Vibrational Analysis of Passive Suspension System for Solar Vehicle

May Mya Darli Cho*, Htay Htay Win**, Aung Ko Latt***

*Department of Mechanical Engineering, Mandalay Technological University, The Republic of the Union of Myanmar
**Department of Mechanical Engineering, Mandalay Technological University, The Republic of the Union of Myanmar
***Department of Mechanical Engineering, Mandalay Technological University, The Republic of the Union of Myanmar

Email: maymyadarliecho@gmail.com
Email: htayhtayw@gmail.com
Email: dr.aungkolat@gmail.com

Abstract- Suspension system in an automobile determines the riding comfort of passengers and the amount of damage to the vehicle. In this paper, the passive suspension system, quarter car model is analyzed to improve handling and ride performance of a vehicle. Two degree and three degree of freedom with and without tire damping is analyzed by using Matlab Simulink. Comparisons between passive suspension system with tire damping and without tire damping are performed by using different damping ratios range from 0.2 to 0.4. As a result, the sprung mass displacement of two degree of freedom suspension system with and without tire damping for front suspension are 0.1503 m and 0.1509 m respectively and rear suspension are 0.1548 m and 0.1564 m. For three degree of freedom, the sprung mass displacements of front suspension for with and without tire damping are 0.1301 m and 0.1351 m respectively. Sprung mass displacement for rear suspension for with and without tire damping are 0.1331 m and 0.1393 m. Seat mass displacement of front suspension for with and without tire damping are 0.1581 m and 0.1836 m. Seat mass displacement of rear suspension for with and without tire damping are 0.1590 m and 0.1854 m. The results show that suspension system of both types with tire damping is more passenger comfortable than suspension system without tire damping. Moreover, it is observed that the greater the damping effect, the more the ride handling and ride comfort of passenger.

Index Terms- Automobile, Damping effect, Riding comfort, Solar vehicle, Suspension system

I. INTRODUCTION

A solar vehicle is an electric vehicle powered completely or significantly by direct solar energy. Usually, photovoltaic (PV) cells contained in solar panels convert the sun’s energy directly into electric energy. The term “solar vehicle” usually implies that solar energy is used to power all or part of a vehicle’s propulsion [9]. The brief study of solar car is efficient in our daily life because now day’s pollution and fuel rate is very big problem and many people having fuel cars. Solar energy is being used for car, besides the control of vehicular pollution in the city, less consumption of fuel, solar cars are effective reducing global warming and environment problem in big frame.

There are many components in a solar car such as solar panel, chassis frame, steering system, transmission system, suspension system, brake system, axle, wheel, motor, etc. A solar car works as the following principle. Firstly, solar panel converts light energy from the sun into the electrical power. Solar controller converts the energy collected from the solar array to the proper system voltage, so that the batteries and motor can use it. Then, motor controller adjusts the amount of energy that flows to the motor. Finally, the motor uses that energy to drive the transmission system.

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Suspension consists of the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other meaning, suspension system is a mechanism that physically separates the car body from the car wheel. The main function of the vehicle suspension system is to minimize the vertical acceleration transmitted to the passenger which directly provides road comfort [5].

At present, three types of vehicle suspensions are used: passive, active and semi-active suspension system. The passive system is the most used type in automobile suspensions. The main reasons are the simplicity, low cost and reliability of this solution. A spring and a damper compose this suspension system, both fixed between the wheel supporting structure (unsprung mass) and the vehicle body (sprung mass). The damper is a cylinder filled with hydraulic oil or a compressed gas. Inside the cylinder there is a piston driven from the outside by a rod. Allowing the piston displacement there is a hole, which permits the fluid pass between the parts of the cylinder. This fluid flow generates a reaction force that is proportional to the relative speed between sprung and unsprung masses. The damping is achieved converting the energy of the oscillations in heat [5].

The active suspension system is also comprises of an actuator, sensors, and a control programming unit (CPU). Actually, the shock absorber is replaced by an active force actuator. The operational conditions of the vehicle are continuously controlled by sensors that measure the velocity of the sprung and un-sprung masses and lead it to the CPU that ensures correct impulses for the actuator, which creates the desired active damping forces when required. Semi-active suspension systems were first proposed in the early 1970’s. The semi-active suspension system is based on passive and active systems. The presented one contains instead a passive shock absorber,
and a variable shock absorber as an active damping force that is automatically controlled by an integrated regulator. In this type of system, the conventional spring element is retained, but the damper is replaced with a controllable damper [6].

In this work, two degrees and three degrees of freedom passive suspension system are designed. Sprung mass displacement for passive suspension system with and without tire damping are analysed by changing different damping effects. Matlab Simulink is used for analysing of the suspension system.

