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Fuzzy A-Super Continuous Mappings

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Abstract- In this paper we extend the concepts of a- super closed sets and a- super continuous mappings due to Baker [2] in fuzzy topological spaces and obtain several results concerning the preservation of fuzzy g- closed sets. Furthermore we characterize fuzzy a-super continuous and fuzzy a-super closed mappings and obtain some of the basic properties and characterization of these mappings.

Index Terms- Fuzzy super closure fuzzy super interior fuzzy super closed set, fuzzy super open set fuzzy g-super closed sets, fuzzy g-super open sets, fuzzy g- super continuous, fuzzy a-super closed, fuzzy a- super continuous and fuzzy gc-irresolute mappings.

I. Introduction

After the introduction of fuzzy sets by Zadeh [12] in 1965 and fuzzy topology by Chang [6] in 1968, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Thakur and Malviya [7,10,11] introduced the concepts of fuzzy g- closed sets , fuzzy g-continuity and fuzzy gc-irresolute mappings in fuzzy topological spaces.

In this paper we introduce the concepts of fuzzy a-super closed and fuzzy a-super continuous mappings using fuzzy g-super closed sets .This definition enables us to obtain conditions under which maps and inverse maps preserve fuzzy g-super closed sets. We also characterize fuzzy $T_{½}$ -spaces in terms of fuzzy a-super continuous and fuzzy a-super closed mappings. Finally some of the basic properties of fuzzy a-super continuous and fuzzy a -super closed mappings are investigated.

II. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y)=\beta$ for y = x and $x_\beta(y) = 0$ for $y \ne x$,$\beta \in [0,1]$ and $y\in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x)>1$. A fuzzy set A is quasi–coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x)>1$.For any two fuzzy sets A and B of X, $A \le B$ if and only if $\rceil(A_qB^c)$[5].A family τ of fuzzy sets of X is called a fuzzy topology [1] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Definition 2.1[4]: Let (X,τ) fuzzy topological space and $A\subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\le A\neq\phi\}$

Definition 2.2[4]: A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a) Fuzzy super closed if $scl(A) \le A$.
(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 2.1[4]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 2.2[4]: Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy super closed.

Remark 2.3[4]: The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 2.3 [6,10]: A fuzzy set A of a fuzzy topological space $(X,\Im)$ is called:

(a) Fuzzy g-super closed if $cl(A) \le O$ whenever $A \le O$ and O is fuzzy super open.
(b) Fuzzy g super open if and only if A^c is fuzzy g-super closed.

Remark 2.4[10]: Every fuzzy super closed set is fuzzy g-super closed but its converse may not be true.

Remark 2.5[6,7,10]: A fuzzy set A of a fuzzy topological space is fuzzy g-super open if and only if $F \le int(A)$ whenever F is fuzzy super closed and $F \le A$.

Remark 2.6 [10]: Let $(X,\Im)$ be a fuzzy topological space and R be the family of fuzzy super closed sets of X .Then $\Im = R$ if and only if every fuzzy subset of X is fuzzy g- super closed.

Definition 2.4 [10]: A fuzzy topological space $(X,\Im)$ is called fuzzy $T_{1/2}$ - Space if every fuzzy g- super closed set in X is fuzzy super closed in X.

Definition 2.5[1,4,5,6,7,11] : Let $(X,\Im)$ and (Y, σ) be two fuzzy topological spaces and let f: $X\rightarrow Y$ be a function. Then f is said to be:

(a) fuzzy super continuous if the pre image of each fuzzy super open set if Y is fuzzy super open set in X.
(b) fuzzy g-super continuous if the pre image of every fuzzy super closed set in Y is fuzzy g-super closed set in X.
(c) fuzzy gc-irresolute if the pre image of every fuzzy g-super closed set in Y is an fuzzy g-super closed set in X.
(d) Fuzzy super closed mapping if and only if the image of each fuzzy super closed set in X is fuzzy super closed set in Y**.**
(e) fuzzy super open if the image of every fuzzy super open set in X is fuzzy super open set in Y.

Remark 2.7 [4,5,6] : Every fuzzy super continuous mapping is fuzzy g-super continuous, but the converse may not be true.

