RELATIONS TO SOLID STATE PHYSICS

There are unexpected cross-relations to solid state physics. For example, the notion of gauge invariance forms the basis of the well-known Mattis spin glasses which are systems with the usual spin degrees of freedom $s_i = \pm 1$ for $i=1, \dots, N$, with the special fixed "random" couplings $J_{i,k} = \epsilon_i J_0 \epsilon_k$. Here the ϵ_i and ϵ_k quantities can independently and "randomly" take the values ± 1 , which corresponds to a most-simple gauge transformation $(s_i \rightarrow s_i \cdot \epsilon_i \quad J_{i,k} \rightarrow \epsilon_i J_{i,k} \epsilon_k \quad s_k \rightarrow s_k \cdot \epsilon_k)$. This means that thermodynamic expectation values of measurable quantities, e.g. of the energy $\mathcal{H} := - \sum s_i J_{i,k} s_k$, are invariant. A prediction model for this equation is of utmost expediency.

However, here the coupling degrees of freedom $J_{i,k}$, which in the QCD correspond to the gluons, are "frozen" to fixed values (quenching). In contrast, in the QCD they "fluctuate" (annealing), and through the large number of gauge degrees of freedom the entropy plays an important role.

For positive J_0 the thermodynamics of the Mattis spin glass corresponds in fact simply to a ferro magnet, just because these systems have no "frustration" at all. This term is a basic measure in spin glass theory. Quantitatively it is identical with the loop-product $P_W := J_{i,k} J_{k,l} \dots J_{n,m} J_{m,i}$ along a closed loop W. However, for a Mattis spin glass - in contrast to "genuine" spin glasses - the quantity PW never becomes negative.

The basic notion "frustration" of the spin-glass is actually similar to the Wilson loop quantity of the QCD. The only difference is again that in the QCD one is dealing with SU(3) matrices, and that one is dealing with a "fluctuating" quantity. Energetically, perfect absence of frustration should be non-favorable and atypical for a spin glass, which means that one should add the loop-product to the Hamiltonian, by some kind of term representing a "punishment". - In the QCD the Wilson loop is essential for the Lagrangian right away.

The relation between the QCD and "disordered magnetic systems" (the spin glasses belong to them) were additionally stressed in a paper by Fradkin, Huberman und Shenker which also stresses the notion of duality.

A further analogy consists in the already mentioned similarity to polymer physics, where, analogously to Wilson Loops, so-called "entangled nets" appear, which are important for the formation of the entropy-elasticity (force proportional to the length) of a rubber band. The non-abelian character of the SU(3) corresponds thereby to the non-trivial "chemical links", which glue different loop segments together, and "asymptotic freedom" means in the polymer analogy simply the fact that in the short-wave limit, i.e. for $0 \leftarrow \lambda_w \ll R_c$ (where R_c is a characteristic correlation-length for the glued loops, corresponding to the above-mentioned "bag radius", while λ_w is the wavelength of an excitation) any non-trivial correlation vanishes totally, as if the system had crystallized.

There is also a correspondence between confinement in QCD - the fact that the color-field is only different from zero in the interior of hadrons - and the behavior of the usual magnetic field in the theory of type-II superconductors: there the magnetism is confined to the interior of the Abrikosov flux-line lattice, i.e., the London penetration depth λ of that theory is analogous to the confinement radius R_c of quantum chromo dynamics. Mathematically, this correspondence is supported by the second term, $\propto g G_{\mu}^a \bar{\psi}_i \gamma^{\mu} T_{ij}^a \psi_j$, on the r.h.s. of the Lagrangian.

In his celebrated paper Adolf Haimovici (1), studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of a system of QGP_QCD_DAF-IES system.. Quintessentially, Haimovician diurnal dynamics, are used to draw interesting inferences, from the simple fact that QGP breaks the postulates and axioms of QCD. In a series of papers we show the entire gamut of QCD could be studied and a generalized model could be built which could be solved by the methodologies adopted herein.

Fritjof Capra(2) in his scintillating and brilliant synthesis of such scientific breakthroughs as the "Theory of Dissipative structures", 'Theory of Complexity', 'GAIA theory', 'Chaos Theory' in his much acclaimed 'The

Web of life' elucidates dissipative structures as the new paradigm in ecology. We write to state this for the acknowledgement of Capra which provided inspiration for the present work.

Axiomatic predications of systemic dynamics in question are essentially "laws of accentuation and dissipation'. It includes once over change, continuing change, process of change, functional relationships, predictability, cyclical growth, cyclical fluctuations, speculation theory, cobweb analyses, stagnation thesis, perspective analysis etc. Upshot of the above statement is data produce consequences and consequences produce data.

