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Pseudo Fuzzy Coset

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Abstract- We introduce here the notion of (i) a fuzzy subgroup (ii) a pseudo fuzzy coset and (iii) a pre class. In this paper we give necessary and sufficient condition for a of pseudo fuzzy coset a fuzzy group.The aim of the paper is to invastigate conjugate fuzzy subgroup of a group from a general point of view.

Index Terms- Fuzzy subgroup, Pseudo Fuzzy Coset, Pre Class.

I. Introduction

The concept of fuzzy set was introduced by Zadeh[2] .Rosenfeld [1] applied this concept to the theory of groupoids and groups .In [3] the notion of a fuzzy subgroup of a fuzzy group was introduced and studied .The purpose of this paper is to continue the study of fuzzy subgroup of a fuzzy group . In this paper we investigate further the theory of fuzzy group and obtain analogs of a number of basic results of fuzzy subgroup .

II. FUZZY SUBGROUP

Definition 2.1: Let X be a non empty set. A fuzzy subset S of the set X is a function $S : X \to [0,1]$.

Definition 2.2: Let G be a group. A fuzzy subset S of a group G is called a fuzzy subgroup of the group G if

(i) $S(x,y) \geq min\{S(x),S(y)\}\ \forall x,y \in G$.

(ii) $S(x^{-1}) = S(x)\ \forall x \in G.$

Definition 2.3: Let S be a fuzzy subset of a set X. For $t \in [0,1]$, the set $S_t = \{x \in X \mid S(x) \geq t\}$ is called a level subset of the fuzzy subset S.

Definition 2.4: Let G be a group. A fuzzy subgroup S of G is called normal if $S(x) = S(y^{-1}xy)\ \forall x,y \in G.$

III. PSEUDO FUZZY COSET

Definition 3.1: Let S be a fuzzy subgroup of group G and $a \in G.$ Then the pseudo fuzzy coset $(as)^p$ is defined by $(as)^p(x)=p(a)s(x)\ \forall x \in G$ and for some $p \in P$.

Definition 3.2: A fuzzy subgroup S of a group G is said to be a positive fuzzy subgroup of G if S is a positive fuzzy subset of the group G.

We need the following results :

Theorem 3.3: Let S be a fuzzy subgroup of a finite group G and $t \in [0,1]$ then $o(G^t_{(as)^p}) \leq o(G^t_s)=o(aG^t_s)$ for any $a \in G$.

Proof: Let G be a finite group and S be a fuzzy subgroup of a G and $\in [0,1]$. To show that $o(G^t_{(as)^p}) \leq o(G^t_s)=o(aG^t_s)$. By definition of level subgroup $G^t_{(as)^p} = \{x \in G:\ (as)^p(x) \geq t\}$. Let $x \in G^t_{(as)^p}$ $\Rightarrow (as)^p(x) \geq t \Rightarrow p(a)s(x) \geq t \Rightarrow s(x) \geq t \Rightarrow x \in G^t_s.$ Therefore $G^t_{(as)^p} \subseteq G^t_s \Rightarrow o(G^t_{(as)^p}) \leq o(G^t_s)$. Again we shall show that $o(G^t_s)= o(aG^t_s)$.We have $G^t_s = \{x \in G:\ s(x) \geq t\}$ and $\{ax \in G:\ x \in G^t_s\}=\{ax \in G: s(x) \geq t\}= G^t_s$. Therefore $o(aG^t_s)=o(G^t_s)$.

Theorem 3.4: Let S and μ be any two fuzzy subgroup of X. Then for $a \in X$, $(as)^p \subseteq (a\mu)^p$ iff $s \subseteq \mu$.

Proof: Let S and μ be any two fuzzy subgroup of X. Then we have to show that $(as)^p \subseteq (a\mu)^p \Leftrightarrow s \subseteq \mu$ for $a \in X$. Now $(as)^p \subseteq (a\mu)^p \Leftrightarrow (as)^p(x) \subseteq (a\mu)^p(x) \Leftrightarrow p(a)s(x) \subseteq p(a)\mu(x) \Leftrightarrow s(x) \subseteq \mu(x) \Leftrightarrow s \subseteq \mu$. Hence $(as)^p \subseteq (a\mu)^p \Leftrightarrow s \subseteq \mu$.

IV. PRE CLASS

Definition 4.1: Let S be a fuzzy subgroup of a group G. Then a fuzzy subset S of a set X is called pre class of a fuzzy binary relation R_s on the set X if $min\{S(x),S(y)\} \leq R_s(x,y)\ \forall x,y \in X$.

Definition 4.2: Let G be a group. Then R_λ is called similarity relation on G if

(i) $R_\lambda(x,x)=1\ \forall x \in G$

(ii) $R_\lambda(x,y)=R_\lambda(y,x)\ \forall x,y \in G$

(iii) $R_\lambda(x,z) \geq min\{R_\lambda(x,y), R_\lambda(y,z)\}\ \forall x,y,z \in G$.

We need the following results :

Theorem 4.3: Let S be a fuzzy subgroup of a group G and R_s: $G \times G \to [0,1]$ be given by $R_s(x,y)=S(xy^{-1})\ \forall x,y \in G$.Then R_s is a similarity relation on G only when S is normalized.

Proof: Given that $R_s(x,y)=S(xy^{-1})\ \forall x,y \in G$.To show that R_s is a similarity relation on G only when S is normalized i.e. $S(e)=1$.

(i) $R_s(x,x)=S(xx^{-1})=S(e)=1$

(ii) $R_s(x,y)=S(xy^{-1})=S(xy^{-1})^{-1}=S(yx^{-1})=R_s(y,x)$

(iii) $R_s(x,z)=S(xz^{-1})=S(xy^{-1}yz^{-1}) \geq \min\{S(xy^{-1}),S(yz^{-1})\}$
$\geq min\{R_s(x,y), R_s(y,z)\}$.

Hence R_s is a similarity relation on G only when S is normalized. .

Theorem 4.4: Let S be a fuzzy subgroup of a group G and R_s: $G \times G \to [0,1]$ be given by $R_s(x,y)=S(xy^{-1})\ \forall x,y \in G$.Then S is a pre class of R_s and in general the pseudo fuzzy coset $(as)^p$ is a pre class of R_s for any $a \in G.$

Proof: Given $R_s(x,y)=S(xy^{-1})\ \forall x,y \in G$. First we shall show that S is a pre class of R_s.By the definition of fuzzy subgroup $min\{S(x),S(y)\} \leq S(xy^{-1}) \leq R_s(x,y)$. Hence S is a pre class of R_s. Again we show that $(as)^p$ is a pre class of R_s. Now $min\{(as)^p(x),(as)^p(y)\}= min\{p(a)s(x),p(a)s(y)\}= p(a)[min\{s(x),s(y)\}] \leq min\{s(x),s(y)\} \leq S(xy^{-1}) \leq R_s(x,y)$. Hence $(as)^p$ is a pre class of R_s.

V. Conclusions

The study of fuzzy set theory has become increasingly important in the wake of fast technological development and increasing complexities in real world decision making problems. The fuzzy set theory technique are now considered as an effective and powerful aid towards solving problems of management decision making, computer science, medical science, artificial intelligence etc.So we define in this paper algebraic data about fuzzy.

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