A Technique for Constructing Even-order Magic Squares using Basic Latin Squares

Tomba I.

Department of Mathematics, Manipur University, Imphal, Manipur (INDIA)
tombairom@gmail.com

Abstract: Tomba (2012) introduced a technique for construction (n x n) magic squares (when n is odd) using basic Latin Squares by fixing the pivot element and arranging other elements in an orderly manner [10]. However, even-order magic squares can't be constructed using the same procedure because of duplication in diagonal elements. In this paper, a technique for constructing (n x n) magic squares (when n is doubly even) using basic Latin square is developed. Doubly even magic squares are made by fixing the column associated with the elements, adjacent to the pivot element and arranging in an orderly manner that generates a magic parametric constant (T, known as Tomba's constant) and sub-magic parametric constants (T_i) and finally derived by minor adjustment on the values of T_i's. The technique can provide weak magic squares for singly even cases. The construction is illustrated with suitable examples.

Key-words: Latin square (basic), singly even and doubly even magic square (normal), weak magic squares, magic parametric constant and sub-magic parametric constants

AMS Classification No: A- 05 and A-22

I. INTRODUCTION

Magic squares are practically important of the properties of equality in the sum of its rows, columns, diagonals. Latin squares and Greco-Latin squares are used in statistical research particularly in agricultural sciences and design of experiments whereas magic squares are used in puzzle games of cubes, pattern recognition and magic carpet constructions, magic square cipher in Cryptology etc.

II. LATIN SQUARES

In a Latin square, Latin letters are seen once in each row and column. In a Latin square, the sums of rows and columns are equal but not the sums of diagonals.

A basic (4 x 4) Latin square can be represented with Latin letters A, B, C and D as:

\[
\begin{bmatrix}
A & B & C & D \\
B & C & D & A \\
C & D & A & B \\
D & A & B & C \\
\end{bmatrix}
\]

represented as

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44} \\
\end{bmatrix}
\]

Where, \( \sum_i a_{ij} = \sum_j a_{ij} \) but \( \sum_i d_{ij} \neq \sum_j d_{ij} \) with diagonal notation \( d_{ij} \)

2.1 Symmetric properties of Basic Latin Squares

Lemma-1: A basic (n x n) Latin square (n is odd) is symmetric and non-duplicated.

Let a (3 x 3) basic Latin square be

\[
\begin{bmatrix}
A & B & C \\
B & C & A \\
C & A & B \\
\end{bmatrix} \Rightarrow \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} \\
\end{bmatrix}
\]

Here \( a_{ij} = a_{ji} \) for all \( i \) and \( j \) but \( a_{11} = A \), \( a_{22} = C \), \( a_{33} = C \)

The diagonal elements are not equal or repeated \( \Rightarrow \) non-duplicated

Lemma-2: A (n x n) basic Latin square (n is even) is symmetric but duplicated

Again, let a (4 x 4) basic Latin square be

\[
\begin{bmatrix}
A & B & C & D \\
B & C & D & A \\
C & D & A & B \\
D & A & B & C \\
\end{bmatrix} \Rightarrow \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44} \\
\end{bmatrix}
\]

Clearly, \( \{a_{ij}\} = \{a_{ji}\} \) for all \( i \) and \( j \) \( \Rightarrow \) Basic Latin squares (of all orders) are symmetric.

but, \( a_{11} = A \), \( a_{22} = C \), \( a_{33} = C \)

The diagonal elements are equal or repeated \( \Rightarrow \) duplicated

Lemma-3: Conversely, a (n x n) square (n is odd), satisfying the symmetric and non duplication properties is a basic Latin square.

Lemma-4: In a basic Latin square (n is odd), one of the sum of diagonal is equal to the sum of rows or columns.

Lemma-5: In a basic Latin square (n is even), the diagonal elements are equal or repeated \( \Rightarrow \) duplicated

III. MAGIC SQUARES (normal)

An arrangement of non repeated integers \( (n \geq 0) \) in an array of equal rows and columns such that the sums of its rows, columns and diagonals are equal.

For a normal magic square, the following properties can be established

(i) Elements or numbers \( (n \geq 0) \) are consecutive and not repeated

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(ii) Sums of the rows, columns and diagonals are equal to the magic sum, $S$
\[ \sum_{i} b_{ij} = \sum_{j} b_{ij} = \sum_{i} d_{ij} = \sum_{j} d_{ij}, \quad i, j = 1, 2, \ldots, n \]  
[4]

(iii) Equality property of the rows, columns and diagonals remain unaltered for rotations and reflections.

