SOME CONTRIBUTIONS TO YANG MILLS THEORY FORTIFICATION –DISSIPATION MODELS

1DR K N PRASANNA KUMAR, 2PROF B S KIRANAGI AND 3PROF C S BAGEWADI

ABSTRACT. We provide a series of Models for the problems that arise in Yang Mills Theory. No claim is made that the problem is solved. We do factorize the Yang Mills Theory and give a Model for the values of LHS and RHS of the Yang Mills theory. We hope these forms the stepping stone for further factorizations and solutions to the subatomic denominations at Planck’s scale. Work also throws light on some important factors like mass acquisition by symmetry breaking, relation between strong interaction and weak interaction, Lagrangian Invariance despite transformations, Gauge field, Noncommutative symmetry group of Gauge Theory and Yang Mills Theory itself.

We take into consideration the following parameters, processes and concepts:

1. Acquisition of mass
2. Symmetry Breaking
3. Strong interaction
4. Unified Electroweak interaction
5. Continuous group of local transformations
6. Lagrangian Variance
7. Group generator in Gauge Theory
8. Vector field or Gauge field
9. Non commutative symmetry group in Gauge Theory
10. Yang Mills Theory (We repeat the same Bank’s example. Individual debits and Credits are conservative so also the holistic one. Generalized theories are applied to various systems which are parameterized. And we live in ‘measurement world’. Classification is done on the parameters of various systems to which the Theory is applied.)

(i) First Term of the Lagrangian of the Yang Mills Theory (LHS)

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4} \text{Tr}(F^2) = -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} \]

(ii) RHS of the Yang Mills Theory

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4} \text{Tr}(F^2) = -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} \]

SYMmetry BREAKING AND ACQUISITION OF MASS:

MODULE NUMBERED ONE

NOTATION :

\( G_{13} \) : CATEGORY ONE OF SYMMETRY BREAKING
$G_{14}$ : CATEGORY TWO OF SYMMETRY BREAKING

$G_{15}$ : CATEGORY THREE OF SYMMETRY BREAKING

$T_{13}$ : CATEGORY ONE OF ACQUISITION OF MASS

$T_{14}$ : CATEGORY TWO OF ACQUISITION OF MASS

$T_{15}$ : CATEGORY THREE OF ACQUISITION OF MASS

----

UNIFIED ELECTROWEAK INTERACTION AND STRONG INTERACTION:

MODULE NUMBERED TWO:

$G_{16}$ : CATEGORY ONE OF UNIFIED ELECTROWEAK INTERACTION

$G_{17}$ : CATEGORY TWO OF UNIFIED ELECTROWEAK INTERACTION

$G_{18}$ : CATEGORY THREE OF UNIFIED ELECTROWEAK INTERACTION

$T_{16}$ : CATEGORY ONE OF STRONG INTERACTION

$T_{17}$ : CATEGORY TWO OF STRONG INTERACTION

$T_{18}$ : CATEGORY THREE OF STRONG INTERACTION

----

LAGRANGIAN INVARIANCE AND CONTINUOUS GROUP OF LOCAL TRANSFORMATIONS:

MODULE NUMBERED THREE:

$G_{20}$ : CATEGORY ONE OF CONTINUOUS GROUP OF LOCAL TRANSFORMATIONS

$G_{21}$ : CATEGORY TWO OF CONTINUOUS GROUP OF LOCAL TRANSFORMATIONS

$G_{22}$ : CATEGORY THREE OF CONTINUOUS GROUP OF LOCAL TRANSFORMATION

$T_{20}$ : CATEGORY ONE OF LAGRANGIAN INVARIANCE

$T_{21}$ : CATEGORY TWO OF LAGRANGIAN INVARIANCE

$T_{22}$ : CATEGORY THREE OF LAGRANGIAN INVARIANCE

----

GROUP GENERATOR OF GAUGE THEORY AND VECTOR FIELD (GAUGE FIELD):

MODULE NUMBERED FOUR:

----
\(G_{24}\) : CATEGORY ONE OF GROUP GENERATOR OF GAUGE THEORY

\(G_{25}\) : CATEGORY TWO OF GROUP GENERATOR OF GAUGE THEORY

\(G_{26}\) : CATEGORY THREE OF GROUP GENERATOR OF GAUGE THEORY

\(T_{24}\) : CATEGORY ONE OF VECTOR FIELD NAMELY GAUGE FIELD

\(T_{25}\) : CATEGORY TWO OF GAUGE FIELD

\(T_{26}\) : CATEGORY THREE OF GAUGE FIELD

---

YANG MILLS THEORY AND NON COMMUTATIVE SYMMETRY GROUP IN GAUGE THEORY:

MODULE NUMBERED FIVE:

---

\(G_{28}\) : CATEGORY ONE OF NON COMMUTATIVE SYMMETRY GROUP OF GAUGE THEORY

\(G_{29}\) : CATEGORY TWO OF NON COMMUTATIVE SYMMETRY GROUP OF GAUGE THEORY

\(G_{30}\) : CATEGORY THREE OF NON COMMUTATIVE SYMMETRY GROUP OF GAUGE THEORY

\(T_{28}\) : CATEGORY ONE OF YANG MILLS THEORY (Theory is applied to various subatomic particle systems and the classification is done based on the parametricization of these systems. There is not a single system known which is not characterized by some properties)

\(T_{29}\) : CATEGORY TWO OF YANG MILLS THEORY

\(T_{30}\) : CATEGORY THREE OF YANG MILLS THEORY

LHS OF THE YANG MILLS THEORY AND RHS OF THE YANG MILLS THEORY, TAKEN TO THE OTHER SIDE THE LHS WOULD DISSIPATE THE RHS WITH OR WITHOUT TIME LAG:

MODULE NUMBERED SIX:

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} \text{Tr}(F^2) - \frac{1}{4} f_{\mu\nu}^{\alpha\beta} F_{\mu\nu}^{\alpha\beta}
\]

---

\(G_{32}\) : CATEGORY ONE OF LHS OF YANG MILLS THEORY
\begin{align*}
G_{33} & : \text{CATEGORY TWO OF LHS OF YANG MILLS THEORY} \\
G_{34} & : \text{CATEGORY THREE OF LHS OF YANG MILLS THEORY} \\
T_{32} & : \text{CATEGORY ONE OF RHS OF YANG MILLS THEORY} \\
T_{33} & : \text{CATEGORY TWO OF RHS OF YANG MILLS THEORY} \\
T_{34} & : \text{CATEGORY THREE OF RHS OF YANG MILLS THEORY (Theory applied to various characterized systems and the systemic characterizations form the basis for the formulation of the classification).}
\end{align*}

\[
\begin{align*}
& (a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{13}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, a_{16}^{(2)}, a_{17}^{(2)}, a_{18}^{(2)}), \\
& (b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}, a_{20}^{(3)}, a_{21}^{(3)}, a_{22}^{(3)}, b_{20}^{(3)}, b_{21}^{(3)}, b_{22}^{(3)}), \\
& (a_{24}^{(4)}, a_{25}^{(4)}, a_{26}^{(4)}, b_{24}^{(4)}, b_{25}^{(4)}, b_{26}^{(4)}, b_{28}^{(5)}, b_{29}^{(5)}, b_{30}^{(5)}), \\
& (a_{28}^{(5)}, a_{30}^{(5)}, a_{32}^{(6)}, a_{33}^{(6)}, a_{34}^{(6)}, b_{32}^{(6)}, b_{33}^{(6)}, b_{34}^{(6)}), \\
& \text{are Accentuation coefficients} \\
& \begin{align*}
& (a_{13}^{(1)}, a_{14}^{(1)}, a_{15}^{(1)}, b_{13}^{(1)}, b_{14}^{(1)}, b_{15}^{(1)}, a_{16}^{(2)}, a_{17}^{(2)}, a_{18}^{(2)}), \\
& (b_{16}^{(2)}, b_{17}^{(2)}, b_{18}^{(2)}, a_{20}^{(3)}, a_{21}^{(3)}, a_{22}^{(3)}, b_{20}^{(3)}, b_{21}^{(3)}, b_{22}^{(3)}), \\
& (a_{24}^{(4)}, a_{25}^{(4)}, a_{26}^{(4)}, b_{24}^{(4)}, b_{25}^{(4)}, b_{26}^{(4)}, b_{28}^{(5)}, b_{29}^{(5)}, b_{30}^{(5)}), \\
& (a_{28}^{(5)}, a_{30}^{(5)}, a_{32}^{(6)}, a_{33}^{(6)}, a_{34}^{(6)}, b_{32}^{(6)}, b_{33}^{(6)}, b_{34}^{(6)}), \\
& \text{are Dissipation coefficients}
\end{align*}
\]

**SYMMETRY BREAKING AND ACQUISITION OF MASS:**

**MODULE NUMBERED ONE**

The differential system of this model is now (Module Numbered one)

\[
\begin{align*}
\frac{dg_{13}}{dt} & = (a_{13}^{(1)} G_{14} - [(a_{13}^{(1)} + (a_{13}^{(1)} \tau_{14} + t))] G_{13} \\
\frac{dg_{14}}{dt} & = (a_{14}^{(1)} G_{13} - [(a_{14}^{(1)} + (a_{14}^{(1)} \tau_{14} + t))] G_{14} \\
\frac{dg_{15}}{dt} & = (a_{15}^{(1)} G_{14} - [(a_{15}^{(1)} + (a_{15}^{(1)} \tau_{14} + t))] G_{15} \\
\frac{dt_{13}}{dt} & = (b_{13}^{(1)} T_{14} - [(b_{13}^{(1)} + (b_{13}^{(1)} \tau_{14} + t)) G, T] T_{13} \\
\frac{dt_{14}}{dt} & = (b_{14}^{(1)} T_{13} - [(b_{14}^{(1)} - (b_{14}^{(1)} \tau_{14} + t))] G, T] T_{14} \\
\frac{dt_{15}}{dt} & = (b_{15}^{(1)} T_{14} - [(b_{15}^{(1)} - (b_{15}^{(1)} \tau_{14} + t))] G, T] T_{15} \\
+(a_{13}^{(1)} \tau_{14} + t) & = \text{First augmentation factor} \\
-(b_{13}^{(1)} G, T) & = \text{First detritions factor}
\end{align*}
\]

**UNIFIED ELECTROWEAK INTERACTION AND STRONG INTERACTION:**

**MODULE NUMBERED TWO**

www.ijsrp.org
The differential system of this model is now (Module numbered two)

\[
\frac{dG_{16}}{dt} = (a_{16}'(t)G_{17} - ((a_{16}')^2 + (a_{16}'')^2)(T_{17}, t))G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17}'(t)G_{16} - ((a_{17}')^2 + (a_{17}'')^2)(T_{17}, t))G_{17}
\]

\[
\frac{dG_{18}}{dt} = (a_{18}'(t)G_{17} - ((a_{18}')^2 + (a_{18}'')^2)(T_{17}, t))G_{18}
\]

\[
\frac{dT_{16}}{dt} = (b_{16}'(t)T_{17} - ((b_{16}')^2 - (b_{16}'')^2)(G_{19}, t))T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17}'(t)T_{16} - ((b_{17}')^2 - (b_{17}'')^2)(G_{19}, t))T_{17}
\]

\[
\frac{dT_{18}}{dt} = (b_{18}'(t)T_{17} - ((b_{18}')^2 - (b_{18}'')^2)(G_{19}, t))T_{18}
\]

\[+(a_{16}'')^2(T_{17}, t) = \text{First augmentation factor}\]

\[-(b_{16}'')^2(G_{19}, t) = \text{First detritions factor}\]

**LAGRANGIAN INVARIANCE AND CONTINUOUS GROUP OF LOCAL TRANSFORMATIONS:**

**MODULE NUMBERED THREE**

The differential system of this model is now (Module numbered three)

\[
\frac{dG_{20}}{dt} = (a_{20}'(t)G_{21} - ((a_{20}')^3 + (a_{20}'')^3)(T_{21}, t))G_{20}
\]

\[
\frac{dG_{21}}{dt} = (a_{21}'(t)G_{20} - ((a_{21}')^3 + (a_{21}'')^3)(T_{21}, t))G_{21}
\]

\[
\frac{dG_{22}}{dt} = (a_{22}'(t)G_{21} - ((a_{22}')^3 + (a_{22}'')^3)(T_{21}, t))G_{22}
\]

\[
\frac{dT_{20}}{dt} = (b_{20}'(t)T_{21} - ((b_{20}')^3 - (b_{20}'')^3)(G_{23}, t))T_{20}
\]

\[
\frac{dT_{21}}{dt} = (b_{21}'(t)T_{20} - ((b_{21}')^3 - (b_{21}'')^3)(G_{23}, t))T_{21}
\]

\[
\frac{dT_{22}}{dt} = (b_{22}'(t)T_{21} - ((b_{22}')^3 - (b_{22}'')^3)(G_{23}, t))T_{22}
\]

\[+(a_{20}'')^3(T_{21}, t) = \text{First augmentation factor}\]

\[-(b_{20}'')^3(G_{23}, t) = \text{First detritions factor}\]

**GROUP GENERATOR OF GAUGE THEORY AND VECTOR FIELD (GAUGE FIELD):**

**MODULE NUMBERED FOUR**

The differential system of this model is now (Module numbered Four)

\[
\frac{dG_{24}}{dt} = (a_{24}'(t)G_{25} - ((a_{24}')^4 + (a_{24}'')^4)(T_{25}, t))G_{24}
\]
\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[ (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t) \right] G_{25}
\]
\[
\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[ (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t) \right] G_{26}
\]
\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[ (b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}, t) \right] T_{24}
\]
\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[ (b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}, t) \right] T_{25}
\]
\[
\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[ (b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}, t) \right] T_{26}
\]
\[
+ (a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}
\]
\[- (b''_{24})^{(4)}(G_{27}, t) = \text{First detritions factor}
\]

**YANG MILLS THEORY AND NON COMMUTATIVE SYMMETRY GROUP IN GAUGE THEORY:**

**MODULE NUMBERED FIVE**

The differential system of this model is now (Module number five)

\[
\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[ (a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right] G_{28}
\]
\[
\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[ (a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) \right] G_{29}
\]
\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[ (a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) \right] G_{30}
\]
\[
\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[ (b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t) \right] T_{28}
\]
\[
\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[ (b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t) \right] T_{29}
\]
\[
\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[ (b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}, t) \right] T_{30}
\]
\[
+ (a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}
\]
\[- (b''_{28})^{(5)}(G_{31}, t) = \text{First detritions factor}
\]

**LHS OF THE YANG MILLS THEORY AND RHS OF THE YANG MILLS THEORY, TAKEN TO THE OTHER SIDE THE LHS WOULD DISSIPATE THE RHS WITH OR WITHOUT TIME LAG:**

**MODULE NUMBERED SIX**
The differential system of this model is now (Module numbered Six)

\[ \frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \]

\[ \frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \]

\[ \frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \]

\[ \frac{dt_{32}}{dt} = (b_{32})^{(6)}T_{32} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{33}, t)]T_{32} \]

\[ \frac{dt_{33}}{dt} = (b_{33})^{(6)}T_{33} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{33}, t)]T_{33} \]

\[ \frac{dt_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{33}, t)]T_{34} \]

\[ + (a'_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor} \]

\[ -(b''_{32})^{(6)}(G_{33}, t) = \text{First detriments factor} \]

**HOLISTIC CONCATENATE SYTEMAL EQUATIONS HENCEFORTH REFERRED TO AS "GLOBAL EQUATIONS"**

We take in to consideration the following parameters, processes and concepts:

1. Acquisition of mass
2. Symmetry Breaking
3. Strong interaction
4. Unified Electroweak interaction
5. Continuous group of local transformations
6. Lagrangian Variance
7. Group generator in Gauge Theory
8. Vector field or Gauge field
9. Non commutative symmetry group in Gauge Theory
10. Yang Mills Theory (We repeat the same Bank's example. Individual debits and Credits are conservative so also the holistic one. Generalized theories are applied to various systems which are parameterized. And we live in 'measurement world'. Classification is done on the parameters of various systems to which the Theory is applied.)

11. First Term of the Lagrangian of the Yang Mills Theory (LHS)

\[ \mathcal{L}^{(6)} = - \frac{1}{4} \text{Tr}(F^{\mu\nu}) = - \frac{1}{4} F^{\mu\nu} F^{\mu\nu} \]

12. RHS of the Yang Mills Theory

\[ \mathcal{L}^{(6)} = - \frac{1}{4} \text{Tr}(F^{\mu\nu}) = - \frac{1}{4} F^{\mu\nu} F^{\mu\nu} \]

\[ \frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{14} - \left[ \frac{+ (a'_{14})^{(1)}(T_{14}, t) + (a''_{14})^{(2,2)}(T_{17}, t) + (a''_{14})^{(3,3)}(T_{21}, t)}{+ (a''_{14})^{(4,4,4)}(T_{25}, t) + (a''_{14})^{(5,5,5)}(T_{29}, t) + (a''_{14})^{(6,6,6,6)}(T_{33}, t)} \right] G_{14} \]
\[
\begin{align*}
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - & \left[ (a_{14})^{(1)} + (a_{14}^{(2)})(T_{14}, t) + (a_{17}^{(1)})(T_{17}, t) + (a_{21}^{(2,3,4)})(T_{21}, t) \right] G_{14} \\
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - & \left[ (a_{15})^{(1)} + (a_{19}^{(2,3,4)})(T_{19}, t) + (a_{17}^{(1)})(T_{17}, t) + (a_{22}^{(2,3,4)})(T_{22}, t) \right] G_{15} \\
\text{Where} \quad & \left[ (a_{14})^{(1)}(T_{14}, t) \right] + \left[ (a_{14}^{(2,3,4)})(T_{14}, t) \right] = \text{first augmentation coefficients for category 1, 2 and 3} \\
\text{second augmentation coefficient for category 1, 2 and 3} \\
\text{third augmentation coefficient for category 1, 2 and 3} \\
\text{fourth augmentation coefficient for category 1, 2 and 3} \\
\text{fifth augmentation coefficient for category 1, 2 and 3} \\
\text{sixth augmentation coefficient for category 1, 2 and 3} \\
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - & \left[ (b_{13})^{(1)} - (b_{15})^{(2,3)}(G_{15}, t) + (b_{19}^{(1,2,3,4)})(G_{19}, t) - (b_{23}^{(2,3,4)})(G_{23}, t) \right] T_{13} \\
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - & \left[ (b_{14})^{(1)} - (b_{15})^{(2,3)}(G_{15}, t) + (b_{19}^{(1,2,3,4)})(G_{19}, t) - (b_{23}^{(2,3,4)})(G_{23}, t) \right] T_{14} \\
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - & \left[ (b_{15})^{(1)} - (b_{15})^{(2,3)}(G_{15}, t) + (b_{19}^{(1,2,3,4)})(G_{19}, t) - (b_{23}^{(2,3,4)})(G_{23}, t) \right] T_{15} \\
\text{first detrition coefficients for category 1, 2 and 3} \\
\text{second detrition coefficients for category 1, 2 and 3} \\
\text{third detrition coefficients for category 1, 2 and 3} \\
\text{fourth detrition coefficients for category 1, 2 and 3} \\
\text{fifth detrition coefficients for category 1, 2 and 3} \\
\text{sixth detrition coefficients for category 1, 2 and 3} \\
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - & \left[ (a_{16})^{(2)} + (a_{17}^{(1,2,3,4)})(T_{17}, t) + (a_{19}^{(1,2,3,4)})(T_{19}, t) + (a_{23}^{(2,3,4)})(T_{23}, t) \right] G_{16} \\
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - & \left[ (a_{17})^{(2)} + (a_{17}^{(1,2,3,4)})(T_{17}, t) + (a_{19}^{(1,2,3,4)})(T_{19}, t) + (a_{23}^{(2,3,4)})(T_{23}, t) \right] G_{17}
\end{align*}
\]
\[
\begin{align*}
\frac{dg_{18}}{dt} &= (a_{18})^{(2)} g_{17} - \left( a'_{18}^{(2)} + (a''_{18})^{(2)} (T_{17}, t) + (a''_{18})^{(1,1)} (T_{14}, t) + (a''_{18})^{(3,3,3)} (T_{21}, t) \right) g_{18} \quad 68 \\
\end{align*}
\]

Where \( + (a''_{18})^{(2)} (T_{17}, t) \), \( + (a''_{18})^{(1,1)} (T_{14}, t) \), and \( + (a''_{18})^{(3,3,3)} (T_{21}, t) \) are first augmentation coefficients for category 1, 2 and 3

\[
\begin{align*}
+ (a''_{18})^{(2)} (T_{17}, t) + (a''_{18})^{(1,1)} (T_{14}, t) + (a''_{18})^{(3,3,3)} (T_{21}, t) \quad 69 \\
\end{align*}
\]

are second augmentation coefficient for category 1, 2 and 3

\[
\begin{align*}
+ (a''_{18})^{(3,3,3)} (T_{21}, t) + (a''_{18})^{(3,3,3)} (T_{21}, t) \quad 69 \\
\end{align*}
\]

are third augmentation coefficient for category 1, 2 and 3

\[
\begin{align*}
+ (a''_{18})^{(4,4,4,4)} (T_{25}, t) + (a''_{18})^{(4,4,4,4)} (T_{25}, t) \quad 70 \\
\end{align*}
\]

are fourth augmentation coefficient for category 1, 2 and 3

\[
\begin{align*}
+ (a''_{18})^{(5,5,5,5,5)} (T_{25}, t) + (a''_{18})^{(5,5,5,5,5)} (T_{25}, t) \quad 70 \\
\end{align*}
\]

are fifth augmentation coefficient for category 1, 2 and 3

\[
\begin{align*}
+ (a''_{18})^{(5,5,5,5,5)} (T_{25}, t) + (a''_{18})^{(5,5,5,5,5)} (T_{25}, t) \quad 70 \\
\end{align*}
\]

are sixth augmentation coefficient for category 1, 2 and 3

\[
\begin{align*}
\frac{d\tau_1}{dt} &= (b_{16})^{(2)} (T_{17} - \left( b'_{16}^{(2)} - b''_{16}^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \right) \quad 71 \\
\end{align*}
\]

where \( - (b''_{16})^{(2)} (G_{19}, t) \), \( - (b''_{16})^{(1,1)} (G_{19}, t) \), and \( - (b''_{16})^{(3,3,3)} (G_{23}, t) \) are first detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are second detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are second detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are third detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are fourth detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are fifth detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
- (b''_{16})^{(2)} (G_{19}, t) - (b''_{16})^{(1,1)} (G_{19}, t) - (b''_{16})^{(3,3,3)} (G_{23}, t) \quad 72 \\
\end{align*}
\]

are sixth detrition coefficients for category 1, 2 and 3

\[
\begin{align*}
\frac{d\sigma_{20}}{dt} &= (a_{20})^{(3)} g_{21} - \left( a'_{20}^{(3)} + (a''_{20})^{(3)} (T_{21}, t) + (a''_{20})^{(2,2)} (T_{17}, t) + (a''_{20})^{(1,1,1)} (T_{14}, t) \right) g_{20} \quad 76 \\
\end{align*}
\]

\[
\begin{align*}
\frac{d\sigma_{21}}{dt} &= \quad 77 \\
\end{align*}
\]

www.ijsrp.org
\[
(a_{21})^{(3)} G_{20} = \begin{bmatrix}
(a_{21})^{(2)} + (a_{21})^{(3)} T_{21}, t \\
+ (a_{21})^{(2)} (T_{17}, t) \\
+ (a_{21})^{(1,1,1)} (T_{14}, t) \\
+ (a_{21})^{(1)} (T_{25}, t) \\
+ (a_{21})^{(1)} (5,5,5,5,5) (T_{29}, t) \\
+ (a_{21})^{(1)} (6,6,6,6,6) (T_{33}, t)
\end{bmatrix} G_{21}
\]

\[
\frac{dG_{23}}{dt} =
\begin{bmatrix}
(a_{22})^{(3)} G_{21} - \\
+ (a_{22})^{(3)} (T_{21}, t) \\
+ (a_{22})^{(2,2,2)} (T_{17}, t) \\
+ (a_{22})^{(1,1,1)} (T_{14}, t) \\
+ (a_{22})^{(1)} (T_{25}, t) \\
+ (a_{22})^{(1)} (5,5,5,5,5) (T_{29}, t) \\
+ (a_{22})^{(1)} (6,6,6,6,6) (T_{33}, t)
\end{bmatrix} G_{22}
\]