II. QUARTER CAR MODEL

A quarter car model consists of the wheel and its attachments, the tire, the suspension elements and quarter chassis and its rigidly connected parts. Quarter car model is very often used for suspension analysis because it is simple and can capture important characteristics of full model. The quarter car model is used extensively in studying the vehicle dynamics. Many researchers consider this model as a two degree of freedom by considering only the tire wheel assembly and the sprung mass. In this work, not only two degree of freedom but also three degree of freedom quarter model is analyzed by considering the passengers and seat. Parameters of quarter car model for front and rear suspension are shown in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Front suspension</th>
<th>Rear suspension</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger seat stiffness</td>
<td>3923.966</td>
<td>6014.868</td>
<td>N/m</td>
</tr>
<tr>
<td>Suspension spring stiffness</td>
<td>17822.932</td>
<td>36049.070</td>
<td>N/m</td>
</tr>
<tr>
<td>Tire stiffness</td>
<td>190000</td>
<td>190000</td>
<td>N/m</td>
</tr>
<tr>
<td>Tire damping coefficient</td>
<td>1150</td>
<td>1150</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Road input</td>
<td>0.1</td>
<td>0.1</td>
<td>m</td>
</tr>
</tbody>
</table>

In design calculation of passive suspension system, firstly, the total sprung weight, unsprung weight and passenger seat weight are calculated. Then, sprung mass displacement of suspension system are analysed by using Matlab program. Table II shows the specification and technical data of a solar vehicle.

<table>
<thead>
<tr>
<th>Technical Category</th>
<th>Dimensions</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle gross weight</td>
<td>490</td>
<td>kg</td>
</tr>
<tr>
<td>Vehicle net weight</td>
<td>310</td>
<td>kg</td>
</tr>
<tr>
<td>Weight of solar panel</td>
<td>46</td>
<td>kg</td>
</tr>
<tr>
<td>Weight of motor</td>
<td>24</td>
<td>kg</td>
</tr>
<tr>
<td>Weight of batteries</td>
<td>168</td>
<td>kg</td>
</tr>
<tr>
<td>Seating capacity</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Weight of person</td>
<td>180</td>
<td>kg</td>
</tr>
<tr>
<td>Weight of seat</td>
<td>5</td>
<td>kg</td>
</tr>
</tbody>
</table>

The design procedure of total sprung weight involves the following steps:

A. Calculation of the unsprung weight

In today's standard size automobile, the weight of unsprung components is normally in the range of 13 to 15 percent of the vehicle net weight.

\[
\text{Unsprung weight} = 0.15 \times \text{vehicle net weight} \quad (1)
\]

B. Calculation of total unsprung weight

Total unsprung weight can be calculated as

\[
\text{Total unsprung weight} = \text{unprung weight} + \text{motor weight} \quad (2)
\]
C. Calculation of the sprung weight

Gross vehicle weight is the sum of unsprung weight and sprung weight. Sprung weight can be calculated as

\[ \text{Sprung weight} = \text{gross weight} - \text{unsprung weight} \]  

(3)

D. Calculation of total sprung weight

The total sprung weight of solar vehicle can be calculated as

\[ \text{Total sprung weight} = \text{sprung weight} + \text{solar weight} \]  

(4)

E. Calculation of passenger and seat weight

The passenger and seat mass of solar vehicle can be calculated as

\[ \text{Passenger and seat weight} = \text{person weight} + \text{seat weight} \]  

(5)

F. Calculation of weight on quarter car model

By using quarter car model approach, load on each suspension is one fourth of the weight on suspension.

\[ \text{Load on each suspension} = \frac{1}{4} \times \text{weight on suspension} \]  

(6)

G. Calculation of critical damping

Critical damping can be calculated as

\[ c_c = 2\sqrt{k \times m} \]  

(7)

H. Calculation of damping coefficient

Suspension damping and seat damping can be calculated as follow:

Damping ratio is 0.3 [2]

\[ c_s = \xi \times c_c \]  

(8)

The result table of weights acting on quarter car model are shown in Table III

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Results</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total sprung weight</td>
<td>116.375</td>
<td>kg</td>
</tr>
<tr>
<td>Total unsprung weight</td>
<td>17.625</td>
<td>kg</td>
</tr>
<tr>
<td>Seat and passenger weight</td>
<td>46.25</td>
<td>kg</td>
</tr>
<tr>
<td>Front suspension damping coefficient</td>
<td>864.113</td>
<td>N/m</td>
</tr>
<tr>
<td>Rear suspension damping coefficient</td>
<td>1228.933</td>
<td>N/m</td>
</tr>
<tr>
<td>Front seat damping coefficient</td>
<td>255.605</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Rear seat damping coefficient</td>
<td>316.461</td>
<td>Ns/m</td>
</tr>
</tbody>
</table>

III. TWO DEGREE OF FREEDOM PASSIVE SUSPENSION

Sprung mass displacement is considered depend on two degree of freedom quarter car model depend on without and without tire damping in this part.

