Jeans Instability of Optically Thick Quantum Plasma under the Influence of Black-Body Radiation

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Abstract- The Jeans instability of infinite homogeneous, self-gravitating quantum plasma is investigated under the assumption of optically thick medium and black body radiation. The general dispersion relation is obtained and the condition of instability is discussed.

Index Terms- Jeans instability, Quantum Plasma, Black Body Radiation, Self-Gravitating and Jeans Wave Length

PACS No. *52.35; 52.40*

I. INTRODUCTION

It was first shown by James Jeans [1] that an infinite homogeneous self-gravitating fluid is unstable for all wave number which is less than critical Jeans wave number k_J and given as

$$k_J = \sqrt{4\pi\rho G / c_S^2}$$

In this formula G is the gravitational constant, ρ is the unperturbed matter density and $c_s = \sqrt{K_B/m}$, is the sound speed for adiabatic perturbation, where K_B is the Boltzmann's constant, T is the physical temperature and *m* is the mass of the particle. A through reconsideration of the Jeans criterion of instability is given by Chandrasekhar [2]. He has pointed out that uniform rotation and magnetic field not alter Jeans criterion whether taken separately or simultaneously. Ebert [3] has studied the problem of an isothermal gas sphere subjected to external pressure and he suggested that disturbances of length scale approximately equal to the Jeans length based on the central density were unstable to gravitational collapse. He also has shown that that O star could form in the centre of an interstellar cloud. Hunter [4] has investigated the growth of perturbations in a gravitationally contracting isothermal gas cloud and he suggested that perturbations with initial scale of the order of or less than the Jeans length grew less rapidly relative to the back ground density than did perturbation of substantially larger dimension. A detailed account of Jeans instability under varying assumptions of hydrodynamics has been studied by various authors [Sharma and Thakur [5], Herrengger [6], Sharma and Trilok Chand [7], Patidar *et al.* [8], Pensia *et al.* [9], Pensia *et al.* [10], Dangarh *et al.* [11], Uberoi [12].

Recently, there has been interest in understanding the role of radiative heat-loss mechanism in the star formation and molecular cloud condensation process in connection with thermal instability. An important point to be noted in this case, that the heat-loss process is the major cause for the condensation of large astrophysical compact objects. In this paper we argue that for the case where the perturbations under heat-loss by black body radiation under the plasma correction, since radiative pressure in the internal area of the star is usually small in comparison with the gaseous pressure, in interior of hot and large plasma clouds like H_{II} regions, etc., where temperature is rather high while the density is slow, it should be important to take into account the radiative processes. The radiative heat-loss functions are similar to those of the cooling functions considered earlier by Chandrashekhar. In this way, many investigators have discussed the gravitational instability of homogeneous plasma considering the effects of heat-loss mechanism in different form {Wolfire *et al.* [13], Shadmehari and Dib [14], Prjapati *et al.* [15], Vranjes and Cadez [16], Kim and Narayan [17], and Stiele *et al.* [18]}

The consideration of quantum correction in with enhanced turbulent fluctuation can be regarded as central problem in attempting to interpret various astrophysical and laboratory plasma phenomena. The quantum plasma was first investigated by Pines [19, 20] and the kinetic model of the quantum electro dynamical properties of non-thermal plasma analyzed by Bezzerides and Du Bios [21]. Covariant Wigner function description of relativistic quantum plasmas is given by Hakim and Hayvaerts [22]. The Jeans instability of self-gravitating astrophysical quantum dusty plasma has been studied by Shukla and Stenflo [23] and this study was extended by Masood *et al.* [24] by considering a multi-component self-gravitating quantum Bohm potential and statistical terms on electrons and ions were considered. The Jeans instability in homogeneous cold quantum dusty plasma in the presence of a magnetic field and quantum correction was examined by Salimullah *et al.* [25].

In this paper the quantum correction is taken to formulate the hot molecular cloud under the black body radiation. To avoid the discussion about the additional convective instability, effects which may occur in such a system if the temperature T is a decreasing function of a coordinate, therefore, we shall assume T to be constant. As the influence of viscosity, electrical conductivity and magnetic field on the gravitational instability is studied by many authors, we are not going to discuss them here. Thus, in present work, we investigate the simultaneous effects of quantum corrections and black body radiation on Jeans instability of gaseous plasma. The result of the present study is applicable to understand the processes of star formation and condensation of molecular cloud.

II. EQUATIONS OF THE PROBLEM

Let us consider an infinite homogeneous, self-gravitating quantum plasma incorporating black body radiation We introduced the quantum effects through the Bohm potential term in the momentum transfer equation. For simplicity of the problem, we ignore the additional convective instability, effects which may occur in such a system if the temperature T is a decreasing function of a coordinate, we shall assume T to be constant. The medium is taken optically thick and the black body radiation is assumed.

The momentum transfer equation for magnetized quantum plasma is

$$\rho \frac{d\vec{u}}{dt} = -\vec{\nabla} p + \rho \vec{\nabla}\phi + \frac{\hbar^2 \rho}{2m_e m_i}\vec{\nabla}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) \quad (1)$$

The equation of continuity is given by

$$\frac{d\rho}{dt} + \rho\vec{\nabla}.\vec{u} = 0 \quad (2)$$

Poisson's equation for the gravitational potential is given as

$$\nabla^2\phi = -4\pi G\rho \quad (3)$$

The radiative heat transfer equation is given as

$$\frac{dT}{T} = (\Gamma - 1)\frac{d\rho}{\rho} \quad (4)$$

where $p = p_g + p_r$ is the total pressure of the medium, $p_g = \rho RT$ is the gas pressure and $p_r = K_B T^4/3$ is the radiation pressure, K_B is the Boltzmann's constant. Also the symbols ρ, γ, $\vec{v}$, ϕ, G, T, Γ are denote the number density, adiabatic index, fluid velocity, gravitational potential, universal gravitational constant, temperature and radiative heat function, respectively. $\hbar$ Planck's constant divided by 2π. m_e, and m_i are the electron and ion mass, respectively.