+ (a_{22})^{(3)} (T_{21}, t) are first augmentation coefficients for category 1, 2 and 3
+ (a_{22})^{(2,2,2)} (T_{17}, t) are second augmentation coefficients for category 1, 2 and 3
+ (a_{22})^{(1,1,1)} (T_{14}, t) are third augmentation coefficients for category 1, 2 and 3
+ (a_{22})^{(4,4,4,4,4)} (T_{25}, t) are fourth augmentation coefficients for category 1, 2 and 3
+ (a_{22})^{(5,5,5,5,5)} (T_{29}, t) are fifth augmentation coefficients for category 1, 2 and 3
+ (a_{22})^{(6,6,6,6,6)} (T_{33}, t) are sixth augmentation coefficients for category 1, 2 and 3

\[
\frac{dG_{20}}{dt} =
\begin{bmatrix}
(b_{20})^{(3)} T_{20} - \\
(b_{20})^{(2,2,2)} (G_{23}, t) \\
(b_{20})^{(1,1,1)} (G_{19}, t) \\
(b_{20})^{(1)} (G_{23}, t) \\
(b_{20})^{(1)} (5,5,5,5,5) (G_{29}, t) \\
(b_{20})^{(1)} (6,6,6,6,6) (G_{33}, t)
\end{bmatrix} T_{20}
\]

\[
\frac{dG_{21}}{dt} =
\begin{bmatrix}
(b_{21})^{(3)} T_{21} - \\
(b_{21})^{(2,2,2)} (G_{23}, t) \\
(b_{21})^{(1,1,1)} (G_{19}, t) \\
(b_{21})^{(1)} (G_{23}, t) \\
(b_{21})^{(1)} (5,5,5,5,5) (G_{29}, t) \\
(b_{21})^{(1)} (6,6,6,6,6) (G_{33}, t)
\end{bmatrix} T_{21}
\]

\[
\frac{dG_{22}}{dt} =
\begin{bmatrix}
(b_{22})^{(3)} T_{22} - \\
(b_{22})^{(2,2,2)} (G_{23}, t) \\
(b_{22})^{(1,1,1)} (G_{19}, t) \\
(b_{22})^{(1)} (G_{23}, t) \\
(b_{22})^{(1)} (5,5,5,5,5) (G_{29}, t) \\
(b_{22})^{(1)} (6,6,6,6,6) (G_{33}, t)
\end{bmatrix} T_{22}
\]

- (b_{20})^{(3)} (G_{23}, t) are first detritions coefficients for category 1, 2 and 3
- (b_{20})^{(2,2,2)} (G_{19}, t) are second detritions coefficients for category 1, 2 and 3
- (b_{20})^{(1,1,1)} (G_{19}, t) are third detritions coefficients for category 1, 2 and 3
- (b_{20})^{(4,4,4,4,4)} (G_{23}, t) are fourth detritions coefficients for category 1, 2 and 3
- (b_{20})^{(5,5,5,5,5)} (G_{29}, t) are fifth detritions coefficients for category 1, 2 and 3
- (b_{20})^{(6,6,6,6,6)} (G_{33}, t) are sixth detritions coefficients for category 1, 2 and 3

www.ijsrp.org
\[
\begin{align*}
\frac{dG_{24}}{dt} &= (a_{24})^4 G_{25} - \\
&= \left( a_{24}'(4) + a_{24}''(4)(T_{25}, t) + a_{28}''(5,5)(T_{29}, t) + a_{32}''(6,6)(T_{33}, t) \right) \ G_{24} \\
\frac{dG_{25}}{dt} &= (a_{25})^4 G_{24} - \\
&= \left( a_{25}'(4) + a_{25}''(4)(T_{25}, t) + a_{29}''(5,5)(T_{29}, t) + a_{33}''(6,6)(T_{33}, t) \right) \ G_{25} \\
\frac{dG_{26}}{dt} &= (a_{26})^4 G_{25} - \\
&= \left( a_{26}'(4) + a_{26}''(4)(T_{25}, t) + a_{30}''(5,5)(T_{30}, t) + a_{34}''(6,6)(T_{34}, t) \right) \ G_{26}
\end{align*}
\]

Where \(a_{24}'(4)(T_{26}, t), a_{24}''(4)(T_{26}, t), a_{25}'(4)(T_{26}, t)\) are first augmentation coefficients for category 1, 2 and 3

\[
\begin{align*}
\frac{dT_{24}}{dt} &= (b_{24})^4 T_{25} - \\
&= \left( b_{24}'(4) - b_{24}''(4)(G_{27}, t) - b_{30}''(5,5)(G_{31}, t) - b_{34}''(6,6)(G_{35}, t) \right) \ T_{24} \\
\frac{dT_{25}}{dt} &= (b_{25})^4 T_{24} - \\
&= \left( b_{25}'(4) - b_{25}''(4)(G_{27}, t) - b_{31}''(5,5)(G_{31}, t) - b_{35}''(6,6)(G_{35}, t) \right) \ T_{25} \\
\frac{dT_{26}}{dt} &= (b_{26})^4 T_{25} - \\
&= \left( b_{26}'(4) - b_{26}''(4)(G_{27}, t) - b_{30}''(5,5)(G_{31}, t) - b_{34}''(6,6)(G_{35}, t) \right) \ T_{26}
\end{align*}
\]

Where \(-b_{24}'(4)(G_{27}, t), -b_{24}''(4)(G_{27}, t), -b_{25}'(4)(G_{27}, t)\) are first detraction coefficients for category 1, 2 and 3

\[
\begin{align*}
-b_{24}''(5,5)(G_{31}, t), -b_{25}''(5,5)(G_{32}, t), -b_{26}''(5,5)(G_{32}, t) \ 
are second detraction coefficients for category 1, 2 and 3
-b_{24}''(6,6)(G_{31}, t), -b_{25}''(6,6)(G_{32}, t), -b_{26}''(6,6)(G_{32}, t) \ 
are third detraction coefficients for category 1, 2 and 3
-b_{24}''(1,1,1)(G_{32}, t), -b_{25}''(1,1,1)(G_{33}, t), -b_{26}''(1,1,1)(G_{33}, t) \ 
are fourth detraction coefficients for category 1, 2 and 3
-b_{24}''(2,2,2)(G_{32}, t), -b_{25}''(2,2,2)(G_{33}, t), -b_{26}''(2,2,2)(G_{33}, t) \ 
are fifth detraction coefficients for category 1, 2 and 3
-b_{24}''(3,3,3)(G_{32}, t), -b_{25}''(3,3,3)(G_{33}, t), -b_{26}''(3,3,3)(G_{33}, t) \ 
are sixth detraction coefficients for category 1, 2 and 3
\end{align*}
\]

www.ijsrp.org
\[
\frac{dg_{28}}{dt} = (a_{28})^{(5)}g_{29} - \left[ (a'_{28})^{(3)} + (a'_{29})^{(3)}(T_{29}, t) + (a''_{24})^{(4,4)}(T_{25}, t) + (a''_{22})^{(6,6)}(T_{33}, t) \right] \frac{dG_{28}}{dt} \]

\[
\frac{dg_{29}}{dt} = (a_{29})^{(5)}g_{28} - \left[ (a'_{29})^{(3)} + (a'_{30})^{(3)}(T_{29}, t) + (a''_{25})^{(4,4)}(T_{25}, t) + (a''_{23})^{(6,6)}(T_{33}, t) \right] \frac{dG_{29}}{dt} \]

\[
\frac{dg_{30}}{dt} = (a_{30})^{(5)}g_{29} - \left[ (a'_{30})^{(3)} + (a'_{31})^{(3)}(T_{29}, t) + (a''_{26})^{(4,4)}(T_{25}, t) + (a''_{24})^{(6,6)}(T_{33}, t) \right] \frac{dG_{30}}{dt} \]

Where \[+ (a'_{28})^{(3)}(T_{29}, t) + (a'_{29})^{(3)}(T_{29}, t) + (a'_{30})^{(3)}(T_{29}, t)\] are first augmentation coefficients for category 1, 2 and 3.

And \[+ (a''_{24})^{(4,4)}(T_{25}, t) + (a''_{22})^{(6,6)}(T_{25}, t)\] are second augmentation coefficients for category 1, 2 and 3.

\[+ (a''_{25})^{(4,4)}(T_{25}, t) + (a''_{23})^{(6,6)}(T_{25}, t)\] are third augmentation coefficient for category 1, 2 and 3.

\[+ (a''_{26})^{(4,4)}(T_{25}, t) + (a''_{24})^{(6,6)}(T_{25}, t)\] are fourth augmentation coefficients for category 1, 2, and 3.

\[+ (a''_{27})^{(4,4)}(T_{25}, t) + (a''_{25})^{(6,6)}(T_{25}, t)\] are fifth augmentation coefficients for category 1, 2, and 3.

\[+ (a''_{28})^{(4,4)}(T_{25}, t) + (a''_{26})^{(6,6)}(T_{25}, t)\] are sixth augmentation coefficients for category 1, 2, and 3.

\[
\frac{dt_{28}}{dt} = (b_{28})^{(5)}G_{29} - \left[ (b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t) - (b''_{29})^{(4,4)}(G_{27}, t) - (b''_{22})^{(6,6)}(G_{35}, t) \right] T_{28} \]

\[
\frac{dt_{29}}{dt} = (b_{29})^{(5)}G_{28} - \left[ (b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t) - (b''_{25})^{(4,4)}(G_{27}, t) - (b''_{23})^{(6,6)}(G_{35}, t) \right] T_{29} \]

\[
\frac{dt_{30}}{dt} = (b_{30})^{(5)}G_{29} - \left[ (b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}, t) - (b''_{26})^{(4,4)}(G_{27}, t) - (b''_{24})^{(6,6)}(G_{35}, t) \right] T_{30} \]

Where \[- (b'_{28})^{(5)}(G_{31}, t) \] for category 1, 2 and 3.

\[+ (b''_{24})^{(4,4)}(G_{27}, t)\] are second detrition coefficients for category 1, 2 and 3.

\[+ (b''_{22})^{(6,6)}(G_{35}, t)\] are third detrition coefficients for category 1, 2 and 3.

\[+ (b''_{26})^{(4,4)}(G_{27}, t)\] are fourth detrition coefficients for category 1, 2, and 3.

\[+ (b''_{24})^{(6,6)}(G_{35}, t)\] are fifth detrition coefficients for category 1, 2, and 3.

\[+ (b''_{28})^{(4,4)}(G_{27}, t)\] are sixth detrition coefficients for category 1, 2, and 3.

www.ijsrp.org
\[
\frac{dG_{32}}{dt} = (a_{32})^6 G_{33} - \left[ (a_{32}''')^6 (T_{33}, t) + (a_{20}'')^5,5,5 (T_{29}, t) + (a_{24}'')^4,4,4 (T_{25}, t) \right] G_{32} + \left[ (a_{14}''')^4,1,1,1,1 (T_{14}, t) + (a_{16}'')^2,2,2,2,2 (T_{17}, t) + (a_{22}'')^3,3,3,3,3 (T_{21}, t) \right] G_{33}
\]

\[
\frac{dG_{33}}{dt} = (a_{33})^6 G_{32} - \left[ (a_{33}''')^6 (T_{33}, t) + (a_{20}'')^5,5,5 (T_{29}, t) + (a_{24}'')^4,4,4 (T_{25}, t) \right] G_{33} + \left[ (a_{14}''')^4,1,1,1,1 (T_{14}, t) + (a_{16}'')^2,2,2,2,2 (T_{17}, t) + (a_{22}'')^3,3,3,3,3 (T_{21}, t) \right] G_{34}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^6 G_{33} - \left[ (a_{34}''')^6 (T_{33}, t) + (a_{20}'')^5,5,5 (T_{29}, t) + (a_{24}'')^4,4,4 (T_{25}, t) \right] G_{34} + \left[ (a_{14}''')^4,1,1,1,1 (T_{14}, t) + (a_{16}'')^2,2,2,2,2 (T_{17}, t) + (a_{22}'')^3,3,3,3,3 (T_{21}, t) \right] G_{35}
\]

\[+ (a_{20}'')^3,5,5 (T_{25}, t) + (a_{20}'')^5,5,5 (T_{29}, t) \text{ are first augmentation coefficients for category 1, 2 and 3}
\]

\[+ (a_{20}'')^4,4,4 (T_{25}, t) + (a_{20}'')^5,5,5 (T_{29}, t) \text{ are second augmentation coefficients for category 1, 2 an}
\]

\[+ (a_{20}'')^4,4,4 (T_{25}, t) + (a_{20}'')^4,4,4 (T_{29}, t) \text{ are third augmentation coefficients for category 1, 2 an}
\]

\[+ (a_{14}''')^4,1,1,1,1 (T_{14}, t) + (a_{16}'')^2,2,2,2,2 (T_{16}, t) \text{ - are fourth augmentation coefficients}
\]

\[+ (a_{22}''')^4,1,1,1,1 (T_{21}, t) + (a_{16}'')^2,2,2,2,2 (T_{17}, t) \text{ - fifth augmentation coefficients}
\]

\[+ (a_{20}'')^4,4,4,4 (T_{21}, t) + (a_{22}''')^4,1,1,1,1 (T_{21}, t) \text{ sixth augmentation coefficients}
\]

\[
\frac{dT_{32}}{dt} = (b_{32})^6 T_{33} - \left[ (b_{32}'')^6 (G_{33}, t) - (b_{29}'')^5,5,5 (G_{31}, t) - (b_{24}'')^4,4,4 (G_{27}, t) \right] T_{32} - \left[ (b_{13}'')^4,1,1,1,1 (G, t) - (b_{16}'')^2,2,2,2,2 (G_{19}, t) - (b_{20}'')^3,3,3,3,3 (G_{23}, t) \right] T_{33}
\]

\[
\frac{dT_{33}}{dt} = (b_{33})^6 T_{32} - \left[ (b_{33}'')^6 (G_{33}, t) - (b_{29}'')^5,5,5 (G_{31}, t) - (b_{24}'')^4,4,4 (G_{27}, t) \right] T_{33} - \left[ (b_{14}'')^4,1,1,1,1 (G, t) - (b_{17}'')^2,2,2,2,2 (G_{19}, t) - (b_{21}'')^3,3,3,3,3 (G_{23}, t) \right] T_{34}
\]

\[
\frac{dT_{34}}{dt} = (b_{34})^6 T_{33} - \left[ (b_{34}'')^6 (G_{33}, t) - (b_{29}'')^5,5,5 (G_{31}, t) - (b_{24}'')^4,4,4 (G_{27}, t) \right] T_{34} - \left[ (b_{15}'')^4,1,1,1,1 (G, t) - (b_{18}'')^2,2,2,2,2 (G_{19}, t) - (b_{22}'')^3,3,3,3,3 (G_{23}, t) \right] T_{35}
\]

www.ijsrp.org
Where we suppose

(A) \((a_i)^{(1)}, (a_i^* )^{(1)}, (b_i)^{(1)}, (b_i^* )^{(1)} > 0, i, j = 13,14,15\)

(B) The functions \((a_i^* )^{(1)}, (b_i^* )^{(1)}\) are positive continuous increasing and bounded.

Definition of \((p_i)^{(1)}, (r_i)^{(1)}:\)

\[(a_i^* )^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\tilde{A}_{13})^{(1)}\]

\[(b_i^* )^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i)^{(1)} \leq (\tilde{B}_{13})^{(1)}\]

(C) \(\lim_{T_{14} \to 0} (a_i^* )^{(1)}(T_{14}, t) = (p_i)^{(1)}\)

\(\lim_{G \to 0} (b_i^* )^{(1)}(G, t) = (r_i)^{(1)}\)

Definition of \((\tilde{A}_{13})^{(1)}, (\tilde{B}_{13})^{(1)}:\)

Where \([\tilde{A}_{13}]^{(1)}(1), (\tilde{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants and \(i = 13,14,15\)

They satisfy Lipschitz condition:

\[|(a_i^* )^{(1)}(T_{14}, t) - (a_i^* )^{(1)}(T_{14}, t)| \leq (\tilde{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-\tilde{A}_{13}^{(1)}t}\]

\[|(b_i^* )^{(1)}(G, t) - (b_i^* )^{(1)}(G, t)| < (\tilde{k}_{13})^{(1)}|G - G'|e^{-\tilde{B}_{13}^{(1)}t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^* )^{(1)}(T_{14}, t)\) and \((a_i^* )^{(1)}(T_{14}, t), (b_i^* )^{(1)}(T_{14}, t)\) and \((T_{14}, t)\) are points belonging to the interval \([\tilde{k}_{13}]^{(1)}, (\tilde{M}_{13})^{(1)}\). It is to be noted that \((a_i^* )^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\tilde{M}_{13})^{(1)} = 1\) then the function \((a_i^* )^{(1)}(T_{14}, t)\), the first augmentation coefficient WOULD be absolutely continuous.

Definition of \((\tilde{M}_{13})^{(1)}, (\tilde{k}_{13})^{(1)}:\)

(D) \((\tilde{M}_{13})^{(1)}, (\tilde{k}_{13})^{(1)}\), are positive constants

\[\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} \leq \frac{(b_j)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1\]

Definition of \((\tilde{P}_{13})^{(1)}, (\tilde{Q}_{13})^{(1)}:\)

www.ijsrp.org
There exists two constants \((\hat{P}_{13})^{(2)}\) and \((\hat{Q}_{13})^{(2)}\) which together with \((\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}\) and \((\hat{B}_{16})^{(2)}\) and the constants \((a_{i})^{(2)}, (a_{i}')^{(2)}, (b_{i})^{(2)}, (b_{i}')^{(2)}, (p_{i})^{(2)}, (r_{i})^{(2)}, i = 13, 14, 15\), satisfy the inequalities
\[
\frac{1}{(\hat{M}_{16})^{(2)}} \left[ (a_{i})^{(1)} + (a_{i}')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(2)} \right] < 1
\]
\[
\frac{1}{(\hat{M}_{16})^{(2)}} \left[ (b_{i})^{(1)} + (b_{i}')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(2)} \right] < 1
\]
Where we suppose
\[
(a_{i})^{(2)}, (a_{i}')^{(2)}, (a_{i}'')^{(2)}, (b_{i})^{(2)}, (b_{i}')^{(2)}, (b_{i}'')^{(2)} > 0, \quad i, j = 16, 17, 18
\]
The functions \((a_{i}'')^{(2)}, (b_{i}'')^{(2)}\) are positive continuous increasing and bounded.

**Definition of \((p_{i})^{(2)}, (r_{i})^{(2)}:\)**

\[
(a_{i}'')^{(2)}(T_{17}, t) \leq (p_{i})^{(2)} \leq (\hat{A}_{16})^{(2)}
\]
\[
(b_{i}'')^{(2)}(G_{19}, t) \leq (r_{i})^{(2)} \leq (\hat{B}_{16})^{(2)}
\]

**Definition of \((\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}:\)**

Where \((\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_{i})^{(2)}, (r_{i})^{(2)}\) are positive constants and \(i = 16, 17, 18\)

They satisfy Lipschitz condition:
\[
|(a_{i}'')^{(2)}(T_{17}, t) - (a_{i}'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'| e^{-(\hat{A}_{16})^{(2)}t}
\]
\[
|(b_{i}'')^{(2)}(G_{19}, t) - (b_{i}'')^{(2)}(G_{19}, t)| \leq (\hat{k}_{16})^{(2)}|G_{19} - G_{19}'| e^{-(\hat{B}_{16})^{(2)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{i}'')^{(2)}(T_{17}, t)\) and \((a_{i}'')^{(2)}(T_{17}, t)\) \(T_{17}, t\). And \((T_{17}, t)\) are points belonging to the interval \([T_{17}, T_{17}']\). It is to be noted that \((a_{i}'')^{(2)}(T_{17}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{16})^{(2)} = 1\) then the function \((a_{i}'')^{(2)}(T_{17}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of \((\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}:\)**

\[
\frac{(a_{i})^{(2)}}{(\hat{M}_{16})^{(2)}} = \frac{(b_{i})^{(2)}}{(\hat{M}_{16})^{(2)}} < 1
\]

**Definition of \((\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}:\)**

There exists two constants \((\hat{P}_{16})^{(2)}\) and \((\hat{Q}_{16})^{(2)}\) which together with \((\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}\) and \((\hat{B}_{16})^{(2)}\) and the constants \((a_{i})^{(2)}, (a_{i}')^{(2)}, (b_{i})^{(2)}, (b_{i}')^{(2)}, (p_{i})^{(2)}, (r_{i})^{(2)}, i = 16, 17, 18\), satisfy the inequalities
\[ \frac{1}{(M_{16})^{2}} [ (a_{i})^{2} + (a'_{i})^{2} + (\tilde{A}_{16})^{2} + (\tilde{P}_{16})^{2} (\tilde{k}_{16})^{2}] < 1 \]

\[ \frac{1}{(M_{16})^{2}} [ (b_{i})^{2} + (b'_{i})^{2} + (\tilde{B}_{16})^{2} + (\tilde{Q}_{16})^{2} (\tilde{k}_{16})^{2}] < 1 \]

Where we suppose

(i) \[ (a_{i})^{(3)}, (a'_{i})^{(3)}, (a''_{i})^{(3)}, (b_{i})^{(3)}, (b'_{i})^{(3)}, (b''_{i})^{(3)} > 0, \quad i, j = 20, 21, 22 \]

The functions \( (a''_{i})^{(3)}, (b''_{i})^{(3)} \) are positive continuous increasing and bounded.

**Definition of** \( (p_{i})^{(3)}, (r_{i})^{(3)} \):

\[ (a''_{i})^{(3)}(T_{21}, t) \leq (p_{i})^{(3)} \leq (\hat{A}_{20})^{(3)} \]

\[ (b''_{i})^{(3)}(G_{23}, t) \leq (r_{i})^{(3)} \leq (\hat{B}_{20})^{(3)} \]

\[ \lim_{T_{21} \to 0} (a''_{i})^{(3)}(T_{21}, t) = (p_{i})^{(3)} \]

\[ \lim_{G_{23} \to 0} (b''_{i})^{(3)}(G_{23}, t) = (r_{i})^{(3)} \]

**Definition of** \( (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)} \):

\[ (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_{i})^{(3)}, (r_{i})^{(3)} \]

They satisfy Lipschitz condition:

\[ \{(a_{i})^{(3)}(T_{21}, t) - (a''_{i})^{(3)}(T_{21}, t) \} \leq (\hat{k}_{20})^{(3)}|T_{21} - T'_{21}|e^{-(\tilde{k}_{20})^{(3)}t} \]

\[ \{(b_{i})^{(3)}(G_{23}, t) - (b''_{i})^{(3)}(G_{23}, t) \} < (\hat{G}_{20})^{(3)}|G_{23} - G'_{23}|e^{-(\tilde{G}_{20})^{(3)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a''_{i})^{(3)}(T_{21}, t) \) and \( (b''_{i})^{(3)}(G_{23}, t) \) . \( (T_{21}, t) \) and \( (G_{23}, t) \) are points belonging to the interval \( (\hat{k}_{20})^{(3)}, (\tilde{k}_{20})^{(3)} \). It is to be noted that if \( (\hat{k}_{20})^{(3)} = 1 \) then the function \( (a''_{i})^{(3)}(T_{21}, t) \) , the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of** \( (\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)} \):

(K) \( (\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)} \) are positive constants

\[ \frac{(a_{i})^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_{i})^{(3)}}{(\hat{M}_{20})^{(3)}} < 1 \]

There exists two constants \( (\hat{P}_{20})^{(3)} \) and \( (\hat{Q}_{20})^{(3)} \) which together with

\( (\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} \) and \( (\hat{B}_{20})^{(3)} \)

satisfy the inequalities

\[ \frac{1}{(\hat{M}_{20})^{(3)}} [ (a_{i})^{(3)} + (b_{i})^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \]

\[ \frac{1}{(\hat{M}_{20})^{(3)}} [ (b_{i})^{(3)} + (b'_{i})^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1 \]

Where we suppose

\( (a_{i})^{(4)}, (a'_{i})^{(4)}, (a''_{i})^{(4)}, (b_{i})^{(4)}, (b'_{i})^{(4)}, (b''_{i})^{(4)} > 0, \quad i, j = 24, 25, 26 \)

(M) The functions \( (a''_{i})^{(4)}, (b''_{i})^{(4)} \) are positive continuous increasing and bounded.
Definition of \((p_i)^{(4)}\), \((r_i)^{(4)}\):

\[
(a_i''(t)T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}
\]
\[
(b_i''(t)(G_{27}), t) \leq (r_i)^{(4)} \leq (\hat{B}_{24})^{(4)}
\]

\(N\)
\[
\lim_{G \rightarrow \infty}(a_i''(t)T_{25}) = (p_i)^{(4)}
\]
\[
\lim_{G \rightarrow \infty}(b_i''(t)(G_{27}), t) = (r_i)^{(4)}
\]

Definition of \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}\):

Where \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}\) are positive constants and \(t = 24, 25, 26\)

They satisfy Lipschitz condition:

\[
|a_i''(t)(T_{25}, t) - a_i''(t)(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}|e^{-(\hat{G}_{24})^{(4)}t}
\]
\[
|b_i''(t)(G_{27}, t) - b_i''(t)(G_{27}, t)| \leq (\hat{k}_{24})^{(4)}|G_{27} - G_{27}|e^{-(\hat{G}_{24})^{(4)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i''(t)(T_{25}, t)\) and\((a_i''(t)(T_{25}, t)\) . \((T_{25}, t)\) and \((T_{25}, t)\) . \((T_{25}, t)\) . It is to be noted that \((a_i''(t)(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{24})^{(4)} = 4\) then the function \((a_i''(t)(T_{25}, t)\), the FOURTH augmentation coefficient WOULD be absolutely continuous.