A. Two Degree of Freedom without Tire Damping

Modeling of two degree of freedom quarter car suspension without tire damping is shown in Figure 1.
Let \( x_1, x_2 \) \( y \)

![Figure 1. Modeling of Two Degree of Freedom without Tire Damping](image)

where,

\[
\begin{align*}
  m_s &= \text{sprung mass} \\
  m_u &= \text{unsprung mass} \\
  k_s &= \text{suspension spring stiffness} \\
  k_t &= \text{tire stiffness} \\
  c_s &= \text{suspension damping coefficient} \\
  y &= \text{road input (height of speed bump)} \\
  x_1 &= \text{sprung mass vertical movement} \\
  x_2 &= \text{unsprung mass vertical movement}
\end{align*}
\]

The equations of motion for sprung and unsprung mass are written as follows:

For sprung mass,

\[
m_s \ddot{x}_1 = -k_s(x_1 - x_2) - c_s(\dot{x}_1 - \dot{x}_2)
\]

For unsprung mass,

\[
m_u \ddot{x}_2 = -k_t(x_2 - y) + k_s(x_1 - x_2) + c_s(\dot{x}_1 - \dot{x}_2)
\]

Let

\[
Z_1 = x_1, \quad Z_2 = \dot{x}_1, \\
Z_3 = x_2, \quad Z_4 = \dot{x}_2
\]

Sprung and unsprung mass equation are become

\[
\begin{align*}
  \dot{Z}_2 &= -\frac{k_s}{m_s} Z_1 + \frac{k_s}{m_s} Z_3 - \frac{c_s}{m_s} Z_2 + \frac{c_s}{m_s} Z_4 \\
  \dot{Z}_4 &= \frac{k_s}{m_u} Z_1 - \frac{k_s}{m_u} Z_3 + \frac{c_s}{m_u} Z_2 - \frac{c_s}{m_u} Z_4 - \frac{k_t}{m_u} Z_3 + \frac{k_t}{m_u} y
\end{align*}
\]

State space model

\[
\begin{align*}
  \dot{Z} &= AZ + Bu \\
  Y &= CZ + Du
\end{align*}
\]

where,

\[
\begin{align*}
  A &= \text{state space matrix} \\
  B &= \text{input matrix} \\
  C &= \text{output matrix} \\
  D &= \text{direct transmission matrix} \\
  u &= \text{input of system}
\end{align*}
\]

State variable are \( Z_1, Z_2, Z_3, Z_4 \)
State space vector matrix can be written as

\[
\begin{bmatrix}
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3 \\
\dot{Z}_4 \\
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 \\
-k_s/m_s & -c_s/m_s & 0 & 0 \\
k_s/m_u & c_s/m_u & (k_s+k_t)/m_u & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4 \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] 

Output matrix can be written as

\[
\begin{bmatrix}
x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4 \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] 

Matlab program can be used to analyze sprung mass displacement and settling time of a two degree of freedom quarter car model.

Figure 2. Sprung Mass Displacement without Tire Damping for Front Suspension

The sprung mass displacement of front suspension without tire damping is shown in Figure 2. According from these figures, maximum overshoot is 0.1559 m and settling time occurs at 2 sec.

Figure 3. Sprung Mass Displacement without Tire Damping for Rear Suspension
The sprung mass displacement of rear suspension without tire damping is shown in Figure 3. According from these figures, maximum overshoot is 0.1564 m and settling time occur at 2 sec.

Sprung mass displacement is also analyzed by using different damping ratios. Figure 4 show sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1624m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

Sprung mass displacement using various damping ratio for rear suspension without tire damping is shown in Figure 5. As a result, maximum displacement is 0.1669m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

B. Two Degree of Freedom Suspension System with Tire Damping

Modeling of two degree of freedom quarter car suspension with tire damping is shown in Figure 6.
The equations of motion for sprung and unsprung mass are written as follows:

For sprung mass,

$$m_s \ddot{x}_1 = -k_s(x_1 - x_2) - c_s(\dot{x}_1 - \dot{x}_2)$$

For unsprung mass,

$$m_u \ddot{x}_2 = k_t(x_1 - x_2) - k_t(x_2 - y_r) - c_t(\dot{x}_2 - \dot{y}_r) + c_s(\dot{x}_1 - \dot{x}_2)$$

Let,

$$\dot{x}_1 = V_1, \quad \dot{x}_2 = V_2, \quad \Delta = (x_1 - x_2), \quad \dot{\Delta} = (V_1 - V_2)$$

From sprung mass equation

$$m_s \dot{V}_1 + k_s \Delta + c_s (V_1 - V_2) = 0$$

$$\dot{V}_1 = -\frac{k_s}{m_s} \Delta - \frac{c_s}{m_s} (V_1 - V_2) \quad (9)$$

From unsprung mass equation

$$m_u \dot{V}_2 = k_t x_2 - c_t V_2 + k_s \Delta + c_s (V_1 - V_2) + k_t y + c_t \dot{y}$$

$$\dot{V}_2 - \frac{c_t}{m_u} \dot{y}_r = -\frac{k_t}{m_u} x_2 + \frac{k_s}{m_u} \Delta + \frac{c_s}{m_u} (V_1 - V_2) - \left( \frac{c_s + c_t}{m_u} \right) V_1 + \frac{k_t}{m_u} y \quad (10)$$

