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Fuzzy aw-Continuous Mappings

Manoj Mishra[1], S. S. Thakur[2]

[2] Department of Applied Mathematics JEC JBP

Abstract- The purpose of present paper introduce the concepts of fuzzy aw- closed mappings and to obtain some of their basic properties and characterizations.

I. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y)=β for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.

Definition 1.1[10]: A fuzzy set A of a fuzzy topological space (X,τ) is said to be:

(a). fuzzy semiopen if $A \leq cl(int(A))$.
(b). fuzzy g-closed if $cl(A) \leq O$ whenever $A \leq O$ and O is fuzzy open set.
(c). fuzzy g-open if 1-A is fuzzy g-closed.
(d). fuzzy w-closed if $cl(A) \leq O$ whenever $A \leq O$ and O is a fuzzy semi open set.
(e). fuzzy w-open if 1-A is fuzzy w-closed.

Remark 1.1[10]: Every fuzzy closed set is fuzzy w-closed and every fuzzy w-closed set is fuzzy g-closed but the converse may not be true.

Defination1.2: A mapping f: $(X,\tau)\to(Y,\sigma)$ is said to be ;

(a). fuzzy w-continuous if the inverse image of every fuzzy closed set of Y is fuzzy closed in X[6].
(b). fuzzy g-continuous if the inverse image of every fuzzy closed set of Y is fuzzy g-closed in X.
(c). fuzzy w-irresolute if the inverse image of every fuzzy w-closed set of Y is fuzzy w-closed in X.

Remark 1.2: Every fuzzy continuous mapping is fuzzy w-continuous and every fuzzy w-continuous mapping is fuzzy g-continuous mapping but the converse may not be true.

The concepts of fuzzy w-irresolute and fuzzy continuous mapping are independent.

Definition 1.2: A fuzzy topological space (X,τ) is said to be fuzzy w-$T_{1/2}$ if every fuzzy w-closed set is fuzzy closed set in X.

II. Fuzzy aw-continuous mappings

Defination2.1: A mapping f: $(X,\tau)\to(Y,\sigma)$ is said to be fuzzy aw- continuous provided that $cl(A) \leq f^{-1}(O)$ whenever O is fuzzy semi open in Y, A is fuzzy w-closed in X and $A \leq f^{-1}(O)$.

Theorem 2.1: Every fuzzy irresolute is fuzzy aw-continuous.

Proof: Let O be a fuzzy semi open set of Y,A is a fuzzy w-closed set of X and $A \leq f^{-1}(O)$ .Now f is fuzzy irresolute $f^{-1}(O)$ is fuzzy semi open set of X. Since A is fuzzy w-closed and $A \leq f^{-1}(O)$ it follows that $cl(A) \leq f^{-1}(O)$.Hence f is fuzzy aw-continuous.

Remark 2.1: The converse of theorem (3.2.1) is not true for,

Example2.1:Let X={a,b} ,Y={x,y} and τ={0,1} and σ={0,A,1} be the fuzzy topologies where
A(x)=0.3 ,A(y)=0.4.Then the mapping f: $(X,\tau)\to(Y,\sigma)$ be a mapping defined by f(a) = x, f(b) = y, is fuzzy aw-continuous but not fuzzy R-map .Now consider the following example.

Example2.2:Let X={a,b} ,Y={x,y} and τ={0,A,1} and σ={0,1} be the fuzzy topologies where A(a)=0.7 ,A(b)=0.5 .Then the mapping f: $(X,\tau)\to(Y,\sigma)$ be a mapping is fuzzy continuous but not fuzzy aw-continuous.

Remark2.2: Example (2.1) and (2.2) asserts that the concepts of fuzzy continuous and fuzzy aw-continuous mappings awe independent.

Theorem 2.2: Let If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy aw-continuous and fuzzy closed mappings then the image of every fuzzy w-closed set of X is fuzzy w-closed in Y.

Proof: Let A be a fuzzy w-closed set of X and $f(A) \leq O$ where O is the fuzzy semi open set in Y then $A \leq f^{-1}(O)$ and hence f is fuzzy aw- continuous $cl(A) \leq f^{-1}(O)$ which implies $f(cl(A)) \leq O$ since f is fuzzy closed we have $cl(f(A)) \leq cl(fcl((A)))=f(cl(A)) \leq O$ .Hence f(A) is fuzzy w-closed set in Y.

Theorem2.3: If (X,τ) is fuzzy w-$T_{1/2}$ then every mapping If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy aw-continuous.

Proof: Let A be a fuzzy w-closed set of X and $A \leq f^{-1}(O)$ where O is fuzzy semi open set in Y. Since X is fuzzy w-$T_{1/2}$, A is fuzzy closed in X therefore cl(A)=A ≤ f^{-1}(O).Hence f is fuzzy aw- continuous.

Theorem 2.4:If FSO(X)=FC(X) then a mapping f: (X,τ)→(Y,σ) is fuzzy aw-continuous if and only if f^{-1}(O) is fuzzy closed in X for every fuzzy semi open set O in Y.

Proof: Necessity: Let f: (X,τ)→(Y,σ) is fuzzy aw-continuous by theorem(1.7) every fuzzy set of X is fuzzy w-closed (and hence fuzzy w-open) Thus for any fuzzy semi open set O of Y, f^{-1}(O). Is fuzzy w-closed in X. Since f^{-1}(O) ≤ f^{-1}(O) and f is fuzzy aw- continuous, cl(f^{-1}(O)) ≤ f^{-1}(O).Hence f^{-1}(O) is fuzzy closed in X.

Sufficiency: Let O be a fuzzy semi open set of Y and A be a fuzzy w-closed set of X such that A≤ f^{-1}(O) then cl(A) ≤ cl(f^{-1}(O))=f^{-1}(O), because by assumption f^{-1}(O) is fuzzy closed in X, hence f is fuzzy aw-continuous.