Definition of \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}\):

\((\hat{M}_{24})^{176^{(4)}}, (\hat{k}_{24})^{(4)}\), are positive constants

\[
\frac{(a_i)}{M_{24}}^{(4)} \cdot \frac{(b_i)}{M_{24}}^{(4)} < 1
\]

Definition of \((\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}\):

\(Q\)

There exists two constants \((\hat{P}_{24})^{(4)}\) and \((\hat{Q}_{24})^{(4)}\) which together with \((\hat{M}_{24})^{(4)}, (\hat{K}_{24})^{(4)}, (\hat{A}_{24})^{(4)}\), \((\hat{B}_{24})^{(4)}\) and \((p_i)^{(4)}, (r_i)^{(4)}\), \(i = 24, 25, 26\), the constants satisfy the inequalities

\[
\frac{1}{(\hat{M}_{24})^{(4)}}[a_i^{(4)} + a_i^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{K}_{24})^{(4)}] < 1
\]
\[
\frac{1}{(\hat{M}_{24})^{(4)}}[b_i^{(4)} + b_i^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{K}_{24})^{(4)}] < 1
\]

We suppose

\[
(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \ i, j = 28, 29, 30
\]

The functions \((a_i''(t)^{(5)}), (b_i''(t)^{(5)})\) are positive continuous increasing and bounded.

Definition of \((p_i)^{(5)}, (r_i)^{(5)}\):
\[(a_i^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}\]

\[(b_i')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}\]

\[(T) \quad \lim_{t \to \infty}(a_i^{(5)}(T_{29}, t) = (p_i)^{(5)}\]

\[
\lim_{t \to \infty}(b_i')^{(5)}(G_{31}, t) = (r_i)^{(5)}
\]

**Definition of** \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}\):

Where \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\) are positive constants and \(i = 28, 29, 30\)

They satisfy Lipschitz condition:

\[|(a_i^{(5)}(T'_{29}, t) - (a_i^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T'_{29}|e^{-((\hat{a}_{28})^{(5)})t}\]

\[|(b_i')^{(5)}((G_{31})', t) - (b_i')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}|((G_{31}) - (G_{31}))|e^{-((\hat{b}_{28})^{(5)})t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(5)}(T_{29}, t), (a_i^{(5)}(T_{29}, t), (T_{29}, t)\) and \((b_i')^{(5)}(T_{29}, t)\) and \((T_{29}, t)\) are points belonging to the interval \([0, \hat{k}_{28}]^{(5)}, (\hat{A}_{28})^{(5)}\). It is to be noted that \((a_i^{(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{28})^{(5)} = 5\) then the function \((a_i^{(5)}(T_{29}, t)\), the fifth augmentation coefficient attributable would be absolutely continuous.

**Definition of** \((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}\):

\[(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}\], are positive constants

\[(a_{28})^{(5)}, (b_{28})^{(5)} < 1\]

**Definition of** \((\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}\):

There exists two constants \((\hat{P}_{28})^{(5)}\) and \((\hat{Q}_{28})^{(5)}\) which together with \((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}\) and \((\hat{B}_{28})^{(5)}\) and the constants \((a_{28})^{(5)}, (a_{28})^{(5)}, (b_{28})^{(5)}, (b_{28})^{(5)}, (p_{28})^{(5)}, (r_{28})^{(5)}\), \(i = 28, 29, 30\), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[a_{i}^{(5)}) + (a_{j}^{(5)} + (A_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1
\]

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[b_{i}^{(5)} + (b_{j}^{(5)} + (B_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1
\]

Where we suppose

\[(a_{i})^{(6)}, (a_{j})^{(6)}, (a_{i}^{(5)})^{(6)}, (b_{i})^{(6)}, (b_{j})^{(6)}, (b_{i}^{(5)})^{(6)} > 0, \quad i, j = 32, 33, 34\]

(W) The functions \((a_{i}^{(5)})^{(6)}, (b_{i}^{(5)})^{(6)}\) are positive continuous increasing and bounded.

**Definition of** \((p_{i})^{(6)}, (r_{i})^{(6)}\):

\[(a_{i}^{(6)}(T, t) \leq (p_{i})^{(6)} \leq (A_{32})^{(6)}\]

\[(b_{i}^{(6)}((G, t) \leq (r_{i})^{(6)} \leq (b_{i}^{(6)} \leq (B_{32})^{(6)}\]

\[(X) \quad \lim_{t \to \infty}(a_{i}^{(6)}(T_{33}, t) = (p_{i})^{(6)}\]

www.ijsrp.org
Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution 191

Definition of $G(0), T(0)$

$G(0) \leq (\hat{A}_2)(\hat{A}_2, \hat{Q}_2), G(0) = G_0 > 0$

$T(0) \leq (\hat{Q}_3)(\hat{Q}_3, \hat{Q}_2), T(0) = T_0 > 0$

There exist two constants $(\hat{Q}_2)(\hat{Q}_2, \hat{Q}_2)$ and $(\hat{Q}_3)(\hat{Q}_3, \hat{Q}_3)$ which together with the constants $(\hat{A}_2)(\hat{A}_2, \hat{A}_2)$ and $(\hat{Q}_2)(\hat{Q}_2, \hat{Q}_2)$ satisfy the inequalities:

$\frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} < 1$

Then $\hat{Q}_2 < T_0, \hat{Q}_3 < T_0$. The SIXTH augmentation coefficient $\hat{G}$ is defined as

$\hat{G} = \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} < 1$

With the Lipschitz condition, we place a restriction on the behavior of functions $(A_{2\nu})^{(0)}(T_0, T_0)$ and $(Q_{\nu})^{(0)}(T_0, T_0)$, which together with the interval $[G_2^{(0)}(T_0, T_0)] = (T_0, T_0)$, is uniformly continuous. In the eventuality of the fact that $(A_{2\nu})^{(0)}(T_0, T_0)$ is uniformly continuous, the SIXTH augmentation coefficient $\hat{G}$ is defined as

$\hat{G} = \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} + \frac{(A_{2\nu})^{(0)}}{(Q_2)^{(0)}} < 1$

They satisfy Lipschitz condition:

$\lim_{\nu \to \infty} (A_{2\nu})^{(0)}(T_0, T_0) = (\hat{A}_2)(\hat{A}_2, \hat{Q}_2)$

where $(\hat{A}_2)(\hat{A}_2, \hat{Q}_2)$ are positive constants and $\hat{Q}_2 = T_0$.
\[ T_i(t) \leq (\hat{\Omega}_{i16})^{(2)}e^{(\hat{\Omega}_{i16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0 \]

\[ G_i(t) \leq (\hat{P}_{20})^{(3)}e^{(\hat{\Omega}_{i20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{20})^{(3)}e^{(\hat{\Omega}_{i20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) : \)

\[ G_i(t) \leq (\hat{P}_{24})^{(4)}e^{(\hat{\Omega}_{i24})^{(4)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{24})^{(4)}e^{(\hat{\Omega}_{i24})^{(4)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) : \)

\[ G_i(t) \leq (\hat{P}_{28})^{(5)}e^{(\hat{\Omega}_{i28})^{(5)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{28})^{(5)}e^{(\hat{\Omega}_{i28})^{(5)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) : \)

\[ G_i(t) \leq (\hat{P}_{32})^{(6)}e^{(\hat{\Omega}_{i32})^{(6)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{32})^{(6)}e^{(\hat{\Omega}_{i32})^{(6)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{13})^{(1)}(1), \quad T_i^0 \leq (\hat{Q}_{13})^{(1)}(1), \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)}(1)e^{(\hat{\Omega}_{i13})^{(1)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)}(1)e^{(\hat{\Omega}_{i13})^{(1)}t} \]

By

\[ \tilde{G}_{13}(t) = G_{13}^0 + \int_{0}^{t} \left[ (a_{13})^{(1)}G_{14}(s_{13}) - \left( (a_{13})^{(1)} + a_{13}''^{(1)}(1)(T_{14}(s_{13}), s_{13})) \right) G_{13}(s_{13}) \right] ds_{13} \]

\[ \tilde{G}_{14}(t) = G_{14}^0 + \int_{0}^{t} \left[ (a_{14})^{(1)}G_{13}(s_{13}) - \left( (a_{14})^{(1)} + (a_{14}')^{(1)}(T_{14}(s_{13}), s_{13))) \right) G_{14}(s_{13}) \right] ds_{13} \]

\[ \tilde{G}_{15}(t) = G_{15}^0 + \int_{0}^{t} \left[ (a_{15})^{(1)}G_{14}(S_{13}) - \left( (a_{15})^{(1)} + (a_{15}')^{(1)}(T_{14}(s_{13}), s_{13))) \right) G_{15}(s_{13}) \right] ds_{13} \]

\[ \tilde{T}_{13}(t) = T_{13}^0 + \int_{0}^{t} \left[ (b_{13})^{(1)}T_{14}(S_{13}) - \left( (b_{13})^{(1)} - (b_{13}')^{(1)}(G(s_{13}), s_{13))) \right) T_{14}(s_{13}) \right] ds_{13} \]
$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left( (b_{14}(1))_1 T_{14}(s) - \left( (b_{14}^1(1)) - (b_{14}^2(1))(G(s), s) \right) T_{14}(s) \right) ds$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left( (b_{15}(1))_1 T_{15}(s) - \left( (b_{15}^1(1)) - (b_{15}^2(1))(G(s), s) \right) T_{15}(s) \right) ds$$

Where $s$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $A^{(2)}$ defined on the space of sextuples of continuous functions $G, T: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (P_i)_2, T_i^0 \leq (\hat{Q}_i)_2,$$

$$0 \leq G_i(t) - G_i^0 \leq (P_i)_2 e^{(\hat{R}_i)_2 t},$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_i)_2 e^{(\hat{R}_i)_2 t},$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left( (a_{16}(1))_1 G_{16}, (s) - \left( (a_{16}^1(1)) + (a_{16}^2(1))(T_{16}(s), s) \right) G_{16}(s) \right) ds$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left( (a_{17}(1))_1 G_{16}, (s) - \left( (a_{17}^1(1)) + (a_{17}^2(1))(T_{16}(s), s) \right) G_{17}(s) \right) ds$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left( (a_{18}(1))_1 G_{16}, (s) - \left( (a_{18}^1(1)) + (a_{18}^2(1))(T_{16}(s), s) \right) G_{18}(s) \right) ds$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left( (b_{16}(1))_1 T_{16}, (s) - \left( (b_{16}^1(1)) - (b_{16}^2(1))(G(s), s) \right) T_{16}(s) \right) ds$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left( (b_{17}(1))_1 T_{17}, (s) - \left( (b_{17}^1(1)) - (b_{17}^2(1))(G(s), s) \right) T_{17}(s) \right) ds$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left( (b_{18}(1))_1 T_{18}, (s) - \left( (b_{18}^1(1)) - (b_{18}^2(1))(G(s), s) \right) T_{18}(s) \right) ds$$

Where $s$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $A^{(3)}$ defined on the space of sextuples of continuous functions $G, T: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (P_i)_3, T_i^0 \leq (\hat{Q}_i)_3,$$

$$0 \leq G_i(t) - G_i^0 \leq (P_i)_3 e^{(\hat{R}_i)_3 t},$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_i)_3 e^{(\hat{R}_i)_3 t},$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left( (a_{20}(1))_1 G_{20}, (s) - \left( (a_{20}^1(1)) + (a_{20}^2(1))(T_{20}, (s), s) \right) G_{20}, (s) \right) ds$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left( (a_{21}(1))_1 G_{20}, (s) - \left( (a_{21}^1(1)) + (a_{21}^2(1))(T_{20}, (s), s) \right) G_{21}, (s) \right) ds$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left( (a_{22}(1))_1 G_{20}, (s) - \left( (a_{22}^1(1)) + (a_{22}^2(1))(T_{20}, (s), s) \right) G_{22}, (s) \right) ds$$
Consider operator $\mathcal{A}^{(a)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\mathcal{P}_{24})^{(a)}, T_i^0 \leq (\mathcal{Q}_{24})^{(a)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\mathcal{P}_{24})^{(a)}e^{(\mathcal{G}_{24})^{(a)}t},$$

$$0 \leq T_i(t) - T_i^0 \leq (\mathcal{Q}_{24})^{(a)}e^{(\mathcal{G}_{24})^{(a)}t}.$$

By

$$\dot{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(a)}G_{24}(s_{24}) - (a_{24}^{(a)})^{(a)}(T_{24}(s_{24}), s_{24}) \right] G_{24}(s_{24}) ds_{24},$$

$$\dot{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(a)}G_{25}(s_{25}) - (a_{25}^{(a)})^{(a)}(T_{25}(s_{25}), s_{25}) \right] G_{25}(s_{25}) ds_{25},$$

$$\dot{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(a)}G_{26}(s_{26}) - (a_{26}^{(a)})^{(a)}(T_{26}(s_{26}), s_{26}) \right] G_{26}(s_{26}) ds_{26},$$

$$\ddot{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(a)}T_{24}(s_{24}) - (b_{24}^{(a)})^{(a)}(G(s_{24}), s_{24}) \right] T_{24}(s_{24}) ds_{24},$$

$$\ddot{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(a)}T_{25}(s_{25}) - (b_{25}^{(a)})^{(a)}(G(s_{25}), s_{25}) \right] T_{25}(s_{25}) ds_{25},$$

$$\ddot{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(a)}T_{26}(s_{26}) - (b_{26}^{(a)})^{(a)}(G(s_{26}), s_{26}) \right] T_{26}(s_{26}) ds_{26},$$

Where $s_{24}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(b)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\mathcal{P}_{24})^{(b)}, T_i^0 \leq (\mathcal{Q}_{24})^{(b)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\mathcal{P}_{24})^{(b)}e^{(\mathcal{G}_{24})^{(b)}t},$$

$$0 \leq T_i(t) - T_i^0 \leq (\mathcal{Q}_{24})^{(b)}e^{(\mathcal{G}_{24})^{(b)}t}.$$

By

$$\dot{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(b)}G_{28}(s_{28}) - (a_{28}^{(b)})^{(b)}(T_{28}(s_{28}), s_{28}) \right] G_{28}(s_{28}) ds_{28},$$

$$\dot{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(b)}G_{29}(s_{29}) - (a_{29}^{(b)})^{(b)}(T_{29}(s_{29}), s_{29}) \right] G_{29}(s_{29}) ds_{29},$$

$$\dot{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(b)}G_{30}(s_{30}) - (a_{30}^{(b)})^{(b)}(T_{30}(s_{30}), s_{30}) \right] G_{30}(s_{30}) ds_{30}.$$
\[ \bar{T}_{28}(t) = T_{28}^0 + \int_0^t (b_{28}^0(T_{28}(s_{28}) - (b_{28}^0 + (b_{28}^0)G(s_{28}), s_{28})) d s_{28} \]

\[ \bar{T}_{29}(t) = T_{29}^0 + \int_0^t (b_{29}^0(T_{29}(s_{29}) - (b_{29}^0 + (b_{29}^0)G(s_{29}), s_{29})) d s_{29} \]

\[ \bar{T}_{30}(t) = T_{30}^0 + \int_0^t (b_{30}^0(T_{30}(s_{30}) - (b_{30}^0 + (b_{30}^0)G(s_{30}), s_{30})) d s_{30} \]

Where \( s_{28} \), \( s_{29} \), and \( s_{30} \) are the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(6)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\bar{P}_{32})^{(6)}, \quad T_i^0 \leq (\bar{Q}_{32})^{(6)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (P_{32})^{(6)}e^{(H_{32})^{(6)}}t, \]

\[ 0 \leq T_i(t) - T_i^0 \leq (Q_{32})^{(6)}e^{(H_{32})^{(6)}}t. \]

By

\[ G_{32}(t) = G_{32}^0 + \int_0^t [(a_{32})^{(6)}G_{33}(s_{32}) - ((a_{32})^{(6)} + a_{32}^{(6)})(T_{33}(s_{32}), s_{32})) G_{32}(s_{32})] d s_{32} \]

\[ G_{33}(t) = G_{33}^0 + \int_0^t [(a_{33})^{(6)}G_{32}(s_{32}) - ((a_{33})^{(6)} + a_{33}^{(6)})(T_{33}(s_{32}), s_{32})) G_{33}(s_{32})] d s_{32} \]

\[ G_{34}(t) = G_{34}^0 + \int_0^t [(a_{34})^{(6)}G_{33}(s_{32}) - ((a_{34})^{(6)} + a_{34}^{(6)})(T_{33}(s_{32}), s_{32})) G_{34}(s_{32})] d s_{32} \]

\[ T_{32}(t) = T_{32}^0 + \int_0^t [(b_{32})^{(6)}G_{33}(s_{32}) - ((b_{32})^{(6)} + (b_{32}^{(6)})(T_{33}(s_{32}), s_{32})) T_{32}(s_{32})] d s_{32} \]

\[ T_{33}(t) = T_{33}^0 + \int_0^t [(b_{33})^{(6)}G_{32}(s_{32}) - ((b_{33})^{(6)} + (b_{33}^{(6)})(T_{33}(s_{32}), s_{32})) T_{33}(s_{32})] d s_{32} \]

Where \( s_{32} \) is the integrand that is integrated over an interval \((0, t)\)

(a) The operator \( \mathcal{A}^{(1)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself.

Indeed it is obvious that

\[ G_{13}(t) \leq G_{13}^0 + \int_0^t [(a_{13})^{(1)}(G_{14}^0 + (\bar{P}_{13}^{(1)})e^{(H_{13})^{(1)}})] d s_{13} = \]

\[ 1 + (a_{13})^{(1)}G_{14}^0 + (a_{13})^{(1)}(\bar{P}_{13}^{(1)})e^{(H_{13})^{(1)}} - 1 \]

From which it follows that

\[ (G_{13}(t) - G_{13}^0)e^{-(H_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}(\bar{P}_{13}^{(1)})G_{14}^0}{(H_{13})^{(1)}} \left[ (\bar{P}_{13}^{(1)})^{(1)} + G_{14}^0e^{-\frac{G_{14}^{(1)}(a_{13})^{(1)}\bar{P}_{13}^{(1)}}{G_{14}^{(1)}}} \right] \]

\( G_{13}^0 \) is as defined in the statement of theorem 1

www.ijsrp.org
Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$.

(b) The operator $A^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)}(t) \left( G_{17}^0 + (\tilde{P}_{16}^{(2)}) e^{(\tilde{R}_{16}^{(2)}) t} \right) \right] \
\times \left( e^{(\tilde{R}_{16}^{(2)}) t} - 1 \right) \quad dS_{16} = \left( 1 + (a_{16})^{(2)}t \right) G_{17}^0 + \frac{(a_{16})^{(2)}(t)}{(\tilde{R}_{16}^{(2)})} \left( e^{(\tilde{R}_{16}^{(2)}) t} - 1 \right)$$

From which it follows that

$$\left( G_{16}(t) - G_{16}^0 \right) e^{-((\tilde{R}_{16}^{(2)}) t)} \leq \frac{(a_{16})^{(2)}(t)}{(\tilde{R}_{16}^{(2)})^2} \left[ \left( (\tilde{P}_{16}^{(2)}) + G_{17}^0 \right) e^{(\tilde{R}_{16}^{(2)}) t} + (\tilde{P}_{16}^{(2)}) \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$.

(a) The operator $A^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)}(t) \left( G_{21}^0 + (\tilde{P}_{20}^{(3)}) e^{(\tilde{R}_{20}^{(3)}) t} \right) \right] \
\times \left( e^{(\tilde{R}_{20}^{(3)}) t} - 1 \right) \quad dS_{20} = \left( 1 + (a_{20})^{(3)}t \right) G_{21}^0 + \frac{(a_{20})^{(3)}(t)}{(\tilde{R}_{20}^{(3)})} \left( e^{(\tilde{R}_{20}^{(3)}) t} - 1 \right)$$

From which it follows that

$$\left( G_{20}(t) - G_{20}^0 \right) e^{-((\tilde{R}_{20}^{(3)}) t)} \leq \frac{(a_{20})^{(3)}(t)}{(\tilde{R}_{20}^{(3)})^2} \left[ \left( (\tilde{P}_{20}^{(3)}) + G_{21}^0 \right) e^{(\tilde{R}_{20}^{(3)}) t} + (\tilde{P}_{20}^{(3)}) \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$.

(b) The operator $A^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)}(t) \left( G_{25}^0 + (\tilde{P}_{24}^{(4)}) e^{(\tilde{R}_{24}^{(4)}) t} \right) \right] \
\times \left( e^{(\tilde{R}_{24}^{(4)}) t} - 1 \right) \quad dS_{24} = \left( 1 + (a_{24})^{(4)}t \right) G_{25}^0 + \frac{(a_{24})^{(4)}(t)}{(\tilde{R}_{24}^{(4)})} \left( e^{(\tilde{R}_{24}^{(4)}) t} - 1 \right)$$

From which it follows that

$$\left( G_{24}(t) - G_{24}^0 \right) e^{-((\tilde{R}_{24}^{(4)}) t)} \leq \frac{(a_{24})^{(4)}(t)}{(\tilde{R}_{24}^{(4)})^2} \left[ \left( (\tilde{P}_{24}^{(4)}) + G_{25}^0 \right) e^{(\tilde{R}_{24}^{(4)}) t} + (\tilde{P}_{24}^{(4)}) \right]$$

($G_0^0$) is as defined in the statement of theorem 1.

(c) The operator $A^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)}(t) \left( G_{29}^0 + (\tilde{P}_{28}^{(5)}) e^{(\tilde{R}_{28}^{(5)}) t} \right) \right] \
\times \left( e^{(\tilde{R}_{28}^{(5)}) t} - 1 \right) \quad dS_{28} = \left( 1 + (a_{28})^{(5)}t \right) G_{29}^0 + \frac{(a_{28})^{(5)}(t)}{(\tilde{R}_{28}^{(5)})} \left( e^{(\tilde{R}_{28}^{(5)}) t} - 1 \right)$$

From which it follows that
Indeed if we denote

\[(G_{28}(t) - G^0_{28})a_{(\mathcal{A}_{28})^{(5)}t} \leq (a_{28})^{(5)} \left[ (\mathcal{P}_{28})^{(5)} + G^0_{29} \frac{e^{-\mathcal{A}_{28}}}{e_{28}^t} + (\mathcal{P}_{28})^{(5)} \right] \]

\((G^0_{1})\) is as defined in the statement of theorem 1

(d) The operator \(\mathcal{A}^{(6)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself.

\[
G_{32}(t) \leq G^0_{32} + \int_0^t \left[ (a_{32})^{(6)} \left( G^0_{33} + (\mathcal{P}_{32})^{(6)}e^{(\mathcal{A}_{32})^{(6)}t} \right) \right] dS_{(32)} =

\left(1 + (a_{32})^{(6)}t\right)G^0_{33} + \frac{(a_{32})^{(6)}(\mathcal{P}_{32})^{(6)}}{(M_{32})^{(6)}} \left( e^{(\mathcal{A}_{32})^{(6)}t} - 1 \right)

From which it follows that

\[
(G_{32}(t) - G^0_{32})e^{-(\mathcal{A}_{32})^{(6)}t} \leq (a_{32})^{(6)} \left[ (\mathcal{P}_{32})^{(6)} + G^0_{33} \frac{e^{-\mathcal{A}_{32}}}{e_{32}^t} + (\mathcal{P}_{32})^{(6)} \right]

\((G^0_{1})\) is as defined in the statement of theorem 6

Analogous inequalities hold also for \(G_{25}, G_{26}, T_{24}, T_{25}, T_{26}\)

It is now sufficient to take \(\frac{(a_{13})^{(1)}(\mathcal{A}_{13})^{(5)}}{(M_{13})^{(5)}} , \frac{(b_{13})^{(1)}}{(M_{13})^{(3)}} < 1\) and to choose \((\mathcal{P}_{13})^{(1)}\) and \((\mathcal{Q}_{13})^{(1)}\) large to have

\[
\frac{(a_{13})^{(1)}(\mathcal{A}_{13})^{(5)}}{(M_{13})^{(3)}} \left[ (\mathcal{P}_{13})^{(1)} + ((\mathcal{P}_{13})^{(1)} + G^0_{j}\frac{e^{-\mathcal{P}_{13}}}{e_{j}^t}) \right] \leq (\mathcal{P}_{13})^{(1)}

\[
\frac{(b_{13})^{(1)}}{(M_{13})^{(3)}} \left[ ((\mathcal{Q}_{13})^{(1)} + T_{j}^0) - \frac{(\mathcal{Q}_{13})^{(1)}}{T_{j}^0} \right] \leq (\mathcal{Q}_{13})^{(1)}

In order that the operator \(\mathcal{A}^{(1)}\) transforms the space of sextuples of functions \(G_i, T_i\) satisfying GLOBAL EQUATIONS into itself

The operator \(\mathcal{A}^{(1)}\) is a contraction with respect to the metric

\[
d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =

\sup_{t \in \mathbb{R}_+} \max_{i} \left| G^{(1)}_i(t) - G^{(2)}_i(t) \right| e^{-\left(\mathcal{A}_{13}\right)^{(1)}t} \sup_{t \in \mathbb{R}_+} \left| T^{(1)}_i(t) - T^{(2)}_i(t) \right| e^{-\left(\mathcal{A}_{13}\right)^{(1)}t}