Let

$$\dot{T} = V_2 - \frac{c_t}{m_u} \dot{y}_r$$

$$T = V_2 - \frac{c_t}{m_u} y$$

$$V_2 = T + \frac{c_t}{m_u} y \quad (11)$$

Substitute Equation (11) in Equation (10)

$$\dot{T} = \frac{k_t}{m_u} x_2 + \frac{k_s}{m_u} \Delta + \frac{c_s + c_t}{m_u} V_1 - \left[ \frac{c_s + c_t}{m_u} \right] T + \left[ -\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2} + \frac{k_t}{m_u} \right] y$$

$$\dot{\Delta} = V_1 - \left[ T + \frac{c_t}{m_u} y \right]$$
From Equation (8)

\[
\dot{V}_1 = -\frac{k_s}{m_s} \Delta - \frac{c_s}{m_s} V_1 + \frac{c_s}{m_s} \left[ T + \frac{c_t}{m_u} y \right]
\]

\[
\dot{V}_1 = -\frac{k_s}{m_s} \Delta - \frac{c_s}{m_s} V_1 + \frac{c_s}{m_s} T + \left( \frac{c_s c_t}{m_s m_u} \right) y
\]

State space model

\[
\dot{Z} = AZ + Bu
\]

\[
Y = CZ + Du
\]

State variable are \( x_2, \Delta, V_1, T \)

State space vector matrix can be written as

\[
\begin{bmatrix}
\dot{x}_2 \\
\Delta \\
\dot{V}_1 \\
T
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & -1 \\
0 & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\
-\frac{k_t}{m_u} & \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\left( \frac{c_s + c_t}{m_u} \right)
\end{bmatrix}
\begin{bmatrix}
x_2 \\
\Delta \\
V_1 \\
T
\end{bmatrix}
+
\begin{bmatrix}
0 \\
\frac{c_t}{m_u} \\
-\frac{c_t}{m_u} \\
\frac{c_s c_t}{m_s m_u} \\
-\frac{c_s c_t}{m_s m_u} - \frac{c_t^2}{m_u} + \frac{k_t}{m_u}
\end{bmatrix}
\begin{bmatrix}
y
\end{bmatrix}
\]

Output matrix

\[
\begin{bmatrix}
x_1 \\
V_1 \\
x_2 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_2 \\
\Delta \\
V_1 \\
T
\end{bmatrix}
+
\begin{bmatrix}
0 \\
0 \\
\frac{c_t}{m_u}
\end{bmatrix}
\begin{bmatrix}
y
\end{bmatrix}
\]

Matlab program can be used to analyze sprung mass displacement and settling time of a two degree of freedom quarter car model.

Figure 7. Sprung Mass Displacement with Tire Damping for Front Suspension

The sprung mass displacement of front suspension with tire damping is shown in Figure 7. In this figure, maximum overshoot is 0.1503 m and settling time occurs at 2 sec.
The sprung mass displacement of rear suspension with tire damping is shown in Figure 8. In this figure, maximum overshoot is 0.1548 m and settling time occurs at 2 sec.

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 9 show sprung mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1617m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

Figure 8. Sprung Mass Displacement with Tire Damping for Rear Suspension

Figure 9. Sprung Mass Displacement using Various Damping Ratio for Front Suspension with Tire Damping

Figure 10. Sprung Mass Displacement using Various Damping Ratio for Rear Suspension with Tire Damping
Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 10 shows sprung mass displacement using various damping ratio for rear suspension with tire damping. As a result, maximum displacement is 0.1651 m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

C. Comparison of Two Degree of Freedom Suspension System without and with Tire Damping

Sprung mass displacement of two DOF suspension system without tire damping is compared with suspension system with tire damping for front and rear suspension. The comparison for front suspension is shown in Table IV.

<table>
<thead>
<tr>
<th>Damping ratio $\xi$</th>
<th>Sprung mass displacement with tire damping, $X_1$, m</th>
<th>Sprung mass displacement without tire damping, $X_1$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.2$</td>
<td>0.1617</td>
<td>0.1624</td>
</tr>
<tr>
<td>$\xi = 0.25$</td>
<td>0.1556</td>
<td>0.1562</td>
</tr>
<tr>
<td>$\xi = 0.3$</td>
<td>0.1503</td>
<td>0.1509</td>
</tr>
<tr>
<td>$\xi = 0.35$</td>
<td>0.1460</td>
<td>0.1464</td>
</tr>
<tr>
<td>$\xi = 0.4$</td>
<td>0.1421</td>
<td>0.1425</td>
</tr>
</tbody>
</table>

In Table IV, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

<table>
<thead>
<tr>
<th>Damping ratio $\xi$</th>
<th>Sprung mass displacement with tire damping, $X_1$, m</th>
<th>Sprung mass displacement without tire damping, $X_1$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.2$</td>
<td>0.1651</td>
<td>0.1669</td>
</tr>
<tr>
<td>$\xi = 0.25$</td>
<td>0.1596</td>
<td>0.1614</td>
</tr>
<tr>
<td>$\xi = 0.3$</td>
<td>0.1548</td>
<td>0.1564</td>
</tr>
<tr>
<td>$\xi = 0.35$</td>
<td>0.1507</td>
<td>0.1523</td>
</tr>
<tr>
<td>$\xi = 0.4$</td>
<td>0.1449</td>
<td>0.1486</td>
</tr>
</tbody>
</table>

The comparison of sprung mass displacement with and without tire damping for rear suspension is shown in Table V. In Table V, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

IV. THREE DEGREE OF FREEDOM PASSIVE SUSPENSION

Seat displacement and sprung mass displacement are also considered depend on three degree of freedom quarter car model.