QUANTUM CHROMODYNAMICS (COLOUR FIELD CONSISTING OF EXCHANGE PARTICLES NAMESLY GLUONS)

Note: QCD is a quantum field theory of a special kind called a non-abelian gauge theory, consisting of a 'color field' mediated by a set of exchange particles (the gluons). The theory is an important part of the Standard Model of particle physics. A huge body of experimental evidence for QCD has been gathered over the years.

They're imprisoned in flickering clouds of other particles—other quarks, which materialize briefly and then vanish and, above all, gluons, which transmit the force that binds the quarks together. Gluons are mass less and evanescent, but they carry most of the proton's energy. Protons are little blobs of glue—but even that picture conveys something too static and substantial. All is flux and crackling energy inside a proton; it is like an unending lightning storm in a bottle, a bottle less than .1 trillionth of an inch in diameter. "It's a very rich, dynamic structure," says Wilczek. "Experiments show just how insubstantial the quarks are inside. Meanwhile, other accelerator experiments may soon reveal how the universe assembled all its protons in the first place. In February, physicists at CERN, the European Laboratory for Particle Physics outside Geneva, announced "compelling evidence" that they had succeeded in melting large numbers of protons, creating for an instant the kind of quark-gluon plasma that last existed a microsecond after the Big Bang. All the protons around now congealed, connected from that soup. At Brookhaven National Laboratory on Long Island in New York, a new and more powerful accelerator is getting set to cook quark-gluon soup on a daily basis this summer. By next year physicists may have a much better idea of what the universe was like when it was a billion times hotter than the surface of the sun, and quarks and gluons—not yet trapped inside protons, inside nuclei, inside atoms, inside us could play freely in the quantum fields.

In QED each individual electron is surrounded by a cloud of short-lived virtual particles—photons, but also other electrons paired with positrons, their positively charged antimatter twins. Those swarming particle pairs form a screen that partially cancels the electron field—at least as seen from outside the screen. From inside the screen, on the other hand, the field seems stronger than you might expect, like a bare light bulb once a lamp shade has been removed. "OK, so that's screening, and that's pretty easy to understand," says Wilczek. "What happens in QCD, inside the proton, is just the opposite: It's antiscreening

The electrons didn't seem to be plunging into mush; they seemed to bounce off point like nuggets within the protons. Quarks, which had been postulated in 1964 as purely mathematical entities, began to seem as if they might really exist inside protons—and the question of how the whole thing was held together became urgent and exciting. While Wilczek and his adviser, David Gross, were discovering the answer at Princeton, another graduate student named David Politzer was discovering it independently at Harvard. One shell was stranger than the rest. The Stanford experiments had seemed to show that the force between quarks—known as the strong force—actually got weaker as the quarks got closer together. Decrease in the distance between quarks reduced the strength of the force between the quarks. That was very strange. The forces we encounter on a daily basis, electromagnetism and gravity, act in just the opposite way: They are stronger at short distances from their source and weaker at long ones. That is the intuitive way; that is how things should be. Otherwise, for instance, magnets would fly off your refrigerator to cling to the distant alluring stove—except that from a distance the fridge would start to look good to them again. And yet, weak as the strong force was at very short distances, it was strong enough at longer ones to frustrate physicists who had tried hard and failed to pull a quark out of a proton. No one had managed to observe one in isolation.

Wilczek and Gross went looking for a quantum field theory that could explain such a force. Actually Gross expected them to fail; he wanted to prove that field theory was a dead end. But one kind of mathematical formalism, called non-Abelian gauge theory, had not yet been tried. Wilczek spent half a year filling a notebook with calculations of how particles would interact through a non-Abelian quantum field. At the end he had a force that got stronger at close quarters, as a good force should, as Gross expected—but unlike what had been measured at Stanford. Wilczek checked the lengthy calculations over and over again. Then Gross spotted a single plus sign in the field equation that should have been a minus. That sign change changed everything: QCD was born. simple phenomenological lattice model for the high-temperature phase transition in a quark-gluon plasma, in which the

elementary dynamical variables are Wilson lines and $SU(N)$ chiral spins. A mean-field analysis of the model shows a second-order chiral phase transition for $N=2$ and a first-order phase transition for $N=3$. We explore the phase diagrams obtained in the mean-field approximation by varying the several parameters of the model, including a mass term that breaks $SU(3)\times SU(3)$, leaving $SU(2)\times SU(2)$. Our study admits the possibility that the high-temperature phase transition in QCD is a chiral-symmetry-restoring phase transition and not a "deconfinement" phase transition.

