Fuzzy Logic Gates in Electronic Circuits

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Abstract- In this paper, we can see a brief description to fuzzy logic operation and Logic gates. Here we see how Fuzzy Logic extends and generalizes classical logic with propositional logic. Proposition logic deals with finding truth values of formulas containing atomic propositions, whose truth value is either zero or one, connected by "and" (Λ) "or" (∨), implication (→) etc...Finally we find truth values of formulas using a t-norm for Λ and a t-co norm for ∨.

Index Terms- Propositional logic, t-norm, t-co norms, Logic gates

I. Introduction

Fuzzy set were first proposed by Lofti.A.Zadeh in his 1965 paper entitled none other than Fuzzy set. This is the foundation for all fuzzy logic that followed by mathematically defining fuzzy set and their properties. The basic assumption upon which classical logic (or two valued logic) is based that every proposition is either true or false has been questioned since Aristotle. In his treatise "on Interpretation", Aristotle discusses the problematic truth status of matters that are future contingent. Propositions about future events, he maintains are neither actually true nor false, but potentially either hence their truth value is undetermined at least prior to the event. It is that proposition whose truth status is problematic are not strictly restricted to future events.

Logic is the analysis of methods of reasoning. The propositional fuzzy Logic is a Logic which deals with proposition. A proposition is a sentence which is either true or false. The "true" and "false" are called truth values. They are denoted by 1 or 0.Then to any sentence is assigned only 1 or 0.The propositional fuzzy Logic based on this pre assumption is said to be the two valued or Classical propositional Logic.

2.Fuzzy Operators

Definition 2.1: A proposition is a declarative sentence that is either true (denoted either T or 1) or false (denoted either F or 0).
Notation: Variables are used to represent propositions. The most common variables used are p and q.

Definition 2.2: Unary Operator Negation: "not p", ¬p

Definition 2.3: Binary Operators

- **(a) Conjunction:** "p and q", p Λ q.
- **(b) Disjunction:** "p or q", p ∨ q.
- **(c) Exclusive or:** "exactly one of p or q", "p xor q", p ⊕ q.
- **(d) Implication:** "if p then q", p → q.
- **(e) Biconditional:** "p if and only if q", p ↔ q.

Definition 2.4: The functions used for intersection of fuzzy sets are called t-norms. A t-norm T is a function z = T (a, b), 0≤ a, b, z ≤ 1, having the following four properties.

1. T (a, 1) = a;
2. T (a, b) = T (b, a);
3. If $b_1 \le b_2$, then $T(a, b_1) \le T(a, b_2)$;
4. T (a, T (b, c)) = T (T (a, b), c).

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Definition 2.5: The functions used for union of fuzzy sets are called t- co norms. A t-co norm C is a function z = C (a, b), $0 \le a, b, z \le 1$, having the following four properties.

1. C (a, 0) = a;
2. C (a, b) = C (b, a);
3. If $b_1 \le b_2$, then C (a, b_1) ≤ C (a, b_2);
4. C (a, C (b, c)) = C (C (a, b), c).

3.TRUTH TABLES AND CONCEPTS OF PROPOSITIONAL FUZZY LOGIC

Example 3.1: Negation

p: I like blue color
¬p : I hate blue color

Table I: Truth Table for Negation

p	¬p
T	F
F	T

Example 3.2 : Conjunction

P: I like blue color q: I will eat ice-cream
p∧q :I like blue color, and I will eat ice-cream.

Table II: Truth Table for Conjunction

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Example 3.3 : Disjunction

P: I like blue color q: I will eat ice-cream
p∧q :I like blue color, or I will eat ice-cream.

Table III: Truth Table for Disjunction

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Example 3.4 : Exclusive or

P: I like blue color q:I will eat ice-cream
p⊕q : Either i like blue color, or I will eat ice-cream, but not both.

Table IV: Truth Table for Exclusive OR

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

Example 3.5 : Implication Operator

P: I like blue color q:I will eat ice-cream
p →q : If I like blue color, then I will eat ice-cream.

Table V: Truth Table for Implication Operator

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

4:Fuzzy Logic Gates:

Fuzzy Logic gates perform basic logical functions and are the fundamental building blocks of digital integrated circuits. Most fuzzy logic gates take an input of two binary values and output a single value of a 1 or 0.Some circuits may have only a few logic gates, while other such as microprocessors may have millions of them. There are seven different types of logic gates.

In the following examples each logic gates except NOT gate has two inputs P and Q , which can either be 1(True) or 0 (False).The resulting output is a single value of 1 if the result is true or 0 if it is false.

1. NOT - True if input is false, false if input is True.
2. AND - True if P and Q are both True.
3. OR - True if either P or Q are True.
4. XOR - True if either P or Q are True, but False if both are True.
5. NAND - AND followed by NOT: False only if P and Q are both True.
6. NOR - OR followed by NOT: True only if P and Q are both false.
7. XNOR - XOR followed by NOT: True if P and Q are both True or both False.

4.1 t-norms:

The basic t-norms are

Tm(a,b) = min(a,b) [Tm is standard intersection]
Tb(a,b) = max(0,a+b-1) [Tb is the bounded sum]
Tp(a,b) = a b [Tp is the algebraic product]
T* (a,b) = {a, if b =1 [T* is the drastic intersection]
b , if a=1
0, otherwise

$T^*(a,b) \leq Tb(a,b) \leq Tp(a,b) \leq Tm(a,b)$, for all a,b in [0,1] then
$T^*(a,b) \leq T(a,b) \leq Tm(a,b)$.