A. Three Degree of Freedom Suspension System without Tire Damping

Modeling of three degree of freedom, quarter car model without tire damping is shown in Figure 11.
Let \( x_1, x_2, x_3 \) y

\[
\begin{align*}
\text{Seat mass:} & \quad m_{se} \quad \frac{d^2 x_1}{dt^2} = -k_{se}(x_1 - x_2) - c_{se}(\dot{x}_1 - \dot{x}_2) \\
\text{Sprung mass:} & \quad m_s \quad \frac{d^2 x_2}{dt^2} = k_s(x_1 - x_2) + c_s(\dot{x}_1 - \dot{x}_2) - k_s(x_2 - x_3) - c_s(\dot{x}_2 - \dot{x}_3) \\
\text{Unsprung mass:} & \quad m_u \quad \frac{d^2 x_3}{dt^2} = k_t(x_3 - y) 
\end{align*}
\]

where,
- \( m_{se} \): passenger and seat mass
- \( k_{se} \): passenger seat stiffness
- \( k_s \): suspension spring stiffness
- \( c_s \): suspension damping coefficient
- \( k_t \): tire stiffness
- \( x_1 \): passenger seat mass vertical movement
- \( x_2 \): sprung mass vertical movement
- \( x_3 \): unsprung mass vertical movement

Equation of motion can be written from the free body diagram:

For seat and person mass:
\[
m_{se} \ddot{x}_1 = -k_{se}(x_1 - x_2) - c_{se}(\dot{x}_1 - \dot{x}_2)
\]

For sprung mass:
\[
m_s \ddot{x}_2 = k_s(x_1 - x_2) + c_s(\dot{x}_1 - \dot{x}_2) - k_s(x_2 - x_3) - c_s(\dot{x}_2 - \dot{x}_3)
\]

For unsprung mass:
\[
m_u \ddot{x}_3 = k_t(x_3 - y) - k_t(x_3 - x_3) - c_t(\dot{x}_3 - \dot{x}_3)
\]

Let \( Z_1 = x_1, \ Z_2 = \dot{x}_1, \ Z_3 = x_2, \ Z_4 = \dot{x}_2, \ Z_5 = x_3, \ Z_6 = \dot{x}_3 \)

From seat and person mass equation
\[
m_{se} \ddot{Z}_2 = -k_{se}(Z_1 - Z_3) - c_{se}(Z_2 - Z_4)
\]

\[
\ddot{Z}_2 = -\frac{k_{se}}{m_{se}} Z_1 - \frac{c_{se}}{m_{se}} Z_2 + \frac{k_{se}}{m_{se}} Z_3 + \frac{c_{se}}{m_{se}} Z_4
\]

From sprung mass equation
\[
m_s \ddot{Z}_4 = k_s(Z_1 - Z_3) + c_s(Z_2 - Z_4) - k_s(Z_3 - Z_5) - c_s(Z_4 - Z_6)
\]

\[
\ddot{Z}_4 = \frac{k_s}{m_s} Z_1 + \frac{c_s}{m_s} Z_2 - \frac{k_s}{m_s} Z_3 - \frac{c_s}{m_s} Z_4 - \frac{k_s}{m_s} Z_3 + \frac{c_s}{m_s} Z_4 - \frac{c_s}{m_s} Z_6
\]

\[
\ddot{Z}_4 = \frac{k_s}{m_s} Z_1 + \frac{c_s}{m_s} Z_2 - \left( \frac{k_s}{m_s} + \frac{k_s}{m_s} \right) Z_3 - \left( \frac{c_s}{m_s} + \frac{c_s}{m_s} \right) Z_4 + \frac{k_s}{m_s} Z_5 + \frac{c_s}{m_s} Z_6
\]
From unsprung mass equation
\[ m_u \ddot{Z}_6 = k_s (Z_3 - Z_5) + c_s (Z_4 - Z_6) - k_t (Z_5 - y) \]
\[ \dot{Z}_6 = \frac{k_s}{m_u} Z_3 - \frac{k_s}{m_s} Z_5 + \frac{c_s}{m_u} Z_4 - \frac{c_s}{m_s} Z_6 - \frac{k_t}{m_u} Z_5 + \frac{k_t}{m_u} y \]

State space model
\[ \dot{Z} = AZ + Bu \]
\[ Y = CZ + Du \]

State variable are \( Z_1, Z_2, Z_3, Z_4, Z_5, Z_6 \)

State space vector matrix can be written as
\[ \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \\ \dot{Z}_5 \\ \dot{Z}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_{se}}{m_{se}} & -\frac{c_{se}}{m_{se}} & \frac{k_{se}}{m_{se}} & \frac{c_{se}}{m_{se}} & 0 & 0 \\ 0 & 0 & -\left(\frac{k_{se}}{m_{se}} + \frac{k_s}{m_s}\right) & -\left(\frac{c_{se}}{m_{se}} + \frac{c_s}{m_s}\right) & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_s} & \frac{c_s}{m_s} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_t \end{bmatrix} \]

Output matrix can be written as
\[ \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

Matlab program can be used to analyze seat mass and sprung mass displacement of a three degree of freedom quarter car model.

