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Modeling, Analysis and Control of Active Suspension System using Sliding Mode Control and Disturbance Observer

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Abstract- The purpose of this paper is to construct an active suspension control for a quarter car model subject to excitation from a road profile using an improved sliding mode control with an observer design. The sliding mode is chosen as a control strategy, and the road profile is estimated by using an observer design. The objective of a car suspension system is to improve the riding quality without compromising the handling characteristic by directly controlling the suspension forces to suit the road and driving conditions. However, the mathematical model obtained suffers from few uncertainties. In order to achieve the desired ride comfort, road handling and to solve the uncertainties, a sliding mode control technique is presented. A nonlinear surface is used to ensure fast convergence of vehicle's vertical velocity. The nonlinear surface changes the system's damping. The effect of sliding surface selection in the proposed controller is also presented. Extensive simulations are performed and the results obtained shows that the proposed controller perform well in improving the ride comfort and road handling for the quarter car model using the hydraulically actuated suspension system. The main motivation for designing an active suspension system is to improve the ride comfort by absorbing the shocks due to a rough and uneven road.

Index Terms- Active suspension System, Disturbance Observer, Sliding Mode Control, Road Profile

I. Introduction

For many years vehicle dynamics engineers have struggled to achieve a compromise between vehicle handling, ride comfort and stability. The results of this are clear in the vehicles we see today. In general, at one extreme are large sedan and luxury cars with excellent ride qualities but only adequate handling behavior. At the other end of the spectrum are sports cars with very good handling but very firm ride quality. In between is any number of variations dictated by the vehicle manufacturer and target customer needs? Every automotive suspension has two goals: passenger comfort and vehicle control. Comfort is provided by isolating the vehicle's passengers from road disturbances like bumps or potholes. Control is achieved by keeping the car body from rolling and pitching excessively, and maintaining good contact between the tire and the road.

Today's vehicle suspensions use hydraulic dampers (shock absorbers) and springs that are charged with the tasks of absorbing bumps, minimizing the car's body motions while accelerating, braking and turning and keeping the tires in contact with the road surface. Typically, these goals are somewhat at odds with each other.

A typical vehicle suspension is made up of two components: a spring and a damper. The spring is chosen based solely on the weight of the vehicle, while the damper is the component that defines the suspensions placement on the compromise curve. Depending on the type of vehicle, a damper is chosen to make the vehicle perform best in its application. Ideally, the damper should isolate passengers from low-frequency road disturbances and absorb high frequency road disturbances. Passengers are best isolated from low-frequency disturbances when the damping is high.

However, high damping provides poor high frequency absorption. Conversely, when the damping is low, the damper offers sufficient high-frequency absorption, at the expense of low-frequency isolation. The need to reduce the effects of this compromise has given rise to several new advancements in automotive suspensions. Three types of suspensions that will be reviewed here are passive, fully active, and semi-active suspensions. A conventional passive suspension is composed of a spring and a damper. The suspension stores energy in the spring and dissipates energy through the damper. Both components are fixed at the design stage.

There has been a widespread interest in using advanced control techniques to improve the performance of vehicle suspension system. Performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Several performance characteristics have to be considered in order to achieve a good suspension system. These characteristics deal with regulation of body movement, regulation of suspension movement and force distribution. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration [1]. During the design of a suspension system, a number of conflicting requirements have to be met [2]. The suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. These objectives can be achieved by minimizing the vertical car body acceleration. Also, optimal contact between wheel and road surface is needed in various driving conditions in order to maximize safety [3]. An early design for automobile suspension systems was focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [4]. Thus the passive suspension system, which approach optimal characteristics had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, and wheel velocity and wheel and body acceleration [5]. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. In real dynamical system, it is impossible to avoid uncertainties due to modeling and due to other external disturbances. So the crucial is a solution to the robust control problem for uncertain systems.