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4.2 t- co norms:

The basic t co -norms are

Cm(a,b) = min(a,b) [Cm is standard union]

Cb(a,b) = max(0,a+b-1) [Cb is the bounded sum]

Cp(a,b) = a b [Cp is the algebraic product]

C* (a,b) ={ a, if b =1 [C* is the drastic union]
b, if a=1
0, otherwise

C* (a,b) ≤ Cb(a,b) ≤ Cp(a,b) ≤ Cm(a,b) , for all a,b in [0,1] then,

Cm(a,b) ≤ C (a,b) ≤ C* (a,b)

5.Applications & Results:

Traditional *Bivalent fuzzy* logic uses the Boolean operators AND, OR, and NOT to perform the intersect, union and complement operations. These operators work well for bivalent sets and can be essentially defined using the following truth table

Table VI: Truth Table for Logic Gates AND, OR & NOT

x	y	x AND y	x OR y	NOT x
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

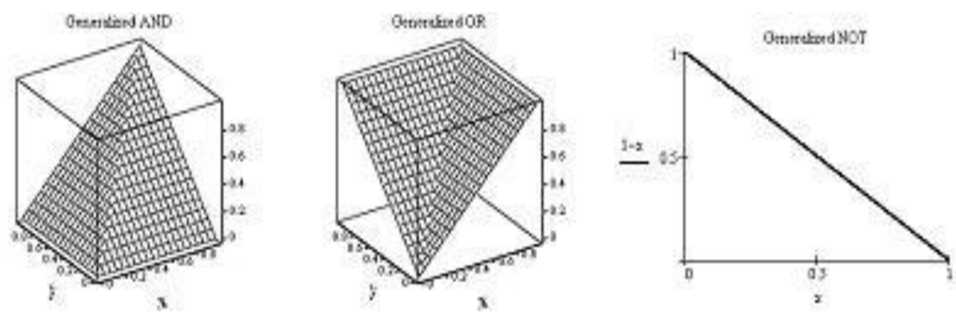

Figure 1: AND, OR & NOT Visualization

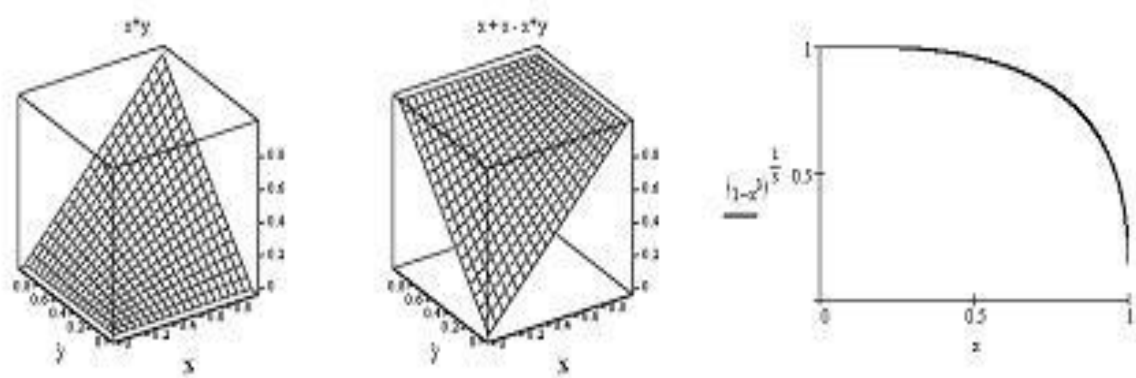

Figure 2: t-norms, t-conorms & negation Visualization

The truth table above works fine for bivalent logic but fuzzy logic does not have a finite set of possibilities for each input; this makes for an infinitely large truth table. The operators need to be defined as functions for all possible fuzzy values, that is, all real numbers from 0 to 1 inclusive. Fuzzy logic is actually a superset of bivalent logic since it includes the bivalent options (0,1) as well as all real's in between, so a generalized form of these operators will be use full. The generalized form for these three operators are given in the table VII.

Table VII: Fuzzy Logic Gates and their Generalized forms

x AND y	min(x,y)
x OR y	max(x,y)
NOT x	1 - x

Using these definitions they can be applied to all of the bivalent combinations above as well as some fuzzy number combinations. The truth table for this can be seen below:

Table VIII: Application examples for Bivalent and Fuzzy Number Combinations

x	y	min(x,y)	max(x,y)	1 - x
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0
0.3	0.7	0.3	0.7	0.7
0.6	0.4	0.4	0.6	0.4
0.9	0.9	0.9	0.9	0.1

CONCLUSION

From the above discussion we can come to a conclusion that fuzzy proposition are equivalent to Logic gates in Electronics Circuits. Mathematical definitions of the AND operator are called triangular norms or t-norms, this name is derived from the shape of the generalized AND. A t-norm is, by definition, a binary operator with both operand and the result in [0,1], is commutative, associative, has 1 as an identity, and is increasing in each variable. Mathematical definitions of the OR operator have all the same properties of t-norms except that they have 0 as an identity; they are called t-conorms. The NOT operator can be redefined as long as it is a continuous, strictly decreasing function within [0,1]

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