![Figure 12. Seat Displacement of Front suspension without Tire Damping](image)

The seat mass displacement of front suspension without tire damping is shown in Figure 12. In this figure, maximum overshoot is 0.1836 m and settling time occurs at 3 sec.
The sprung mass displacement of front suspension without tire damping is shown in Figure 13. In this figure, maximum overshoot is 0.1351 m and settling time occurs at 2 sec.

The seat mass displacement of rear suspension without tire damping is shown in Figure 14. In this figure, maximum overshoot is 0.1854 m and settling time occurs at 3 sec.

The sprung mass displacement of rear suspension without tire damping is shown in Figure 15. In this figure, maximum overshoot is 0.1393 m and settling time occurs at 2 sec.
Seat mass and sprung mass displacement for front and rear suspension are analyzed by using different damping ratio from 0.2 to 0.4.

![Figure 16. Seat Mass Displacement of Different Damping Ratio for Front Suspension without Tire Damping](image1)

Figure 16 shows seat mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.2039 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

![Figure 17. Sprung Mass Displacement of Different Damping Ratio for Front Suspension without Tire Damping](image2)

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 17 shows sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1419m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.
Seat mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 18 shows seat mass displacement using various damping ratio for rear suspension without tire damping. As a result, maximum displacement is 0.2049 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 17 shows sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1466 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

B. Three Degree of Freedom Suspension System with Tire Damping

Seat displacement and sprung mass displacement are also considered depend on three degree of freedom quarter car model. Modeling of quarter car with tire damping is shown in Figure 18.
Let \( x_1 \), \( x_2 \), \( x_3 \) \( y \)

\[ \begin{array}{c}
\text{Seat mass} \\
\begin{array}{c}
\mathbf{v}_1 \\
\mathbf{v}_2 \\
\mathbf{v}_3 \\
\mathbf{y}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\text{Sprung mass} \\
\begin{array}{c}
k_s \\
\mathbf{c}_s \\
\mathbf{m}_s
\end{array}
\end{array} \]

\[ \begin{array}{c}
\text{Unsprung mass} \\
\begin{array}{c}
k_t \\
\mathbf{c}_t \\
\mathbf{m}_u
\end{array}
\end{array} \]

Figure 18. Modeling of Three Degree of Freedom with Tire Damping

Equation of motion can be written from the free body diagram;

For passenger and seat mass:
\[
m_{se} \ddot{x}_1 = -k_se (x_1 - x_2) - c_se (\dot{x}_1 - \dot{x}_2)
\]

For sprung mass:
\[
m_s \ddot{x}_2 = k_se (x_1 - x_2) + c_se (\dot{x}_1 - \dot{x}_2) - k_s (x_2 - x_3) - c_s (\dot{x}_2 - \dot{x}_3)
\]

For unsprung mass:
\[
m_u \ddot{x}_3 = k_s (x_2 - x_3) + c_s (\dot{x}_2 - \dot{x}_3) - k_t (x_3 - y) - c_t (\dot{x}_3 - \dot{y})
\]

Let \( \dot{x}_1 = V_1 \), \( x_2 = V_2 \), \( x_3 = V_3 \)
\[
\Delta_1 = (x_1 - x_2) \\
\Delta_1 = (V_1 - V_2)
\]
\[
\Delta_2 = (x_2 - x_3) \\
\Delta_2 = (V_2 - V_3)
\]

From seat and person equation,
\[
m_{se} \dot{V}_1 = -k_se \Delta_1 - c_se (V_1 - V_2)
\]
\[
\dot{V}_1 = -\frac{k_se}{m_{se}} \Delta_1 - \frac{c_se}{m_{se}} (V_1 - V_2)
\] (12)

For sprung mass equation
\[
m_s \dot{V}_2 = k_se \Delta_1 + c_se (V_1 - V_2) - k_s \Delta_2 - c_s (V_2 - V_3)
\]
\[
\dot{V}_2 = \frac{k_se}{m_s} \Delta_1 + \frac{c_se}{m_s} (V_1 - V_2) - \frac{k_s}{m_s} \Delta_2 - \frac{c_s}{m_s} (V_2 - V_3)
\] (13)