Remark 2.8 [4,5,6,11]: Every fuzzy gc- irresolute mapping is fuzzy g-super continuous, but the converse may not be true. The concepts of fuzzy gc-irresolute and fuzzy super continuous mapping are independent.

III. Fuzzy a-super continuous and Fuzzy a-super closed mappings

Definition 3.1: A mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is said to be fuzzy approximately super continuous , (written as a-super continuous) provided that $cl(F) \leq f^{-1}(O)$ whenever F is fuzzy g-super closed set in X, O is an fuzzy super open set in Y and $F \leq f^{-1}(O)$.

Theorem 3.1: Every fuzzy super continuous mapping is fuzzy a- super continuous.

Proof: Let f: $(X,\Im) \rightarrow (Y, \sigma)$ be a fuzzy super continuous mapping. Let O be a fuzzy super open set of Y and F is a fuzzy g-super closed set of X such that $F \leq f^{-1}(O)$. Now since f is fuzzy super continuous, $f^{-1}(O)$ is fuzzy super open set in X. Since F is fuzzy g-super closed and $F \leq f^{1}(O) \Rightarrow cl(F) \leq f^{-1}(O)$. Hence f is fuzzy a-super continuous.

Remark 3.1: The converse of theorem 3.1 may not be true. For,

Example 3.1: Let X = { a ,b } and $\Im$ ={0,U,1 } be fuzzy topology on X , where U be a fuzzy set on X defined by U(a)= 0.5, and U(b)= 0.4 . Then the mapping $f : (X, \Im) \rightarrow (X, \Im)$ defined by $f(a) = b$ and $f(b) = a$ is fuzzy a-super continuous but it is not fuzzy super continuous.

Definition 3.2: A mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is said to be fuzzy approximately super closed (written as a-super closed) provided that $f(F) \leq int(A)$ whenever F is fuzzy super closed set in X, A is an fuzzy g- super open set in Y and $f(F) \leq A$**.**

Theorem 3.2: Every fuzzy super closed mapping is fuzzy a- super closed.

Proof: Let $f : (X,\Im) \rightarrow (Y, \sigma)$ be an fuzzy super closed mapping . Let F be fuzzy super closed set in X and A is an fuzzy g- super open set in Y such that $f(F) \leq A$ .Since f is super closed f(F) is fuzzy super closed set in Y. Now A is fuzzy g-super open and $f(F) \leq A \Rightarrow f(F) \leq int(A)$. Hence f is fuzzy a-super closed.

Remark 3.2: The converse of theorem 3.2 may not be true. For,

Example 3.2: Let X = { a ,b } and $\Im$ ={ **0** , **1** ,U } be fuzzy topology on X .Let U be a fuzzy set defined as follows U(a)=0.6, U(b)=0.3 be an fuzzy set on X .Then the mapping $f : (X, \Im) \rightarrow (X, \Im)$ defined by $f(a) = b$ and $f(b) = a$ is fuzzy a-super closed but it is not fuzzy super closed.

IV. Preserving Fuzzy g- super closed sets

In this section the concepts of fuzzy a-super continuous and fuzzy a-super closed mappings are used to obtain some results on preservation of fuzzy g- super closed sets.

Theorem 4.1: If a mapping f: $(X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g-super continuous and fuzzy a-super closed then $f^{-1}(A)$ is fuzzy g-super closed set in X whenever A is fuzzy g-super closed set in Y.

Proof: Suppose that $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g- super continuous and fuzzy a-super closed .Let A be a fuzzy g-super closed set in Y such that $f^{-1}(A) \leq O$ where O be a fuzzy super open set in X. Then $1-O \leq f^{-1}(1 - A)$ which implies that $f(1-O) \leq int (1-A) = 1-cl(A)$. Hence $f^{-1}(cl(A)) \leq O$. Since f is fuzzy g-super continuous and $f^{-1}(cl(A))$ is fuzzy g-super closed in X. Therefore $cl (f^{-1}(cl(A))) \leq O$ which implies that $cl (f^{-1}(A)) \leq O$. Hence $f^{-1}(A)$ is fuzzy g-super closed set in X.