Adiabatic changes in an enclosure containing matter and radiation are given by equation (4) Chandrasekhar [26] has defined Γ as

$$\Gamma = 1 + \frac{\Gamma_1 - b}{4-3b} \quad (5)$$

$$\Gamma_1 = \frac{b+(4-3b)^2(\gamma-1)}{b+(1-b)(\gamma-1)} \quad (6)$$

$$b = \frac{p_g}{p_g + p_r} \quad (7)$$

If the radiation is negligible then, $\Gamma = \Gamma_1 = \gamma$, while in case of $p_g \ll p_r$ is $\Gamma = \Gamma_1 = 4/3$.

III. LINEARIZED PERTURBATION EQUATIONS

In the linearization, we write the space and time dependent physical quantities p, ρ, $\vec{v}$, and ϕ, in the form of the sum of the equilibrium and perturbed part as

$$\rho = \rho_0 + \delta\rho, p = p_0 + \delta p, \phi = \phi_0 + \delta\phi,$$
$$\vec{v} = \vec{v}_o + \delta\vec{v} \quad (8)$$

The terms with subscript '0' denote the equilibrium part of the physical quantities. Perturbation in fluid velocity, fluid pressure, fluid density and gravitational potential are given as $\delta\vec{v} = (0,0,v_z)$, δp, $\delta\rho$, and $\delta\phi$, respectively.

Using equation (8) in equation (1) to (7) we write the linearized perturbation equations of infinitely conducting quantum plasma, by removing '0' from subscript in the equilibrium quantities, for simplicity. Thus, we obtain linearized perturbation equation of the considered system as

$$\frac{\partial\delta\vec{v}}{\partial t} + \frac{RT}{\rho}\frac{\partial\delta\rho}{\partial z} + R(1+4R_p)\frac{\partial\delta T}{\partial z} - \frac{\partial\delta\phi}{\partial z} - \frac{\hbar^2}{4m_e m_i}\frac{\partial}{\partial z}\frac{(\nabla^2\delta\rho)}{\rho} = 0 \quad (9)$$

$$\frac{\partial\delta\rho}{\partial t} + \rho\frac{\partial\delta\vec{v}}{\partial z} = 0 \quad (10)$$

$$\frac{\partial^2\delta\phi}{\partial z^2} = -4\pi G\delta\rho \quad (11)$$

$$\frac{\partial\delta T}{\partial t} - (\Gamma-1)\frac{T}{\rho}\frac{\partial\delta\rho}{\partial t} = 0 \quad (12)$$

where $v \equiv v_z$, $R_p = p_{r0}/p_{g0}$ denote the ratio of the radiation pressure and gas pressure, and Γ is assumed to be constant.

IV. DISPERSION RELATION

Let us assume the perturbation of all the quantities very as

$$\exp\{-i\omega t + ikz\} \quad (13)$$

where ω is the frequency of harmonic perturbations and k is the wave number in z- direction. Combining equations (9) to (13) we get the dispersion relation

$$\omega^2 - k^2 RT[1 + (1+4R_p)(\Gamma-1)] - \frac{\hbar^2 k^4}{4m_e m_i} + 4\pi G\rho = 0 \quad (14)$$

This dispersion relation shows the combined influence of black body radiation and quantum correction on Jeans instability of optically thick quantum plasma. if we ignore the quantum correction then this dispersion relation reduces to Vranjes and Cadez [15]. Thus dispersion relation is modified by the presence of quantum correction. This dispersion relation will be able to predict the complete information about the waves and instability of radiative quantum plasma considered the constant term of equation (14) has at least one positive root, this means that at least one value of ω is positive and this gives instability. Thus, from equation (14) we note that the condition of instability of the system is

$$k^2 RT[1 + (1+4R_p)(\Gamma-1)] + \frac{\hbar^2 k^4}{4m_e m_i} - 4\pi G\rho < 0 \quad (15)$$

Thus the system is unstable if

$$\frac{\lambda_J}{\lambda_c} \geq \left[\frac{1}{\gamma}\{1 + (1+4R_p)(\Gamma-1)\} + \frac{\hbar^2}{4\lambda_c^2 m_e m_i}\right]^{1/2} \quad (16)$$

where $\lambda_c = 2\pi/k$ is the critical wave length and $\lambda_J = \sqrt{\pi c_S^2/\rho G}$ is critical Jeans wave length.

Above equation (16) represents the quantum corrected condition of radiative instability. In the absence of quantum correction the above condition of instability is identical to Vranjes and Cadaze [15]. This is new condition of Jeans instability found in the present problem. It is noted here that the role of quantum correction is to stabilize the considered system by decreasing the value of critical Jeans wave length.

V. CONCLUSION

We have analyzed the Jeans instability of an infinite homogeneous self-gravitating optically thick quantum plasma under the influence of black body radiation. The general dispersion relation is obtained with the help of relevant linearized perturbation equation. We find that black body radiation and quantum affect the Jeans instability. The black body radiation has stabilizing influence on the system which is further increased by quantum correction.

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ISSN 2250-3153

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