Indeed if we denote

**Definition of \(\mathcal{G}, \mathcal{T}\):**

\[ (\mathcal{G}, \mathcal{T}) = \mathcal{A}^{(1)}(G,T) \]
It results

\[ |G^{(1)}_{13} - G^{(2)}_{13}| \leq \int_0^t (a_{13})^{(1)} |G^{(1)}_{14} - G^{(2)}_{14}| e^{-(\mathcal{R}_{13})^{(1)} s_{(13)}} e^{(\mathcal{R}_{13})^{(1)} x_{(13)}} dS_{(13)} + \]

\[ \int_0^t ((a_{13}''')^{(1)} (T_{14}^{(1)}, s_{(13)})) |G^{(1)}_{13} - G^{(2)}_{13}| e^{-(\mathcal{R}_{13})^{(1)} s_{(13)}} e^{(\mathcal{R}_{13})^{(1)} x_{(13)}} + \]

\[ (a_{13}''')^{(1)} (T_{14}^{(1)}, s_{(13)})) |G^{(1)}_{13} - G^{(2)}_{13}| e^{-(\mathcal{R}_{13})^{(1)} s_{(13)}} e^{(\mathcal{R}_{13})^{(1)} x_{(13)}} + \]

\[ G^{(2)}_{13} ((a_{13}''')^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a_{13}''')^{(1)} (T_{14}^{(2)}, s_{(13)})) e^{-(\mathcal{R}_{13})^{(1)} s_{(13)}} e^{(\mathcal{R}_{13})^{(1)} x_{(13)}} dS_{(13)} \]

Where \( s_{(13)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ |G^{(1)} - G^{(2)}| e^{-(\mathcal{R}_{13})^{(1)} t} \leq \frac{1}{(\mathcal{R}_{13})^{(1)}} ((a_{13})^{(1)} + (a_{13}''')^{(1)} + (A_{13})^{(1)} + (P_{13})^{(1)} (K_{13})^{(1)}) d \left( (G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{13}''')^{(1)} \) and \((b_{13}''')^{(1)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{13})^{(1)} e^{(\mathcal{R}_{13})^{(1)} t}\) and \((Q_{13})^{(1)} e^{(\mathcal{R}_{13})^{(1)} t}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_i^{(1)}),(b_i^{(1)}) i=13,14,15\) depend only on \( T_{14} \) and respectively on \( G \) (and not on \( t \)) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

\[ G_i(t) \geq G_i^0 e^{\left[ -\int_0^t ((a_i^{(1)} - (a_i'''^{(1)} (T_{14}^{(1)}, s_{(13)}))) dS_{(13)} \right]} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{\left( -(b_i^{(1)} t) \right)} > 0 \text{ for } t > 0 \]

**Definition of** \((\mathcal{M}_{13})^{(1)}\), and \((\mathcal{M}_{13})^{(1)}\) :  

**Remark 3:** If \( G_{13} \) is bounded, the same property will also \( G_{14} \) and \( G_{15} \). Indeed if

\[ G_{13} < (\mathcal{M}_{13})^{(1)} \text{ it follows } \frac{dG_{13}}{dt} \leq (\mathcal{M}_{13})^{(1)} - (a_{14}^{(1)} G_{14} \text{ and by integrating } \]

\[ G_{14} \leq ((\mathcal{M}_{13})^{(1)})^2 = G_{14}^0 + 2(a_{14}^{(1)} ((\mathcal{M}_{13})^{(1)})^t / (a_{14}^{(1)}) \]

In the same way, one can obtain

\[ G_{15} \leq ((\mathcal{M}_{13})^{(1)})^3 = G_{15}^0 + 2(a_{15}^{(1)} ((\mathcal{M}_{13})^{(1)})^t / (a_{15}^{(1)}) \]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13} \), \( G_{15} \) and \( G_{13} \), \( G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} ((b_i^{(1)})^1 (G(t), t)) = (b_i^{(1)} \text{ then } T_{14} \to \infty. \)

**Definition of** \((m)^{(1)}\) and \( \epsilon_1 \):

www.ijsrp.org
Indeed let \( t_1 \) be so that for \( t > t_1 \)
\[
(b_{14}^{(1)}(1) - b_{15}''(1)) G(t, t) < \varepsilon_1, \; T_{13}(t) > (m)^{(1)}
\]
Then \( \frac{d\gamma_1}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14} \) which leads to
\[
T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}
\]
If we take \( t \) such that \( e^{-\varepsilon_1 t} = \frac{1}{2} \) it results
\[
T_{14} \geq \frac{(a_{14})^{(1)}(m)^{(1)}}{2}, \; t = \log \frac{2}{\varepsilon_1}
\]
By taking now \( \varepsilon_1 \) sufficiently small one sees that \( T_{14} \) is unbounded. The same property holds for \( T_{15} \) if \( \lim_{t \to \infty} (b_{15}''(1)) G(t, t) = (b_{15}'(1)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_j)^{(2)}}{(\delta t)^{(2)}} \frac{(b_j)^{(2)}}{(\delta t)^{(2)}} < 1 \) and to choose
\[
(\bar{P}_{16})^{(2)} \quad \text{and} \quad (\bar{Q}_{16})^{(2)}
\]
large to have
\[
\frac{(a_j)^{(2)}}{(\delta t)^{(2)}} \left[ (\bar{P}_{16})^{(2)} + (\bar{P}_{16})^{(2)} + G_j^{(2)} e^{-\frac{(\bar{Q}_{16})^{(2)}(t) + \eta}{t_j}} \right] \leq (\bar{P}_{16})^{(2)}
\]
\[
\frac{(b_j)^{(2)}}{(\delta t)^{(2)}} \left[ (\bar{Q}_{16})^{(2)} + T_j^{(2)} e^{-\frac{(\bar{Q}_{16})^{(2)}(t) + \eta}{t_j}} + (\bar{Q}_{16})^{(2)} \right] \leq (\bar{Q}_{16})^{(2)}
\]

In order that the operator \( \mathcal{A}^{(2)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

The operator \( \mathcal{A}^{(2)} \) is a contraction with respect to the metric
\[
d \left( (G_{19}^{(1)}), (T_{19}^{(1)}), (G_{19}^{(2)}), (T_{19}^{(2)}) \right) =
\sup \max \left| G_i^{(1)}(t) - G_{i}^{(2)}(t) e^{-(\delta t)^{(2)}}, \max \left| T_i^{(1)}(t) - T_i^{(2)}(t) e^{-(\delta t)^{(2)}}, \max \left| T_i^{(1)}(t) - T_i^{(2)}(t) e^{-(\delta t)^{(2)}} \right| \right|
\]
Indeed if we denote

**Definition of** \( \bar{G}_{19}^{(1)}, \bar{T}_{19}^{(1)} : \quad G_{19}^{(1)}, T_{19}^{(1)} = \mathcal{A}^{(2)}(G_{19}^{(1)}, T_{19}^{(1)}) \)

It results
\[
|G_{16}^{(1)} - G_{16}^{(2)}| \leq \int_0^t (a_{16}^{(2)}) |G_i^{(1)} - G_{i}^{(2)}| e^{-(\delta t)^{(2)}} s_{(16)} \quad +
\int_0^t \left( (a_{16}^{(2)}) |G_i^{(1)} - G_{i}^{(2)}| e^{-(\delta t)^{(2)}} s_{(16)} \quad +
\left( (a_{16}^{(2)}) |T_i^{(1)}(s_{(16)}) - T_i^{(2)}(s_{(16)})| e^{-(\delta t)^{(2)}} s_{(16)} \quad +
\left( (a_{16}^{(2)}) |T_i^{(1)}(s_{(16)}) - T_i^{(2)}(s_{(16)})| e^{-(\delta t)^{(2)}} s_{(16)} \quad +
\end{array}
\]

Where \( s_{(16)} \) represents integrand that is integrated over the interval \( [0, t] \)

From the hypotheses it follows

www.ijsrp.org
\[ |(G_{19}^{(1)}) - (G_{19}^{(2)})| e^{-(\mathbb{R}^+)^2 t} \leq \frac{1}{(M_{16})^2 \gamma^2 \left( (\alpha_{16})^{(2)} + (\beta_{16})^{(2)} + (A_{16})^{(2)} \right) + (P_{16})^{(2)} (T_{16})^{(2)} \delta \left( (G_{19}^{(1)}), (T_{19})^{(1)}; (G_{19}^{(2)}), (T_{19})^{(2)} \right)} \]

And analogous inequalities for \( G_1 \) and \( T_1 \). Taking into account the hypothesis the result follows.

**Remark 1:** The fact that we supposed \((a_{16}')(2)\) and \((b_{16}')(2)\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{16})^{(2)} e^{(M_{16})^{(2)} t}\) and \((Q_{16})^{(2)} e^{(M_{16})^{(2)} t}\) respectively on \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{16}')(2)\) and \((b_{16}')(2)\), \( i = 16,17,18 \) depend only on \( T_{17} \) and respectively on \( (G_{19}) \) (not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_1 (t) = 0 \) and \( T_1 (t) = 0 \).

From 19 to 24 it results

\[ G_1 (t) \geq G_0^0 \left[ \gamma_1 \left( (\mathbb{M}_{16})^{(2)} \right)_1 \right] - (a_{17})^{(2)} G_{17} \]

\[ T_1 (t) \geq T_0^0 e^{-((b_{17}')(2)) t} > 0 \text{ for } t > 0 \]

**Definition of** \(( (\mathbb{M}_{16})^{(2)} )_1, ( (\mathbb{M}_{16})^{(2)} )_2 \) and \(( (\mathbb{M}_{16})^{(2)} )_3 \):

**Remark 3:** if \( G_{16} \) is bounded, the same property have also \( G_{17} \) and \( G_{18} \). Indeed if

\[ G_{16} < ( (\mathbb{M}_{16})^{(2)} )_1 \]

it follows \( \frac{dG_{12}}{dt} \leq ( (\mathbb{M}_{16})^{(2)} )_1 - (a_{17})^{(2)} G_{17} \) and by integrating

\[ G_{17} \leq ( (\mathbb{M}_{16})^{(2)} )_2 = G_0^0 + 2(a_{17})^{(2)} ( (\mathbb{M}_{16})^{(2)} )_1 / (a_{17})^{(2)} \]

In the same way, one can obtain

\[ G_{18} \leq ( (\mathbb{M}_{16})^{(2)} )_3 = G_0^0 + 2(a_{18})^{(2)} ( (\mathbb{M}_{16})^{(2)} )_2 / (a_{19})^{(2)} \]

If \( G_{17} \) or \( G_{19} \) is bounded, the same property follows for \( G_{16}, G_{18} \) and \( G_{16}, G_{17} \) respectively.

**Remark 4:** If \( G_{16} \) is bounded, from below, the same property holds for \( G_{17} \) and \( G_{18} \). The proof is analogous to the preceding one. An analogous property is true if \( G_{17} \) is bounded from below.

**Remark 5:** If \( T_{16} \) is bounded from below and \( \lim_{t \to \infty} ((b_{17}')^{(2)} ((G_{19}) (t), t)) = (b_{17}')^{(2)} \) then \( T_{17} \to \infty \).

**Definition of** \((m)^{(2)}\) and \( \varepsilon_2 \):

Indeed let \( t_2 \) be so that for \( t > t_2 \)

\[ (b_{17})^{(2)} - (b_{17}')^{(2)} ((G_{19}) (t), t) < \varepsilon_2, T_{16} (t) > (m)^{(2)} \]

Then \( \frac{dT_{17}}{dt} \geq (a_{17})^{(2)} (m)^{(2)} - \varepsilon_2 T_{17} \) which leads to

\[ T_{17} \geq \frac{(a_{17})^{(2)} (m)^{(2)} \varepsilon_2}{1 - e^{-\varepsilon_2 t} + T_{17} e^{-\varepsilon_2 t} } \] If we take \( t \) such that \( e^{-\varepsilon_2 t} = \frac{1}{2} \) it results

\[ T_{17} \geq \frac{(a_{17})^{(2)} (m)^{(2)} \varepsilon_2}{2} \]

\[ t = \log \frac{2}{\varepsilon_2} \] By taking now \( \varepsilon_2 \) sufficiently small one sees that \( T_{17} \) is unbounded. The same property holds for \( T_{18} \) if \( \lim_{t \to \infty} (b_{18}')^{(2)} ((G_{19}) (t), t) = (b_{18}')^{(2)} \)

www.ijsrp.org
We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_2)^{(3)}}{(M_20)^{(3)}} \), \( \frac{(b_2)^{(3)}}{(M_20)^{(3)}} < 1 \) and to choose \( (\bar{P}_{20})^{(3)} \) and \( (\bar{Q}_{20})^{(3)} \) large to have

\[
\frac{(a_2)^{(3)}}{(M_20)^{(3)}} \left[ (\bar{P}_{20})^{(3)} + \left( (\bar{P}_{20})^{(3)} + G_j^{(3)} e^{-\frac{(P_{20})^{(3)} + G_j^{(3)}}{G_j^{(3)}}} \right) \right] \leq (\bar{P}_{20})^{(3)}
\]

\[
\frac{(b_2)^{(3)}}{(M_20)^{(3)}} \left[ (\bar{Q}_{20})^{(3)} + T_j^{(3)} e^{-\frac{(Q_{20})^{(3)} + T_j^{(3)}}{T_j^{(3)}}} \right] \leq (\bar{Q}_{20})^{(3)}
\]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(3)} \) is a contraction with respect to the metric

\[
d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) =
\]

\[
\sup_{t \in \mathbb{R}^+} \max \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-(\bar{M}_{20})^{(3)} t} + \max \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-(\bar{M}_{20})^{(3)} t}
\]

Indeed if we denote

**Definition of** \( \bar{G}_{23}, \bar{T}_{23} : (\bar{G}_{23}, (\bar{T}_{23})) = \mathcal{A}^{(3)}((G_{23}), (T_{23})) \)

It results

\[
\left| \bar{G}_2^{(2)} - \bar{G}_1^{(2)} \right| \leq \int_0^t (a_{20})^{(3)} \left| G_{21}^{(1)} - G_{21}^{(2)} \right| e^{-(\bar{M}_{20})^{(3)} s_{20}} e^{(\bar{M}_{20})^{(3)} s_{20}} ds_{20} + \int_0^t (a_{20})^{(3)} \left| e_{20}^{(1)} - e_{20}^{(2)} \right| e^{-(\bar{M}_{20})^{(3)} s_{20}} e^{(\bar{M}_{20})^{(3)} s_{20}} ds_{20} + \int_0^t (a_{20})^{(3)} \left| G_{20}^{(2)} - G_{20}^{(2)} \right| e^{-(\bar{M}_{20})^{(3)} s_{20}} e^{(\bar{M}_{20})^{(3)} s_{20}} ds_{20} + \int_0^t (a_{20})^{(3)} \left| (\bar{T}_{21})^{(1)} s_{20} - (\bar{T}_{21})^{(2)} s_{20} \right| e^{-(\bar{M}_{20})^{(3)} s_{20}} e^{(\bar{M}_{20})^{(3)} s_{20}} ds_{20} \]

Where \( s_{20} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| G^{(1)} - G^{(2)} \right| e^{-(\bar{M}_{20})^{(3)} t} \leq \frac{1}{(\bar{M}_{20})^{(3)}} \left( (a_{20})^{(3)} + (a_{20})^{(3)} + (\bar{A}_{20})^{(3)} + (\bar{P}_{20})^{(3)} \right) d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( (a_{20})^{(3)} \) and \( (b_{20})^{(3)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}} \) and \( (Q_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{'})^{(3)} \) and \( (b_i^{'})^{(3)} \), \( i = 20, 21, 22 \) depend only on \( T_{21} \) and respectively on
Remark 2: There does not exist any $t$ where $G_1(t) = 0$ and $T_1(t) = 0$.

From 19 to 24 it results

$$G_1(t) \geq G_1^0 e^{- \int_0^t \left( a_1'(s) - \frac{a_1''(s)}{a_1'(s)} \left( \frac{e}{a_1'(s)} \right) ds \right)} \geq 0$$

$$T_1(t) \geq T_1^0 e^{- \left( a_1'(s) - \frac{a_1''(s)}{a_1'(s)} \left( \frac{e}{a_1'(s)} \right) ds \right)} > 0 \quad \text{for } t > 0$$

Definition of $(\overline{M}_{20})^{(3)}_1$, $(\overline{M}_{20})^{(3)}_2$ and $(\overline{M}_{20})^{(3)}_3$:

Remark 3: If $G_{20}$ is bounded, the same property holds for $\overline{G}_{21}$ and $\overline{G}_{22}$ - indeed if $G_{20} < (\overline{M}_{20})^{(3)}_1$ it follows $\frac{dG_{21}}{dt} \leq \left( (\overline{M}_{20})^{(3)}_1 - (a_{21})^{(3)} \right) G_{21}$ and by integrating

$$G_{21} \leq \left( (\overline{M}_{20})^{(3)}_1 - (a_{21})^{(3)} \right) G_{21} + Z(a_{21})^{(3)}(\overline{M}_{20})^{(3)}_1 / (a_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq \left( (\overline{M}_{20})^{(3)}_2 - (a_{22})^{(3)} \right) G_{22} + Z(a_{22})^{(3)}(\overline{M}_{20})^{(3)}_2 / (a_{22})^{(3)}$$

If $G_{21}$ or $G_{22}$ is bounded, the same property follows for $G_{20}$, $G_{22}$ and $G_{20}$, $G_{21}$ respectively.

Remark 4: If $G_{20}$ is bounded, from below, the same property holds for $G_{21}$ and $G_{22}$. The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below.

Remark 5: If $T_{20}$ is bounded from below and $\lim_{t \to \infty} \left( (b_3''(t) \overline{G}_{23}(t), t) \right) = (b_{21}''(3))$ then $T_{21} \to \infty$.

Definition of $(m)^{(3)}$ and $\varepsilon_3$:

Indeed let $t_3$ be so that for $t > t_3$

$$(b_{21})''(3) - (b_3''(3) \overline{G}_{23}(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$

If we take $t$ such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) , \quad t = \log \frac{2}{\varepsilon_3}$$

By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded. The same property holds for $T_{22}$ if $\lim_{t \to \infty} (b_{22}'(3)) \left( (G_{23}(t), t) \right) = (b_{22}''(3))$.

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_{21})^{(4)}}{(M_{24})^{(4)}}$, $\frac{(b_3')^{(4)}}{(M_{24})^{(4)}} < 1$ and to choose

$$(\overline{P}_{24})^{(4)}$$ and $$(\overline{Q}_{24})^{(4)}$$ large to have

$$\left( \frac{(a_{21})^{(4)}}{(M_{24})^{(4)}} \right) \left( (\overline{P}_{24})^{(4)} + \left( (\overline{P}_{24})^{(4)} + G_j^0 \right) e^{- \frac{(\overline{P}_{24})^{(4)} + e_0^2}{e_0^2}} \right) \leq \left( \overline{P}_{24} \right)^{(4)}$$

www.ijsrp.org
\[
\frac{\langle b_i \rangle}{\langle a_{24} \rangle} \left[ (\tilde{Q}_{24})^4 + T_i^4 \right] e^{-\left(\frac{\langle Q_{24} \rangle - \langle a_{24} \rangle}{T_i} \right)^2} \leq (\tilde{Q}_{24})^4
\]

In order that the operator \( \mathcal{A} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying IN to itself

The operator \( \mathcal{A} \) is a contraction with respect to the metric

\[
d \left( (G_{27})^{(1)}, (T_{27})^{(1)}, (G_{27})^{(2)}, (T_{27})^{(2)} \right) =
\]

\[
\sup \{ \max_i \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\langle \mathcal{H}_{24} \rangle^4 t}, \max_i |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-\langle \mathcal{H}_{24} \rangle^4 t} \}
\]

Indeed if we denote

**Definition of** \( (\bar{G}_{27}), (\bar{T}_{27}) : (G_{27}), (T_{27}) = \mathcal{A}((G_{27}), (T_{27})) \)

It results

\[
|\bar{G}_{24}^{(1)} - \bar{G}_{24}^{(2)}| \leq \int_0^t |a_{24}'(t)| \left| G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-\langle \mathcal{H}_{24} \rangle^4 s(t)} e^{\langle \mathcal{H}_{24} \rangle^4 s(t)} ds(t) +
\]

\[
\int_0^t \left| (a_{24}''(t)) \left| G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-\langle \mathcal{H}_{24} \rangle^4 s(t)} e^{\langle \mathcal{H}_{24} \rangle^4 s(t)} +
\]

\[
(a_{24}''(t)) \left| (T_{25}^{(1)} - S_{24}(t)) \left| G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-\langle \mathcal{H}_{24} \rangle^4 s(t)} e^{\langle \mathcal{H}_{24} \rangle^4 s(t)} +
\]

\[
G_{24}^{(2)} |a_{24}''(t) S_{24}(t) - (a_{24}''(t) T_{25}^{(2)} + S_{24}(t)) | e^{-\langle \mathcal{H}_{24} \rangle^4 s(t)} e^{\langle \mathcal{H}_{24} \rangle^4 s(t)} ds(t)
\]

Where \( S_{24} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
|G_{27}^{(1)} - G_{27}^{(2)}| e^{-\langle \mathcal{H}_{24} \rangle^4 t} \leq
\]

\[
\frac{1}{\langle a_{24} \rangle} \left( (a_{24})^4 + (a_{24})^4 + (\mathcal{H}_{24})^4 +
\]

\[
(P_{24})^4 \left( \mathcal{H}_{24} \right)^4 d \left( (G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)} \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1**: The fact that we supposed \( (a_{24}''(t)) \) and \( (b_{24}''(t)) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{24})^4 e^{\langle \mathcal{H}_{24} \rangle^4 t} \) and \( (\mathcal{Q}_{24})^4 e^{\langle \mathcal{H}_{24} \rangle^4 t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_{24}''(t)), (b_{24}''(t)) \), \( i = 24, 25, 26 \) depend only on \( T_{25} \) and respectively on \( (G_{27})(and not on t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2**: There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

www.ijsrp.org
\[ G_i(t) \geq G_i^0 e^{-\int_{i}^{t}(a_i'(s) - (a_i''(s)^2)(r_2(s) \cdot s_2(s) d(2s)))} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{-\int_{i}^{t}(a_i'(s)^2)} > 0 \quad \text{for } t > 0 \]

**Definition of** \((\{ M_{2k}\}^{(4)}), (\{ M_{2k}\}^{(4)})_1, (\{ M_{2k}\}^{(4)})_2, (\{ M_{2k}\}^{(4)})_3: \)

**Remark 3:** If \( G_{24} \) is bounded, the same property have also \( G_{25} \) and \( G_{26} \). Indeed if

\[ G_{24} < (\{ M_{2k}\}^{(4)}) \text{ it follows } \frac{dG_{24}}{dt} \leq (\{ M_{2k}\}^{(4)})_1 - (a_{25}'(s)^2)G_{25} \text{ and by integrating } \]

\[ G_{25} < (\{ M_{2k}\}^{(4)})_2 = G_{25}^0 + 2(a_{25}'(s)^2)(\{ M_{2k}\}^{(4)})_2/(a_{25}'(s)^2) \]

In the same way, one can obtain

\[ G_{26} < (\{ M_{2k}\}^{(4)})_3 = G_{26}^0 + 2(a_{26}'(s)^2)(\{ M_{2k}\}^{(4)})_2/(a_{26}'(s)^2) \]

If \( G_{25} \) or \( G_{26} \) is bounded, the same property follows for \( G_{24}, G_{25} \) and \( G_{24}, G_{25} \) respectively.

**Remark 4:** If \( G_{24} \) is bounded, from below, the same property holds for \( G_{25} \) and \( G_{26} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{25} \) is bounded from below.