Theorem 2.5: If FSO(X)=FC(Y) then a mapping f: (X,τ)→(Y,σ) is fuzzy aw- continuous if and only if it is an fuzzy irresolute.

Proof: Necessity: Let O be a fuzzy semi open set of Y, then by theorem(2.4) f^{-1}(O) is fuzzy closed in X and so f^{-1}(O) is fuzzy semi open in X and hence f is fuzzy is an irresolute.

Sufficiency: Let A be a fuzzy w-closed of X and O be a fuzzy semi open set of Y and A ≤ f^{-1}(O).By hypothesis f^{-1}(O) is fuzzy semi open and thus fuzzy semi closed, cl(A) ≤ cl(f^{-1}(O))=f^{-1}(O) hence f is fuzzy aw-continuous.

Theorem 2.6:If f: (X,τ)→(Y,σ) is fuzzy aw- continuous and g:(Y,σ)→(z,ω) is a fuzzy irresolute then g of: (Y,σ))→(z,ω) is fuzzy aw-continuous.

Proof: Let A be a fuzzy w-closed of X and O be a fuzzy semi open sub set of Z such that A ≤ $(gof)^{-1}$(O). Since g is a fuzzy irresolute g^{-1}(O) is fuzzy semi open in Y. Since f is fuzzy aw- continuous cl(A) ≤ f^{-1}(g^{-1}(O))=$(gof)^{-1}$(O).Hence (gof) is fuzzy aw-continuous.

Definition 2.2:A mapping f: (X,τ)→(Y,σ) is called fuzzy aw-closed provided that f(A) ≤ int(O) whenever A is fuzzy semi closed in X O is fuzzy w-open in Y and f(A) ≤ O.

Theorem 2.7: Every fuzzy w-continues and fuzzy aw-closed mappings awe fuzzy w- irresolute.

Proof: A mapping f: (X,τ)→(Y,σ) is called fuzzy w-continuous and fuzzy aw-closed let A is fuzzy semi closed in Y and f^{-1}(A) ≤ O where O is fuzzy semi -open in X. Then 1-O ≤ f^{-1}(1-A) which implies f(1-O) ≤ (1-A).Since f is fuzzy aw-closed f(1-O) ≤ int(1-A)=1-cl(A).Hence f^{-1}(cl(A)) ≤ O. Since f is fuzzy w-continuous f^{-1}(cl(A)) is fuzzy w-closed set in X. Therefore cl(f^{-1}(cl(A))) ≤ O which implies that cl(f^{-1}(A) ≤ O .Hence f^{-1}(A) is fuzzy w-closed in X.

Theorem 2.8: A mapping f: (X,τ)→(Y,σ) is fuzzy continuous and fuzzy aw-closed mapping then it is fuzzy w-irresolute.

Proof: Follows from theorem (2.7).

Theorem 2.9: If (Y,σ) is fuzzy w-$T_{1/2}$ then every mapping f: (X,τ)→(Y,σ) is fuzzy aw-closed.

Proof: Obvious

Theorem 2.10: If FSO(X) = FC(Y) then a mapping f: (X,τ)→(Y,σ) is fuzzy aw-closed if and only if f(O) is open for every fuzzy semi closed subset O of X.

Proof: **Necessity**: Let A mapping f: (X,τ)→(Y,σ) is fuzzy continuous and fuzzy aw-closed mapping by theorem (1.7) ,all fuzzy set of Y are fuzzy w-closed and hence all are fuzzy w-open .Thus for any fuzzy semi closed subset O of X. f(O) is fuzzy w-open in Y. Since f is aw-closed f(O) ≤ intf(O).Hence f(O) is fuzzy open.

Sufficiency: Let O be a fuzzy semi closed set of X and A be a fuzzy w-open set of Y and f(O) ≤ A. By hypothesis f(O) is fuzzy open in Y and so f^{-1}(O)=int(f(O)) ≤ int(A) hence f is fuzzy aw-closed.

Definition 2.3: A mapping f: (X,τ)→(Y,σ) is fuzzy pre semi closed if the image of every fuzzy semi closed set of X is fuzzy semi closed in Y.

Theorem 2.11: If FSO(X) =FC(Y) then a mapping f: (X,τ)→(Y,σ) is fuzzy aw-closed if and only if f is fuzzy pre semi closed .

Proof: Necessity: Let O be a semi closed set of X then by theorem (2.10) f(O) is fuzzy open and fuzzy closed and hence it is fuzzy semi closed and f is fuzzy pre semi closed.

Sufficiency: Let O be a fuzzy semi closed set of X and A be a fuzzy w-open set of Y and f(O) ≤ A. By hypothesis f(O) is fuzzy semi closed in Y. Thus f(O) is fuzzy semi .Hence f(O)=int(f(O)) ≤ int(A) and f is fuzzy aw-closed.

Theorem 2.12: Let mapping f: (X,τ)→(Y,σ) is fuzzy pre semi closed and g :(Y,σ) →(Z, ω) is fuzzy aw-closed then (g of) is fuzzy aw-closed.

Proof: Let O be a fuzzy semi closed set of X and A be a fuzzy w-open set of Z, such that (g of)(O) ≤ A since f is fuzzy pre semi closed f(O) is fuzzy semi closed in Y. Therefore (g of)(O) ≤ int(A) because g is fuzzy aw-closed and hence (g of) is fuzzy aw-closed.

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AUTHORS

First Author – Manoj Mishra
Second Author – S. S. Thakur, 2Department of Applied Mathematics JEC JBP, E-mail: drmkm1969@rediffmail.com