II. ACTIVE SUSPENSION SYSTEM NONLINEAR MODEL

The concept of active suspension system was introduced as early as 1958.Since the vibration suppression capabilities of the traditional passive and semi-active suspension systems are restricted, an active suspension system with additional control force to suppress the oscillations is one of the major development fields in recent vehicle industry. The difference compare to conventional suspension is active suspension system able to inject energy into vehicle dynamic system via actuators rather than dissipate energy. In active suspension system actuator are placed between the unsprung mass and the sprung mass to produce control force to cope with a variety of road disturbances in real time. The main motivation for designing an active suspension system is to improve the ride comfort by absorbing the shocks due to a rough and uneven road. In the active suspension system, the force actuator is able to both add and dissipate energy from the system. This Results in the capability of the suspension system to control the attitude of the vehicle, to reduce the effects of braking and to reduce the vehicle roll during cornering and braking in addition to increasing the ride comfort and vehicle handling. Figure 1 shows a quarter car model for active suspension system.

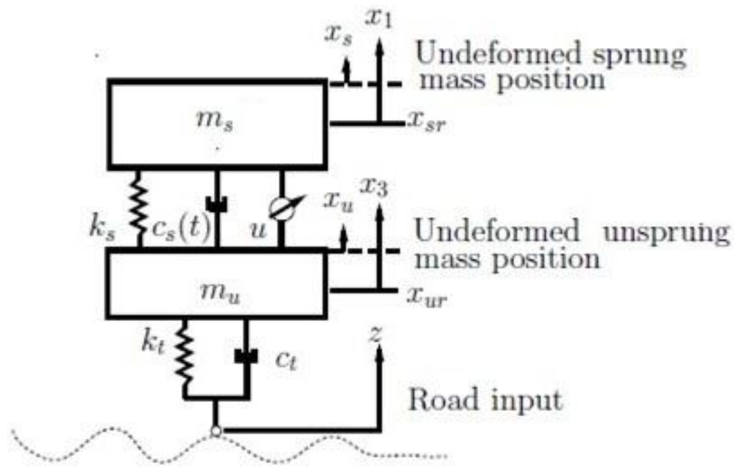

Figure 1: A quarter-car model of active suspension system.

The sprung mass m_s represents the mass of the car body, frame, internal components that are supported by the suspension. The unsprung mass m_u is the mass of the assembly of the axle and the wheel, k_s and $c_s(t)$ are respectively the spring and time-varying damper coefficients of the passive components of the suspension system. The spring coefficient k_s comprise of linear stiffness coefficient k_{1s} and nonlinear stiffness coefficient k_{2s}. The coefficient k_s and $c_s(t)$ are unknown because k_s changes as per the loading condition and $c_s(t)$ is a time varying damping coefficient. The coefficients k_t and c_t are respectively the unknown linear spring and damper coefficients of the tyre model. The control force generated by actuator connected between sprung and unsprung masses is denoted by u while z denotes the road disturbance input acting on the unsprung mass. The vertical displacements of the sprung mass and unsprung mass with respect to their undeformed suspension positions are denoted by x_s and x_u respectively.

III. MATHEMATICAL MODELING OF ACTIVE SUSPENSION SYSTEM

According to Newton's second law and free body diagram approach, the equations of motion for the system are written as
For sprung mass-

$$m_s\ddot{x}_s = -k_{1s}(x_s - x_u) - k_{2s}(x_s - x_u)^3 - c_s(t)(\dot{x}_s - \dot{x}_u) + u - m_s gr \quad (1)$$

For unsprung mass-

$$m_u\ddot{x}_u = k_{1s}(x_s - x_u) + k_{2s}(x_s - x_u)^3 + c_s(t)(\dot{x}_s - \dot{x}_u) - k_t(x_u - z) - c_t(\dot{x}_u - \dot{z}) - u - m_u g_r \quad (2)$$

Where g_r is the acceleration due to gravity. The static tyre deflection due to the nominal road denoted by x_{ur} is given by

$$x_{ur} = -\frac{(m_u + m_{sm})}{k_t} g_r \quad (3)$$

The reference position of the unsprung mass is considered to be x_{ur} while the reference position of the sprung mass denoted by x_{sr} is defined as

$$x_{sr} = x_{ur} + \delta_0 \quad (4)$$

Where $\delta_0 < 0$ is the static spring deflection.