**Remark 5:** If \( T_{24} \) is bounded from below and \( \lim_{t \to \infty} ((b_{27}')^{(4)}((G_{27})'(t), t)) = (b_{25}')^{(4)} \) then \( T_{25} \to \infty. \)

**Definition of** \((m)^{(4)} \) and \( \epsilon_4 : \)

Indeed let \( t_4 \) be so that for \( t > t_4 \)

\[ (b_{25}')^{(4)} - (b_{27}')^{(4)}((G_{27})'(t), t) < \epsilon_4, T_{24}(t) > (m)^{(4)} \]

Then \[ \frac{d(b_{25}')^{(4)}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \epsilon_4 T_{25} \]

which leads to

\[ T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{\epsilon_4} (1 - e^{-\epsilon_4 t}) + T_{25}^0 e^{-\epsilon_4 t} \]

If we take \( \epsilon_4 \) such that \( e^{-\epsilon_4 t} = \frac{1}{2} \) it results

\[ T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{\frac{2}{\epsilon_4}}, t = \frac{\log 2}{\epsilon_4} \]

By taking now \( \epsilon_4 \) sufficiently small one sees that \( T_{25} \) is unbounded. The same property holds for \( T_{26} \) if \[ \lim_{t \to \infty} ((b_{26}')^{(4)}((G_{27})'(t), t)) = (b_{26}')^{(4)} \]

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{29}, G_{30}, T_{29}, T_{29}, T_30 \)

It is now sufficient to take \[ \frac{(a_j)^{(5)}}{(M_{2k})^{(5)}}, \frac{(b_j)^{(5)}}{(M_{2k})^{(5)}} < 1 \] and to choose

\[ (\bar{P}_{28})^{(5)} \text{ and } (\bar{Q}_{28})^{(5)} \text{ large to have } \]

\[ \frac{(a_j)^{(5)}}{(M_{2k})^{(5)}} [(\bar{P}_{28})^{(5)} + ((\bar{P}_{28})^{(5)} + G_j^0 e^{-(\bar{P}_{28})^{(5)} + G_j^0})] \leq (\bar{P}_{28})^{(5)} \]

\[ \text{www.ijsrp.org} \]
From GLOBAL EQUATIONS it results

Indeed if we denote

\[
\left( \frac{(b_1)^{(5)}}{(28)^{(5)}} \right) \left( ( \dot{Q}_{28} )^{(5)} + T_j^{(5)} e^{-\left( \frac{(Q_{28})^{(5)}}{T_j^{(5)}} \right)} + ( \ddot{Q}_{28} )^{(5)} \right) \leq ( \ddot{Q}_{28} )^{(5)}
\]

In order that the operator \( \mathcal{A}^{(5)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(5)} \) is a contraction with respect to the metric

\[
d \left( \left( (G_{31}^{(1)}, (T_{31}^{(1)})), \left( (G_{31}^{(2)}, (T_{31}^{(2)})) \right) \right) = \right.
\]

\[
\sup \{ \max \limits_{t \in \mathbb{R}^+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) T_j^{(5)}} \max \limits_{t \in \mathbb{R}^+} \left| T_j^{(1)}(t) - T_j^{(2)}(t) \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) t} \}
\]

Indeed if we denote

Definition of \( (\ddot{G}_{31}), (\ddot{T}_{31}) \) : \( \left( (\ddot{G}_{31}), (\ddot{T}_{31}) \right) = \mathcal{A}^{(5)}((G_{31}), (T_{31})) \)

It results

\[
\left| G_{28}^{(1)} - G_i^{(2)} \right| \leq \int_0^t \left( a_{28}^{(5)} \right) \left| G_{29}^{(1)} - G_i^{(2)} \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) T_j^{(5)}} e^{\left( \mathcal{H}_{28}^{(5)} \right) t} ds_{(28)} +
\]

\[
\int_0^t \left( a_2^{(5)} \right) \left| G_{29}^{(1)} - G_i^{(2)} \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) T_j^{(5)}} e^{\left( \mathcal{H}_{28}^{(5)} \right) t} ds_{(28)} +
\]

\[
\left( G_{28}^{(2)} \right) e^{\left( \mathcal{H}_{28}^{(5)} \right) t} \left( s_{(28)} \right) \left| G_{29}^{(1)} - G_i^{(2)} \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) T_j^{(5)}} e^{\left( \mathcal{H}_{28}^{(5)} \right) t} ds_{(28)} +
\]

Where \( s_{(28)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| (G_{31})^{(1)} - (G_{31})^{(2)} \right| e^{-\left( \mathcal{H}_{28}^{(5)} \right) t} \leq \frac{1}{(\mathcal{H}_{28}^{(5)})} \left( a_{28}^{(5)} + (a_2^{(5)}) + (a_2^{(5)}) + (a_2^{(5)}) + (a_2^{(5)}) \right) ds_{(28)} +
\]

\[
(\mathcal{P}_{28}^{(5)}) \left( \mathcal{H}_{28}^{(5)} \right) \left( (G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)} \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed \( (a_2^{(5)}) \) and \( (b_2^{(5)}) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\mathcal{P}_{28}^{(5)}) e^{\left( \mathcal{H}_{28}^{(5)} \right) t} \) and \( (\ddot{Q}_{28}^{(5)}) e^{\left( \mathcal{H}_{28}^{(5)} \right) t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{(5)}) \) and \( (b_i^{(5)}) \), \( i = 28, 29, 30 \) depend only on \( T_{29} \) and respectively on \( (G_{31}) \) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \)

From GLOBAL EQUATIONS it results
\[ G_1(t) \geq G_1^0 \left[ - \int_{t_1}^{t} (a_1^0(t') - (a_1^0(t')) (r_1(s_1(s_2(s_3 s_4))) ds_4(s_3)) ds_2(s_1) \right] \geq 0 \]

\[ T_1(t) \geq T_1^0 e^{-(a_1^0(t))} > 0 \quad \text{for} \quad t > 0 \]

**Definition of** \(( (\mathcal{M}_{29})^{(5)} )_1, ( (\mathcal{M}_{29})^{(5)} )_2 \) and \(( (\mathcal{M}_{29})^{(5)} )_3 \):

**Remark 3:** if \( G_{29} \) is bounded, the same property have also \( G_{29} \) and \( G_{30} \). Indeed if

\[ G_{29} < ( (\mathcal{M}_{29})^{(5)} ) \text{ it follows } \frac{dG_{29}}{dt} \leq \left( ( (\mathcal{M}_{29})^{(5)} )_1 - (a_{29}^0)^{(5)} G_{29} \right) \text{ and by integrating } \]

\[ G_{29} \leq \left( ( (\mathcal{M}_{29})^{(5)} )_1 \right)_t = G_{29}^0 + 2(a_{29}^0) \left( ( (\mathcal{M}_{29})^{(5)} )_1 / (a_{29}^0) \right) \]

In the same way, one can obtain

\[ G_{30} \leq \left( ( (\mathcal{M}_{29})^{(5)} )_3 \right)_t = G_{30}^0 + 2(a_{30}^0) \left( ( (\mathcal{M}_{29})^{(5)} )_3 / (a_{30}^0) \right) \]

If \( G_{29} \) or \( G_{30} \) is bounded, the same property follows for \( G_{29} \), \( G_{30} \) and \( G_{28} \), \( G_{29} \) respectively.

**Remark 4:** if \( G_{29} \) is bounded, from below, the same property holds for \( G_{29} \) and \( G_{30} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{29} \) is bounded from below.

**Remark 5:** if \( T_{29} \) is bounded from below and \( \lim_{t \to \infty} \left( (b_{29}''')^{(5)} \left( (G_{31})(t), t \right) \right) = (b_{29}')^{(5)} \) then \( T_{29} \to \infty \).

**Definition of** \((m)^{(5)}\) and \( \varepsilon_5 \):

Indeed let \( t_5 \) be so that for \( t > t_5 \)

\[ (b_{29}^{(5)}) - (b_{29}''')^{(5)} \left( (G_{31})(t), t \right) < \varepsilon_5, T_{29}(t) > (m)^{(5)} \]

Then \( \frac{dT_{29}}{dt} \geq (a_{29}^{(5)})(m)^{(5)} - \varepsilon_5 T_{29} \) which leads to

\[ T_{29} \geq \frac{(a_{29}^{(5)})(m)^{(5)}}{\varepsilon_5} \left( 1 - e^{-\varepsilon_5} \right) + T_{29}^0 e^{-\varepsilon_5 t} \]

If we take \( t < \frac{1}{2} \) it results

\[ T_{29} \geq \frac{(a_{29}^{(5)})(m)^{(5)}}{\varepsilon_5} \]

\[ t = log \frac{2}{\varepsilon_5} \]

By taking now \( \varepsilon_5 \) sufficiently small one sees that \( T_{29} \) is unbounded. The same property holds for \( T_{39} \) if \( \lim_{t \to \infty} (b_{39}''')^{(5)} \left( (G_{31})(t), t \right) = (b_{39}')^{(5)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions.

Analogous inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \frac{(a_{2}^{(6)})}{(\mathcal{M}_{32})^{(6)}} < 1 \) and to choose \( (P_{32})^{(6)} \) and \( (Q_{32})^{(6)} \) large to have

www.ijsrp.org
From the hypotheses it follows

Indeed if we denote

\[ \text{Definition of } (\overline{G}_{35}, \overline{T}_{35}) : \quad (\overline{G}_{35}, \overline{T}_{35}) = \mathcal{A}^{(6)} ((G_{35}), (T_{35})) \]

It results

\[ |G_{32}^{(1)}(t) - G_{32}^{(2)}(t)| \leq \int_0^t (a_{32}^{(6)}) |G_{33}^{(1)} - G_{33}^{(2)}| e^{-\left( R_{32}^{(6)}(s_{32}) + R_{32}^{(6)}(s_{32}) \right) ds_{32}} + \]

\[ \int_0^t (a_{32}^{(6)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-\left( R_{32}^{(6)}(s_{32}) + R_{32}^{(6)}(s_{32}) \right) ds_{32}} + \]

\[ (a_{32}^{(6)}) |T_{33}^{(1)}(t), s_{32})| e^{-\left( R_{32}^{(6)}(s_{32}) + R_{32}^{(6)}(s_{32}) \right) ds_{32}} - \]

\[ G_{32}^{(2)} |(a_{32}^{(6)}) |T_{33}^{(2)}(t), s_{32})| e^{-\left( R_{32}^{(6)}(s_{32}) + R_{32}^{(6)}(s_{32}) \right) ds_{32}} \]

Where \( s_{32} \) represents integrand that is integrated over the interval \([0, t] \)

From the hypotheses it follows

\[ |(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-\left( R_{32}^{(6)}(t) \right) t} \leq \]

\[ \frac{1}{(a_{32}^{(6)}) + (a_{32}^{(6)}) + (A_{32}^{(6)}) + (P_{32}^{(6)}) (Q_{32}^{(6)})} \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1**: The fact that we supposed \( (a_{32}^{(6)})^{(6)} \) and \( (b_{32}^{(6)})^{(6)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{32}^{(6)}) e^{(R_{32})^{(6)} t} \) and \( (Q_{32}^{(6)}) e^{(R_{32})^{(6)} t} \) respectively of \( R_+ \).

If instead of proving the existence of the solution on \( R_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{(6)})^{(6)} \) and \( (b_i^{(6)})^{(6)} \), \( i = 32, 33, 34 \) depend only on \( T_{33} \) and respectively on \( (G_{35}) \) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2**: There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \)
From 69 to 32 it results
\[ G_i(t) \geq G^0_i e^{-(a_i''(6)) t} \leq 0 \]
\[ T_i(t) \geq T^0_i e^{-(a_i''(6)) t} > 0 \quad \text{for } t > 0 \]

**Definition of** \((\overline{M}_{32})^{(6)}_2, (\overline{M}_{32})^{(6)}_1\) and \((\overline{M}_{32})^{(6)}_3\)

**Remark 3:** If \(G_{32}\) is bounded, the same property holds for \(G_{33} \) and \(G_{34}\). Indeed if
\[ G_{32} < (\overline{M}_{32})^{(6)} \]
then \(\frac{dG_{32}}{dt} \leq \left((\overline{M}_{32})^{(6)} - (a_{33}''(6)) G_{33}\right)\]
and by integrating
\[ G_{33} \leq \left((\overline{M}_{32})^{(6)}_2 = G^0_{33} + 2(a_{33}''(6))(\overline{M}_{32})^{(6)}_1/(a_{33}'(6)) \right. \]
In the same way, one can obtain
\[ G_{34} \leq \left((\overline{M}_{32})^{(6)}_3 = G^0_{34} + 2(a_{34}''(6))(\overline{M}_{32})^{(6)}_2/(a_{34}'(6)) \right. \]
If \(G_{33}\) or \(G_{34}\) is bounded, the same property follows for \(G_{32}, G_{34}\) and \(G_{32}, G_{33}\) respectively.

**Remark 4:** If \(G_{32}\) is bounded, from below, the same property holds for \(G_{33}\) and \(G_{34}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{33}\) is bounded from below.

**Remark 5:** If \(T_{32}\) is bounded from below and \(\lim_{t \to \infty} ((b''_i)^{(6)}((G_{35}(t), t)) = (b''_{33})^{(6)}\) then \(T_{33} \to \infty\).

**Definition of** \((m)^{(6)}\) and \(\varepsilon_6\):

Indeed let \(t_6\) be so that for \(t > t_6\)
\[ (b_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}(t), t)) < \varepsilon_6, T_{32}(t) > (m)^{(6)} \]

Then \(\frac{dT_{32}}{dt} \geq (a_{33})''(6) (m)^{(6)} - \varepsilon_6 T_{33}\) which leads to
\[ T_{33} \geq \left((a_{33}''(6)(m)^{(6)})/\varepsilon_6\right) (1 - e^{-\varepsilon_6 \tau}) + T^0_{33} e^{-\varepsilon_6 \tau} \quad \text{if we take } t \quad \text{such that } e^{-\varepsilon_6 \tau} = \frac{1}{2} \quad \text{it results} \]
\[ T_{33} \geq \left((a_{33}''(6)(m)^{(6)})/2\right), t = \log \frac{2}{t_6} \quad \text{By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is unbounded. The same property holds for } T_{34} \text{ if } \lim_{t \to \infty} (b''_{34})^{(6)}((G_{35}(t), t), t) = (b''_{34})^{(6)} \]

We now state a more precise theorem about the behaviors at infinity of the solutions

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}\):

(a) \((\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}\) four constants satisfying
- (σ2)(1) ≤ -(a′13)(1) + (a′14)(1) - (a′′13)(1)(T14, t) + (a′′14)(1)(T14, t) ≤ -(σ1)(1)
- (τ2)(1) ≤ -(b′13)(1) + (b′14)(1) - (b′′13)(1)(G, t) - (b′′14)(1)(G, t) ≤ -(τ1)(1)

**Definition of** (v1)(1), (v2)(1), (u1)(1), (u2)(1), v(1), u(1):

(b) By (v1)(1) > 0, (v2)(1) < 0 and respectively (u1)(1) > 0, (u2)(1) < 0 the roots of the equations

(a14)(1)(v(1))² + (σ1)(1)v(1) - (a13)(1) = 0 and (b14)(1)(u(1))² + (τ1)(1)u(1) - (b13)(1) = 0

**Definition of** (v̅1)(1), (v̅2)(1), (u̅1)(1), (u̅2)(1):

By (v̅1)(1) > 0, (v̅2)(1) < 0 and respectively (u̅1)(1) > 0, (u̅2)(1) < 0 the roots of the equations

(a14)(1)(v̅(1))² + (σ1)(1)v̅(1) - (a13)(1) = 0 and (b14)(1)(u̅(1))² + (τ1)(1)u̅(1) - (b13)(1) = 0

**Definition of** (m1)(1), (m2)(1), (μ1)(1), (μ2)(1), (v0)(1):

(c) If we define (m1)(1), (m2)(1), (μ1)(1), (μ2)(1) by

(m2)(1) = (v0)(1), (m1)(1) = (v1)(1), if (v0)(1) < (v1)(1)

(m̅2)(1) = (v̅(1)), (m̅1)(1) = (v̅1)(1), if (v̅(1)) < (v̅1)(1),

and

(m2)(1) = (v0)(1), (m1)(1) = (v1)(1), if (v0)(1) < (v1)(1)

and analogously

(μ2)(1) = (u0)(1), (μ1)(1) = (u1)(1), if (u0)(1) < (u1)(1)

(μ̅2)(1) = (u̅(1)), (μ̅1)(1) = (u̅1)(1), if (u̅(1)) < (u̅1)(1),

and

(μ2)(1) = (u0)(1), (μ1)(1) = (u1)(1), if (u0)(1) < (u1)(1) where (u0)(1), (u1)(1)

are defined respectively

Then the solution satisfies the inequalities

\[ G_{13}^0 e^{(s_1(1)-p_{13}(1))t} \leq G_{14}(t) \leq G_{13}^0 e^{(s_1(1))t} \]

where \( (p_1)(1) \) is defined

\[ \frac{1}{(m_1)(1)} \frac{G_{13}^0 e^{(s_1(1)-p_{13}(1))t}}{G_{13}^0 e^{(s_1(1))t}} \leq G_{14}(t) \leq \frac{1}{(m_2)(1)} \frac{G_{13}^0 e^{(s_1(1))t}}{G_{13}^0 e^{(s_1(1)-p_{13}(1))t}} \]

\[ \frac{(a_{13}(1)+G_{13}^0 e^{(s_1(1)-p_{13}(1))t})}{(m_1)(1) ((s_1(1)-p_{13}(1))t)} \frac{G_{13}^0 e^{-(s_1(1))t}}{+ G_{13}^0 e^{-(s_1(1))t}} \leq G_{15}(t) \leq \]

\[ \frac{1}{(m_2)(1) ((s_1(1)-a_{13}(1)))} \frac{G_{13}^0 e^{-(s_1(1))t}}{+ G_{13}^0 e^{-(a_{13}(1))t}} \]

\[ T_{13}^0 e^{(r_1(1))t} \leq T_{13}(t) \leq T_{13}^0 e^{((r_1(1)+r_{13}(1)))t} \]

\[ \frac{1}{(μ_1)(1)} \frac{T_{13}^0 e^{(r_1(1))t}}{T_{13}(t)} \leq T_{13}(t) \leq \frac{1}{(μ_2)(1)} \frac{T_{13}^0 e^{((r_1(1)+r_{13}(1)))t}}{T_{13}(t)} \]
\[
\frac{(R_{13})^{(1)}}{(\mu_{2})^{(3)}}\left[e^{(R_{13})^{(1)}t} - e^{-(b_{13})^{(1)}t}\right] + T_{15}^{(0)}e^{-((b_{13})^{(1)}t)} \leq T_{15}(t) \leq \frac{(a_{15})^{(3)}}{(\mu_{2})^{(3)}}\left[e^{((R_{13})^{(1)}+p_{15})^{(1)}t} - e^{-(b_{13})^{(1)}t}\right] + T_{15}^{(0)}e^{-((b_{13})^{(1)}t)}
\]

**Definition of \((S_{1})^{(1)}, (S_{2})^{(1)}, (R_{1})^{(1)}, (R_{2})^{(1)}: \)**

Where

\[
\begin{align*}
(S_{1})^{(1)} &= (a_{13})^{(1)}(m_{2})^{(1)} - (a_{13})^{(1)} \\
(S_{2})^{(1)} &= (a_{13})^{(1)} - (p_{15})^{(1)} \\
(R_{1})^{(1)} &= (b_{13})^{(1)}(\mu_{2})^{(1)} - (b_{13})^{(1)} \\
(R_{2})^{(1)} &= (b_{13})^{(1)} - (r_{13})^{(1)}
\end{align*}
\]

**Behavior of the solutions**

If we denote and define

\[
\begin{align*}
\sigma \quad : & = \frac{\sigma}{Gb_{36}} \\
\tau \quad : & = \frac{\tau}{Gb_{36}} \\
\nu \quad : & = \frac{\nu}{Gb_{36}} \\
\end{align*}
\]

\[
\begin{align*}
(\sigma_{1})^{(2)}, (\sigma_{2})^{(2)}, (\tau_{1})^{(2)}, (\tau_{2})^{(2)} \quad : & \text{ four constants satisfying} \\
-\sigma(2) \leq & -\left((a'_{16})^{(2)} + (a''_{17})^{(2)} - (a'_{16})^{(2)}(T_{17}^{(1)}, t) + (a''_{17})^{(2)}(T_{17}^{(1)}, t) \leq -\sigma(2)
\end{align*}
\]

\[
\begin{align*}
-\tau(2) \leq & -\left((b'_{16})^{(2)} + (b''_{17})^{(2)} - (b'_{16})^{(2)}((G_{19}^{(1)}, t) - (b''_{17})^{(2)}((G_{19}^{(1)}, t) \leq -\tau(2)
\end{align*}
\]

**Definition of \((v_{1})^{(2)}, (v_{2})^{(2)}, (u_{1})^{(2)}, (u_{2})^{(2)}: \)**

By \((v_{1})^{(2)} > 0, (v_{2})^{(2)} < 0 \) and respectively \((u_{1})^{(2)} > 0, (u_{2})^{(2)} < 0 \) the roots

\[
\begin{align*}
\sigma(2) \quad : & = \sigma(2) \quad \nu(2) \quad : = \nu(2) \\
(\sigma_{1})^{(2)}v(2) + (\sigma_{2})^{(2)}v(2) - (a_{16})^{(2)} = 0 \\
(\sigma_{1})^{(2)}u(2) + (\tau_{1})^{(2)}u(2) - (b_{16})^{(2)} = 0
\end{align*}
\]

**Definition of \((\tilde{v}_{1})^{(2)}, (\tilde{v}_{2})^{(2)}, (\tilde{u}_{1})^{(2)}, (\tilde{u}_{2})^{(2)}: \)**

By \((\tilde{v}_{1})^{(2)} > 0, (\tilde{v}_{2})^{(2)} < 0 \) and respectively \((\tilde{u}_{1})^{(2)} > 0, (\tilde{u}_{2})^{(2)} < 0 \) the roots of the equations

\[
\begin{align*}
(\sigma_{1})^{(2)}v(2) + (\sigma_{2})^{(2)}v(2) - (a_{16})^{(2)} = 0 \\
(\tau_{2})^{(2)}u(2) + (\tau_{2})^{(2)}u(2) - (b_{16})^{(2)} = 0
\end{align*}
\]

**Definition of \((m_{1})^{(2)}, (m_{2})^{(2)}, (\mu_{1})^{(2)}, (\mu_{2})^{(2)}: \)**

\[
(m_{2})^{(2)} = (v_{0})^{(2)}, (m_{1})^{(2)} = (v_{1})^{(2)} \quad \text{if} \ (v_{0})^{(2)} < (v_{1})^{(2)}
\]

\[
(m_{2})^{(2)} = (v_{1})^{(2)}, (m_{1})^{(2)} = (v_{0})^{(2)} \quad \text{if} \ (v_{1})^{(2)} < (v_{0})^{(2)} \quad < (v_{1})^{(2)}
\]

\[
\begin{align*}
(v_{0})^{(2)} &= \frac{c_{01}^{*}}{c_{11}^{*}} \\
(m_{2})^{(2)} = (v_{1})^{(2)}, (m_{1})^{(2)} = (v_{0})^{(2)} \quad \text{if} \ (v_{1})^{(2)} < (v_{0})^{(2)}
\end{align*}
\]

and analogously
\((\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_3)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}\)

\((\mu_2)^{(2)} = (u_2)^{(2)}, (\mu_3)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_2)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}\)

and \(\left(\frac{u_0}{u_0}\right)^{(2)} = \frac{\mu_0}{\mu_0}\)

\((\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_3)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}\)

Then the solution satisfies the inequalities

\[ G_{16}^0 e^{(S_1)^{(2)} - (p_{16})^{(2)} t} \leq G_{16}(t) \leq G_{16} e^{(S_2)^{(2)} t} \]

\((p_i)^{(2)}\) is defined

\[ \frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)} t)} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_2)^{(2)} t} \]

\[ G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)} t)} \leq G_{16}(t) \leq G_{16} e^{(S_2)^{(2)} t} \]

\[ T_{16}^0 e^{((R_1)^{(2)} + (R_2)^{(2)}) t} \leq T_{16}(t) \leq T_{16} e^{((R_1)^{(2)} + (R_2)^{(2)}) t} \]

\[ \frac{1}{(m_1)^{(2)}} \frac{1}{(m_2)^{(2)}} \frac{(p_{16})^{(2)} e^{(R_1)^{(2)} t} - e^{(R_2)^{(2)} t}}{e^{((R_1)^{(2)} + (R_2)^{(2)}) t} + T_{16}^0 e^{(R_2)^{2} t}} \leq T_{16}(t) \leq \frac{1}{(m_1)^{(2)}} \frac{1}{(m_2)^{(2)}} \frac{(p_{16})^{(2)} e^{(R_1)^{(2)} t} - e^{(R_2)^{(2)} t}}{e^{((R_1)^{(2)} + (R_2)^{(2)}) t} + T_{16}^0 e^{(R_2)^{2} t}} \]

**Definition of \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\):**

\[ (S_1)^{(2)} = (a_{16})^{(2)} (m_2)^{(2)} - (a'_{16})^{(2)} \]

\[ (S_2)^{(2)} = (a_{16})^{(2)} - (p_{16})^{(2)} \]

\[ (R_1)^{(2)} = (b_{16})^{(2)} (\mu_3)^{(2)} - (b'_{16})^{(2)} \]

\[ (R_2)^{(2)} = (b'_{16})^{(2)} - (r_{16})^{(2)} \]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):**

(a) \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[ -(\sigma_2)^{(3)} \leq -(a''_{20})^{(3)} + (a''_{21})^{(3)} - (a''_{20})^{(3)} (T_{21}, t) + (a''_{21})^{(3)} (T_{21}, t) \leq -(\sigma_1)^{(3)} \]

\[ -(\tau_2)^{(3)} \leq -(b''_{20})^{(3)} + (b''_{21})^{(3)} - (b''_{20})^{(3)} (G, t) - (b''_{21})^{(3)} (G, t) \leq -(\tau_1)^{(3)} \]

**Definition of \((v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}\):**

(b) By \((v_1)^{(3)} > 0, (v_2)^{(3)} < 0\) and respectively \((u_1)^{(3)} > 0, (u_2)^{(3)} < 0\) the roots of the
equations \((a_{21})^3(v^{(3)})^2 + (\sigma_{1})^{(3)}(v^{(3)}) - (a_{20})^{(3)} = 0\) and 
\((b_{21})^3(u^{(3)})^2 + (\tau_{1})^{(3)}(u^{(3)}) - (b_{20})^{(3)} = 0\) and 

By \((\tilde{v}_{1})^{(3)} > 0, (\tilde{v}_{2})^{(3)} < 0\) and respectively \((\tilde{u}_{1})^{(3)} > 0, (\tilde{u}_{2})^{(3)} < 0\) the 
roots of the equations \((a_{21})^3(v^{(3)})^2 + (\sigma_{2})^{(3)}(v^{(3)}) - (a_{20})^{(3)} = 0\) and 
\((b_{21})^3(u^{(3)})^2 + (\tau_{2})^{(3)}(u^{(3)}) - (b_{20})^{(3)} = 0\)

**Definition of** \((m_{1})^{(3)}, (m_{2})^{(3)}, (\mu_{1})^{(3)}, (\mu_{2})^{(3)}\) :-

(c) If we define \((m_{1})^{(3)}, (m_{2})^{(3)}, (\mu_{1})^{(3)}, (\mu_{2})^{(3)}\) by 

