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An integral involving general class of polynomials with I-function

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Abstract- In the present paper we derive an integral involving general class of polynomials whose integrand contains product of I-function and a general class of polynomiyals.The integral evaluated is quite general in nature and we can derive from them by a large number of integrals involving simpler functions as their particular cases.

Index Terms- I-function, general class of polynomials, Fox H-function.

I. Introduction

The I-function of one variable introduced by Saxena [1982],will be represented and defined in the following manner:

$$I^{m,n}_{p_i,q_i:r}[x|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]} = \frac{1}{2\pi i}\int_L \theta(s) z^s ds. \tag{1.1}$$

Where

$\omega = \sqrt{(-1)}, (z \neq 0)$ is a complex variable and

$$z^s = exp[s\{log|z| + \omega arg z\}$$

in which log |z|represent the natural logarithm of |z| and arg |z| is not necessarily the principal value. An empty product is interpreted as unity.

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{pi}\Gamma(a_{ji}-\alpha_{ji}s)]}$$

z is not equal to zero and an empty product is interpreted as unity; $m, nandp_i \forall i \in (1,...,r)$ are non -negative integer

satisfying $0 \leq n \leq p_i, 0 \leq m \leq q_i$: forall $(i = 1,...,r) a_{ji}(j = 1,...,p_i; i = 1,...,r)$ and

$\beta_{ji}(j = 1,...,q_i; i = 1,...,r)$ are assumed to be positive quantities for standardization purpose.

Also $a_{ji} = (j = 1,...p_i; i = 1,..,r)$ and $b_{ji}(j = 1,...,q_i; i = 1,...,r)$ complex numbers.

A general class of polynomial $S_V^U[x]$ occurring in the paper was introduced by Srivastava [4] is defined and represented in the following manner:

$S_V^U[x] = \sum_{k=0}^{[V/U]} \frac{(-V)_{UK} A(V,K)}{K!} x^k,$ V = 0, 1, 2, ... where U is an arbitrary positive integer and coefficients $A(V,K)(V,K \geq 0)$ are arbitrary constant, real or complex.

II. MAIN RESULT

$$\int_0^\infty y^{\lambda-1}[y+t+(y^2+2ty)^{1/2}]^{-\nu} I_{p_i,q_i:r}^{m,n}[x\{y+t+(y^2+2ty\}^{-\gamma}]$$

$$S_V^U[Z\{y+t+(y^2+2ty)^{1/2}\}^{-\xi}]dx = 2a^{-\nu}(\frac{a}{2})^\lambda \Gamma(2\lambda) \sum_{k=0}^{V/U}(-V)_{UK} A(V,K)$$

$$(\frac{Z/a^\alpha)^K}{K!} I_{p_i+2,q_i+2:r}^{m,n+2}[xa^{-\gamma}|_{......,.....,(1-\nu-wk,r),(-\nu-wk-\lambda,r)}^{(-\nu-wk,r),(1+\lambda-\nu-wk,r),...,......,...}] \qquad (2.1)$$

Where

$$(1)\gamma > 0, Re(\lambda,\nu,w) > o \quad (2)Re(\lambda) - Re(\nu) - \nu min Re(\frac{b_j}{\beta_j}) < 0$$

We shall require the following elementary integral for the evaluation of our main integral

$$\int_0^\infty x^{z-1}[x+a+(x^2+2ax)\frac{1}{2}]^{-\nu}dx = 2\nu a^{-\nu}(\frac{1}{2}a)^z[\Gamma(1+\nu+z)]^{-1}\Gamma(2z)\Gamma(\nu-z)$$

$$0 < Re(z) < \nu$$

Proof :To obtain the result (2.1) , we first express I-function involved in its left -hand side in terms of contour integral using eq.(1.1) and general class of polynomiyals $S_V^U[x]$ given by in eq.(1.2) and Interchanging the order of integration and summation (which is permissible under the conditions stated) with (2.1) and evaluating the y-integral with the help of the result from given by equation (2.2) so we arrive at the main result (2.1) after a little simplification:

5 Particular case : It may be noted that on account of generalized nature of the I -function,several new intersting result can be drived with proper choice parameters. Hence formulae estaiblish in this paper are of general character.

Two particular cases should be mentoined here:

(i) In equation (2.1) setting r=1 and using (1.2) we obtained the result integrals involving Fox H-function.

(ii) In equation (2.1) by taking r=1 and $\alpha_j = \beta_j = 1$ so we obtained the result integrals involving Meijer's G-function.

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