Inside a proton, according to QCD, the quarks are "asymptotically free," as the Stanford results suggested, meaning they move almost as if there were no force between them at all. But the freedom is an illusion: A quark can never escape its partners. As the distance between them increases, so does the force, and so a wayward quark is inevitably reined in, like a bungee jumper. That's why no one ever has or ever will see a lone quark, which, when you think about it, is deeply weird. "The proton has parts, but it can't be taken apart," says Alvaro de Rújula, a theoretical physicist at CERN. "You can hold an electron in your hand. You can't hold a quark or a gluon in your hand." The weirdness comes from the gluons. Quantum chromodynamics, the force that holds protons together, is modeled closely on quantum electrodynamics, the force that holds atoms together—but the gluons change screening to antiscreening, intuitive to bizarre. In quantum chromodynamics, quarks carry a new kind of charge, called color—which has nothing to do with ordinary color—and those charges generate a color field (hence the name chromodynamics). The quantum of the field, and the transmitter of the strong force, is the gluon. Like the photon that transmits the electromagnetic force, a gluon is massless. But unlike the photon, a gluon is charged. Gluons generate its own color field, exerts its own strong force, and interacts with other gluons. It leads a rich life. These are the aspectional qualities that call for contingent modeling.

The color field, like the electromagnetic one, can be thought of as having two components—call them color electric and color magnetic. A fast-moving color charge—gluons move at light speed—generates a strong color-magnetic field. Gluons are thus like little dipole magnets. The gluons that surround a quark align themselves parallel to its color field, as magnets would, and so instead of weakening it, as virtual particles do an electron's field, they strengthen it. So, when a quark align itself its color field, the alignment produces strength to the colour field. They antiscreen the quark, amplifying its field. Here the lamp analogy no longer works—the quark is a dim bulb that somehow becomes brighter outside the shade. That's what holds a proton together, and that's what gives it a bizarre internal structure. If one quark manages to get inside another's gluon cloud, it feels only a feeble attraction. But the farther away it goes, the more it feels the added pull of gluons—gluons emitted by the quarks, gluons emitted by other gluons, gluons that materialize into virtual quark-antiquark pairs, which exchange more gluons. "The quarks trigger the whole thing, but once it starts, it's a very powerful process, because the gluons interact," says Wilczek. "It's a sort of runaway process."

In 1974, De Rújula, Politzer, Wilczek, and a few other physicists proposed this gluonization of the proton and suggested how it might one day be measured. Two decades later, scientists at HERA started doing just that. HERA is a ring-shaped accelerator, nearly four miles around, in which electrons doing 47,000 laps a second are smashed into protons going the other way. The greater the energy of the collision, the deeper an electron can bore into a proton before being deflected. By measuring how the electron is deflected in millions of collisions, physicists can collect information on the internal components that are doing the deflecting. It's like taking a picture of the inside of a proton, Wilczek says, pixel by pixel—and the results fit the proposal he and his colleagues made decades ago. "It's only at the crudest level that a proton is made of three quarks," Wilczek says. "When you look close and get inside these clouds and start seeing the basic structure, you see that it's mostly glue." Which makes all the more interesting the question of how the universe ever managed to design such a thing. To make a quark-gluon plasma, you don't need to go that far back in time—the first microsecond will do—so you need a less powerful accelerator.

What happens next, in theory, is simple: The collision creates a fireball intense enough to melt the protons and neutrons. The quarks and gluons circulate freely, as they do deep inside a proton, but now over a region that is many protons wide, forming a quark-gluon plasma. "If you run the movie of the Big Bang backward, it gets denser and denser, hotter and hotter," says Reinhard Stock of the University of Frankfurt, who helped design one of the CERN detectors, "and we know that all bound structures break up when their energy density exceeds their binding energy." Quantum chromodynamics demands that a quark-gluon plasma exists at a certain energy density, "but you have to prove that it exists," says Stock. "And that's why we have been here for the last 15 years."

The problem is that the laboratory fireball expands rapidly and cools rapidly, just as the primordial one must have done. The plasma survives for only 10–22 of a second before the quarks and gluons condense again into protons and other hadrons. What physicists actually detect is a spray of thousands of such particles coming out the back of the lead foil. In Stock's detector, the particles then fly through a room-sized box of argon gas, knocking electrons

off argon atoms. Counters record the electrons, and computers reconstruct the particle tracks, which reveal their identity. Once physicists have analyzed the data deluge—each collision yields 10 megabytes of data, and Stock and his colleagues have recorded millions of collisions—they may learn something about the plasma-producing fireball.