3.1: Odd order Magic Squares, derived from basic Latin Squares

For any $n$ (n is odd), a magic square constructed using basic Latin square, the following theorem holds

**Theorem:** A $(n \times n)$ matrix $\{b_{ij}\}$, developed by using basic Latin square format when the pivot element is fixed and rearranging in an orderly manner represents a magic square of order $n$ (n is odd). [For details; See IJSRP, May 2012: Tomba I.

3.2 Other Magic squares

There exists different $(n \times n)$ magic square not satisfying these properties. Examples of such magic squares, not satisfying the above properties are: magic squares (special or random, prime numbers etc.)

Examples: (i) MS (special)

\[
\begin{bmatrix}
1 & 14 & 14 & 4 \\
11 & 7 & 6 & 9 \\
8 & 10 & 10 & 5 \\
13 & 2 & 3 & 15
\end{bmatrix}
\]

(ii) MS (prime numbers)

\[
\begin{bmatrix}
17 & 39 & 71 \\
113 & 59 & 5 \\
47 & 29 & 101
\end{bmatrix}
\]

It satisfies: $\sum_{i} b_{ij} = \sum_{j} b_{ij} = \sum_{i} d_{ij} = \sum_{j} d_{ij}$ however these magic squares are not normal because (i) the elements are repeated and non-consecutive and in (ii) the numbers (prime) are not repeated but non-consecutive.

3.3 Weak magic squares (normal)

A $(n \times n)$ array $\{b_{ij}\}$ with diagonal notation $d_{ij}$ satisfying the properties

(i) Elements or numbers $(n \geq 0)$ are consecutive and not repeated

(ii) Sums of diagonals are equal to the magic sum, $S$ \[ \sum_{i} d_{ij} = \sum_{j} d_{ij}, \quad i, j = 1, 2, \ldots, n \]

(iii) Sums of the rows, columns are equal to the magic sum $(S)$, except for some $i$ and $j$
\[ \sum_{i} b_{ij} = \sum_{j} b_{ij}, \quad i, j = 1, 2, \ldots, n \]  
[6]

(iv) Equality property of the rows, columns and diagonals remain unaltered for rotations and reflections.

Then the matrix $\{b_{ij}\}$ satisfying the above properties can be treated as weak magic squares

**Lemma 6:** Magic squares are weak magic squares but the converse is not true.

Since the elements are consecutive, non-repeated and diagonal sums are equal in both magic squares and weak magic squares. Sums of the rows and columns are equal (except for some rows and columns) in case of weak magic square, whereas sums of rows and columns are equal for the magic squares, hence the lemma holds.

3.5 More Properties

(i) For the consecutive numbers $(n \geq 1)$, then magic sum, $S$
\[ S = \frac{1}{2} \left(n(n^2 + 1)\right) \]

(ii) If the consecutive number $(n \geq 0)$, then it gives the lowest magic square.

(iii) If the consecutive number starts from $s+1$ where $s \geq 1$, then $S = n\left(s + \frac{(n^2+1)}{2}\right)$

(iv) The magic parametric constant, sub-magic parametric constants can be determined

3.6 Alternate Structures of $(n \times n)$ magic squares

Let $\{a_{ij}\}$ be a magic square satisfying the properties (a) to (d).

Equality in the sums of rows, columns and diagonals will remain unchanged for rotations and reflections Hence alternate structures of a magic square can be expressed (if the rotation is clockwise or anticlockwise for $(k \div 2)$; $k = \pm 1, \pm 2, \ldots, \pm m$) as $\{a_{ij}(k)\}$

where $a_{ij} = a_{ij}(k)$ for all $i = 0, 4, 8, \ldots$.

3.7 More properties on alternative structures

Infinite number of magic squares can be generated by multiplying or adding by a number $p \geq 1$ to each element of the given magic square.

If the minimum element/number is 0, then $\{a_{ij}\}$ gives the lowest magic square

Sum of two magic squares in the same rotation/reflection gives a magic square

Sum of two magic squares in different rotation are not magic squares.

