ISSN 2250-3153

Fuzzy Spanning Tree Flow-Equivalent Algorithm

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Abstract- By introducing, the new concept fuzzy edge cut-set, multiple sources and sinks of fuzzy graphs and with arc and vertex capacities of fuzzy graphs , simple variations of the maximal flow problem is introduced. A maximum flow matrix is constructed by a spanning tree using vertex condensation process. Also, in this paper a fuzzy spanning tree with maximum flow is constructed, which is equivalent to an undirected graph by algorithmic representation. In general, the flow equivalent fuzzy tree is not unique.

Index Terms- Fuzzy Edge cut-sets- arc and vertex capacity- Fuzzy equivalent-flow.

I. Introduction

The purpose of a network is to implement a flow of water, oil, electricity, traffic, or whatever the network is designed to carry. Mathematically, a flow in a network N is a function that assigns to each edge (i, j) of N a nonnegative number f_{ij} that does not exceed arc capacities q_{ij}. Intuitively, f_{ij} represents the amount of flow passing through the edge (i, j) when the flow is f. Informally, we refer to f_{ij} as the flow through edge (i ,j) . We also require that for each node other than the source and sink, the sum of the f_{kj} on edges leaving node k. This means that flow cannot accumulate, be created, dissipate, or be lost any node other than the source or the sink. This is called conservation of flow. A consequence of this requirement is that the sum of the flows entering the sink. This sum is called the value of the flow.

In some cases, for example in the case where the graph represents a telephone network with vertices corresponding to stations and the arcs corresponding to the telephone cables, what may be required is the maximum rate of communication between any two stations s and t, assuming that only a single pair of stations can communicate with each other at any one time. In this example the capacity of a cable is the number of independent calls that can be routed through it. A similar situation exists in the case where the graph represents a road network and where, say, a system of one-way streets is proposed. What is then required is to determine how then one-way streets affects the maximum rate of traffic flow between any two areas, when those areas are considered in isolation. This problem could be solved by simply considering the (s to t) maximum flow problem for every pair of vertices (s,t).

II. Definitions

Definition 2.1. A *fuzzy graph* with V as the underlying set is a pair G: (A, Γ) where A: V ⟶ [0,1] is a fuzzy subset, Γ : VxV ⟶ [0,1] is a fuzzy relation on the fuzzy subset A, such that $\Gamma(u,v) \le A(u) \cap A(v)$ for all u,v ϵ V.

Definition 2.2. ***A flow of a fuzzy graph f*** is an assignment of an integer value ***f* (e)** to each edge ***e*** that satisfies the following properties:

Capacity Rule: For each edge ***e***, $0 \le f(e) \le q(e)$

Conservation Rule: For each vertex **v ≠ s,t**

$$\sum_{e \,\epsilon\, \Gamma_+(v)} f(e) = \sum_{e \,\epsilon\, \Gamma_-(v)} f(e)$$

Where, Γ - (v) and Γ +(v) are the incoming and outgoing arcs of v, respectively. The value of a flow ***f***, denoted |***f***|, is the total flow from the source, which is the same as the total flow into the sink.

Definition 2.3. A flow of a fuzzy graph N is said to be **maximum** if its value is the largest of all flows for N.

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Definition 2.4. If the vertices of a undirected fuzzy graph G=(A, Γ) are partitioned into two sets A_0 and $\tilde{A}_0$ (where A_0 C A and $\tilde{A}_0$ is the complement of A_0 in A), then the set of links of fuzzy graph G whose terminal vertices lie one in A_0 and the other in $\tilde{A}_0$, is called a ***edge cut-set of fuzzy graph G***. The set of links in the cut-set can be represented by the vertex set doublet $(A_0, \tilde{A}_0)$.