Corollary 4.1:If a mapping f: $(X,\Im) \rightarrow (Y,\sigma)$ is fuzzy g-super continuous and fuzzy super closed then $f^{-1}(A)$ is fuzzy g-super closed set in X whenever A is fuzzy g-super closed set in Y.

Corollary 4.2: If a mapping f: $(X,\Im) \rightarrow (Y, \sigma)$ is fuzzy super continuous and fuzzy super closed then $f^{-1}(A)$ is fuzzy g-super closed set in X whenever A is fuzzy g-super closed set in Y.

Theorem 4.2: If a mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g- super continuous and fuzzy a-super closed then $f^{-1}(A)$ is fuzzy g-super open in X whenever A is fuzzy g-super open in Y.

Proof: Suppose that $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g- super continuous and fuzzy a-super closed mapping. Let A is fuzzy g- super open in Y. Then 1-A is fuzzy g-super closed in Y. Hence by theorem 4.1, $f^{-1}(1 - A)$ is fuzzy g- super closed in X. Since $f^{-1}(1 - A) = 1 - f^{-1}(A)$ for every fuzzy set A of Y . Hence $1- (f^{-1}(A))$ is fuzzy g-super closed set in X. Therefore $f^{-1}(A)$ is fuzzy g-super open set in X.

Corollary 4.3 : If a mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g- super continuous and fuzzy super closed then $f^{-1}(A)$ is fuzzy g-super open in X whenever A is fuzzy g-super open in Y.

Corollary 4.4 : If a mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy super continuous and fuzzy super closed then $f^{-1}(A)$ is fuzzy g-super open in X whenever A is fuzzy g-super open in Y.

Theorem 4.3: If $f: (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy a- super continuous and fuzzy super closed mapping then the image of every fuzzy g- super closed set of X is fuzzy g- super closed in Y.

Proof: Let B be a fuzzy g super closed set of X, and $f(B) \leq$ O. where O is fuzzy super open set in Y. Then $B \leq f^{-1}(O)$ and since f is fuzzy a- super continuous $cl(B) \leq f^{-1}(O)$ which implies that $f (cl(B)) \leq O$. Since f is fuzzy super closed mapping we have $cl(f(B)) \leq cl (f (cl(B)) = f (cl(B)) \leq O$. Hence f(B) is fuzzy g- super closed in Y.

Corollary 4.5[7] :If $f: (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy super continuous and fuzzy super closed mapping then the image of every fuzzy g- super closed set of X is fuzzy g- super closed in Y.

V. A CHARACTERIZATION OF FUZZY T1/2 - SPACES.

In the following theorems we give a characterization of a class of Fuzzy $T_{1/2}$ -spaces by using the concepts of fuzzy a-super closed and fuzzy a-super continuous mapping.

Theorem 5.1 An fuzzy topological space $(X,\Im)$ is fuzzy $T_{1/2}$- space if and only if every mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy a- super continuous.

Proof: Necessity: Let $f: (X,\Im) \rightarrow (Y, \sigma)$ be a fuzzy mapping. Let A be a fuzzy g-super closed set of X and $A \leq f^{-1}(O)$ where O is a fuzzy super open set of Y. Since X is fuzzy $T_{1/2}$ -space, A is fuzzy super closed set in X. Therefore $cl(A) = A \leq f^{-1}(O)$.Hence A is fuzzy a- super continuous .

Sufficiency: Let A be a non empty fuzzy g-super closed set in X and let Y be the set X with the fuzzy topology $\sigma= \{0, A, 1\}$Finally let $f: (X,\Im) \rightarrow (Y, \sigma)$ be identity mapping. By assumption f is fuzzy a- super continuous. Since A is fuzzy g-super closed in X and fuzzy super open in Y and $A \leq f^{-1}(A)$,it follows that $cl(A) \leq f^{-1}(A) =A$. Hence A is fuzzy super closed in X and therefore X is fuzzy $T_{1/2}$ -space.
An analogous argument proves the following result for fuzzy a-super closed mapping.

Theorem 5.2: A fuzzy topological space $(X,\Im)$ is fuzzy $T_{1/2}$ -space if and only if every mapping $f: (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy a-super closed.