Product of two magic squares is not a magic square

Magic squares in the same rotation/reflection are additive

IV. METHODOLOGY

For constructing $(n \times n)$ magic square using basic Latin squares when $n$ is even

The technique of constructing doubly even magic square using basic Latin square principle can be expressed as follows:

Let the $n^2$ matrix $\{a_{ij}\}; \quad i, j = 1, 2, \ldots, n$ with the consecutive elements/numbers be arranged in Basic Latin square format as;
Step-3: Retaining the diagonal elements unchanged, make transformations of \( \{b_{ij}\} \) to \( \{b_{ji}\} \), to construct the extreme corner blocks of (2 x 2).

Step-4: Reverting rows and columns in a systematic manner, a magic parametric constant (T) and a set of sub-magic parametric constants \((T_1, T_2,...)\) can be determined.

The reverse process can be made as follows: Depending upon \( n \)
(i) \( n \) (singly even), \( 2 \leq \frac{16}{4} + 2k \leq (n - 2) \) for \( k = 0, 1, 2 \ldots n \)
(ii) \( n \) (doubly even), \( 2 \leq \frac{n^2}{4} + 2k \leq (n - 2) \) for \( k = 0, 1, 2 \ldots n \)

Step-5: Main adjustments should be made on the elements corresponding to the magic parametric constant (T), whereas minor adjustments should be made on other elements of sub-magic parametric constants to get the magic square \( \{b_{ij}\} \); \( i, j = 1,2,...n \) satisfying the properties of \( \sum_i b_{ij} = \sum_j b_{ij} = \sum_i b_{ij} = \sum_j d_{ij} \)

The technique can be used for constructing doubly even magic squares. For singly even cases, it can provide weak magic squares only. Illustrations are shown as follows:

## V. Numerical Examples (doubly even cases)

### 5.1: To construct \((4 \times 4)\) magic square using basic Latin square

Step-1: Let the numbers be \((1 \text{ to } 16)\) arranged in square format be

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
6 & 7 & 8 & 5 \\
11 & 12 & 9 & 10 \\
16 & 13 & 14 & 15 \\
\end{array}
\]

The arrangement gives the column totals equal, \( \sum_j a_{ij} = 34 \) for all \( i \)

Here, \( S = \frac{1}{2} \left( n(n^2 + 1) \right) = 34 \) for \( n = 4 \)

and \( \frac{1}{2} \left( (1 + 16) \right) \) lies between 8 and 9. \([10]\)

Step-2: Select the row associated with these elements (say 3, 8, 9, 14) and assign it as diagonal elements.

Rearranging the other elements in an orderly manner to get a new \((4 \times 4)\) array \( \{b_{ij}\} \) satisfying \( \sum_i d_{ij} = \sum_j d_{ij} = 34 \)
5.2 To construct (8 x 8) magic square

Step-1: Let the consecutive numbers be (1, 2, 3, ..., 64), arranged in 8 rows and 8 columns be rearranged in Latin Square format as:

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Step-3: Retaining the diagonal elements un-changed, make transformations of \( b_{ij} \) to \( b_{ji} \) to generate the extreme corner blocks of (2 x 2).

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The extreme corner blocks: 
\[
\begin{bmatrix}
15 & 6 \\
12 & 8 \\
14 & 7 \\
11 & 2
\end{bmatrix}
\]

No central block is created.

Step-4: Retaining the diagonal elements unchanged, make transformations of \( b_{ij} \) to \( b_{ji} \) to construct the extreme corner blocks and central block of (2 x 2) each.

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Step-5: No minor adjustment needed and therefore the construction of (4 x 4) magic squares is completed in Step-3.

The extreme corner blocks: 
\[
\begin{bmatrix}
15 & 6 & 10 & 3 \\
12 & 8 & 14 & 7 \\
11 & 2 & 54 & 19 \\
33 & 56 & 49 & 50
\end{bmatrix}
\]

Central block: 
\[
\begin{bmatrix}
33 & 25 \\
40 & 32
\end{bmatrix}
\]

Step-2: Select the row associated with the element (32 and 33) and assign this row as diagonal elements. Rearranging the other elements in an orderly manner to get a new matrix giving the diagonal sums equal satisfying

\[
\sum d_j = \sum d_{\bar{j}} = S \text{ where } S = 260
\]
Step-6: Here T = 65 (sum of the elements in 4th column and 5th column or 4th row and 5th row).