Definition 2.5. Consider a fuzzy graph G = (A, Γ) , with s and t being of G chosen at random. Let us say that $(A_0, \tilde{A}_0)$ is the minimum cut-set corresponding to this maximum flow and consider two vertices x_i and x_j which are both in A_0(or both in $\tilde{A}_0$).If we now wish to find f_{ij}, the maximum flow from x_i to x_j, then all vertices of $\tilde{A}_0$ (or A_0 if x_i and x_j ϵ$\tilde{A}_0$) may be "condensed " into a single vertex x_c, say, as far as this flow calculation is concerned. The condensation is such that links (x_a,x_b), x_aϵ A_0 and x_b ϵ $\tilde{A}_0$ are replaced by links (x_a,x_c), and any parallel links between the same pair of vertices (which may result) are replaced by a single link of capacity equal to the sum of the capacities of the parallel links. Figures 2.1(a), 2.1(b) and 2.1(c) illustrate the ***Fuzzy Vertex Condensation*** process.

Minimum (s to t) cut-set

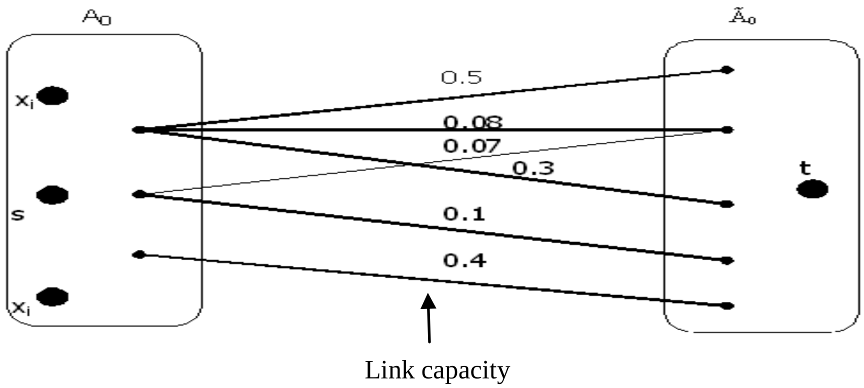

Fig 2.1(a)

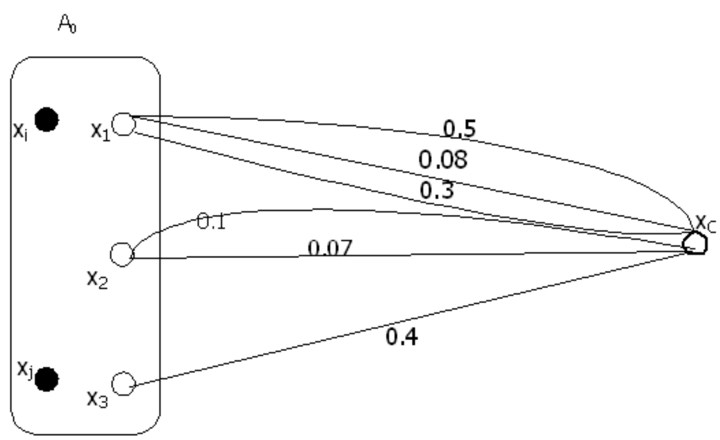

Fig.2.1(b): Fuzzy vertex condensation for x_i to x_j flow calculation

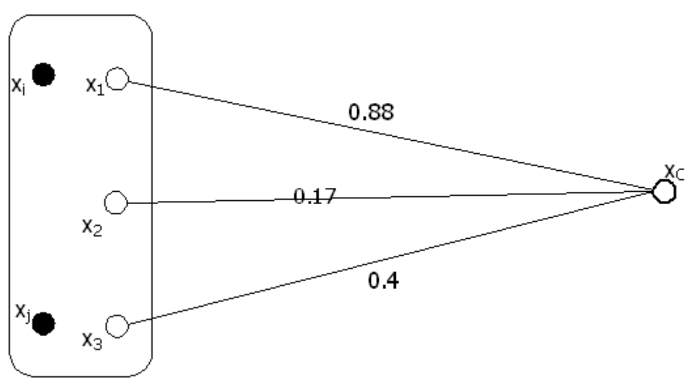

Fig. 2.1(c): Condensed Fuzzy graph

ISSN 2250-3153

III. FUZZY GRAPHS WITH MULTIPLE SOURCES AND SINKS

Consider a fuzzy graph with N_s sink vertices and assume that flow can go from any sources to any sink. The problem of finding the maximum total flow from all the sources to all the sinks can be converted to the simple (s to t) maximum flow problem by adding a new artificial source vertex s and a new artificial sink vertex t with added arcs leading from s to each of the real source vertices and from every real sink to t.