VI. PROPERTIES OF FUZZY A- SUPER CONTINUOUS AND FUZZY A-SUPER CLOSED MAPPINGS

In this section we investigate some of the properties of fuzzy a-super closed and fuzzy a-super continuous mappings.

Theorem 6.1: Every fuzzy g- super continuous and fuzzy a-super closed mapping is fuzzy gc-irresolute.

Proof: Suppose that $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy g- super continuous and fuzzy a-super closed mapping and A is fuzzy g-super closed set in Y. Let $f^{-1}(A) \leq O$ where O is a fuzzy super open set in X. Then $1-O \leq f^{-1}(1-A)$ which implies that $f(1-O)\leq int(1-A)=1 - (cl(A))$. Hence $f^{-1}(cl(A)) \leq O$. since f is fuzzy g-super continuous $f^{-1}(cl(A))$ is fuzzy g-super closed in X .Therefore $cl (f^{-1}(cl(A))) \leq O$ which implies that $cl(f^{-1}(A)) \leq O$. Hence $f^{-1}(A)$ is fuzzy g-super closed set in X. Therefore f is fuzzy gc-irresolute.

Theorem 6.2: If $f: (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy super continuous and fuzzy a-super closed mapping then it is fuzzy gc-irresolute.

Proof: It follows from Remark 2.2 and theorem 6.1.

Theorem 6.3 : If $f : (X,\Im) \rightarrow (Y, \sigma)$ is a mapping for which f(F) is fuzzy super open set in Y for every fuzzy super closed set F of X then f is fuzzy a-super closed mapping.

Proof: Let $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy mapping , F is fuzzy super closed in X , A is fuzzy g-super open in Y and $f(F) \leq A$. By hypothesis f(F) is fuzzy super open in X. Therefore, $f (F) = int f(F) \leq int(A)$. Hence f is fuzzy a-super closed.

Theorem 6. 4 : If $f : (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy mapping for which $f^{-1}(V)$ is fuzzy super closed in X for every fuzzy super open set V of Y, then f is fuzzy a-super continuous mapping.

Proof: Let $f: (X,\Im) \rightarrow (Y, \sigma)$ is fuzzy mapping. Let F is fuzzy g-super closed set in X and V is fuzzy super open set of Y such that $F \leq f^{-1}(V)$.By hypothesis $f^{-1}(V)$ is fuzzy super closed in X. Hence $cl(f^{-1}(V) = f^{-1}(V)$.Therefore $cl(F)\leq cl(f^{-1}(V))=f^{-1}(V)$.Hence f is fuzzy a-super continuous.
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Remark 6.1: Since the identity mapping on any fuzzy topological space is both fuzzy a- super continuous and fuzzy a- super closed, it is clear that the converse of theorems 6.3 and 6.4 does not hold.

Theorem 6.5 :If the families of fuzzy super open and fuzzy super closed sets of Y are coincide ,then the mapping *f* : (X,ℑ) → (Y, σ) is fuzzy a-super closed if and only if *f*(F) is fuzzy super open set in Y, for every fuzzy super closed set F of X.

Proof: **Necessity:** Let *f* : (X,ℑ) → (Y, σ) is fuzzy a-super closed mapping. By Theorem 2.2, every fuzzy set of Y are fuzzy g-super closed and hence all are fuzzy g-super open. Thus for any fuzzy super closed set F of X, f(F) is fuzzy g-super open in Y. Since f is fuzzy a-super closed, f(F) ≤ int (f(F)) and then f(F)= int(f(F)) .Hence f(F) is fuzzy super open.

Sufficiency: Let F be an fuzzy super closed set of X and A be an g-super open set of Y and f(F) ≤ A .By hypothesis f(F) is fuzzy super open in Y and f(F)=int(f(F)) ≤ int(A).Hence f is fuzzy a-super closed.

Corollary 6.1: If the families of fuzzy super open and fuzzy super closed sets of Y are coincide, then the mapping f*:* (X,ℑ) → (Y, σ) is fuzzy a-super closed if and only if it is fuzzy super closed.

Theorem 6.6: If f : (X,ℑ) → (Y, σ) is fuzzy super closed and *g*:(Y, σ) → (Z, Φ) is fuzzy a-super closed mapping, then gof : (X,ℑ) →(Z, Φ) is fuzzy a-super closed .