Next we express the dynamic model of the suspension system (1) and (2) into the state variable form. Let the state $x=[x_1\ x_2\ x_3\ x_4]^T$ be defined as,

$$x_1 = x_s - x_{sr}$$
$$x_2 = \dot{x}_s$$
$$x_3 = x_u - x_{ur}$$
$$x_4 = \dot{x}_u \quad (5)$$

For convenience of expression and interpretation of the equation, first we define function f(x, t) and g(t) given by the equations,

$$f(x,t) = \frac{1}{m_s}[-k_{1s}(x_1 - x_3 + \delta_0) - k_{2s}(x_1 - x_3 + \delta_0)^3] + \frac{1}{m_s}[-c_s(t)(x_2 - x_4) - m_s g_r] \quad (6)$$

$$g(t) = \frac{1}{m_s} \quad (7)$$

It may be noted that these functions are uncertain and that the function f(x, t) depends on all the four states of the suspension system.

$$g(t) = g_m\,\Delta g \quad (8)$$

Where g_m is the nominal value and Δg is the multiplicative uncertainty. The bounds of g(t) are not known.
hence

$$\dot{x}_1 = x_2 \quad (9)$$

$$\dot{x}_2 = f(x,t) + g(t)u \quad (10)$$

$$\dot{x}_3 = x_4 \quad (11)$$

$$\dot{x}_4 = \frac{m_s}{m_u}(f(x,t) + g_r) - \frac{1}{m_u}(u + k_t(x_3 + x_{ur} - z) + c_t(x_4 - \dot{z}) + m_u g_r) \quad (12)$$

The parameters of the passive suspension are unknown and nonlinear. Further, the vehicle is subjected to uneven road profile about which no a priori information is available. We assume that even the bounds of these uncertainties are not known. The objective is to control the deflection of the sprung mass in order get good ride comfort using the measurements of x_1 and x_2 only. The motivation for using only these measurements is to simplify the implementation by avoiding the deployment of sensors on the wheel and tyre assembly.

IV. SLIDING SURFACE AND CONTROL DESIGN

In this section we select sliding surface as a function of x1 and x2 and design a scheme of control using disturbance observer which estimates the lumped uncertainty.

a. Sliding Surface

In this section we select sliding surface as a function of x1 and x2 and design a scheme of control using disturbance observer which estimates the lumped uncertainty.

$$\sigma = Sx_1 + x_2 \quad (13)$$

Where S is a user chosen constant.

$$\dot{\sigma} = Sx_2 + + f(x,t) + g(t)u \quad (14)$$

By using (4.8) in (4.14) and rearranging,

$$\dot{\sigma} = Sx_2 + f(x,t) + g_m(\Delta g - 1)u + g_m u \quad (15)$$
$$= Sx_2 + e(x,t) + g_m u \quad (16)$$

Where e(x, t) is the lumped uncertainty, given by the equation

$$e(x,t) = f(x,t) + g_m(\Delta g - 1)u \quad (17)$$

Assumption 1: The lumped uncertainty e(x, t) is such that

$$\left|\frac{de(x,t)}{dt}\right| < \mu \quad (18)$$

Where μ is a small positive number.

To compensate for the uncertainty and to get good ride comfort, sliding mode control in combination with disturbance observer which estimates the lumped uncertainty e(x, t) is used. In the sequel, at times we denote e(x, t) by simply e.

b. Design of Control

We split control u to be designed into two parts viz. u_{eq} to compensate for known terms and u_n to compensate for the estimate of the lumped uncertainty. The scheme is to estimate the uncertainty by disturbance observer and then to use opposite of it in u_n.

$$u = u_{eq} + u_n \quad (19)$$

With

$$u_{eq} = -\frac{1}{g_m}(Sx_2 + K\sigma) \quad (20)$$

where K is a positive number. Using (19) and (20) in (16)

$$\dot{\sigma} = -K\sigma + e + g_m u_n \quad (21)$$

Now selecting

$$u_n = -\frac{1}{g_m}\hat{e} \quad (22)$$

Where $\hat{e}$ be the estimate of uncertainty e. substituting (22) in (21), we get

$$\dot{\sigma} = -K\sigma + e - \hat{e} \quad (23)$$

Define the estimation error as

$$\tilde{e} = e - \hat{e} \quad (24)$$

And substituting in (4.23), we get

$$\dot{\sigma} = -K\sigma + \tilde{e} \quad (25)$$

If the estimate $\hat{e}$ is such that $\tilde{e}$ goes to zero, sliding variable σ will go close to zero, thereby reducing the displacement of the sprung mass and improving the ride comfort.