\((m_{2})^{(3)} = (v_{0})^{(3)}, (m_{1})^{(3)} = (v_{1})^{(3)}, if (v_{0})^{(3)} < (v_{1})^{(3)}\) 

\((m_{2})^{(3)} = (v_{1})^{(3)}, (m_{1})^{(3)} = (v_{1})^{(3)}, if (v_{1})^{(3)} < (v_{0})^{(3)} < (v_{1})^{(3)},\) and 
\((V_{0})^{(3)} = \frac{\sigma_{20}}{\sigma_{21}}\)

\((m_{2})^{(3)} = (v_{1})^{(3)}, (m_{1})^{(3)} = (v_{0})^{(3)}, if (\tilde{v}_{1})^{(3)} < (v_{0})^{(3)}\)

and analogously

\((\mu_{2})^{(3)} = (u_{0})^{(3)}, (\mu_{1})^{(3)} = (u_{1})^{(3)}, if (u_{0})^{(3)} < (u_{1})^{(3)}\)

\((\mu_{2})^{(3)} = (u_{1})^{(3)}, (\mu_{1})^{(3)} = (u_{1})^{(3)}, if (u_{1})^{(3)} < (u_{0})^{(3)} < (u_{1})^{(3)},\) and 
\((U_{0})^{(3)} = \frac{\tau_{20}}{\tau_{21}}\)

\((\mu_{2})^{(3)} = (u_{1})^{(3)}, (\mu_{1})^{(3)} = (u_{0})^{(3)}, if (\tilde{u}_{1})^{(3)} < (u_{0})^{(3)}\)

Then the solution satisfies the inequalities

\(G_{20}^{0}(S_{1})^{(3)} - (p_{20})^{(3)}t \leq G_{20}^{0}(S_{1})^{(3)}t\)

\((p_{i})^{(3)}\) is defined

\[-\frac{1}{(m_{1})^{(3)}}G_{20}^{0} e^{((S_{1})^{(3)} - (p_{20})^{(3)}t)} \leq G_{21}^{0}(t) \leq \frac{1}{(m_{2})^{(3)}}G_{20}^{0} e^{((S_{1})^{(3)}t)}\]

\[-\frac{(a_{22})^{(3)}(a_{20})^{(3)} \left[ e^{((S_{1})^{(3)} - (p_{20})^{(3)}t)} - e^{(-(S_{2})^{(3)}t)} \right] + G_{22}^{0} e^{-(S_{2})^{(3)}t} \leq G_{22}^{0}(t) \leq \frac{(a_{22})^{(3)}(a_{20})^{(3)}}{(m_{2})^{(3)}((S_{1})^{(3)} - (a_{22})^{(3)})} [ e^{(S_{1})^{(3)}t} - e^{-((a_{22})^{(3)}t)} ] + G_{22}^{0} e^{-(a_{22})^{(3)}t} \]

\[-T_{20}^{0} e^{(R_{1})^{(3)}t} \leq T_{20}^{0}(t) \leq T_{20}^{0} e^{(R_{1})^{(3)} + (R_{2})^{(3)}t} \]

\[-\frac{(R_{20})^{(3)} G_{20}^{0} \left[ e^{((R_{1})^{(3)} - (b_{20})^{(3)}t)} - e^{-(b_{22})^{(3)}t} \right] + T_{22}^{0} e^{-(b_{22})^{(3)}t} \leq T_{22}^{0}(t) \leq \frac{(R_{20})^{(3)} G_{20}^{0}}{(R_{1})^{(3)}((R_{1})^{(3)} + (R_{20})^{(3)} + (R_{2})^{(3)})} [ e^{((R_{1})^{(3)} + (R_{20})^{(3)}t)} - e^{-((R_{1})^{(3)}t)} ] + T_{22}^{0} e^{-(R_{2})^{(3)}t} \]

**Definition of** \((S_{1})^{(3)}, (S_{2})^{(3)}, (R_{1})^{(3)}, (R_{2})^{(3)}\) :-

Where \((S_{1})^{(3)} = (a_{20})^{(3)}(m_{2})^{(3)} - (a_{20})^{(3)}\)

www.ijsrp.org
Behavior of the solutions

If we denote and define

**Definition of** \( (\sigma_1)^{(d)}, (\sigma_2)^{(d)}, (\tau_1)^{(d)}, (\tau_2)^{(d)} : \)

(d) \( (\sigma_1)^{(d)}, (\sigma_2)^{(d)}, (\tau_1)^{(d)}, (\tau_2)^{(d)} \) four constants satisfying

\[-(\sigma_2)^{(d)} \leq -(a_{24}^{(d)} + (a_{25}^{(d)} - (a_{25}^{(d)})(T_{25}, t) + (a_{29}^{(d)}(T_{25}, t) \leq -(\sigma_1)^{(d)} \]

and

\[-(\tau_2)^{(d)} \leq -(b_{24}^{(d)} + (b_{29}^{(d)} - (b_{29}^{(d)}((G_{27}, t) - (b_{29}^{(d)}((G_{27}, t) \leq -(\tau_1)^{(d)} \]

**Definition of** \( (\nu_1)^{(d)}, (\nu_2)^{(d)}, (u_1)^{(d)}, (u_2)^{(d)}, (v)^{(d)}, u^{(d)} : \)

(e) By \( (\nu_2)^{(d)} > 0, (\nu_2)^{(d)} < 0 \) and respectively \( (u_1)^{(d)} > 0, (u_2)^{(d)} < 0 \) the roots of the equations \( (a_{25}^{(d)})(v)^{(d)} + (\sigma_1)^{(d)}v^{(d)} - (a_{24}^{(d)} = 0 \]

and

\[(b_{25}^{(d)})(u)^{(d)} + (\tau_1)^{(d)}u^{(d)} - (b_{24}^{(d)} = 0 \]

**Definition of** \( (\bar{\nu}_1)^{(d)}, (\bar{\nu}_2)^{(d)}, (\bar{u}_1)^{(d)}, (\bar{u}_2)^{(d)} : \)

By \( (\bar{\nu}_1)^{(d)} > 0, (\bar{\nu}_2)^{(d)} < 0 \) and respectively \( (\bar{u}_1)^{(d)} > 0, (\bar{u}_2)^{(d)} < 0 \) the roots of the equations \( (a_{25}^{(d)})(v)^{(d)} = (\sigma_1)^{(d)}v^{(d)} - (a_{24}^{(d)} = 0 \]

and

\[(b_{25}^{(d)})(u)^{(d)} + (\tau_1)^{(d)}u^{(d)} - (b_{24}^{(d)} = 0 \]

**Definition of** \( (m_1)^{(d)}, (m_2)^{(d)}, (\mu_1)^{(d)}, (\mu_2)^{(d)}, (v_0)^{(d)} : \)

(f) If we define \( (m_1)^{(d)}, (m_2)^{(d)}, (\mu_1)^{(d)}, (\mu_2)^{(d)} \) by

\[(m_2)^{(d)} = (v_0)^{(d)}, (m_1)^{(d)} = (v_1)^{(d)}, if (v_0)^{(d)} < (v_1)^{(d)} \]

\[(m_2)^{(d)} = (v_1)^{(d)}, (m_1)^{(d)} = (\bar{v}_1)^{(d)}, if (v_2)^{(d)} < (v_0)^{(d)} < (\bar{v}_1)^{(d)} \]

and

\[
(m_2)^{(d)} = (v_0)^{(d)}, (m_1)^{(d)} = (v_0)^{(d)}, if (v_2)^{(d)} < (v_0)^{(d)}
\]

and analogously

\[ (\mu_2)^{(d)} = (u_0)^{(d)}, (\mu_1)^{(d)} = (u_1)^{(d)}, if (u_0)^{(d)} < (u_1)^{(d)} \]

\[ (\mu_2)^{(d)} = (u_1)^{(d)}, (\mu_1)^{(d)} = (\bar{u}_1)^{(d)}, if (u_1)^{(d)} < (u_0)^{(d)} < (\bar{u}_1)^{(d)} \]

and

\[
(\mu_2)^{(d)} = (u_0)^{(d)}, (\mu_1)^{(d)} = (u_0)^{(d)}, if (u_0)^{(d)} < (u_0)^{(d)} \]

are defined by 59 and 64 respectively
Then the solution satisfies the inequalities

\[ G_2^0 e^{((s_1)^{(4)} - (p_{24})^{(4)}_t) t} \leq G_2^0 e^{(s_1)^{(4)}_t} \leq G_2^0 e^{(s_2)^{(4)}_t} \]

where \((p_i)^{(4)}\) is defined

\[ \frac{1}{(m_1)^{(4)}} G_2^0 e^{((s_1)^{(4)} - (p_{24})^{(4)}_t) t} \leq G_2^{(4)}(t) \leq \frac{1}{(m_2)^{(4)}} G_2^0 e^{((s_2)^{(4)}_t) - (s_1)^{(4)}_t)} + G_2^0 e^{-(s_2)^{(4)}_t} \leq G_2^{(4)}(t) \leq \]

\[ \frac{(p_{26})^{(4)} G_2^{(4)}}{(m_1)^{(4)} ((s_1)^{(4)} - (p_{24})^{(4)}_t) - (s_2)^{(4)}_t)} \left[ e^{((s_1)^{(4)} - (p_{24})^{(4)}_t) t} - e^{-(s_2)^{(4)}_t)} + G_2^0 e^{-(s_2)^{(4)}_t} \left] + \frac{(p_{26})^{(4)} G_2^{(4)}}{(m_2)^{(4)} ((s_2)^{(4)}_t) - (s_2)^{(4)}_t)} \left[ e^{((s_2)^{(4)}_t - (p_{24})^{(4)}_t) t} - e^{-(s_2)^{(4)}_t)} + \right] \right] \]

Definition of \((S_1)^{(4)}_t, (S_2)^{(4)}_t, (R_1)^{(4)}_t, (R_2)^{(4)}_t) : \]

Where \((S_1)^{(4)}_t = (a_{24})^{(4)}_t (m_2)^{(4)} - (a_2)^{(4)}_t \)

\[(S_2)^{(4)}_t = (a_{26})^{(4)}_t - (p_{26})^{(4)} \]

\[(R_1)^{(4)}_t = (b_{24})^{(4)}_t (\mu_2)^{(4)}_t - (b_2)^{(4)}_t \]

\[(R_2)^{(4)}_t = (b_2)^{(4)}_t - (r_{26})^{(4)}_t \]

Behavior of the solutions

If we denote and define

Definition of \((\sigma_1)^{(5)}_t, (\sigma_2)^{(5)}_t, (r_1)^{(5)}_t, (r_2)^{(5)}_t) : \]

\(g\) \((\sigma_1)^{(5)}_t, (\sigma_2)^{(5)}_t, (r_1)^{(5)}_t, (r_2)^{(5)}_t\) four constants satisfying

\[-(\sigma_2)^{(5)}_t \leq -(a_{28}^′)^{(5)}_t + (a_{29}^′)^{(5)}_t - (a_{28}^{′′})^{(5)}_t (T_{29}, t) + (a_{29}^{′′})^{(5)}_t (T_{29}, t) \leq -(\sigma_2)^{(5)}_t \]

\[-(r_2)^{(5)}_t \leq -(b_{28}^′)^{(5)}_t + (b_{29}^′)^{(5)}_t - (b_{28}^{′′})^{(5)}_t (G_{31}, t) - (b_{29}^{′′})^{(5)}_t (G_{31}, t) \leq -(r_2)^{(5)}_t \]

Definition of \((v_1)^{(5)}_t, (v_2)^{(5)}_t, (u_1)^{(5)}_t, (u_2)^{(5)}_t, v^{(5)}_t, u^{(5)}_t) : \]

\(h\) By \((v_2)^{(5)}_t > 0, (v_2)^{(5)}_t < 0\) and respectively \((u_2)^{(5)}_t > 0, (u_2)^{(5)}_t < 0\) the roots of the equations \((a_{29})^{(5)}_t v^{(5)}_t = 0\) and \((b_{29})^{(5)}_t u^{(5)}_t = 0\) and

Definition of \((\bar{v}_1)^{(5)}_t, (\bar{v}_2)^{(5)}_t, (\bar{u}_1)^{(5)}_t, (\bar{u}_2)^{(5)}_t) : \]

By \((\bar{v}_1)^{(5)}_t > 0, (\bar{v}_2)^{(5)}_t < 0\) and respectively \((\bar{u}_1)^{(5)}_t > 0, (\bar{u}_2)^{(5)}_t < 0\) the
roots of the equations \((a_{2g})^{(S)}(v^{(S)})^2 + (\sigma_v^{(S)})v^{(S)} - (a_{2g'})^{(S)} = 0\)
and \((b_{2g})^{(S)}(u^{(S)})^2 + (\tau_u^{(S)})u^{(S)} - (b_{2g'})^{(S)} = 0\)

**Definition of** \((m_1)^{(S)}, (m_2)^{(S)}, (\mu_1)^{(S)}, (\mu_2)^{(S)}, (v_0)^{(S)}\):

(i) If we define \((m_1)^{(S)}, (m_2)^{(S)}, (\mu_1)^{(S)}, (\mu_2)^{(S)}\) by

\[
(m_1)^{(S)} = (v_0)^{(S)}, \quad (m_2)^{(S)} = (v_1)^{(S)}, \quad \text{if} \quad (v_0)^{(S)} < (v_1)^{(S)}
\]

\[
(m_2)^{(S)} = (v_1)^{(S)}, \quad (m_1)^{(S)} = (\bar{v}_1)^{(S)}, \quad \text{if} \quad (v_0)^{(S)} < (\bar{v}_1)^{(S)}
\]

and \(v_0^{(S)} = \frac{g_{26}}{g_{24}}\)

\[(m_2)^{(S)} = (v_0)^{(S)}, \quad (m_1)^{(S)} = (v_0)^{(S)}, \quad \text{if} \quad (v_0)^{(S)} < (v_0)^{(S)}\]

and analogously

\[
(\mu_2)^{(S)} = (u_0)^{(S)}, \quad (\mu_1)^{(S)} = (u_1)^{(S)}, \quad \text{if} \quad (u_0)^{(S)} < (u_1)^{(S)}
\]

\[
(\mu_2)^{(S)} = (u_1)^{(S)}, \quad (\mu_1)^{(S)} = (\bar{u}_1)^{(S)}, \quad \text{if} \quad (u_0)^{(S)} < (\bar{u}_1)^{(S)}
\]

and \(u_0^{(S)} = \frac{y_{26}}{y_{24}}\)

\[(\mu_2)^{(S)} = (u_0)^{(S)}, \quad (\mu_1)^{(S)} = (u_0)^{(S)}, \quad \text{if} \quad (u_0)^{(S)} < (u_0)^{(S)}\]

where \((p_1)^{(S)}\) is defined

\[
\frac{1}{(m_3)^{(S)}} G_{28}^{y} e^{((S_3)^{(S)}-(p_{28})^{(S)})t} \leq G_{29}^{y} (t) \leq G_{28}^{y} e^{((S_2)^{(S)})t}
\]

where \((p_1)^{(S)}\) is defined

\[
\frac{1}{(m_3)^{(S)}} G_{28}^{y} e^{((S_3)^{(S)}-(p_{28})^{(S)})t} \leq G_{29}^{y} (t) \leq G_{28}^{y} e^{((S_2)^{(S)})t}
\]

**Definition of** \((S_1)^{(S)}, (S_2)^{(S)}, (R_1)^{(S)}, (R_2)^{(S)}\):

Where \((S_1)^{(S)} = (a_{28})^{(S)}(m_2)^{(S)} - (a_{28})^{(S)}\)

\[(S_2)^{(S)} = (a_{30})^{(S)} - (p_{30})^{(S)}\]

\[(R_1)^{(S)} = (b_{28})^{(S)}(\mu_2)^{(S)} - (b_{28})^{(S)}\]

www.ijsrp.org
\[(R_2)^{(5)} = (b_{30}^{'}(5)) - (r_{30}^{(5)})\]

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}\):

\[(j) \quad (\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)} \quad \text{four constants satisfying}
-(\sigma_2)^{(6)} \leq -a_{32}^{(6)} + a_{33}^{(6)}(T_{33}, t) + (a_{32}^{(6)})(T_{33}, t) \leq -(\sigma_1)^{(6)}
-(\tau_2)^{(6)} \leq -b_{32}^{(6)} + b_{33}^{(6)} - (b_{32}^{(6)})(G_{33}, t) - (b_{32}^{(6)})(G_{35}, t) \leq -(\tau_1)^{(6)}
\]

**Definition of** \((v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}\):

\[(k) \quad \text{By} \quad (v_1)^{(6)} > 0, (v_2)^{(6)} < 0 \quad \text{and respectively} \quad (u_1)^{(6)} > 0, (u_2)^{(6)} < 0 \quad \text{the roots of the equations}
(a_{33}^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32}^{(6)}) = 0
\]
and \((b_{33}^{(6)}u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32}^{(6)}) = 0\)

**Definition of** \((\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}\):

\[\text{By} \quad (\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0 \quad \text{and respectively} \quad (\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0 \quad \text{the roots of the equations}
(a_{33}^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32}^{(6)}) = 0
\]
and \((b_{33}^{(6)}u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32}^{(6)}) = 0\)

**Definition of** \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}\):

\[(l) \quad \text{If we define} \quad (m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)} \quad \text{by}
(m_2)^{(6)} = (v_0)^{(6)} + (m_1)^{(6)} = (v_1)^{(6)}, \quad \text{if} \quad (v_0)^{(6)} < (v_1)^{(6)}
\]
\[(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \quad \text{if} \quad (v_1)^{(6)} < (v_0)^{(6)} < (v_2)^{(6)}
\]
and \((v_0)^{(6)} = \frac{\delta_3}{\delta_2}\)

\[(m_2)^{(6)} = (v_2)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \quad \text{if} \quad (v_2)^{(6)} < (v_0)^{(6)}\)

and analogously

\[(\mu_2)^{(6)} = (u_0)^{(6)} + (\mu_1)^{(6)} = (u_1)^{(6)}, \quad \text{if} \quad (u_0)^{(6)} < (u_1)^{(6)}
\]
\[(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \quad \text{if} \quad (u_1)^{(6)} < (u_0)^{(6)} < (u_2)^{(6)}
\]
and \((u_0)^{(6)} = \frac{\delta_3}{\delta_2}\)

\[(\mu_2)^{(6)} = (u_2)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \quad \text{if} \quad (u_2)^{(6)} < (u_0)^{(6)} \quad \text{where} \quad (u_1)^{(6)}, (\bar{u}_1)^{(6)} \quad \text{are defined respectively}
\]

Then the solution satisfies the inequalities

\[G_{32}^{0}((\tau_1)^{(6)} - (p_{32})^{(6)}) \leq G_{32}^{0}(t) \leq G_{32}^{0}((\tau_1)^{(6)})
\]

where \((p_3)^{(6)}\) is defined

\[
\frac{1}{(m_1)^{(6)}}G_{32}^{0}((\tau_1)^{(6)} - (p_{32})^{(6)}) \leq G_{33}^{0}(t) \leq \frac{1}{(m_2)^{(6)}}G_{32}^{0}((\tau_1)^{(6)})
\]

www.ijsrp.org
\[
\begin{align*}
\left(\frac{a_{13}}{a_{23}}\right)^{(6)}e^{\frac{b_{13}}{a_{23}}} & = e^{\frac{(b_{13} - b_{13}')(6)}{a_{23} - a_{23}'(6)}}t - e^{\frac{(b_{13} - b_{13}')(6)}{a_{23} - a_{23}'(6)}}t + G_3^0e^{-(s_2)^{(6)}t} \\
& \leq G_3^0e^{-(s_2)^{(6)}t} \\
\end{align*}
\]
\[
T_3^0e^{(s_2)^{(6)}t} \leq T_3^0e^{((r_1)^{(6)} + (r_2)^{(6)})t}
\]
\[
\frac{1}{(\mu_3)^6}T_3^0e^{(s_2)^{(6)}t} \leq T_3^0e^{((r_1)^{(6)} + (r_2)^{(6)})t}
\]
\[
\frac{1}{(\mu_3)^6}T_3^0e^{(s_2)^{(6)}t} \leq T_3^0e^{((r_1)^{(6)} + (r_2)^{(6)})t} \\
\]
\[
\begin{align*}
\text{Definition of } (S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}: & \\
(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32}')^{(6)} \\
(S_2)^{(6)} = (a_{32})^{(6)} - (p_{34})^{(6)} \\
(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32}')^{(6)} \\
(R_2)^{(6)} = (b_{32}')^{(6)} - (r_{34})^{(6)}
\end{align*}
\]
\[
\text{Proof: From GLOBAL EQUATIONS we obtain}
\]
\[
\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a_{13}')^{(1)} - (a_{13}''^{(1)})(T_{14},t) \right) - (a_{14}'^{(1)})(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}
\]
\[
\text{Definition of } v^{(1)} :- \quad \frac{v^{(1)}}{G_{13}} = \frac{G_{13}}{G_{14}}
\]
\[
\text{It follows}
\]
\[
- \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)
\]
\[
\text{From which one obtains}
\]
\[
\text{Definition of } (\tilde{v}_1)^{(1)}, (\tilde{v}_0)^{(1)} :-
\]
\[
\begin{align*}
\text{(a) For } 0 < \frac{(v_0)^{(1)}}{G_{14}} < (v_1)^{(1)} < (\tilde{v}_1)^{(1)} \\
\end{align*}
\[
\begin{align*}
\frac{v^{(1)}}{G_{13}} & \geq \frac{(v_0)^{(1)}(G_{14})^{(1)}}{(v_1)^{(1)}(G_{14})^{(1)}}e^{-(a_{14})^{(1)}[(v_1)^{(1)}(v_1)^{(1)}]t} - (a_{14})^{(1)}[(v_1)^{(1)}(v_1)^{(1)}]t]}{1 + (C)^{(1)}e^{-(a_{14})^{(1)}[(v_1)^{(1)}(v_1)^{(1)}]t]} \quad , \quad \frac{(C)^{(1)} = (v_1)^{(1)}(v_1)^{(1)} - (v_0)^{(1)}(v_0)^{(1)}]}{(v_0)^{(1)}(v_1)^{(1)} - (v_0)^{(1)}(v_0)^{(1)}]}
\end{align*}
\]
\[
\text{it follows} \quad (v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}
\]
\[
\text{In the same manner, we get}
\]
\[ v^{(1)}(t) \leq \frac{v_2^{(1)} + (\bar{C}^{(1)})(v_2^{(1)})}{v_1^{(1)} - v_2^{(1)}} \leq v^{(1)}(t) \leq \bar{v}^{(1)}(t) \]

From which we deduce \( v_0^{(1)}(t) \leq v^{(1)}(t) \leq \bar{v}^{(1)}(t) \)

(b) If \( 0 < v_0^{(1)} > v_0^{(1)} = \frac{g_0}{g_{14}} < (\bar{v}_0)^{(1)} \) we find like in the previous case,

\[ v_1^{(1)}(t) \leq \frac{v_2^{(1)} + (\bar{C}^{(1)})(v_2^{(1)})}{v_1^{(1)} - v_2^{(1)}} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}(t) \]

(c) If \( 0 < v_1^{(1)} \leq \bar{v}_1^{(1)} \leq \frac{g_0}{g_{14}} \), we obtain

\[ v_1^{(1)}(t) \leq v^{(1)}(t) \leq v_0^{(1)}(t) = \frac{g_{13}(t)}{g_{14}(t)} \]

And so with the notation of the first part of condition (c), we have

**Definition of \( v^{(1)}(t) \):**

\[ (m_0)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{g_{13}(t)}{g_{14}(t)} \]

In a completely analogous way, we obtain

**Definition of \( u^{(1)}(t) \):**

\[ (\mu_0)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( (a''_0)^{(1)} = (a''_1)^{(1)}, then (\sigma_1)^{(1)} = (\sigma_2)^{(1)} \) and in this case \( v_1^{(1)} = (\bar{v}_1)^{(1)} \) if in addition \( v_0^{(1)} = v_2^{(1)}(t) = (\bar{v}_0)^{(1)} \) and as a consequence \( g_{13}(t) = (v_0)^{(1)}g_{14}(t) \) this also defines \( v_0^{(1)} \) for the special case

Analogously if \( (b''_0)^{(1)} = (b''_1)^{(1)}, then (\tau_1)^{(1)} = (\tau_2)^{(1)} \) and then

\( u_0^{(1)} = (\bar{u}_0)^{(1)}, \) if in addition \( u_0^{(1)} = (u_0)^{(1)} \) then \( T_{13}(t) = (v_0)^{(1)}T_{14}(t) \) This is an important consequence of the relation between \((\bar{v}_1)^{(1)} \) and \((\bar{v}_0)^{(1)} \) and definition of \((u_0)^{(1)} \).

we obtain

\[ \frac{dv^{(2)}}{dt} = (a''_0)^{(2)} - (a''_1)^{(2)}(T_{14}(t) - (a''_0)^{(2)}(T_{14}(t)u^{(2)} - (a''_1)^{(2)}u^{(2)}) \]

www.ijsrp.org
Definition of \( \psi_1(2) \) :-

\[
\psi_1(2) = \frac{G_{14}}{G_{17}}
\]