Proof : Let F be an fuzzy super closed set of X and A is fuzzy g-super open set of Z for which gof (F) ≤ A since f : (X,ℑ) →(Y, σ) is fuzzy super closed mapping , f(F) is fuzzy super closed set of Y. Now since g:(Y,σ)→(Z, Φ) is fuzzy a-super closed mapping, then g(f(F)) ≤ int(A).Hence gof : (X,ℑ) →(Z, Φ) is fuzzy a-super closed mapping.

Theorem 6.7 : If f : (X,ℑ) → (Y, σ) is fuzzy a-super closed and g: (Y, σ)→ (Z, Φ) is fuzzy super open and fuzzy gc-irresolute then gof : (X,ℑ) →(Z, Φ) is fuzzy a-super closed.

Proof: Let F be a fuzzy super closed set of X and A is fuzzy g-super open set of Z for which gof (F) ≤ A. Then f(F) ≤ f^{-1}(A) .Since g is gc-irresolute ,g^{-1}(A) is fuzzy g-super open in X and *f* :(X,ℑ) → (Y, σ) is fuzzy a-super closed mapping .It follows that f(F) ≤ int (g $^{-1}$(A)). Thus (gof)(F) =g(f(F) ≤ g(int(g $^{-1}$(A)) ≤ int(g((g $^{-1}$(A)) ≤ int(A).Hence gof : (X,ℑ) →(Z, Φ) is fuzzy a-super closed.

Theorem 6.9: If f: (X,ℑ) → (Y, σ) is fuzzy a-super continuous and *g:*(Y, σ) → (Z, Φ) is fuzzy super continuous mapping, then gof: (X,ℑ) →(Z, Φ) is fuzzy a-super continuous.

Proof: Let A be a fuzzy g- super closed set of X and V is fuzzy super open set of Z for which A ≤ $(gof)^{-1}$(V). Now since g : (Y, σ)→ (Z, Φ) is fuzzy super continuous mapping, g $^{-1}$(V) is fuzzy super open set of Y. Because *f*:(X,ℑ)→(Y,σ) is fuzzy a-super continuous, cl(A)≤ f $^{-1}$(g^{-1}(V)) = (gof) $^{-1}$(V). Hence gof: (X,ℑ) →(Z, Φ) is fuzzy a-super continuous mapping.

REFERENCES

[1] C.L. Chang Fuzzy Topological Spaces, J. Math. Anal. Appl. 24(1968), 182-190.
[2] C.W. Baker on Preserving g-super closed sets Kyungpook Math. J. 36(1996), 195-199.
[3] M.K. Mishra,et all on " Fuzzy super closed set" International Journal International Journal of Mathematics and applied Statistics Accepted.
[4] M.K. Mishra et all on " Fuzzy super continuity" International Review in Fuzzy Mathematics ISSN : 0973-4392July –December2012.
[5] M.K. Mishra M. Shukla M. Fuzzy Regular Generalized Super Closed Set" Accepted for publication in International Journal of Scientfic and Research Publication ISSN2250-3153. July December 2012.(Accepted).
[6] N. Levine Generalized super closed sets in topology Rend. Cerc. Mat. Palermo 19(2) (1970), 571-599.
[7] R. Malviya on certain concepts in fuzzy topology. Ph.D. Dissertation, R.D.V.V. Jabalpur (1997).
[8] P. M. Pu and Y. M. Liu Fuzzy topology I Neighbourhood structure of a fuzzy point and More-Smith Convergence. J. Math. Anal. Appl. 76(1980) 571-594.
[9] P. M. Pu and Y. M. Liu Fuzzy topology II Product and quotient spaces J. Math. Anal. Appl. 77(1980) 20-37.
[10] S. S. Thakur and R. Malviya Generalized super closed sets in fuzzy topology , Math Notae 38(1995) 137-140
[11] S. S. Thakur and R. Malviya Fuzzy gc-Irresolute Mappings Proc. Math. Soc. BHU 11(1995),184-186 .
[12] L. A. Zadeh, Fuzzy Sets, Information and Control, 18(1965),338-353

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