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Global and Factor Domination in Fuzzy Graph

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Abstract- The purpose of this paper is to introduce the concept of Global Domination number in fuzzy graph .Then we introduce the concept of Factor Domination number in fuzzy graph and theorems based on Global and Factor domination in fuzzy graphs are proved.

Index Terms- Fuzzy Global Domination Number, Fuzzy Factor Domination Number

I. Introduction

The study of dominating sets in graphs was begun by ore and berge, the domination number is introduced by cockayne and Hedetniemi. Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A Somasundram and S. Somasundram discussed domination in fuzzy graph. They defined domination using effective edges in fuzzy graph. Nagoor Gani and Chandrasekaran discussed domination in fuzzy graph using strong arc. We also discuss the domination number γ (G) of the fuzzy graph G, the fuzzy factor domination number of the fuzzy graph G and the fuzzy global domination number of the fuzzy graph are discussed in this paper.

Preliminaries1.1

- A fuzzy subset of a nonempty set V is a mapping σ: V $\rightarrow$ [0,1]
- A fuzzy relation on V is a fuzzy subset of V $\times$ V.
- A fuzzy graph G = (σ , μ) is a pair of function σ: V$\rightarrow$[0,1] and μ :V$\times$V$\rightarrow$[0,1] where μ(u, v)$\leq\sigma$(u)Λ σ(v) for u ,v ϵ V.
- The underlying crisp graph of G= (σ , μ) is denoted by G*=(V,E) where V={u ϵ V :σ(u)>0}and E={(u , v) ϵ V $\times$ V : (u ,v)>0}.
- The order p and size q of the fuzzy graph G = (σ , μ) are defined by
$$p= \sum_{v \epsilon V}\sigma(v) \text{ and } q= \sum_{(u, v)\epsilon E}\mu (u ,v) .$$
- Let G be a fuzzy graph on V and S$\subseteq$V, then the fuzzy cardinality of S is defined to be $\sum_{v \epsilon S}\sigma(v)$.
- The strength of the connectedness between two nodes u , v in a fuzzy graph G is μ^{∞}(u , v)=sup{μ^k(u ,v):k=1,2,3…..},where μ^k(u ,v)=sup{$\mu(u,u_1)\Lambda$ $\mu(u_1,u_2)$………Λ $\mu(u_{k-1},v)$}.
- An arc (u, v) is said to be a strong arc or strong edge, if μ (u, v) $\geq$ μ ∞ (u ,v) and the node v is said to be a strong neighbor of u.
- A node u is said to be isolated if μ (u , v)=0 for all u $\neq$ v.
- In a fuzzy graph, every arc is a strong arc then the graph is called strong arc fuzzy graph.
- A path in which every arc is a strong arc then the path is called strong path and the path contains n strong arcs is denoted by p_n
- Let u be a node in a fuzzy graph G then N(u)={v: (u , v) is a strong arc} is called neighborhood of u and N[u]=N(u)U{u} is called closed neighborhood of u.
- Let G be a fuzzy graph and S be a subset of V.A node v is said to be fuzzy private neighbor of uϵ S with respect to S or S- private neighbor of u, if N[v]$\cap$ S= {u}.
- Define a fuzzy private neighborhood of u ϵ S with respect to S to be P N [u , S] = {v: N[v]$\cap$S={u}} In other words P N[u,S]=N[u]-N[S-{u}].
- If u ϵ P N [u , S], then u is an isolated node in G[S]. It is also stated that u is its own private neighbor.

II. Fuzzy Global and Factor Domination Number

Definition 2.1:

The complement of a fuzzy graph G=(σ , μ) is a fuzzy graph $\overline{G}$=($\overline{\sigma}$, $\overline{\mu}$) where $\overline{\sigma}$=σ and $\overline{\mu}$(u ,v)=σ(u) Λ σ(v)-μ(u ,v) for all u , v in V.