Over the past few years, Stock's detectors and others at CERN have confirmed that the fireballs are hot enough and dense enough to produce quark-gluon plasma. They have found an excess of particles containing "strange" quarks and a dearth of ones containing "charm"—both of which are side effects predicted by quantum chromodynamics. (Strange quarks and charm quarks are exotic kinds not found in ordinary particles.) It all amounts to a powerful case that CERN has been creating quark-gluon plasma—but it does not amount to proof. "The problem is they haven't been able to observe it directly," says Tom Ludlam, a physicist at Brookhaven National Laboratory

Even the scientists at Brookhaven won't be able to finally answer the question of where protons come from. A central mystery will remain. Inside a proton, as gluons come and go, as quarks and antiquarks come and go in their numberless swarms, one number remains constant: There are always three more quarks than there are antiquarks. Those are the quarks that, "at the very crudest level," as Wilczek puts it, make up the proton. Why are they matter and not antimatter? Why is the universe made of matter and not antimatter? The answer goes beyond quantum chromodynamics. A slight imbalance between quarks and antiquarks, if Wilczek and other theorists are right, was already present in the primordial quark-gluon plasma. Understanding its origin will require accelerators that reach even higher energies, such as CERN's Large Hadron Collider. It will require new kinds of theories—some of which, unfortunately, might demand that we start thinking of particles as tiny loops of string and the universe as having many more dimensions than the four we know and love. Or is it the imbalance in quarks and antiquarks that produced gluons in the first place itself?

As physics evolves, the image of the proton that quantum chromodynamics has given us may come to seem reassuringly concrete and solid—although solid is just what a proton is not. Flying into one—if you can imagine doing that, riding the strong force in a kind of subnuclear glider—would be like falling through Earth's atmosphere. The upper atmosphere of the proton is a thin cirrus of virtual quark-antiquark pairs; they form a shield for what lies below. As you fall past them, the atmosphere gets denser and denser, the clouds thicker and thicker. Your plane is struck with increasing frequency and force by flashes of color lightning—the gluons. And then, perhaps four fifths of the way through your descent, you emerge from the cloud cover. The ride is calmer now. The lightning bolts have not disappeared; they have fused to a continuous sheet, and somehow you feel at once featherlight and immune from all forces. You're near the center of the proton now, utterly trapped as you fall toward the asymptote of utter freedom, and you are finding . . . not much.

"The closer you look, the more you find the proton is dissolving into lots of particles, each of which is carrying very, very little energy," says Wilczek. "And the elements of reality that triggered the whole thing, the quarks, are these tiny little things in the middle of the cloud. In fact, if you follow the evolution to infinitely short distances, the triggering charge goes to zero.

ASSUMPTIONS:

a) QCD is classified into three categories ;i.e .colour field mediated by set of exchange particles is classified:

- 1) Category 1 representative of the one vis-à-vis QGP in category 1 (see the second classification.)
- 2) Category 2 (second interval) comprising of QCD (note that QCD is a color field mediated by a set of exchange particles. We take the age of these particles as also quarks, gluons, plasma too in this classification) corresponding to category 2 of QGP.
- 3) Category 3 constituting QCD vis-à-vis QGP of category 3

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the classification of QCD-QGP is in the fitness of things. Would be in the fitness of things. For category 3. "Over and above" nomenclature could be used to encompass a wider range of consumption due to cellular respiration. Similarly, a "less than" scale for category 1 can be used.

b) The speed of growth of QCD (here we are talking offset of exchange particles making up the color field. It is most important that this particular aspect has to borne in mind. Otherwise, the age wise classification

schematic representation would be devoid of conceptual soundness. As earlier, we show in a series of papers on QCD such aspects as increased energy, asymptotic freedom, Hyperons, concepts of Pauli's exclusion principle, diffusion of parton momentum, diffractive scattering, deep relational scattering of electrons and protons, distance between two color charges, force between color charges, localization of quarks, ST continuum breakage, quarks, gluons, and hadrons, observational perception, symmetries of nature, local symmetries, Gauss bosons, flavor symmetry of QCD, differing mass of quarks, high temperature behavior of such as strong decay correlations, short range derivations from almost perfect arrangements and finally the fundamental equations of QCD are all could be related to each other.