c. Modified Control

It can be seen from (4.25) that addition of a discontinuous component to the control (4.19) will help bring σ closer to 0. However in view of the Assumption 1, a pure discontinuous component will not work. Therefore, as commonly done in sliding mode control, we add a discontinuous component u_s with its smooth approximation. We propose a control

$$u = u_{eq} + u_n + u_s \quad (26)$$

With u_s given by

$$u_s = -k_{st}\,\mathrm{sat}(\sigma) \quad (27)$$

Where k_{st} is a positive constant to be chosen by the designer. Under the modified control, the dynamics of σ is given by

$$\dot{\sigma} = -K\sigma - k_{st}\,\mathrm{sat}(\sigma) + \tilde{e} \quad (28)$$

V. DISTURBANCE OBSERVER

We use the term 'disturbance Observer' for an estimator that gives an estimate of the lumped uncertainty e(x, t). Next, we design a disturbance observer.

Let the estimate of the lumped uncertainty e(x, t) be given by

$$\hat{e} = \hat{d}(t) + p(\sigma) \qquad (29)$$

Where p(σ) is some linear or nonlinear scalar function of σ. Now, $\hat{d}(t)$ is to be updated in such a way that the estimation error $\tilde{e} = e - \hat{e}$ goes to zero. Differentiating (29) we get,

$$\dot{\hat{e}} = \dot{\hat{d}}(t) + \frac{\partial p}{\partial \sigma}\dot{\sigma} \qquad (30)$$

Substituting $\dot{\sigma}$ from (21)

$$\dot{\hat{e}} = \dot{\hat{d}}(t) + \frac{\partial p}{\partial \sigma}(-K\sigma + e + g_m u_n) \qquad (31)$$

This suggests an update law for $\hat{d}(t)$ as

$$\dot{\hat{d}}(t) = -\frac{\partial p}{\partial \sigma}(-K\sigma + \hat{e} + g_m u_n) \qquad (32)$$

Giving

$$\dot{\hat{e}} = \frac{\partial p}{\partial \sigma}\tilde{e} \qquad (33)$$

Subtracting both sides of (33) from $\dot{e}$, we get

$$\dot{\tilde{e}} = -\frac{\partial p}{\partial \sigma}\tilde{e} + \dot{e} \qquad (34)$$

This suggests that the choice of p(σ) be such that $\frac{\partial p}{\partial \sigma}$ is a positive function.

A. Stability of the System during Sliding Mode

During the sliding mode, the uncertain system with mismatched condition is stable provided the following theorem is satisfied.

Theorem 1:

Consider a candidate Lyapunov function

$$V(\sigma, \tilde{e}) = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{e}^2 \qquad (35)$$

Taking the derivative of $V(\sigma, \tilde{e})$ along (25) and (34)

$$\dot{V}(\sigma, \tilde{e}) = -K\sigma^2 + \sigma\tilde{e} - \frac{\partial p}{\partial \sigma}\tilde{e}^2 + \tilde{e}\dot{e} \qquad (36)$$

Using young's inequality $ab \le \frac{1}{2}(a^2 + b^2)$ and the assumption 1 and simplifying

$$\dot{V}(\sigma, \tilde{e}) \le -\left(K - \frac{1}{2}\right)\sigma^2 - \left(\frac{\partial p}{\partial \sigma} - 1\right)\tilde{e}^2 + \frac{1}{2}\mu^2 \qquad (37)$$

The control parameters can be chosen so that $\left(K - \frac{1}{2}\right) > 0$ and $\frac{\partial p}{\partial \sigma} - 1 > 0$. From (37) it can be seen that the dynamics of σ and the estimation error $\tilde{e}$ is not asymptotically stable but is ultimately bounded.

VI. RESULTS AND CONCLUSION

Figure 2 and 3 shows the response of passive suspension system for unit step and two bump road profile respectively. The Response of passive suspension system shows that the system is stable but need some time to settle down for unit step input and in case of two bump road disturbances; the passive suspension system could not attenuate the given force. This cause sprung mass deflection occur in the system.