For unsprung mass:
\[
m_u \dot{V}_3 = k_s \Delta_2 + c_s (V_2 - V_3) - k_t (x_3 - y) - c_t (V_3 - y)
\]
\[
\dot{V}_3 = \frac{k_s}{m_u} \Delta_2 + \frac{c_s}{m_u} V_2 - \left[ \frac{c_t}{m_u} + \frac{k_t}{m_u} \right] V_3 - \frac{k_t}{m_u} x_3 + \frac{k_t}{m_u} y
\]
\[
\dot{V}_3 = \frac{c_t}{m_u} \dot{y} = \frac{k_s}{m_u} \Delta_2 + \frac{c_s}{m_u} V_2 - \left[ \frac{c_t}{m_u} + \frac{k_t}{m_u} \right] V_3 - \frac{k_t}{m_u} x_3 + \frac{k_t}{m_u} y
\] (14)
Let $\hat{T} = \dot{V}_3 - \frac{c_{t}}{m_u} \dot{y}$

$$T = V_3 - \frac{c_{t}}{m_u} y$$

$$V_3 = T + \frac{c_{t}}{m_u} y \quad \text{(15)}$$

Equation (15) substitute in equation (14)

$$\hat{T} = \frac{k_{s}}{m_u} \Delta_2 + \frac{c_{s}}{m_u} V_2 - \left[ \frac{c_{s}}{m_u} + \frac{c_{t}}{m_u} \right] T + \frac{c_{t}}{m_u} y - \frac{k_t}{m_u} x_3 + \frac{k_t}{m_u} \dot{y}$$

$$\hat{T} = \frac{k_{s}}{m_u} \Delta_2 + \frac{c_{s}}{m_u} V_2 - \left[ \frac{c_{s} c_{t}}{2 m_u^2} + \frac{c_{t}}{m_u} \right] T + \frac{c_{s} c_{t}}{2 m_u^2} y - \frac{k_t}{m_u} x_3 + \frac{k_t}{m_u} \dot{y} \quad \text{(16)}$$

$$\dot{\Delta}_2 = V_2 - V_3$$

$$\dot{\Delta}_2 = V_2 - \left[ T + \frac{c_{t}}{m_u} y \right] \quad \text{(17)}$$

Equation (15) substitute in equation (13)

$$\dot{V}_2 = \frac{k_{s e}}{m_s} \Delta_1 + \frac{c_{s e}}{m_s} (V_1 - V_2) - \frac{k_{s}}{m_s} \Delta_2 - \frac{c_{s}}{m_s} (V_2 - V_3)$$

$$\dot{V}_2 = \frac{k_{s e}}{m_s} \Delta_1 + \frac{c_{s e}}{m_s} V_1 - \frac{c_{s e}}{m_s} V_2 - \frac{k_{s}}{m_s} \Delta_2 - \frac{c_{s}}{m_s} V_2 + \frac{c_{s}}{m_s} \left[ T + \frac{c_{t}}{m_u} y \right]$$

$$\dot{V}_2 = \frac{k_{s e}}{m_s} \Delta_1 + \frac{c_{s e}}{m_s} V_1 - \left[ \frac{c_{s e}}{m_s} + \frac{c_{s}}{m_s} \right] V_2 - \frac{k_{s}}{m_s} \Delta_2 - \frac{c_{s}}{m_s} T + \frac{c_{s} c_{t}}{m_s m_u} y$$

State variable are $x_3, \Delta_1, \Delta_2, V_1, V_2, T$

State space vector matrix can be written as
Output matrix is

\[
\begin{bmatrix}
  x_1 \\
  \dot{x}_1 \\
  x_2 \\
  \dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
  1 & 1 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  1 & -1 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix} \begin{bmatrix}
  x_3 \\
  \Delta_1 \\
  \Delta_2 \\
  V_1 \\
  V_2 \\
  T \\
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
\end{bmatrix} [y] 
\]

Matlab program can be used to analyze seat mass and sprung mass displacement of a three degree of freedom quarter car model.

![Figure 19. Seat Mass Displacement of Front Suspension for with Tire Damping](image)

The seat mass displacement of front suspension without tire damping is shown in Figure 19. In this figure, maximum overshoot is 0.1581 m and settling time occurs at 3 sec.

![Figure 20. Sprung Mass Displacement of Front Suspension for with Tire Damping](image)

The sprung mass displacement of front suspension with tire damping is shown in Figure 20. In this figure, maximum overshoot is 0.1301 m and settling time occurs at 2 sec.

The seat mass displacement of rear suspension with tire damping is shown in Figure 21. In this figure, maximum overshoot is 0.1590 m and settling time occurs at 3 sec.
The sprung mass displacement of rear suspension with tire damping is shown in Figure 22. In this figure, maximum overshoot is 0.1331 m and settling time occurs at 2 sec.

Figure 22. Sprung Mass Displacement of Rear Suspension for with Tire Damping

The sprung mass displacement of rear suspension with tire damping is shown in Figure 22. In this figure, maximum overshoot is 0.1331 m and settling time occurs at 2 sec.
Figure 23 shows seat mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1698 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

Figure 24. Sprung Mass Displacement of Different Damping Ratio for Front suspension with Tire Damping

Figure 24 shows sprung mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1375 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

Figure 25. Seat Mass Displacement of Different Damping Ratio for Rear Suspension with Tire Damping

Figure 25 shows seat mass displacement using various damping ratio for rear suspension with tire damping. As a result, maximum displacement is 0.1714 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

Sprung mass displacement using various damping ratio for rear suspension with tire damping is shown in Figure 26. According from this figure, the maximum displacement is 0.1392 m and maximum settling time is 4s which occur at the smallest damping ratio. The minimum displacement is 0.1296 m and minimum settling time is 2 s which occurs at the greatest damping ratio. Therefore, the greater the damping ratio, the more comfortable the passenger.
C. Comparison of Three Degree of Suspension System without and with Tire Damping

Seat mass and sprung mass displacement of three DOF suspension system without tire damping is compared with suspension system with tire damping for front and rear suspension. The comparison of seat mass displacement for front suspension is shown in Table VI.