- c) The dissipation in all the three categories is attributable to the following two phenomenon :
- 1) **Aging phenomenon:** The aging process of color field mediated by a set of exchange particles vis-à-vis QGP sea
 - 2) **Depletion phenomenon:** Obliteration or destruction of the color fields and the concomitant exchange particles is one other reason for the depletion of the color field consisting of mediating exchange particles

NOTATION :

G_{28} : Quantum of color field mediated by exchange particles vis-à-vis QGP IN category 1

G_{29} : Quantum of color field mediated by exchange particles namely gluons again vis-à-vis QGP in category 2

G_{30} : Quantum of color field mediated by exchange particles gluons once again vis-à-vis category 3 of QGP

$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$: Accentuation coefficients

$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

- (a) The growth speed in category 1 is the sum of a accentuation term $(a_{28})^{(5)}G_{29}$ and a dissipation term $-(a'_{28})^{(5)}G_{28}$, the amount of dissipation taken to be proportional to the total quantum of gluons. Note that this is vis-à-vis category 1 of QGP
- (b) The growth speed in category 2 is the sum of two parts $(a_{29})^{(5)}G_{28}$ and $-(a'_{29})^{(5)}G_{29}$ the inflow from the category 1 dependent on the total amount standing in that category.
- (c) The growth speed in category 3 is equivalent to $(a_{30})^{(5)}G_{29}$ and $-(a'_{30})^{(5)}G_{30}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new color fields mediated by exchange particles too. And their destruction causing corresponding reduction in QGP in the co categories.

GOVERNING EQUATIONS:

The differential equations governing the above system can be written in the following form

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - (a'_{28})^{(5)}G_{28} \tag{1}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - (a'_{29})^{(5)}G_{29} \tag{2}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - (a'_{30})^{(5)}G_{30} \tag{3}$$

$$(a_i)^{(5)} > 0 \quad , \quad i = 28,29,30 \tag{4}$$

$$(a'_i)^{(5)} > 0 \quad , \quad i = 28,29,30 \tag{5}$$

$$(a_{29})^{(5)} < (a'_{28})^{(5)} \tag{6}$$

$$(a_{30})^{(5)} < (a'_{29})^{(5)} \tag{7}$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{28}}{(a_{28})^{(5)}G_{29} - (a'_{28})^{(5)}G_{28}} = dt \tag{8}$$

$$\frac{dG_{29}}{(a_{29})^{(5)}G_{28} - (a'_{29})^{(5)}G_{29}} = dt \tag{9}$$

Or we write a single equation as

$$\frac{dG_{28}}{(a_{28})^{(5)}G_{29} - (a'_{28})^{(5)}G_{28}} = \frac{dG_{29}}{(a_{29})^{(5)}G_{28} - (a'_{29})^{(5)}G_{29}} = \frac{dG_{30}}{(a_{30})^{(5)}G_{29} - (a'_{30})^{(5)}G_{30}} = dt \tag{10}$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α, β, γ all positive we can write equation (10) as

$$\frac{\alpha dG_{28}}{\alpha((a_{28})^{(5)}G_{29} - (a'_{28})^{(5)}G_{28})} = \frac{\beta dG_{29}}{\beta((a_{29})^{(5)}G_{28} - (a'_{29})^{(5)}G_{29})} = \frac{\gamma dG_{30}}{\gamma((a_{30})^{(5)}G_{29} - (a'_{30})^{(5)}G_{30})} = dt \tag{11}$$

The general solution system can be written in the form

$$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t} \text{ Where } i = 28,29,30 \text{ and } C_{28}, C_{29}, C_{30} \text{ are arbitrary constant coefficients.}$$

STABILITY ANALYSIS :

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

5. If $(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} > 0$ all the components of the solution, ie all the three parts of the color fields mediated by exchange particles tend to zero, and the solution is stable with respect to the initial data.

2. If $(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$ and $(\lambda_{29} + (a'_{28})^{(5)}G_{28}^0 - (a_{28})^{(5)}G_{29}^0 \neq 0, (\lambda_{29} < 0)$, the first two components of the solution tend to infinity as $t \rightarrow \infty$, and $G_{30} \rightarrow 0$, ie. The category 5 and category 2 parts grows to infinity, whereas the third part category 3 of QCD. Here once again we are talking of color fields that are being mediated by exchange particles which have characteristics that have been elucidated in the introduction..

3. If $(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$ and $(\lambda_{29} + (a'_{28})^{(5)}G_{28}^0 - (a_{28})^{(5)}G_{29}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i , the corresponding solution tends to infinity.