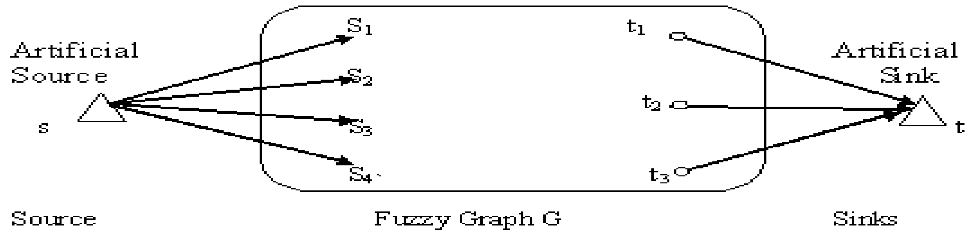

Fig. 3.1

The above figure shows how multiple sources and sinks can be reduced to a single source and a single sink.

3.1 Fuzzy Graphs With Arc and Vertex Capacities

For a fuzzy graph G let the arcs have capacities q_{ij}, and in addition let the vertices have capacities w_j, say(j=1,2,3….n) so that the total flow entering vertes x_j must have a value less than w_j.

$$\text{(ie)} \sum_{x_i \in \Gamma^{-1}(x_j)} \xi_{ij} \le w_j \quad \text{for all } x_j$$

Let the maximum flow between vertices s and t for such a fuzzy graph be required. Let us define a fuzzy graph G_0 so that every vertex x_j of fuzzy graph G corresponds to two vertices x_j^+ and x_j^- in the fuzzy graph G_0 , in such a way so that for every arc (x_i,x_j) of G incident at x_j corresponds an arc (x_i^-,x_j^+) of G_0 incident at $x_j^{+,}$ and for every arc (x_i,x_j) of G emanating from x_j corresponds an arc (x_i^-,x_j^+) of G_0 emanating from x_j^-. moreover , we introduce an arc between x_j^+ and x_j^- of capacity w_j , (ie) equal to the capacity of vertex x_j.

EXAMPLE- 3.1

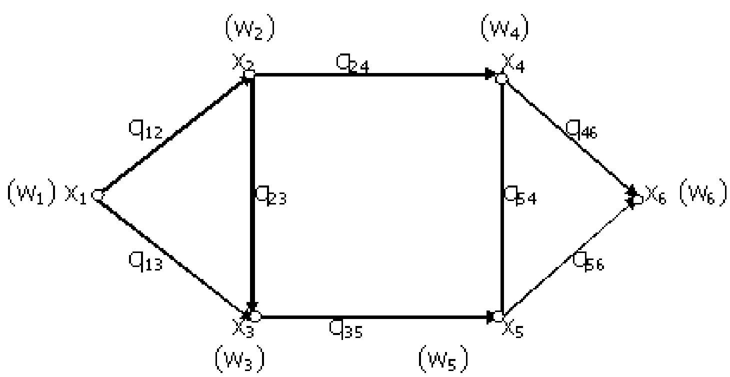

Fig 3. 2: ***Fuzzy graph with both vertex and arc capacities***

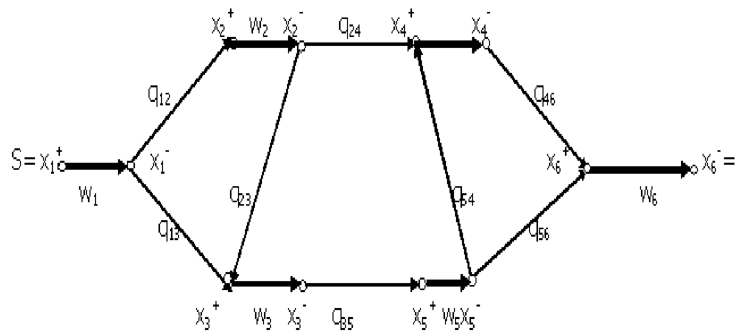

Fig. 3.3: ***Equivalent fuzzy graph with arc capacities only*.**

Since, the total flow entering a vertex x_i^+ must, by necessity, travel along the arc (x_i^+,x_j^-) whose capacity is w_j, the maximum flow in the fuzzy graph G with vertex capacities , is equal to the maximum flow in the graph G_0 which has only arc capacities.