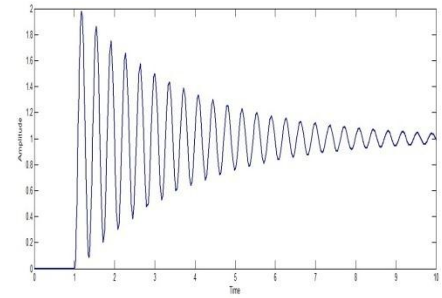

Figure 2: Sprung mass displacement of passive suspension system for unit step road profile

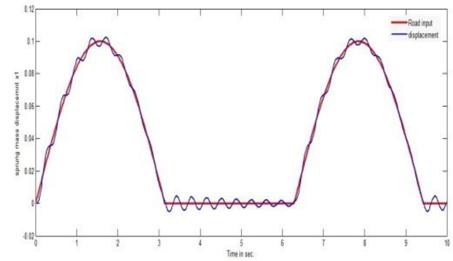

Figure 3: Sprung mass displacement of passive suspension system for two bump road profile

Next, we investigate the performance of active suspension system. Figure 4 and figure 5 shows the response of active suspension system for unit step and two bump road profile. The figure shows that the proposed system is successful in attenuating the deflection and acceleration of the sprung mass which demonstrates the efficacy of the system in improving the ride comfort.

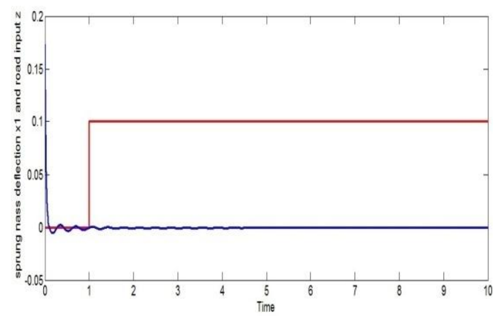

Figure 4: Sprung mass deflection x1 and road profile z

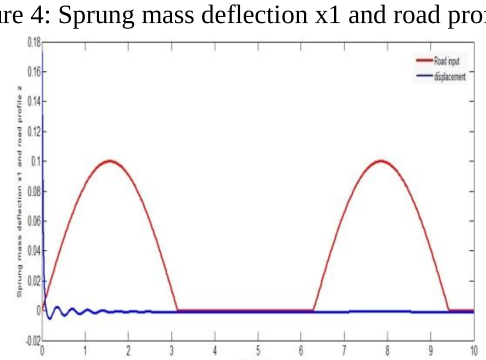

Figure 5: Sprung mass deflection x1 and road profile z

Figure 6 and 7 shows the control during sliding mode control for unit step and two bump road profile.

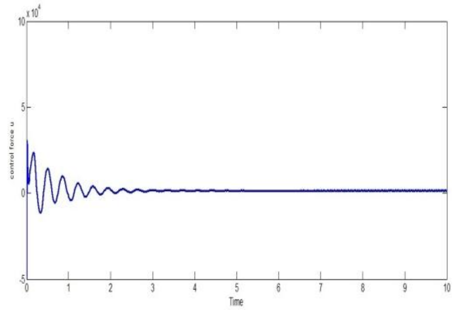

Figure 6: Control Force u for unit step disturbance

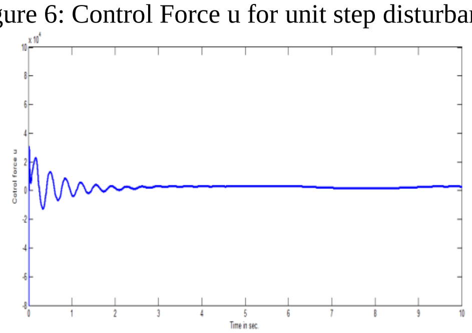

Figure 7: Control Force u for two bump disturbance

From the results it is proven that by using disturbance observer, active suspension will response much better than without observer and also passive suspension system. Passive suspension is the weakest suspension to absorb any disturbance exerted to the system. In this chapter the performance of proposed observer and SMC with disturbance observer has been investigated. It has been shown that the SMC with disturbance observer improved the ride comfort and road handling performances of quarter car active suspension system compared to passive suspension system with disturbance observer control technique. The proposed sliding mode control with disturbance observer is robust to various types of disturbance. Furthermore, the chattering problem has been overcome by using the continuous switching function and the boundary layer that can be adjusted by varying the sliding gain in the controller.

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