Example2.2

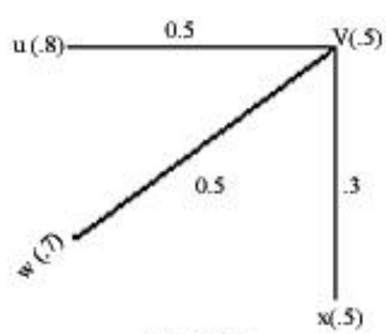
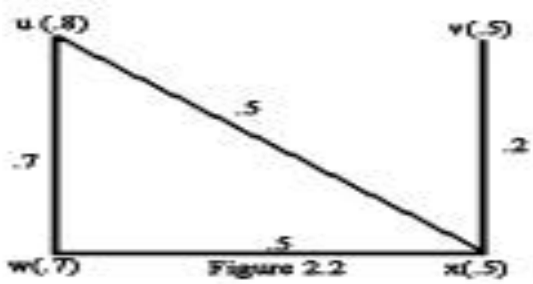


Figure 2.1



Figures 2.1, 2.2 be a fuzzy graph and its complement

Definiton2.3

*A Fuzzy graph is **self complementray** if* $G = \overline{G}$

Definition2.4

Let G=(σ , μ) be a fuzzy graph. A subset D of V is said to be **fuzzy dominating set** of G if for every v ϵ V-D there exists u ϵD such that (u,v) is a strong arc.

Definition 2.5

A fuzzy graph H=(σ,μ) is said to have a **fuzzy t-factoring** into factors $F(H)=\{G_1\ G_2,G_3,......G_t\}$ if each fuzzy graph $G_i=(\sigma_i,\mu_i)$ such that $\sigma_i=\sigma$ and the set $\{\mu_1,\mu_2,\mu_3........\mu t\}$ form a fuzzy partition of μ.

Definition 2.6

Given A t-factoring F of H, a subset Df⊆V is a **fuzzy factor dominating set** if Df is a fuzzy dominating set of Gi, for1≤i≤t.

Example2.7

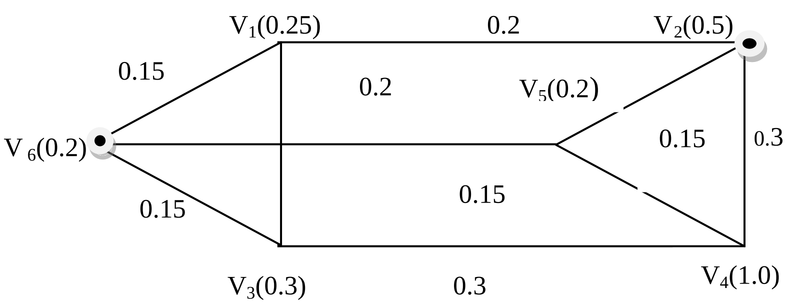


Figure2.3: H

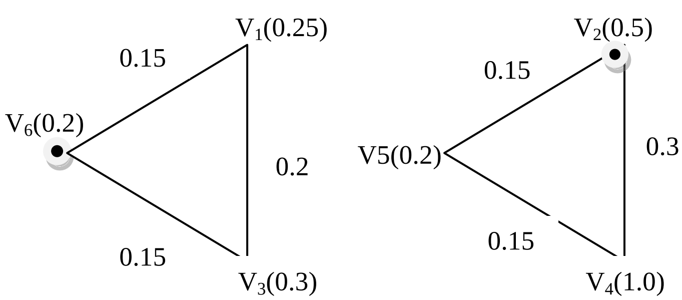
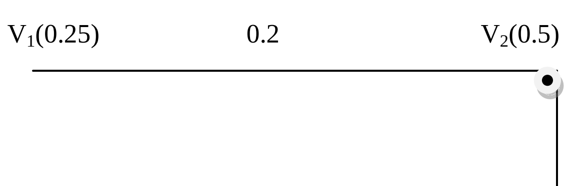


Figure 2.4: G_1



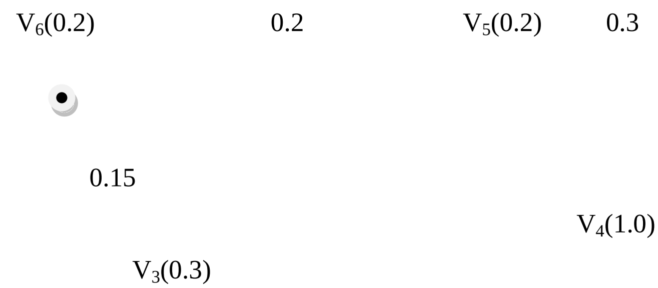


Figure 2.5 G_2

The fuzzy t-factoring and a fuzzy factor dominating set in given in figures 2.3,2.4 and 2.5 the set of vertices $\{v_2(0.5),v_6(0.2)\}$ in H dominate each of the fuzzy factor G_1 and G_2, while no smaller fuzzy factor dominating set exists, since G_1 and G_2has two fuzzy components that is at least two vertices are required to dominate G_1 and G_2