Remark: 3.1

If the minimum edge cut-set of G_0 does not contain arcs of the form (x_i^+,x_j^-), then the vertex capacities in G are inactive and superfluous;

If on the other hand the minimum edge cut-set of G_0 contains arcs of the form (x_i^+,x_j^-), then the corresponding vertices of G are saturated by the **maximum** flow solution

3.2. Fuzzy Spanning Tree Algorithm

The algorithm that is given in this section generates a tree T* which is flow-equivalent to the undirected fuzzy graph G. The maximum flow f_{ij} between two vertices x_i and x_j of the fuzzy graph G can then be found from this tree as:

$$f_{ij} = \min [q'_{ik1}, q'_{k1k2}, q'_{k2k3}, \ldots\ldots q'_{kpj}] \text{---------------} \quad (3.1)$$

where $(x'_i, x'_{k1}, x'_{k2}, \ldots\ldots, x'_j)$ is the unique path along links of T* which leads from x'_I to x'_j. Every vertex x'_k of T* corresponds to a vertex x_k of G and q'_{kl} is the capacity of link (x'_k, x'_l) of T*. Since the algorithm generated T* gradually and since at any one of the (n-1) stages of the algorithm the "vertices " of

T* may in fact be sets composed of vertices of G, we will, in order to avoid confusion , refer to the vertices of G as G –vertices and to the vertices of T* as T*-vertices.

Step 1: Set a set $\zeta 1$ = A, N= 1. At any stage, T* is the graph defined by N T*-vertices $\zeta 1, \zeta_2, \ldots, \zeta_N$ each one of which corresponds to a set of G- vertices. T* is initialized to be the single vertex ζ_1 .

Step 2: Find a set $\zeta^* \epsilon \{ \zeta_1, \zeta_2, \ldots \zeta_N\}$ which contains more than one G-vertex in it. If none exist go to step 6, otherwise go to step 3.

Step 3: If ζ^* were to be removed from T*, the fuzzy tree would in general be reduced to a number of fuzzy sub-trees (connected components).Condense the T*-vertices in each sub-tree into single vertices to form the condensed fuzzy graph. Take any two vertices x_i and x_j $\epsilon\ \zeta^*$ and calculate the minimum cut-set $(A_0 ,\tilde{A}_0)$ of G separating x_i from x_j by doing an (x_i to x_j) maximum flow calculation.

Step 4: Remove the T*-vertex ζ^* together with its incident links from T*, and replace it by two T*-vertices composed of the G-vertex sets $\zeta^* \cap A_0$ and $\zeta^* \cap \tilde{A}_0$, and a link of capacity $v(A_0,\tilde{A}_0)$ between them. Also, for all T*-vertices ζ_i which were incident on before the replacement, add the link (ζ_i , $\zeta^* \cap A_0$) to T* if $\zeta_i \subset A_0$; or add the link (ζ_i , $\zeta^* \cap \tilde{A}_0$) to T* if $\zeta_i \subset \tilde{A}_0$. The capacities of the links being taken as c_i^o(link of capacity) regardless of which one is added.

Step 5: Set N=N+1. The vertices of T* are now the sets of G-vertices $\zeta 1, \zeta_2, \ldots, \zeta^* \cap A_0, \zeta^* \cap \tilde{A}_0, \ldots \zeta_N$ where ζ^* has been replaced by the two T*-vertices $\zeta^* \cap A_0$ and $\zeta^* \cap \tilde{A}_0$ as explained earlier. Go to step 2.