It follows

\[
- \left( (a_{17})^{(2)}(\psi_1(2) + (a_{16})^{(2)}(\psi_1(2) - (a_{14})^{(2)} \right) \leq \frac{d\psi_1(2)}{dt} \leq - \left( (a_{17})^{(2)}(\psi_1(2) + (a_{16})^{(2)}(\psi_1(2) - (a_{14})^{(2)} \right)
\]

From which one obtains

Definition of \( (\psi_1(2), (\psi_2(2)) \) :-

(d) For \( 0 < (\psi_0(2) = \frac{G_{14}}{G_{17}} < (\psi_1(2) < (\psi_1(2)

\[
\psi_2(2)(t) \geq \frac{(\frac{(\psi_1(2)(t) + (\psi_2(2)(t)[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t)]}{1 + (G_{17})^{(2)}[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t])\right)}
\]

it follows \( (\psi_0(2) \leq \psi_2(2)(t) \leq (\psi_1(2)

In the same manner, we get

\[
\psi_2(2)(t) \leq \frac{(\frac{(\psi_1(2)(t) + (\psi_2(2)(t)[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t)]}{1 + (G_{17})^{(2)}[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t])\right)}
\]

From which we deduce \( (\psi_0(2) \leq \psi_2(2)(t) \leq (\psi_1(2)

(e) If \( 0 < (\psi_1(2) < (\psi_0(2) = \frac{G_{14}}{G_{17}} < (\psi_1(2) \) we find like in the previous case,

\[
(\psi_1(2) \leq \frac{(\frac{(\psi_1(2)(t) + (\psi_2(2)(t)[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t)]}{1 + (G_{17})^{(2)}[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t])\right)} \leq \psi_2(2)(t) \leq \frac{(\psi_1(2)(t) + (\psi_2(2)(t)[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t)]}{1 + (G_{17})^{(2)}[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t])\right)}
\]

(f) If \( 0 < (\psi_1(2) < (\psi_0(2) = \frac{G_{14}}{G_{17}} < (\psi_1(2) \) we obtain

\[
(\psi_1(2) \leq \psi_2(2)(t) \leq \frac{(\frac{(\psi_1(2)(t) + (\psi_2(2)(t)[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t)]}{1 + (G_{17})^{(2)}[(a_{17})^{(2)}(\psi_1(2)(t) - (\psi_2(2)(t])\right)} \leq (\psi_0(2)
\]

And so with the notation of the first part of condition (c), we have

Definition of \( \psi_2(2)(t) \) :-

\[
(m_2(2) \leq \psi_2(2)(t) \leq (m_2(2), \psi_2(2)(t) = \frac{G_{14}}{G_{17}}(t)
\]

In a completely analogous way, we obtain

Definition of \( u_2(t) \) :-

\[
(\mu_2(2) \leq u_2(t) \leq (\mu_2(2), \mu_2(t) = \frac{7c}{2}(t)
\]

Particular case :
If \((a''_0)(2) = (a''_1)(2)\), then \((\sigma_1)(2) = (\sigma_2)(2)\) and in this case \((v_1)(2) = (\nu_1)(2)\) if in addition \((v_0)(2) = (v_1)(2)\) then \(v(2)(t) = (v_0)(2)\) and as a consequence \(G_{16}(t) = (v_0)(2)G_{17}(t)\)

Analogously if \((b''_0)(2) = (b''_1)(2)\), then \((\tau_1)(2) = (\tau_2)(2)\) and then

\[\langle u_1 \rangle(2) = \langle \bar{u}_1 \rangle(2)\] if in addition \((u_0)(2) = \langle u_1 \rangle(2)\) then \(T_{16}(t) = (u_0)(2)T_{17}(t)\). This is an important consequence of the relation between \((v_1)(2)\) and \((\nu_1)(2)\).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv(3)}{dt} = (a_2(3)) - (a'_2(3) - (a_2(3) + (a''_0)(3)(T_{21}, t)) - (a''_1)(3)(T_{21}, t)v(3) - (a_2(3)v(3))
\]

**Definition of \(v(3)\):**

\[
v(3) = \frac{g_2}{g_1}
\]

It follows

\[- \left( (a_2(3)(v(3))^2 + (\sigma_2)(3)v(3) - (a_2(3)) \right) \leq \frac{dv(3)}{dt} \leq - \left( (a_2(3)(v(3))^2 + (\sigma_1)(3)v(3) - (a_2(3)) \right)
\]

From which one obtains

(a) For \(0 < (v_0)(3) = \frac{g_0}{g_2} < (v_1)(3) < (\nu_1)(3)\)

\[
v(3)(t) \geq \frac{(y_2(3) + (C_3)y_3(3)e^{-[(a_2(3)(v_3(3) - (v_0)(3))]}t}{1 + (C_3)e^{-[(a_2(3)(v_3(3) - (v_0)(3))]}t}, \quad (C_3) = \frac{(v_2(3) - (v_0)(3)}{(v_0)(3) - (v_2)(3))
\]

it follows \((v_0)(3) \leq v(3)(t) \leq (v_1)(3)\)

In the same manner, we get

\[
v(3)(t) \leq \frac{(y_3(3) + (C_3)y_2(3)e^{-[(a_2(3)(v_1(3) - (v_2(3)))]}t}{1 + (C_3)e^{-[(a_2(3)(v_1(3) - (v_2(3)))]}t}, \quad (C_3) = \frac{(v_1(3) - (v_0)(3)}{(v_0)(3) - (v_2)(3))
\]

**Definition of \((\nu_1)(3)\):**

From which we deduce \((v_0)(3) \leq v(3)(t) \leq (\nu_1)(3)\)

(b) If \(0 < (v_1)(3) < (v_0)(3) = \frac{g_0}{g_2} < (\nu_1)(3)\) we find like in the previous case,

\[
(v_1)(3) \leq \frac{(y_2(3) + (C_3)y_3(3)e^{-[(a_2(3)(v_3(3) - (v_2(3)))]}t}{1 + (C_3)e^{-[(a_2(3)(v_3(3) - (v_2(3)))]}t} \leq v(3)(t) \leq \frac{(y_3(3) + (C_3)y_2(3)e^{-[(a_2(3)(v_1(3) - (v_2(3)))]}t}{1 + (C_3)e^{-[(a_2(3)(v_1(3) - (v_2(3)))]}t} \leq (\nu_1)(3)
\]

(c) If \(0 < (v_1)(3) \leq (\nu_1)(3) \leq (v_0)(3) = \frac{g_0}{g_2} \), we obtain
\[ (v_1^{(3)}) \leq v^{(3)}(t) \leq \frac{C_{(3)}^{(3)} + (C_{(4)}^{(4)})(v_1^{(4)})}{1 + (C_{(4)}^{(4)})(v_1^{(4)})} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(3)}(t) := \)

\[ (m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \]
\[ v^{(3)}(t) = \frac{a_2(t)}{a_1(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(3)}(t) := \)

\[ (\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \]
\[ u^{(3)}(t) = \frac{T_2(t)}{T_1(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( (a_2^{(3)}) = (a_2^{(3)}), then (\sigma_1^{(3)}) = (\sigma_2^{(3)}) \) and in this case \( (v_1^{(3)}) = (\bar{v}_1^{(3)}) \)

If in addition \( (v_0^{(3)}) = (v_1^{(3)}) \) then \( v^{(3)}(t) = (v_0^{(3)}) \) and as a consequence \( G_{20}(t) = (v_0^{(3)})G_{21}(t) \)

Analogously if \( (b_2^{(3)}) = (b_2^{(3)}), then (r_1^{(3)}) = (r_2^{(3)}) \) and then

\( (u_0^{(3)}) = (\bar{u}_1^{(3)}) \) if in addition \( (u_0^{(3)}) = (u_1^{(3)}) \) then \( T_{20}(t) = (u_0^{(3)})T_{21}(t) \) This is an important consequence of the relation between \( (v_1^{(3)}) \) and \( (\bar{v}_1^{(3)}) \)

**From GLOBAL EQUATIONS we obtain**

\[ \frac{du^{(4)}}{dt} = a_2^{(4)} - \left( a_2^{(4)} - a_2^{(4)} + (a_2^{(4)})(T_{25},t) - (a_2^{(4)})(T_{25},t)v^{(4)} - (a_2^{(4)})v^{(4)} \right) \]

**Definition of** \( v^{(4)} := \)

\[ \frac{a_2^{(4)}}{a_2^{(4)}} \]

It follows

\[ \left( a_2^{(4)}(v^{(4)})^2 + (\sigma_2^{(4)})v^{(4)} - (a_2^{(4)})v^{(4)} \right) \leq \frac{du^{(4)}}{dt} \leq - \left( a_2^{(4)}(v^{(4)})^2 + (\sigma_4^{(4)})v^{(4)} - (a_2^{(4)})v^{(4)} \right) \]

From which one obtains

**Definition of** \( (\bar{v}_1^{(4)}), (v_0^{(4)}) := \)

\( (v_0^{(4)}) = \frac{a_2^{(4)}}{a_2^{(4)}} < (v_1^{(4)}) < (\bar{v}_1^{(4)}) \)

\( v^{(4)}(t) \geq \frac{v_1^{(4)} + (C_{(4)}^{(4)})(v_0^{(4)})}{1 + (C_{(4)}^{(4)})(v_0^{(4)})} \)

\( (C_{(4)}^{(4)}) = \frac{(v_2^{(4)} - v_0^{(4)})}{(v_0^{(4)} - v_2^{(4)})} \)

It follows \( (v_0^{(4)}) \leq v^{(4)}(t) \leq (v_1^{(4)}) \)

In the same manner, we get
\( \nu^{(4)}(t) \leq \frac{(v_1^{(4)}) + (\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}}{4 + (\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}} \), \quad (\tilde{c})^{(4)} = \frac{(v_1^{(4)}) - (v_0^{(4)})}{(v_0^{(4)}) - (v_1^{(4)})}

From which we deduce \( \nu_0^{(4)} \leq \nu^{(4)}(t) \leq (\tilde{v}_1)^{(4)} \)

(e) If \( 0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}}{G_{25}} \) we find like in the previous case,

\[
(v_1)^{(4)} \leq \frac{(v_1^{(4)})+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}}{1+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}} \leq \nu^{(4)}(t) \leq \frac{(v_1^{(4)})+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}}{1+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}} \leq (\tilde{v}_1)^{(4)}
\]

(f) If \( 0 < \nu^{(4)}(t) \leq (\tilde{v}_1)^{(4)} \leq \frac{(v_0)^{(4)} = \frac{G_{24}}{G_{25}}}{(v_0)^{(4)}} \), we obtain

\[
(v_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(v_1^{(4)})+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}}{1+(\tilde{c})^{(4)}(v_2^{(4)})e^{-(a_{25}^{(4)})(v_1^{(4)} - v_2^{(4)})t}} \leq (v_0)^{(4)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \( \nu^{(4)}(t) \) :-

\[
(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \( u^{(4)}(t) \) :-

\[
(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \( (a_0^{''})^{(4)} = (a_0^{''})^{(4)} \), then \( (\sigma_1)^{(4)} = (\sigma_2)^{(4)} \) and in this case \( (v_1)^{(4)} = (\tilde{v}_1)^{(4)} \) if in addition \( (v_0)^{(4)} = (v_1)^{(4)} \) then \( \nu^{(4)}(t) = (v_0)^{(4)} \) and as a consequence \( G_{24}(t) = (v_0)^{(4)}G_{25}(t) \) this also defines \( (v_0)^{(4)} \) for the special case.

Analogously if \( (b_1^{''})^{(4)} = (b_1^{''})^{(4)} \), then \( (\tau_1)^{(4)} = (\tau_2)^{(4)} \) and then \( (u_1)^{(4)} = (u_4)^{(4)} \) if in addition \( (u_0)^{(4)} = (u_1)^{(4)} \) then \( T_{24}(t) = (u_0)^{(4)}T_{25}(t) \) This is an important consequence of the relation between \( (v_1)^{(4)} \) and \( (\tilde{v}_1)^{(4)} \), and definition of \( (u_0)^{(4)} \).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - (a_{28})^{(5)}(a_{28})^{(5)} + (a_{28})^{(5)}(T_{29} \nu^{(5)}) - (a_{28})^{(5)}(T_{29} \nu^{(5)})\nu^{(5)} - (a_{28})^{(5)}\nu^{(5)}
\]

**Definition of** \( \nu^{(5)} \) :-

\[
\nu^{(5)} = \frac{G_{28}}{G_{29}}
\]

It follows

\[
-(a_{28})^{(5)}\nu^{(5)} + (a_{28})^{(5)}\nu^{(5)} - (a_{28})^{(5)} \leq \frac{dv^{(5)}}{dt} \leq -(a_{28})^{(5)}\nu^{(5)} + (a_{28})^{(5)}\nu^{(5)} - (a_{28})^{(5)}
\]
From which one obtains

**Definition of** \((\bar{v}_1)^{(S)}, (v_0)^{(S)}\) :

\[(g)\] For \(0 < \frac{(v_0)^{(S)}}{(v_1)^{(S)}} = \frac{G_{28}}{G_{29}} < (\bar{v}_1)^{(S)}\)

\[v^{(S)}(t) \geq \frac{(v_0)^{(S)}(v_1)^{(S)}}{1+G_{28}} e^{-\frac{(a_{20})(v_1)^{(S)}(v_0)^{(S)}}{1+G_{28}} t}, \quad (C)^{(S)} = \frac{(v_1)^{(S)}(v_0)^{(S)}}{(v_0)^{(S)}(v_2)^{(S)}}\]

it follows \((v_0)^{(S)} \leq v^{(S)}(t) \leq (v_1)^{(S)}\)

In the same manner, we get

\[v^{(S)}(t) \leq \frac{(v_1)^{(S)}(v_2)^{(S)}}{1+G_{28}} e^{-\frac{(a_{20})(v_1)^{(S)}(v_2)^{(S)}}{1+G_{28}} t}, \quad (\bar{C})^{(S)} = \frac{(v_2)^{(S)}(v_0)^{(S)}}{(v_0)^{(S)}(v_2)^{(S)}}\]

From which we deduce \((v_0)^{(S)} \leq v^{(S)}(t) \leq (\bar{v}_2)^{(S)}\)

\[(h)\] If \(0 < (v_1)^{(S)} < (v_0)^{(S)} = \frac{G_{28}}{G_{29}} < (\bar{v}_2)^{(S)}\) we find like in the previous case,

\[(v_1)^{(S)} \leq \frac{(v_1)^{(S)}(v_2)^{(S)}}{1+G_{28}} e^{-\frac{(a_{20})(v_1)^{(S)}(v_0)^{(S)}}{1+G_{28}} t} \leq v^{(S)}(t) \leq \frac{(v_2)^{(S)}(v_1)^{(S)}}{1+G_{28}} e^{-\frac{(a_{20})(v_0)^{(S)}(v_2)^{(S)}}{1+G_{28}} t} \leq (\bar{v}_1)^{(S)}\]

\[(i)\] If \(0 < (v_1)^{(S)} \leq (\bar{v}_1)^{(S)} \leq (v_0)^{(S)} = \frac{G_{28}}{G_{29}}\), we obtain

\[(v_1)^{(S)} \leq v^{(S)}(t) \leq \frac{(v_1)^{(S)}(v_2)^{(S)}}{1+G_{28}} e^{-\frac{(a_{20})(v_1)^{(S)}(v_0)^{(S)}}{1+G_{28}} t} \leq (v_0)^{(S)}\]

And so with the notation of the first part of condition \(c\), we have

**Definition of** \(v^{(S)}(t)\) :

\[m_2)^{(S)} \leq v^{(S)}(t) \leq (m_1)^{(S)}, \quad v^{(S)}(t) = \frac{G_{28}(t)}{G_{29}(t)}\]

In a completely analogous way, we obtain

**Definition of** \(u^{(S)}(t)\) :

\[m_2)^{(S)} \leq u^{(S)}(t) \leq (m_1)^{(S)}, \quad u^{(S)}(t) = \frac{G_{28}(t)}{G_{29}(t)}\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case** :

If \((a_{20})^{(S)} = (a_{29})^{(S)}\), then \((\sigma_1)^{(S)} = (\sigma_2)^{(S)}\) and in this case \((v_1)^{(S)} = (\bar{v}_1)^{(S)}\) if in addition \((v_0)^{(S)} = (v_2)^{(S)}\) then \(v^{(S)}(t) = (v_0)^{(S)}\) and as a consequence \(G_{28}(t) = (v_0)^{(S)}G_{29}(t)\) this also defines \((v_0)^{(S)}\) for the special case.
Analogously if \((b_{29}^{(5)}) = (b_{29}^{(5)})\), then \((\tau_1)^{(5)} = (\tau_2)^{(5)}\) and then
\((u_1)^{(5)} = (u_1)^{(5)}\). If in addition \((u_0)^{(5)} = (u_0)^{(5)}\) then \(T_{29}(t) = (u_0)_{(5)}T_{29}(t)\). This is an important
consequence of the relation between \((v_1)^{(5)}\) and \((\tilde{v}_1)^{(5)}\), and definition of \((u_0)^{(5)}\), we obtain

\[
\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a''_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) - (a''_{32})^{(6)}(T_{33}, t)v^{(6)} - (a_{32})^{(6)}v^{(6)}\right)
\]

**Definition of** \(v^{(6)}\) :

\[
v^{(6)} = \frac{a_{32}}{a_{33}}
\]

It follows

\[
-\left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)
\]

From which one obtains

**Definition of** \((\tilde{v}_1)^{(6)}\), \((v_0)^{(6)}\) :

\[
(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}
\]

\[
\frac{(v_1)^{(6)} + (v_2)^{(6)}}{1 + (C)^{(6)}} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}
\]

It follows \((v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}\)

In the same manner, we get

\[
\frac{(v_1)^{(6)} + (C)^{(6)} + (v_2)^{(6)}}{1 + (C)^{(6)}} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}
\]

From which we deduce \((v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}\)

\[
(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}
\]

\[
\frac{(v_1)^{(6)} + (v_0)^{(6)}}{1 + (C)^{(6)}} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}
\]

we find like in the previous case,

\[
(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (v_0)^{(6)}}{1 + (C)^{(6)}}e^{-[(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6})]t} \leq v^{(6)}(t) \leq \frac{(v_1)^{(6)} + (v_0)^{(6)}}{1 + (C)^{(6)}}e^{-[(a_{33})^{(6)}(v_0)^{(6)} - (v_2)^{(6})]t} \leq \tilde{v}_1^{(6)}
\]

\[
(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(v_1)^{(6)} + (v_0)^{(6)}}{1 + (C)^{(6)}}e^{-[(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6})]t} \leq \tilde{v}_1^{(6)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(6)}(t)\) :

www.ijsrp.org
In a completely analogous way, we obtain

\[ (m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)} \]

Definition of \[u^{(6)}(t) := \]

\[ (\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_{32})^{(6)} = (a_{33})^{(6)}, \text{then } (\sigma_1)^{(6)} = (\sigma_2)^{(6)}\) and in this case \((v_1)^{(6)} = (\bar{v}_1)^{(6)}\) if in addition \((v_0)^{(6)} = (v_1)^{(6)}\) then \(v^{(6)}(t) = (v_0)^{(6)}\) and as a consequence \(G_{32}(t) = (v_0)^{(6)}G_{33}(t)\) this also defines \((v_0)^{(6)}\) for the special case.

Analogously if \((b_{32})^{(6)} = (b_{33})^{(6)}, \text{then } (\tau_1)^{(6)} = (\tau_2)^{(6)}\) and then \((u_0)^{(6)} = (u_1)^{(6)}\) if in addition \((u_0)^{(6)} = (u_1)^{(6)}\) then \(T_{32}(t) = (u_0)^{(6)}T_{33}(t)\) This is an important consequence of the relation between \((v_1)^{(6)}\) and \((\bar{v}_1)^{(6)}\), and definition of \((u_0)^{(6)}\).

We can prove the following

**Theorem 3:** If \((a_i^{(1)})^{(1)} \text{and } (b_i^{(1)})^{(1)}\) are independent on \(t\), and the conditions

\[ (a_{13})^{(1)}(a_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0 \]
\[ (a_{13})^{(1)}(a_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0 \]
\[ (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 \]
\[ (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} + (b_{13})^{(1)}(r_{14})^{(1)} + (b_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0 \]

with \((p_{14})^{(1)}, (r_{14})^{(1)}\) as defined, then the system

If \((a_i^{(2)})^{(2)} \text{and } (b_i^{(2)})^{(2)}\) are independent on \(t\), and the conditions

\[ (a_{16})^{(2)}(a_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \]
\[ (a_{16})^{(2)}(a_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \]
\[ (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 \]
\[ (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(r_{17})^{(2)} - (b_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \]

with \((p_{16})^{(2)}, (r_{17})^{(2)}\) as defined are satisfied, then the system

If \((a_i^{(3)})^{(3)} \text{and } (b_i^{(3)})^{(3)}\) are independent on \(t\), and the conditions

\[ (a_{20})^{(3)}(a_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0 \]
\[ (a_{20})^{(3)}(a_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0 \]
\[ (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0 \]
If \((a''_r)^{(4)}\) and \((b''_r)^{(4)}\) are independent on \(t\), and the conditions
\[
(a'_{24})^{(4)}(a_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0
\]
\[
(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0
\]
\[
(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0,
\]
\[
(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0
\]
with \((p_{24})^{(4)}, (r_{25})^{(4)}\) as defined are satisfied, then the system

If \((a''_r)^{(5)}\) and \((b''_r)^{(5)}\) are independent on \(t\), and the conditions
\[
(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0
\]
\[
(a'_{28})^{(5)}(a_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0
\]
\[
(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0,
\]
\[
(b'_{28})^{(5)}(b_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0
\]
with \((p_{28})^{(5)}, (r_{29})^{(5)}\) as defined satisfied, then the system

If \((a''_r)^{(6)}\) and \((b''_r)^{(6)}\) are independent on \(t\), and the conditions
\[
(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0
\]
\[
(a'_{32})^{(6)}(a_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0
\]
\[
(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0,
\]
\[
(b'_{32})^{(6)}(b_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0
\]
with \((p_{32})^{(6)}, (r_{33})^{(6)}\) as defined are satisfied, then the system

\[
(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0
\]
\[
(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0
\]
\[
(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0
\]
\[
(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0
\]
\[
(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0
\]
\[
(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0
\]
has a unique positive solution , which is an equilibrium solution for the system

\[
(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0
\]
\[
(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0
\]

www.ijsrp.org
\[(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0\]

\[(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0\]

\[(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0\]

\[(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0\]

\[(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0\]

\[(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0\]

\[(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0\]

\[(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0\]

\[(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution

\[(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0\]

\[(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0\]

\[(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0\]

\[(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27})]T_{24} = 0\]

\[(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27})]T_{25} = 0\]

\[(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27})]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0\]

\[(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0\]

\[(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0\]

\[(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0\]

\[(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0\]

\[(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0\]

has a unique positive solution, which is an equilibrium solution for the system
\[(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0\]

\[(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0\]

\[(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0\]

\[(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0\]

\[(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0\]

\[(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

(a) Indeed the first two equations have a nontrivial solution \(G_{13}, G_{14}\) if

\[F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} + (a'_{13})^{(1)}(a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{13}) + (a_{14})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14}) + (a''_{14})^{(1)}(T_{14}) = 0\]

(a) Indeed the first two equations have a nontrivial solution \(G_{16}, G_{17}\) if

\[F(T_{16}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(a'_{17})^{(2)}(T_{17}) + (a_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17}) + (a''_{17})^{(2)}(T_{17}) = 0\]

(a) Indeed the first two equations have a nontrivial solution \(G_{20}, G_{21}\) if

\[F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(a'_{21})^{(3)}(T_{21}) + (a_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21}) + (a''_{21})^{(3)}(T_{21}) = 0\]

(a) Indeed the first two equations have a nontrivial solution \(G_{24}, G_{25}\) if

\[F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(a'_{25})^{(4)}(T_{25}) + (a_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25}) + (a''_{25})^{(4)}(T_{25}) = 0\]

(a) Indeed the first two equations have a nontrivial solution \(G_{28}, G_{29}\) if

\[F(T_{31}) = (a_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a'_{29})^{(5)} + (a_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29}) + (a''_{29})^{(5)}(T_{29}) = 0\]
(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{32}) = (a_{32})^{(6)}(a_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(a_{33})^{(6)}(T_{33}) + (a_{33})^{(6)}(T_{33}) + (a_{32})^{(6)}(T_{33}) = 0$$

**Definition and uniqueness of $T_{14}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T_{14}$ for which $f(T_{14}) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(3)}G_{14}}{[(a_{13})^{(3)}+(a_{13})^{(3)}(T_{14})]} , \quad G_{15} = \frac{(a_{12})^{(3)}G_{14}}{[(a_{12})^{(3)}+(a_{12})^{(3)}(T_{14})]}$$

**Definition and uniqueness of $T_{17}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T_{17}$ for which $f(T_{17}) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a_{16})^{(2)}+(a_{16})^{(2)}(T_{17})]} , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a_{18})^{(2)}+(a_{18})^{(2)}(T_{17})]}$$

**Definition and uniqueness of $T_{21}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T_{21}$ for which $f(T_{21}) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a_{20})^{(3)}+(a_{20})^{(3)}(T_{21})]} , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a_{22})^{(3)}+(a_{22})^{(3)}(T_{21})]}$$