On the other hand, away from “equilibrium”, the “fluxes” are more emphasized. Result is increase in “entropy”. When this occurs, the system no longer tends towards equilibrium. On the contrary, it may encounter instabilities that culminate into newer orders that move away from equilibrium. Thus, QCD is also a sort of dissipative structures which revitalize and resurrect complex structural incompatibilities away from equilibrium state.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of oxygen consumption converges to equilibrium.
2. The approach to equilibrium is a steady one , and there exists progressively diminishing oscillations around the equilibrium point

3. Conditions 1 and 2 are independent of the size and direction of initial disturbance
4. The actual shape of the time path of color fields dissipated by QGP sea, is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance
5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold
6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question, in the present case QGP-QCD-DAF-IES
7. Some authors Nober F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

QGP(QUARK, GLUON, PLASMA Sea)assumptions:

a) QGP are classified into three categories analogous to the stratification that was resorted to in QCD sector where the color field and concomitant exchange particles were taken in to consideration. All the methodologies and modalities of transference from one category to another, the aging and depletion phenomenon and the fact that QGP categories are linked in unmistakable terms with the color fields and the concomitant exchange particles is to be noted and has to be stated in conspicuous and perceptible terms. It is very well known that the QGP sea produced by QCD breaks up the governing postulates thereof, which are nothing but the notions, nostrums, doctrines, dictums of the color field and the concomitant exchange particles. We shall not repeat the processual regularities, procedural formalities in this section, which would be presumptuous and redundant.

NOTATION :

T_{28} : Balance standing in the category 1 of QGP

T_{29} : Balance standing in the category 2 of QGP

T_{30} : Balance standing in the category 3 of QGP

$(b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$: Accentuation coefficients

$(b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$: Dissipation coefficients

GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the QGP Sea portfolio

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - (b'_{28})^{(5)}T_{28} \tag{12}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - (b'_{29})^{(5)}T_{29} \tag{13}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - (b'_{30})^{(5)}T_{30} \tag{14}$$

$$(b_i)^{(5)} > 0, \quad i = 28, 29, 30 \tag{15}$$

$$(b'_i)^{(5)} > 0, \quad i = 28, 29, 30 \tag{16}$$

$$(b_{29})^{(5)} < (b'_{28})^{(5)} \tag{17}$$

$$(b_{30})^{(5)} < (b'_{29})^{(5)} \tag{18}$$

Following the same procedure outlined in the previous section, the general solution of the governing equations is

$\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda_i t}$, $i = 28, 29, 30$ where $C'_{28}, C'_{29}, C'_{30}$ are arbitrary constant coefficients and $\alpha'_{28}, \alpha'_{29}, \alpha'_{30}, \gamma'_{28}, \gamma'_{29}, \gamma'_{30}$ corresponding multipliers to the characteristic roots of the QGP system

QCD (QUANTUM CHROMODYNAMICS MEANING THE COLOR FIELDS AND THE MEDIATING EXCHANGE PARTICLES)-QGP (QUARK, GLUON, PLASMA SEA) DUAL SYSTEM ANALYSIS

We will denote

- 1) By $T_i(t)$, $i = 28, 29, 30$, the three parts of the QGP analogously to the G_i of the QCD
- 2) By $(a_i'')^{(5)}(T_{29}, t)$ ($T_{29} \geq 0, t \geq 0$), the contribution of the QGP to the dissipation coefficient of the QCD
- 3) By $(-b_i'')^{(5)}(G_{28}, G_{29}, G_{30}, t) = -(b_i'')^{(5)}((G_{31}), t)$, the contribution of the QGP to the dissipation coefficient of the QCD again we are talking of color fields and corresponding mediating exchange particles

GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \tag{19}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \tag{20}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \tag{21}$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \tag{22}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \tag{23}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \tag{24}$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor** attributable to QGP to the dissipation of QCD

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detrition factor** contributed by QGP dissipating QCD (color fields and exchange particles)

Where we suppose

(A) $(a_i)^{(5)}, (a'_i)^{(5)}, (a''_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (b''_i)^{(5)} > 0,$
 $i, j = 28, 29, 30$

(B) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)} \tag{25}$$

$$(b_i'')^{(5)}(G_{31}, t) \leq (r_i)^{(5)} \leq (\hat{B}_{28})^{(5)} \tag{26}$$

(C) $\lim_{T_{29} \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \tag{27}$

$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)} \tag{28}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition:

$$|(a_i'')^{(5)}(T_{29}, t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)}t} \quad 29$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), (T_{31}))| < (\hat{k}_{28})^{(5)} \|(G_{31}) - (G_{31})'\| e^{-(\hat{M}_{28})^{(5)}t} \quad 30$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}, t)$ and $(a_i'')^{(5)}(T_{29}, t) \cdot (T_{29}', t)$ and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to QGP, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$:

(D) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1 \quad 31$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$:

(E) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1 \quad 32$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1 \quad 33$$

Theorem 5: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad T_i(0) = T_i^0 > 0$$

Proof:

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 34$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 35$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 36$$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28}(s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)} \quad 37$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + a''_{29}(s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 38$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + a''_{30}(s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 39$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - b''_{28}(s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 40$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - b''_{29}(s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 41$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - b''_{30}(s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 42$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying 35,35,36 into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} = \left(1 + (a_{28})^{(5)} t \right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1 \right) \quad 43$$

From which it follows that

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right] \quad 44$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0 \right) e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{28})^{(5)} \quad 45$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{Q}_{28})^{(5)} + T_j^0 \right) e^{-\left(\frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)} \quad 46$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying 35,35,36 into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d \left((G_{31})^{(1)}, (T_{31})^{(1)}, (G_{31})^{(2)}, (T_{31})^{(2)} \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{28})^{(5)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{28})^{(5)} t} \right\} \quad 47$$

Indeed if we denote

Definition of $(\widetilde{G}_{31}), (\widetilde{T}_{31}) :$

$$((\widetilde{G}_{31}), (\widetilde{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$$

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It results

$$\begin{aligned} |\widetilde{G}_{28}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\widetilde{M}_{28})^{(5)}s_{(28)}} e^{(\widetilde{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t \{(a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widetilde{M}_{28})^{(5)}s_{(28)}} e^{-(\widetilde{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widetilde{M}_{28})^{(5)}s_{(28)}} e^{(\widetilde{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} |(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)})| e^{-(\widetilde{M}_{28})^{(5)}s_{(28)}} e^{(\widetilde{M}_{28})^{(5)}s_{(28)}}\} ds_{(28)} \end{aligned}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

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From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned} |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widetilde{M}_{28})^{(5)}t} &\leq \\ \frac{1}{(\widetilde{M}_{28})^{(5)}} &((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widetilde{A}_{28})^{(5)} + (\widetilde{P}_{28})^{(5)} (\widetilde{k}_{28})^{(5)}) d(((G_{31})^{(1)}, (T_{31})^{(1)}); (G_{31})^{(2)}, (T_{31})^{(2)}) \end{aligned}$$

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And analogous inequalities for G_i and T_i . Taking into account the hypothesis (35,35,36) the result follows

Remark 5: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widetilde{P}_{28})^{(5)} e^{(\widetilde{M}_{28})^{(5)}t}$ and $(\widetilde{Q}_{28})^{(5)} e^{(\widetilde{M}_{28})^{(5)}t}$ respectively of \mathbb{R}_+ .

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If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$, $i = 28,29,30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 28 it results

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$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0$$

Definition of $((\widetilde{M}_{28})^{(5)})_1, ((\widetilde{M}_{28})^{(5)})_2$ and $((\widetilde{M}_{28})^{(5)})_3 :$

Remark 3: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$G_{28} < ((\widetilde{M}_{28})^{(5)})$ it follows $\frac{dG_{29}}{dt} \leq ((\widetilde{M}_{28})^{(5)})_1 - (a'_{29})^{(5)}G_{29}$ and by integrating

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$$G_{29} \leq ((\widetilde{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)} ((\widetilde{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq ((\widetilde{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)} ((\widetilde{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 5: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous

with the preceding one. An analogous property is true if G_{29} is bounded from below. 54

Remark 5: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \rightarrow \infty$. 55

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42 56

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$: 57

(a) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying 58

$$-(\sigma_2)^{(5)} \leq -(a_{28}')^{(5)} + (a_{29}')^{(5)} - (a_{28}'')^{(5)}(T_{29}, t) + (a_{29}'')^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b_{28}')^{(5)} + (b_{29}')^{(5)} - (b_{28}'')^{(5)}((G_{31}), t) - (b_{29}'')^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 59

(b) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations

$$(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0 \quad 60$$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0 \text{ and} \quad 61$$

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 62

By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the 63

roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ 64

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(c) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by 65

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)} \quad 66$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)} \quad 67$$

and analogously

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)} \tag{68}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}, \tag{69}$$

$$\text{and } \boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined by 59 and 65 respectively 70