Sub-magic parametric constants are T1 = 49 (3rd column and 4th column), T2 = 81 (5th column and 6th column), T3 = 59 (3rd row and 4th row), T4 = 71 (5th row and 6th row).

Main adjustments on the values of T and then on the values of sub-parametric constants

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Secondary adjustments are made on the values of the sub-magic parametric constants as shown above and finally the necessary magic square of 8x8 is available as follows;

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Reverting the 4th column and 5th column, keeping the diagonal elements and extreme corner blocks unchanged, sub-parametric constants derived are :

T1 (3rd row + 4th row), T2 (5th row + 6th row),
T3 (3rd column + 4th column) and T4 (5th column + 6th column).

Here, T1 = 49 , T2 = 81, T3 = 59, T4 = 71.

And the parametric constant can be determined indirectly as T = 65 (3rd row + crossed 5th row or 4th row + crossed 5th row or 3rd column +crossed 5th column or 4th column + crossed 5th column)

Step-6: Since T1 = 49, T2 = 81, T3 = 59, T4 = 71.

Minor adjustments are made on the values of these sub-magic parametric constants as shown above and finally the necessary magic square of (8 x 8) is available as follows:

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Alternative approach

Step-5: Suppose the central block is assumed as \[
\begin{bmatrix}
32 & 40 \\
25 & 33
\end{bmatrix}
\]
5.3 To construct a (12 X 12) magic square

Step-1: Let the consecutive numbers be (1, 2, 3,... 144), be arranged in 12 rows and 12 columns in basic Latin square format. Selecting the column associated with 72 and 73 as diagonal element, rearranging in an orderly manner, a new matrix is available which satisfies the property

\[ \sum_{i,j} d_{ij} = \sum_{j} d_{ij} = S \]

where, \( S = 870 \)

\[ \sum_{i,j} \sum_{i,j} \]

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<tr>
<th>139</th>
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Step-2: Retaining the diagonal elements unchanged make symmetric transformations \( \{b_{ij}\} \) to \( \{b_{ji}\} \) to construct the extreme corner blocks and central block of (2 x 2) each.

**Extreme corner blocks:**

\[
\begin{bmatrix}
    139 & 18 \\
    5 & 128 \\
\end{bmatrix}
\begin{bmatrix}
    126 & 7 \\
    137 & 8 \\
\end{bmatrix}
\begin{bmatrix}
    125 & 10 \\
    138 & 140 \\
\end{bmatrix}
\begin{bmatrix}
    17 & 6 \\
\end{bmatrix}
\]

**Central block:**

\[
\begin{bmatrix}
    33 & 25 & 30 & 23 \\
    40 & 32 & 27 & 24 \\
\end{bmatrix}
\begin{bmatrix}
    139 & 18 & 5 & 128 \\
    126 & 7 & 137 & 8 \\
    125 & 10 & 138 & 140 \\
    17 & 6 \\
\end{bmatrix}
\]

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Step-3 Reverting 3rd, 5th, 7th and 9th columns, 3rd, 5th, 7th and 9th rows retaining the diagonal elements, extreme corner blocks and central block unchanged, magic parametric constant and sub-magic parametric constants can be determined:

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Step-4: Main adjustments are made on the elements of T (corresponding to the central block) and secondary adjustments are made on other elements as:

Finally the required (12x12) magic square is obtained as:

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VI  NUMERICAL EXAMPLES (Singly even cases)

For singly even cases, the technique generates weak magic squares only

6.1  To construct a (6 x 6) weak magic square

Step-1: Let the consecutive numbers be (1, 2, ...36) arranged in 6 rows and 6 columns be arranged in Latin square format as:

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
8 & 9 & 10 & 11 & 12 & 7 \\
15 & 16 & 17 & 18 & 19 & 20 \\
22 & 23 & 24 & 25 & 26 & 27 \\
29 & 30 & 31 & 32 & 33 & 28 \\
36 & 31 & 32 & 33 & 34 & 35 \\
\end{array}
\]

Satisfying \( \sum a_{ij} = S \) for all \( i \), where, \( S = 111 \). Since, \( \frac{1}{2}(1+16) \) lies between 18 and 19

Step-2: Select the row associated with the element (18, 19) as (4, 11, 18, 19, 26, 33) and assign this row as diagonal elements

Rearrange the other elements in an orderly manner to get a new matrix \( \{b_{ij}\} \) to make the diagonal sums equal.

\[
\begin{array}{cccccc}
34 & 28 & 22 & 16 & 10 & 4 \\
35 & 29 & 23 & 17 & 11 & 5 \\
36 & 30 & 24 & 18 & 12 & 6 \\
31 & 25 & 19 & 13 & 7 & 1 \\
32 & 26 & 20 & 14 & 8 & 2 \\
33 & 27 & 21 & 15 & 9 & 3 \\
\end{array}
\]

Step-3: Retaining the diagonal elements unchanged, make symmetric transformations of \( \{b_{ij}\} \) to \( \{b_{ji}\} \), to construct the extreme corner and central blocks of (2 x 2) each.