Definition2.8

The **fuzzy factor domination number** $\gamma_{ft}(F(H))$ is the minimum cardinality of a fuzzy factor dominating set of F(H).

The fuzzy factor domination number is that $\max_{1\le i\le t}\{\gamma_i\}\le\gamma_{ft}\le\sum_{i=1 \text{ to } t}\gamma_i$

Definition2.9

In a fuzzy graph G=(σ, μ) the minimum cardinality set D of G which is the fuzzy dominating set of both G and $\overline{G}$. is called a **fuzzy global dominating set.**

Definiton2.10

The minimum cardinality of a fuzzy global dominating set is denoted by $\gamma_g(G)$ is called the **fuzzy global domination number.**

Definition2.11

A **vertex cover of a fuzzy graph G i**s a set of vertices that covers all the edges and an **edge cover of a fuzzy graph** is a set of edges that covers all the vertices. The minimum cardinality of vertex cover is $\alpha_0(G)$ and the minimum cardinality of edge cover is$\alpha_1(G)$.

Theorem2.1

If I is the set of vertices in V(H) which are isolated in atleast one fuzzy graph G_i, then $\gamma_{ft}\le\alpha_0(H)+|I|$

Proof.

Let S be a minimum vertex cover for a fuzzy graph H and let v be a non isolated vertex of G_i with an incident edge μ. since μ is an edge in H, at least one vertex, say u, covers μ in H, and therefore this vertex u dominates v in G_i . It follows that the set SUIis a fuzzy factor dominating set F(H)

Theorem2.2

For any fuzzy graph H, $\gamma_{ft}\ge t$ if $t\le\Delta(H)$. $\gamma_{ft}=n$ otherwise

Proof.

Let D_f be a minimum fuzzy factor dominating set for F(H). If $t\leq\Delta(H)$, then any vertex v in $V(H)-D_f$ must have in H at least μ_t edges to D_f so it can be dominated in each G_i. Hence $|D_f|\geq t$.If $t>\Delta(H)$, no such vertex v can exist and $H-D_f$ must be empty, that is $\gamma_{ft}=n$

Theorem2.3

For any fuzzy graph H,$\gamma_{ft}\leq\gamma+t-2$ if $t\leq\Delta(H)$: $\gamma_{ft}=n$ otherwise

Proof.

The proof of theorem 2.2 shows that for $t>\Delta(H)$,$\gamma_{ft}=n$. Therefore, assume that $t\leq\Delta(H)$ and let D_f be a minimum fuzzy factor dominating set. If $D_f=V(H)$ then $\gamma_{ft}=n\geq\gamma+\Delta(H)\geq\gamma+t>\gamma+t-2$.If $D_f\neq V(H)$,let $v\epsilon V(H)-D_f$. since v must be adjacent to at least t vertices of D_f. let $X\subseteq D_f$ be a set of t-1 vertices contained in N(v)in H. In H every vertex of $V(H)-(D_f U\{v\})$is adjacent to at least one vertex of D_f-X. Thus$(D_f-X)U\{v\}$ dominates H.so $\gamma\leq|D_f-X|+1=\gamma_{ft}-(t-1)+1$, or $\gamma_{ft}\leq\gamma+t-2$

Theorem2.4

For any fuzzy graph H and any fuzzy t-factoring $F(H)=\{G_1,G_2,G_3,\ldots.G_t\}$of H,$\gamma_{ft}+\epsilon_{ft}=n$

Proof.