Step 6: Stop. T* is the required flow-equivalent tree of G and its T*-vertices are now single G-vertices. The link capacities of T* corresponds to the values of the (n-1) independent cut-sets of G. equations (3.1) can then be used to calculate the f_{ij} (for any x_i, x_j ϵ A), directly from T*.

EXAMPLE: 3.2

Consider an undirected fuzzy graph G shown in figure 3.2 where the link capacities are shown next to the links. It is required to find the maximum flow between every pair of vertices of G.

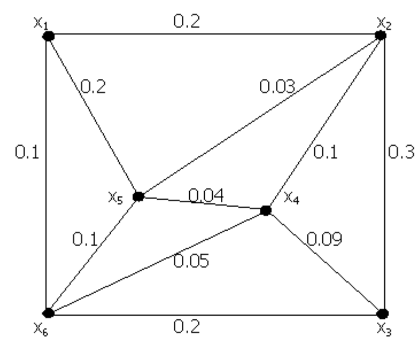

Fig. 3.2: Fuzzy graph.

From the above algorithm we get.

Step 1: $\zeta_1 = \{x_1, x_2, x_3, x_4, x_5, x_6 \}$; N=1

Step 2: $\zeta^* = \zeta_1$

Step 3: The fuzzy graph cannot be condensed. Take $x_i = x_1$ and $x_j = x_2$ arbitrarily. From an (x_1 to x_2) maximum flow calculation we find the minimum cut-set to be $(A_0 ,\tilde{A}_0)$ where $A_0=\{ x_1, x_5, x_6, \}$ and $\tilde{A}_0 =\{ x_2, x_3, x_4\}$, the value of the cut-set being 0.52.

Step 4: The fuzzy tree T* and its link capacities are now shown in figure 3.2(a)

Step 5: N=2 (i.e. T* now has 2 vertices ζ_1 and ζ_2 as shown in figure 3.2(a)).

Similarly, proceeding to step 2 and continuing the process till $\zeta^* = \zeta_3$ we get the following figure:-

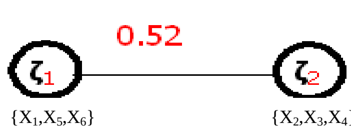

Fig. 3.2(a): T* after the first stage

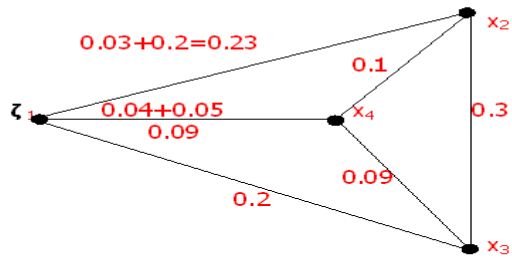

Fig.3.2 (b and d): The condensed fuzzy graph after the 2nd (b) and 3rd stages (d)

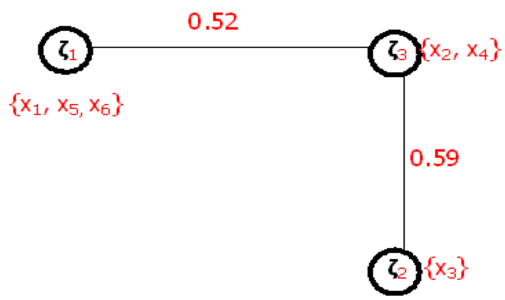

Fig .3.2(c): T* after the 2nd stage

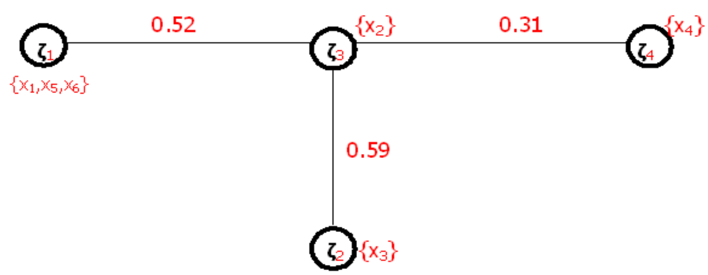

Fig .3.2(e) : T* after the 3rd stage

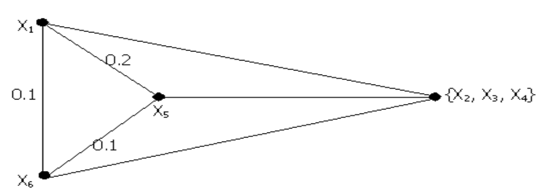

Fig .3.2(f and h): The condensed fuzzy graph after the 4th (f) and 5th (h) stages