The extreme corner blocks
\[
\begin{bmatrix}
34 & 9 \\
2 & 29
\end{bmatrix}
\begin{bmatrix}
27 & 4 \\
11 & 32
\end{bmatrix}
\begin{bmatrix}
5 & 26 \\
33 & 10
\end{bmatrix}
\begin{bmatrix}
8 & 35 \\
28 & 3
\end{bmatrix}
\]

And central block
\[
\begin{bmatrix}
24 & 18 \\
19 & 13
\end{bmatrix}
\]

A magic parametric constant, \( T (3^{rd} \text{ row } + 4^{th} \text{ row}) \) or \( (3^{rd} \text{ column } + 4^{th} \text{ column}) \) (except for the central block of 2x2) is determined as \( T = 37 \).

Step-4: Reverting the 3rd row and 3rd column, retaining the diagonal elements and central block unchanged, a (6 x 6) matrix is available, where \( T = 37 \) (3rd row + 4th row or 3rd column + 4th column) as:

\[
\begin{array}{cccccc}
34 & 9 & 16 & 21 & 27 & 4 \\
2 & 29 & 17 & 20 & 11 & 32 \\
31 & 25 & 24 & 18 & 7 & 1 \\
6 & 12 & 19 & 13 & 30 & 36 \\
5 & 26 & 14 & 23 & 8 & 35 \\
33 & 10 & 15 & 22 & 28 & 3 \\
111 & 111 & 111 & 111 & 111 & 111 \\
\end{array}
\]

Here, \( T = 37 \) (36+1 or 30+7 or 15+22 or 16 +21 or 20+17 or 14+23, or 24+13 or 19+18)

\[
\begin{array}{cccccc}
34 & 9 & 16 & 21 & 27 & 4 \\
2 & 29 & 17 & 14 & 11 & 32 \\
31 & 30 & 24 & 18 & 7 & 1 \\
6 & 12 & 19 & 13 & 25 & 36 \\
5 & 26 & 20 & 23 & 8 & 35 \\
33 & 10 & 15 & 22 & 28 & 3 \\
111 & 116 & 111 & 111 & 111 & 111 \\
\end{array}
\]

Step-5: After simple adjustments in the value of the magic parametric constants, we get the weak magic square, generated from Latin Squares (taking the central block as \( \begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix} \))
Adjustment to make row and column sums equal will affect the sum of the diagonals or adjustment to make diagonal sums equal will affect the sum of the rows and columns, generating only weak magic squares.

Depending upon the central block, different forms of weak magic squares can be generated

(a) Central block $\begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix}$

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(b) Central block $\begin{bmatrix} 13 & 19 \\ 18 & 24 \end{bmatrix}$

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(c) Central block $\begin{bmatrix} 18 & 13 \\ 24 & 19 \end{bmatrix}$

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(d) Making rows and columns sums equal effects the diagonal sums (unequal)
Here, the diagonal sums are affected. It gives the diagonal sums as 105 and 117 (not equal).

VII. CONCLUSION

The technique can be used for finding magic squares from basic Latin Squares of any order \((n \geq 1\), for \(n\) is doubly even). The construction is done by fixing the column, associated with the adjacent elements to the pivot element and, assigning it as diagonal element and arranging other elements in an orderly manner with minor adjustments on the elements corresponding to the magic parametric constant and sub parametric constants.

However, for singly even cases, the technique can generate weak magic squares only and therefore separate techniques are to be applied further for making magic squares. For crypto-logical studies, magic squares and weak magic squares in different structures can be applied.

The application of magic squares, derived from Latin squares (when \(n\) is singly even) is expected to provide more security against using the actual magic squares, derived for any \(n\) (odd or doubly even).

VIII. REFERENCES


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