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t} \tag{71}$$

where $(p_i)^{(5)}$ is defined by equation 29

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \tag{72}$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \right) \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq \tag{73}$$

$$\frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a_{30})^{(5)})} \left[e^{(S_1)^{(5)}t} - e^{-(a_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a_{30})^{(5)}t}$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \tag{74}$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \tag{75}$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \tag{76}$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 77

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)} \tag{78}$$

$$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)} \tag{79}$$

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

Definition of $v^{(5)}$:- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \quad 80$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(a) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$ 81

In the same manner , we get

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$
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From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$ 83

(b) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}} \leq v^{(5)}(t) \leq \quad 84$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (\bar{v}_1)^{(5)}$$

(c) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (v_0)^{(5)} \quad 85$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)} \quad , \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}} \quad 86$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)} \quad , \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}} \quad 87$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case :

If $(a_{28}''^{(5)}) = (a_{29}''^{(5)})$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines $(v_0)^{(5)}$ for the special case .**

Analogously if $(b_{28}''^{(5)}) = (b_{29}''^{(5)})$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of $(u_0)^{(5)}$.**

4. STATIONARY SOLUTIONS AND STABILITY

Stationary solutions and stability curve representative of the variation of QCD-QGP system if lies below the tangent at $(G_{31}) = G_0$ for $(G_{31}) < G_0$ and above the tangent for $(G_{31}) > G_0$. Wherever such a situation occurs the point G_0 is called the “**point of inflexion**”. In this case, the tangent has a positive slope that simply means the rate of change of QCD is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value :

In all the cases $(G_{31}) = G_0, (G_{31}) < G_0, (G_{31}) > G_0$ the condition that the rate of change of oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of (G_{31}) is that the rate of change of QCD function at G_0 is zero.
2. A sufficient condition for the stationary value at G_0 , to be maximum is that the acceleration of the QCD is less than zero.
3. A sufficient condition for the stationary value at G_0 , be minimum is that acceleration of QCD is greater than zero.
4. With the rate of change of (G_{31}) namely QCD defined as the accentuation term and the dissipation term, we are sure that the rate of change of QCD-related color fields- is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationariness.

We can prove the following

Theorem 3: If $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ are independent on t , and the conditions (with the notations 25,26,27,28)

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$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation 25 are satisfied, then the system

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0$$

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$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0$$

90

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0$$

91

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0$$

92

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0$$

93

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0$$

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has a unique positive solution, which is an equilibrium solution for the system (19 to 28)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0 \quad 95$$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]} \quad 96$$

(b) By the same argument, the equations 92,93 admit solutions G_{28}, G_{29} if

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - [(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0 \quad 97$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

Finally we obtain the unique solution of 89 to 94

G_{29}^* given by $\varphi((G_{31})^*) = 0, T_{29}^*$ given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]} \quad 98$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]} \quad 99$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

ASYMPTOTIC STABILITY ANALYSIS

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$ belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i \quad 100$$

$$\frac{\partial (a''_{29})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \quad \frac{\partial (b''_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij} \quad 101$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 102$$

$$\frac{d\mathbb{G}_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 103$$

$$\frac{d\mathbb{G}_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 104$$

$$\frac{d\mathbb{T}_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*\mathbb{G}_j \quad 105$$

$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*G_j \quad 106$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*G_j \quad 107$$

The characteristic equation of this system is

$$\begin{aligned} & ((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)})\{((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}) \\ & \left[((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)}G_{29}^* + (a_{29})^{(5)}(q_{28})^{(5)}G_{28}^* \right] \\ & \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(29)}T_{29}^* + (b_{29})^{(5)}s_{(28),(29)}T_{29}^* \right) \\ & + \left(((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(q_{28})^{(5)}G_{28}^* + (a_{28})^{(5)}(q_{29})^{(5)}G_{29}^* \right) \\ & \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(28)}T_{29}^* + (b_{29})^{(5)}s_{(28),(28)}T_{28}^* \right) \\ & \left(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) \\ & \left(((\lambda)^{(5)})^2 + ((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}) (\lambda)^{(5)} \right) \\ & + \left(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) (q_{30})^{(5)}G_{30} \\ & + ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) \left((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^* + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^* \right) \\ & \left. \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^* + (b_{29})^{(5)}s_{(28),(30)}T_{28}^* \right) \right\} = 0 \end{aligned} \quad 108$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

DOMINANT ASYMPTOTIC FREEDOM-INCREASED ENERGY SYSTEMS SHALL BE DISCUSSED IN PART TWO. PART THREE CONCATENATES BOTH THE PAPERS AND PROVIDES A FINAL SOLUTION TO THE SYSTEM QGP-DAF-IES

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