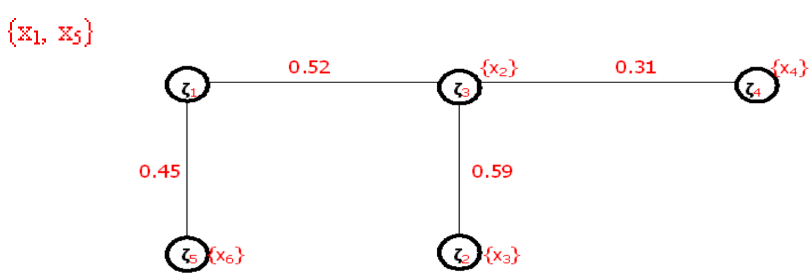

Fig. 3.2(g): T* after the 4th stage

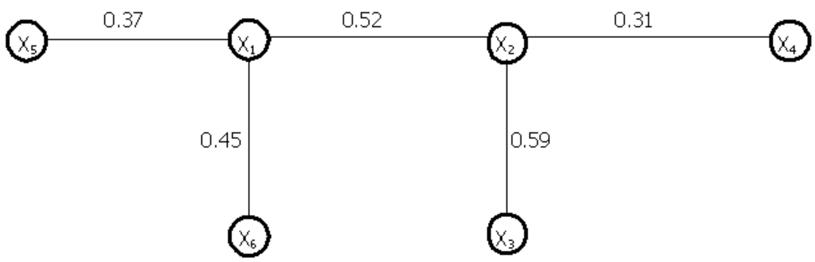

Fig. 3.2(i): Final flow – equivalent fuzzy tree T*

Continuing in the same way and taking $\zeta^* = \zeta_1$ next, the development of the flow-equivalent fuzzy tree is illustrated above. Also, the maximum flow matrix $[f_{ij}]$ of the original fuzzy graph can be calculated as follows:

	X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**
X_1	-	0.52	0.52	0.31	0.37	0.45
X_2	0.52	-	0.59	0.31	0.37	0.45
X_3	0.52	0.59	-	0.31	0.37	0.45
X_4	0.31	0.31	0.31	-	0.31	0.31
X_5	0.37	0.37	0.37	0.31	-	0.37
X_6	0.45	0.45	0.45	0.31	0.37	-

Maximum Flow Matrix

The five fuzzy cut-sets (in general n-1) of fuzzy graph G corresponding to the links of the fuzzy tree T* are shown in dotted figure 3.3 .

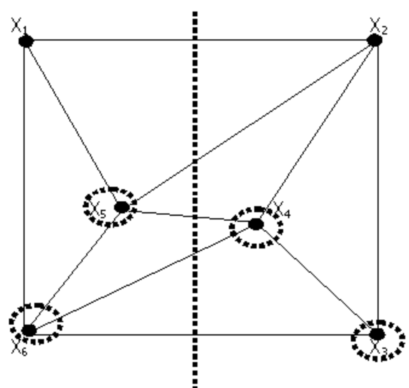

Fig.3.3: Cut-sets forming fuzzy tree T*

In general, the flow-equivalent fuzzy tree T* is not unique. One such fuzzy tree is shown in following Figure 3.4.

Fig. 3.4: Another flow-equivalent fuzzy tree

IV. CONCLUSION

Using fuzzy graphs with multiple sources and sinks, and with arc and vertex capacities we can construct a fuzzy spanning tree with maximum flow using condensation process which is flow equivalent to an undirected graph by algorithmic representation. The paper concludes that the flow equivalent fuzzy tree is not unique.

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