**Definition and uniqueness of $T_{25}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T_{25}$ for which $f(T_{25}) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a_{24})^{(4)}+(a_{24})^{(4)}(T_{25})]} , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a_{26})^{(4)}+(a_{26})^{(4)}(T_{25})]}$$

**Definition and uniqueness of $T_{29}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}$ for which $f(T_{29}) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a_{28})^{(5)}+(a_{28})^{(5)}(T_{29})]} , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a_{30})^{(5)}+(a_{30})^{(5)}(T_{29})]}$$

**Definition and uniqueness of $T_{33}^*$:**

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i''')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}$ for which $f(T_{33}) = 0$. With this value, we obtain from the three first equations

www.ijsrp.org
\[ G_{32} = \frac{(a_{23})(b_{13})}{[a_{12}^2 + (a_{23}^2)(r_{23})]} \quad ; \quad G_{34} = \frac{(a_{24})(b_{14})}{[a_{14}^2 + (a_{24}^2)(r_{23})]} \]

(e) By the same argument, the equations 92, 93 admit solutions \( G_{13}, G_{14} \) if

\[ \varphi(G) = (b_{13}^4)(b_{14}^4) - (b_{14}^3) - \]

\[ (b_{13}^4)(b_{14}^4) + (b_{14}^3)(b_{13}^4) + (b_{14}^2)(b_{13}^2) = 0 \]

Where in \( G(G_{13}, G_{14}, G_{15}) \), \( G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) if it follows that there exists a unique \( G_{14} \) such that \( \varphi(G^{*}) = 0 \)

(f) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if

\[ \varphi(G_{19}) = (b_{16}^4)^2(b_{17}^4)^2 - (b_{16}^3)^2(b_{17}^3)^2 - \]

\[ (b_{16}^4)^2(b_{17}^4)^2 + (b_{16}^3)^2(b_{17}^3)^2 + (b_{16}^2)^2(b_{17}^2)^2 = 0 \]

Where in \( G(G_{19}, G_{17}, G_{18}) \), \( G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14} \) such that \( \varphi(G^{*}) = 0 \)

(g) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if

\[ \varphi(G_{23}) = (b_{20}^4)^3(b_{21}^4)^3 - (b_{20}^3)^3(b_{21}^3)^3 - \]

\[ (b_{20}^4)^3(b_{21}^4)^3 + (b_{20}^3)^3(b_{21}^3)^3 + (b_{20}^2)^3(b_{21}^2)^3 = 0 \]

Where in \( G(G_{20}, G_{21}, G_{22}, G_{20}, G_{22}) \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{21} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{21} \) such that \( \varphi(G^{*}) = 0 \)

(h) By the same argument, the equations of modules admit solutions \( G_{24}, G_{25} \) if

\[ \varphi(G_{27}) = (b_{24}^4)^4(b_{25}^4)^4 - (b_{24}^3)^4(b_{25}^4)^4 - \]

\[ (b_{24}^4)^4(b_{25}^4)^4 + (b_{24}^3)^4(b_{25}^4)^4 + (b_{24}^2)^4(b_{25}^4)^4 = 0 \]

Where in \( G(G_{24}, G_{25}, G_{26}, G_{24}, G_{26}) \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{25} \) such that \( \varphi(G^{*}) = 0 \)

(i) By the same argument, the equations (modules) admit solutions \( G_{26}, G_{29} \) if

\[ \varphi(G_{31}) = (b_{26}^5)^5(b_{29}^5)^5 - (b_{26}^4)^5(b_{29}^5)^5 - \]

\[ (b_{26}^5)^5(b_{29}^5)^5 + (b_{26}^4)^5(b_{29}^5)^5 + (b_{26}^3)^5(b_{29}^5)^5 = 0 \]

Where in \( G(G_{26}, G_{29}, G_{30}, G_{26}, G_{30}) \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{29} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{29} \) such that \( \varphi(G^{*}) = 0 \)

(j) By the same argument, the equations (modules) admit solutions \( G_{32}, G_{33} \) if
\[ \varphi(G_{35}) = (b'_{32})^{(6)}(b_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - \]
\[ \left[ (b'_{32})^{(6)}(b_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b_{32})^{(6)}(G_{35}) \right] + (b'_{32})^{(6)}(G_{35})(b_{33})^{(6)}(G_{35}) = 0 \]

Where in \((G_{35})(G_{32}, G_{33}, G_{34})\), \(G_{32}, G_{33}\) must be replaced by their values. It is easy to see that \(\varphi\) is a decreasing function in \(G_{33}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{33}\) such that \(\varphi(G^*) = 0\).

Finally we obtain the unique solution of 89 to 94

\[ G_{14}^* \text{ given by } \varphi(G^*) = 0, \quad T_{14}^* \text{ given by } f(T_{14}^*) = 0 \text{ and} \]
\[ G_{13}^* = \frac{(a_{13}]^{(1)}G_{13}^{*}}{[a_{13}]^{(1)}G_{14}^{*} + [a_{13}]^{(1)}G_{14}^{*}(T_{14}^*)}, \quad G_{15}^* = \frac{(a_{15}]^{(1)}G_{15}^{*}}{[a_{15}]^{(1)}G_{14}^{*} + [a_{15}]^{(1)}G_{14}^{*}(T_{14}^*)} \]
\[ T_{13}^* = \frac{(b_{13}]^{(1)}T_{13}^{*}}{[b_{13}]^{(1)}T_{13}^{*} - (b_{13}]^{(1)}G^*)}, \quad T_{15}^* = \frac{(b_{15}]^{(1)}T_{15}^{*}}{[b_{15}]^{(1)}T_{15}^{*} - (b_{15}]^{(1)}G^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution

\[ G_{17}^* \text{ given by } \varphi((G_{19})^*) = 0, \quad T_{17}^* \text{ given by } f(T_{17}^*) = 0 \text{ and} \]
\[ G_{16}^* = \frac{(a_{16}]^{(2)}G_{16}^{*}}{[a_{16}]^{(2)}G_{17}^{*} + [a_{16}]^{(2)}G_{17}^{*}(T_{17}^*)}, \quad G_{18}^* = \frac{(a_{18}]^{(2)}G_{18}^{*}}{[a_{18}]^{(2)}G_{17}^{*} + [a_{18}]^{(2)}G_{17}^{*}(T_{17}^*)} \]
\[ T_{16}^* = \frac{(b_{16}]^{(2)}T_{16}^{*}}{[b_{16}]^{(2)}T_{16}^{*} - (b_{16}]^{(2)}((G_{19})^*)}, \quad T_{18}^* = \frac{(b_{18}]^{(2)}T_{18}^{*}}{[b_{18}]^{(2)}T_{18}^{*} - (b_{18}]^{(2)}((G_{19})^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution

\[ G_{21}^* \text{ given by } \varphi((G_{23})^*) = 0, \quad T_{21}^* \text{ given by } f(T_{21}^*) = 0 \text{ and} \]
\[ G_{20}^* = \frac{(a_{20}]^{(3)}G_{20}^{*}}{[a_{20}]^{(3)}G_{21}^{*} + [a_{20}]^{(3)}G_{21}^{*}(T_{21}^*)}, \quad G_{22}^* = \frac{(a_{22}]^{(3)}G_{22}^{*}}{[a_{22}]^{(3)}G_{21}^{*} + [a_{22}]^{(3)}G_{21}^{*}(T_{21}^*)} \]
\[ T_{20}^* = \frac{(b_{20}]^{(3)}T_{20}^{*}}{[b_{20}]^{(3)}T_{20}^{*} - (b_{20}]^{(3)}((G_{23})^*)}, \quad T_{22}^* = \frac{(b_{22}]^{(3)}T_{22}^{*}}{[b_{22}]^{(3)}T_{22}^{*} - (b_{22}]^{(3)}((G_{23})^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution

\[ G_{25}^* \text{ given by } \varphi(G_{27}) = 0, \quad T_{25}^* \text{ given by } f(T_{25}^*) = 0 \text{ and} \]
\[ G_{24}^* = \frac{(a_{24}]^{(4)}G_{24}^{*}}{[a_{24}]^{(4)}G_{25}^{*} + [a_{24}]^{(4)}G_{25}^{*}(T_{25}^*)}, \quad G_{26}^* = \frac{(a_{26}]^{(4)}G_{26}^{*}}{[a_{26}]^{(4)}G_{25}^{*} + [a_{26}]^{(4)}G_{25}^{*}(T_{25}^*)} \]
\[ T_{24}^* = \frac{(b_{24}]^{(4)}T_{24}^{*}}{[b_{24}]^{(4)}T_{24}^{*} - (b_{24}]^{(4)}((G_{27})^*)}, \quad T_{26}^* = \frac{(b_{26}]^{(4)}T_{26}^{*}}{[b_{26}]^{(4)}T_{26}^{*} - (b_{26}]^{(4)}((G_{27})^*)} \]

Obviously, these values represent an equilibrium solution.

Finally we obtain the unique solution
Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i')^{(1)}$ and $(b_i')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of $G_i, T_i :-**

\[
G_i = G_i^* + \mathcal{G}_i, \quad T_i = T_i^* + \mathcal{T}_i
\]

\[
\frac{\partial (a_i')^{(1)}}{\partial T_{14}} (T_{14}^*) = (q_{14})^{(1)}, \quad \frac{\partial (b_i')^{(1)}}{\partial G_i} (G^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{13}}{dt} = -((a_{13}')^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^* T_{14}
\]

\[
\frac{dG_{14}}{dt} = -((a_{14}')^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^* T_{14}
\]

\[
\frac{dG_{15}}{dt} = -((a_{15}')^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^* T_{14}
\]

\[
\frac{dG_{16}}{dt} = -((b_{13}')^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} (\xi_{13}(j))T_{14}^* \mathcal{G}_j
\]

\[
\frac{dT_{14}}{dt} = -((b_{14}')^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15} (\xi_{14}(j))T_{14}^* \mathcal{G}_j
\]

\[
\frac{dT_{15}}{dt} = -((b_{15}')^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15} (\xi_{15}(j))T_{14}^* \mathcal{G}_j
\]

If the conditions of the previous theorem are satisfied and if the functions $(a_i')^{(2)}$ and $(b_i')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Definition of $G_i, T_i :-**
If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$ belong to $C^3(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Denote

**Definition of** $G_i, T_i :-$

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[ \frac{\partial (G''_{21})^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial (b''_{23})^{(3)}}{\partial G_j}(G_{23}^*) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{20}}{dt} = -((a''_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \]

\[ \frac{dG_{21}}{dt} = -((a''_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{21} - (q_{21})^{(3)}G_{21}^*T_{21} \]

\[ \frac{dG_{22}}{dt} = -((a''_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \]

\[ \frac{dT_{20}}{dt} = -((b''_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{20} s_{(20)(j)}T_{20}^*G_j \]

\[ \frac{dT_{21}}{dt} = -((b''_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{21} + \sum_{j=20}^{21} s_{(21)(j)}T_{21}^*G_j \]

\[ \frac{dT_{22}}{dt} = -((b''_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{22} + \sum_{j=20}^{22} s_{(22)(j)}T_{22}^*G_j \]

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(4)}$ and $(b''_i)^{(4)}$ belong to $C^4(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Denote
Definition of $G_i, T_i$ :

$G_i = G_i^* + \mathcal{G}_i, T_i = T_i^* + \mathcal{T}_i$

$\frac{\partial (a_{25}^{(4)}(T_{25}))}{\partial T_{25}} = (q_{25}^{(4)}), \quad \frac{\partial (b_{25}^{(4)}(T_{25}))}{\partial T_{25}} = (G_{25}^*)(s_{ij})$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$\frac{dG_{24}}{dt} = -((a_{24}^{(4)} + (p_{24}^{(4)})G_{24} + (a_{25}^{(4)}G_{25} - (q_{24}^{(4)}G_{24}^{*}T_{25})

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$\frac{dG_{29}}{dt} = -((a_{29}^{(5)} + (p_{29}^{(5)})G_{29} + (a_{30}^{(5)}G_{29} - (q_{29}^{(5)}G_{29}^{*}T_{29})

$\frac{dG_{30}}{dt} = -((a_{30}^{(5)} + (p_{30}^{(5)})G_{30} + (a_{30}^{(5)}G_{29} - (q_{30}^{(5)}G_{30}^{*}T_{29})

$\frac{dG_{28}}{dt} = -((a_{28}^{(5)} + (p_{28}^{(5)})G_{28} + (a_{29}^{(5)}G_{28} - (q_{28}^{(5)}G_{28}^{*}T_{29})

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{(n)})$ and $(b_i^{(n)})$
Belong to $C^{(n)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Definition of $G_i, T_i$

$G_i = G_i^* + \mathcal{G}_i, T_i = T_i^* + \mathcal{T}_i$

$\frac{\partial (a_{29}^{(5)}(T_{29}))}{\partial T_{29}} = (q_{29}^{(5)}), \quad \frac{\partial (b_{29}^{(5)}(T_{29}))}{\partial T_{29}} = (G_{29}^{*})(s_{ij})$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$\frac{dG_{28}}{dt} = -((a_{28}^{(5)} + (p_{28}^{(5)})G_{28} + (a_{29}^{(5)}G_{28} - (q_{28}^{(5)}G_{28}^{*}T_{29})

$\frac{dG_{29}}{dt} = -((a_{29}^{(5)} + (p_{29}^{(5)})G_{29} + (a_{30}^{(5)}G_{29} - (q_{29}^{(5)}G_{29}^{*}T_{29})

$\frac{dG_{30}}{dt} = -((a_{30}^{(5)} + (p_{30}^{(5)})G_{30} + (a_{30}^{(5)}G_{29} - (q_{30}^{(5)}G_{30}^{*}T_{29})

$\frac{dG_{28}}{dt} = -((a_{28}^{(5)} + (p_{28}^{(5)})G_{28} + (a_{29}^{(5)}G_{28} - (q_{28}^{(5)}G_{28}^{*}T_{29})

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{(n)})$ and $(b_i^{(n)})$
Belong to $C^{(n)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

www.ijsrp.org
Denote

**Definition of G_i, T_i :**

\[ G_i = G^*_i + G_i \quad , \quad T_i = T^*_i + T_i \]

\[ \frac{\partial (a_{ij}^{(6)}(x))}{\partial T_{ij}} (T_{ij}) = (q_{33})^{(6)} \quad , \quad \frac{\partial (a_{ij}^{(6)}(x))}{\partial G_{ij}} (G_{33}^*) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{32}}{dt} = -((a'_{32})^{(6)} + (p'_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^* T_{33} \]

\[ \frac{dG_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^* T_{33} \]

\[ \frac{dG_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^* T_{33} \]

\[ \frac{dT_{32}}{dt} = -((b_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=3}^{34} (x)_{32(j)} T_{32}^* G_{ij} \]

\[ \frac{dT_{33}}{dt} = -((b_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=3}^{34} (x)_{33(j)} T_{33}^* G_{ij} \]

\[ \frac{dT_{34}}{dt} = -((b_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=3}^{34} (x)_{34(j)} T_{34}^* G_{ij} \]

The characteristic equation of this system is

\[ ((\lambda)^{(1)} + (a'_{13})^{(1)} - (r_{13})^{(1)})((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \]

\[ \left[ ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}) (q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)}G_{13} \right] \]

\[ ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}) s_{14(14)} T_{14} + (b_{14})^{(1)} s_{13(14)} T_{14}^* \]

\[ + ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}) (q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)} (q_{14})^{(1)}G_{14} \]

\[ ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}) s_{13(13)} T_{13} + (b_{14})^{(1)} s_{13(13)} T_{13}^* \]

\[ ((\lambda)^{(1)})^2 + ((a'_{14})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \]

\[ ((\lambda)^{(1)})^2 + ((b'_{14})^{(1)} + (b_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \]

\[ + ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} (q_{13})^{(1)}G_{15} \]

\[ + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{13})^{(1)} (q_{14})^{(1)}G_{14} + (a_{14})^{(1)} (a_{13})^{(1)} (q_{13})^{(1)}G_{13}^* \]

\[ ((\lambda)^{(1)} + (b_{13})^{(1)} - (r_{13})^{(1)}) s_{14(15)} T_{14} + (b_{14})^{(1)} s_{13(15)} T_{13}^* \] = 0

\[ + ((\lambda)^{(2)} + (b_{10})^{(2)} - (r_{10})^{(2)}) ((\lambda)^{(2)} + (a_{10})^{(2)} + (p_{10})^{(2)}) \]
\[
\left[ (\lambda)^{(2)} + (a_{16})^{(2)} + (p_{16})^{(2)}(q_{17})^{(2)}G_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G_{16} \right] \\
\left[ (\lambda)^{(2)} + (b_{16})^{(2)} - (r_{16})^{(2)}s_{(17),(17)}T_{17} + (b_{17})^{(2)}s_{(16),(17)}T_{17} \right] \\
+ \left[ (\lambda)^{(2)} + (a_{17})^{(2)} + (p_{17})^{(2)}(q_{16})^{(2)}G_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G_{17} \right] \\
\left[ (\lambda)^{(2)} + (b_{16})^{(2)} - (r_{16})^{(2)}s_{(17),(16)}T_{17} + (b_{17})^{(2)}s_{(16),(16)}T_{16} \right] \\
\left[ (\lambda)^{(2)} + (a_{16})^{(2)} + (a_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} (\lambda)^{(2)} \right] \\
\left[ (\lambda)^{(2)} + (b_{16})^{(2)} + (b_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} (\lambda)^{(2)} \right] \\
+ \left[ (\lambda)^{(2)} + (a_{16})^{(2)} + (a_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} (\lambda)^{(2)} \right] (q_{16})^{(2)}G_{16} \\
+ \left[ (\lambda)^{(2)} + (a_{16})^{(2)} + (p_{16})^{(2)} \right] (a_{18})^{(2)}(q_{17})^{(2)}G_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16} \\
\left[ (\lambda)^{(2)} + (b_{16})^{(2)} - (r_{16})^{(2)}s_{(17),(16)}T_{17} + (b_{17})^{(2)}s_{(16),(16)}T_{16} \right] = 0
\]
\[
\left((\lambda)^{4} + \left(b_{24}^{(4)} - (r_{24})^{(4)}\right)S_{(25),(26)}^{*}T_{25}^{*} + (b_{25})^{(4)}S_{(24),(25)}^{*}T_{25}^{*}\right)
\]
\[
+ \left((\lambda)^{4} + \left(a_{25}^{(4)} + (p_{26})^{(4)}\right)G_{24}^{*} + (a_{26})^{(4)}G_{25}^{*}\right)
\]
\[
\left((\lambda)^{4} + \left(b_{24}^{(4)} - (r_{24})^{(4)}\right)S_{(25),(26)}^{*}T_{25}^{*} + (b_{25})^{(4)}S_{(24),(25)}^{*}T_{25}^{*}\right)
\]
\[
\left((\lambda)^{4} - (r_{24})^{(4)}\right)S_{(25),(26)}^{*}T_{25}^{*} + (b_{25})^{(4)}S_{(24),(25)}^{*}T_{25}^{*}\right) = 0
\]

\[
+ \left((\lambda)^{5} + \left(b_{30}^{(5)} - (r_{30})^{(5)}\right)\left((\lambda)^{5} + (a_{30})^{(5)} + (p_{30})^{(5)}\right)\right)
\]
\[
\left((\lambda)^{5} + \left(a_{29}^{(5)} + (p_{28})^{(5)}\right)q_{29}^{(5)}G_{29}^{*} + (a_{28})^{(5)}q_{28}^{(5)}G_{28}^{*}\right)
\]
\[
\left((\lambda)^{5} + \left(b_{28}^{(5)} - (r_{28})^{(5)}\right)S_{(29),(29)}^{*}T_{29}^{*} + (b_{29})^{(5)}S_{(28),(28)}^{*}T_{28}^{*}\right)
\]
\[
\left((\lambda)^{5} - (r_{28})^{(5)}\right)S_{(29),(29)}^{*}T_{29}^{*} + (b_{29})^{(5)}S_{(28),(28)}^{*}T_{28}^{*}\right) = 0
\]

\[
+ \left((\lambda)^{6} + \left(b_{34}^{(6)} - (r_{34})^{(6)}\right)\left((\lambda)^{6} + (a_{34})^{(6)} + (p_{34})^{(6)}\right)\right)
\]
\[
\left((\lambda)^{6} + \left(a_{33}^{(6)} + (p_{32})^{(6)}\right)q_{33}^{(6)}G_{33}^{*} + (a_{32})^{(6)}q_{32}^{(6)}G_{32}^{*}\right)
\]
\[
\left((\lambda)^{6} + \left(b_{32}^{(6)} - (r_{32})^{(6)}\right)S_{(33),(33)}^{*}T_{33}^{*} + (b_{33})^{(6)}S_{(32),(33)}^{*}T_{33}^{*}\right) = 0
\]
+ \left( (\lambda)^{6} + (a_{32})^{6} + (p_{32})^{6} \right) (q_{32})^{6} G_{32}^{*} + (a_{32})^{6} (q_{33})^{6} G_{33}^{*} \\
\left( (\lambda)^{6} + (b_{32})^{6} - (r_{32})^{6} \right) s_{(33),(32)} T_{33}^{*} + (b_{33})^{6} s_{(32),(32)} T_{32}^{*} \\
\left( (\lambda)^{6} \right)^{2} + \left( (a_{32})^{6} + (a_{33})^{6} + (p_{32})^{6} + (p_{33})^{6} \right) (\lambda)^{6} \\
\left( (\lambda)^{6} \right)^{2} + \left( (b_{32})^{6} + (b_{33})^{6} - (r_{32})^{6} + (r_{33})^{6} \right) (\lambda)^{6} \\
+ \left( (\lambda)^{6} + (a_{32})^{6} + (p_{32})^{6} \right) \left( (a_{34})^{6} (q_{33})^{6} G_{33}^{*} + (a_{33})^{6} (a_{34})^{6} (q_{32})^{6} G_{32}^{*} \right) \\
\left( (\lambda)^{6} + (b_{32})^{6} - (r_{32})^{6} \right) s_{(33),(34)} T_{33}^{*} + (b_{33})^{6} s_{(32),(34)} T_{32}^{*} \right) = 0

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Acknowledgments:
The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature ’s Letters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, Ask a Physicist Column, Deliberations with Professors, the the the the eventuality of the fact that there has been any act of omission on the part of the authors, eventuality of the fact that wwwwe reg e reg e reg e regret with ret with ret with ret with

REFERENCES


3. A HAIMOVICI: “On the growth of a two species ecological system divided on age groups”. Tensor, Vol 37 (1982),Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday


(9)\(^{a^b^c}\) Einstein, A. (1905), "Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?", *Annalen der Physik* 18: 639 Bibcode 1905AnP...323..639E, DOI:10.1002/andp.19053231314. See also the English translation.


(12)\(^\) In F. Fernflores. The Equivalence of Mass and Energy. Stanford Encyclopedia of Philosophy

(13)\(^\) Note that the relativistic mass, in contrast to the rest mass \(m_0\), is not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity \(\frac{d\tau}{dt}\), which is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between \(d\tau\) and \(dt\).


(20)\(^\) [2] Cockcroft-Walton experiment

(21)\(^{a^b^c}\) Conversions used: 1956 International (Steam) Table (IT) values where one calorie = 4.1868 J and one BTU = 1055.05585262 J. Weapons designers' conversion value of one gram TNT

www.ijsrp.org
≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a $C_p$ of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average $C_p$ of 25.8, 5.134 moles of metal, and 132 J.K$^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.


(25)^*) Earth's gravitational self-energy is $4.6 \times 10^{10}$ that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.


(41)^ Ives, Herbert E. (1952), "Derivation of the mass-energy relation", *Journal of the Optical Society of America* 42 (8): 540–543, DOI:10.1364/JOSA.42.000540


(48) UIBK.ac.at


(54) ^a^ See Abraham Pais' account of this period as well as L. Susskind's "Superstrings, Physics World on the first non-abelian gauge theory" where Susskind wrote that Yang–Mills was "rediscovered" only because Pauli had chosen not to publish.


www.ijsrp.org


First Author: 1Mr. K. N.Prasanna Kumar has three doctorates one each in Mathematics, Economics, Political Science. Thesis was based on Mathematical Modeling. He was recently awarded D.litt. for his work on ‘Mathematical Models in Political Science’--- Department of studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India Corresponding Author:drknpkumar@gmail.com

Second Author: 2Prof. B.S Kiranagi is the Former Chairman of the Department of Studies in Mathematics, Manasa Gangotri and present Professor Emeritus of UGC in the Department. Professor Kiranagi has guided over 25 students and he has received many encomiums and laurels for his contribution to Cohomology Groups and Mathematical Sciences. Known for his prolific writing, and one of the senior most Professors of the country, he has over 150 publications to his credit. A prolific writer and a prodigious thinker, he has to his credit several books on Lie Groups, Cohomology Groups, and other mathematical application topics, and excellent
Third Author: Prof. C.S. Bagewadi is the present Chairman of Department of Mathematics and Department of Studies in Computer Science and has guided over 25 students. He has published articles in both national and international journals. Professor Bagewadi specializes in Differential Geometry and its wide-ranging ramifications. He has to his credit more than 159 research papers. Several Books on Differential Geometry, Differential Equations are coauthored by him--- Chairman, Department of studies in Mathematics and Computer science, Jnanasahyadri Kuvempu University, Shankarghatta, Shimoga district, Karnataka, India

=================================================================================================