**TABLE VI**

<table>
<thead>
<tr>
<th>Damping ratio $\xi$</th>
<th>Seat mass displacement with tire damping, $X_{1}, m$</th>
<th>Seat mass displacement without tire damping, $X_{1}, m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.2$</td>
<td>0.1698</td>
<td>0.2039</td>
</tr>
<tr>
<td>$\xi = 0.25$</td>
<td>0.1636</td>
<td>0.1928</td>
</tr>
<tr>
<td>$\xi = 0.3$</td>
<td>0.1581</td>
<td>0.1836</td>
</tr>
<tr>
<td>$\xi = 0.35$</td>
<td>0.1531</td>
<td>0.1758</td>
</tr>
<tr>
<td>$\xi = 0.4$</td>
<td>0.1488</td>
<td>0.1692</td>
</tr>
</tbody>
</table>

In Table VI, the seat mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest seat mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest seat mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

**TABLE VII**

<table>
<thead>
<tr>
<th>Damping ratio $\xi$</th>
<th>Sprung mass displacement with tire damping, $X_{2}, m$</th>
<th>Sprung mass displacement without tire damping, $X_{2}, m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.2$</td>
<td>0.1375</td>
<td>0.1419</td>
</tr>
<tr>
<td>$\xi = 0.25$</td>
<td>0.1335</td>
<td>0.1381</td>
</tr>
<tr>
<td>$\xi = 0.3$</td>
<td>0.1301</td>
<td>0.1351</td>
</tr>
<tr>
<td>$\xi = 0.35$</td>
<td>0.1134</td>
<td>0.1325</td>
</tr>
<tr>
<td>$\xi = 0.4$</td>
<td>0.1258</td>
<td>0.1303</td>
</tr>
</tbody>
</table>

The comparison of sprung mass displacement for front suspension is shown in Table VII. In this table, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.
The comparison of seat mass displacement for rear suspension is shown in Table VIII. In this table, the seat mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest seat mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest seat mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

![Table VIII: Comparison of Seat Mass Displacement for Rear Suspension](image)

The comparison of sprung mass displacement for rear suspension is shown in Table VII. In this table, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

![Table VII: Comparison of Sprung Mass Displacement for Rear Suspension](image)

V. Conclusion

In the present work, vibration analysis of two degree of freedom and three degree of freedom quarter car model are performed. Sprung mass displacement and seat mass displacement of both type are analyze depend on two types of conditions such as without tire damping and with tire damping. Matlab software is used to analyze the two types of suspension system. As a results, the sprung mass displacement of two degree of freedom suspension system with and without tire damping for front suspension are 0.1503 m and 0.1509 m respectively. Two degree of freedom rear suspension with and without tire damping are 0.1548 m and 0.1564 m. For three degree of freedom, the sprung mass displacement of front suspension for with and without tire damping are 0.1301 m and 0.1351 m. Sprung mass displacement for rear suspension for with and without tire damping are 0.1331 m and 0.1393 m. Seat mass displacement of front suspension for with and without tire damping are 0.1581 m and 0.1836 m. Seat mass displacement for rear suspension for with and without tire damping are 0.1590 m and 0.1854 m. The results show that suspension system of both types with tire damping is more passenger comfortable than suspension system without tire damping. For front suspension, the sprung mass displacement of three degree of freedom suspension system with tire damping is 13% more comfortable than two degree of freedom suspension system. For rear suspension, the sprung mass displacement of three degree of freedom suspension system with tire damping is 14% more comfortable than two degree of freedom suspension system. Moreover, it is observed that the greater the damping effect, the more the ride handling and ride comfort of passenger.

Acknowledgements

The author would like to acknowledge the support and encouragement of Dr. Sint Soe, Rector of Mandalay Technological University. The author owes a debt of gratitude to her supervisor, Dr. Htay Htay Win, Professor and Head, Department of Mechanical Engineering, Mandalay Technological University, for her encouragement, patient guidance, invaluable supervision, kindly permission and suggestions throughout the course of challenging study. The author would like to express her heartfelt gratitude to all teachers for their supports, valuable suggestions and discussions during the presentation of the paper.
REFERENCES


AUTHORS

First Author – May Mya Darli Cho, Ph.D. candidate, Mandalay Technological University, maymyadarliecho@gmail.com
Second Author – Htay Htay Win, Professor, Mandalay Technological University, htayhtayw@gmail.com.
Third Author – Aung Ko Latt, Associate Professor, Mandalay Technological University, dr.aungkolat@gmail.com