If $t>\Delta(H)$, then $\gamma_{ft}=n$ and $\epsilon_{ft}=0$ and the equality holds .Assume therefore, that $t\leq\Delta$ and $\gamma_{ft}<n$. Let D_f be a minimum fuzzy factor dominating set. In each fuzzy factor G_i, arbitrarily select for each vertex $v\epsilon V(G_i)-D_f$ one edge between v and a vertex in D_f. The fuzzy sub graph of G_i formed by these selected edges is a union of stars centered at vertices of D_f and forms a fuzzy spanning forest of G_i. Notice that in each of these fuzzy spanning forests, the vertices in $V(G_i)-D_f$ are end vertices and therefore, independent. Thus, $\epsilon_{ft}\geq|V(G_i)-D_f|=n-\gamma_{ft}$.

Conversely, let S be a set of vertices satisfying the definition of ϵ_{ft}. clearly, the vertices in V(H)-S form a fuzzy factor dominating set hence $\gamma_{ft}\leq|V(H)-S|=n-\epsilon_{ft}$.

Theorem 2.5

If either G or $\overline{G}$ is a disconnected fuzzy graph, then $\gamma_g=\max\{\gamma,\overline{\gamma}\}$

Proof.

Assume that the fuzzy graph G is disconnected, that is G has at least two components. Then it follows that $\gamma(\overline{G})=2$ and any fuzzy dominating set of G is automatically a dominating set of $\overline{G}$. Since a fuzzy dominating set of G must contain at least one vertex from each component. Therefore, $\gamma_g(G)=\gamma(G)=\max\{\gamma,\gamma\}$.

Theorem 2.6

If both G and $\overline{G}$ are fuzzy connected graphs, then

(a) $\gamma_g=\max\{\gamma,\overline{\gamma}\}$ if $diam(G)+diam(\overline{G})\geq7$.
(b) $\gamma_g\leq\max\{3,\gamma,\overline{\gamma}\}+1$ if $diam(G)+diam(\overline{G})=6$
(c) $\gamma_g\leq\max\{\gamma,\overline{\gamma}\}+2$ if $diam(G)+diam(\overline{G})=5$.
(d) $\gamma_g\leq\min\{\delta,\overline{\delta}\}+1$ if $diam(G)+diam(\overline{G})=2$

Proof

We can assume that neither G nor $\overline{G}$ is a complete fuzzy graph, and therefore $diam(G)\geq diam(\overline{G})\geq2$.let vertices u and v be distance diam(G) apart in G.

(a) It is known that if diam(G) >3, then $diam(\overline{G})<3$. thus if $diam(G)+diam(\overline{G})\geq7$, then we can assume that $diam(G)\geq5$and $diam(\overline{G})\leq2$. In This case vertices u and v dominates $\overline{G}$, since $N_G[u]\cap N_G[v]=\Phi$.Therefore $\gamma=2$and$\gamma^g=\gamma=\max\{\gamma,\gamma\}$,since any fuzzy minimum dominating set for G must contain at least one vertex from N[u]and one from N[v].

(b) In this case ,either $diam(G)=diam(\overline{G})=3$ or $diam(G)=4$ and $diam(\overline{G})=2$ But in any fuzzy graph G with diam(G)=3,any two vertices at distance three dominate $\overline{G}$ .In this case,$\gamma=\overline{\gamma}=2$ and therefore $\gamma_g\leq4$.In casediam(G)=4 and diam(G)=2 as in (a) above ,any γ-sets of G must contain at least one vertex from N[u] and one from N[v]. If neither u and v are in D, then DU{u}will suffice to dominate $\overline{G}$.

(c) In this case, we can assume that diam(G)=3 and $diam(\overline{G})=2$. It follows that any two vertices at distance three in G will dominates G. Therefore, if D is a γ-set of G, then DU{u,v} is a fuzzy global dominating set of G

(d) In a fuzzy graph of diameter two,for each xϵv, N[x] is a fuzzy global dominating set.

III. Results

- We have introduced the fuzzy factor dominating set, fuzzy factor domination number.
- We have introduced the fuzzy global domination number and theorems based on fuzzy global domination number, fuzzy factor dominance number.

IV. CONCLUSION

For graphical research the fuzzy global domination number and fuzzy factor domination number are very useful for solving very wide range problems. We can impose additional restriction .This will lead us to a new notion for fuzzy graph. From a practical point of view the fuzzy factor and fuzzy global domination number may be built from many different kinds of functions

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