An Inventory System with Retrial Demands and Working Vacation

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Abstract- This article considers a continuous review retrial inventory system at a service facility, wherein an item demanded by a customer is issued after performing service on the item. The arrival time points of customers form a Poisson process. The inventory replenished according to an (s,Q) policy and the lead times are assumed to follow an exponential distribution. The demands that occur during the stock out period or the server busy (regular or working vacation) are permitted to enter into the orbit of infinite size. When the inventory level is zero or no demands in the system or both, server goes to a working vacation which is exponentially distributed. If the server is in working vacation or the inventory level is zero, the impatience occurs in orbiting customers, that follows an exponential distribution. The joint probability distribution of the number of demands in the orbit, the inventory level and the server status is obtained in the steady state case. Some system performance measures are derived, the long-run total expected cost rate is calculated and the results are illustrated numerically.

Index Terms- Continuous review inventory system, (s,Q) Policy, Positive leadtime, Retrial demand, Working vacation.

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I. Introduction

The concept of server vacation in inventory with two servers was first introduced by Daniel and Ramanarayanan [1]. Also they have studied an inventory system in which the server takes rest when the level of the inventory is zero in [2]. They assumed that the demands that occurred during stock-out period are assumed to be lost. The inter-occurrence times between successive demands, the lead times, and the rest times are assumed to follow mutually independent general distributions. Using renewal and convolution techniques they obtained the state transition probabilities.

The various types of vacation models in queueing systems have been widely studied in the literature. We refer the reader to Doshi [4]. Working vacation is a kind of semi-vacation policy that was introduced by Servi and Finn[9]. In the classical vacation queuing models, during the vacation period the server doesn't continue on the original work and such policy may cause the loss or dissatisfaction of the customers. For the working vacation policy, the server can still work during the vacation and may accomplish other assistant work simultaneously. So, the working vacation more reasonable that the classical vacation in some cases. Tien Van Do[8] studied, M/M/1 retrial queue with working vacations. Paul Manual et al. [10] analyzed a service facility inventory system with impatient customers. The authors have assumed the customers arrive in Poisson fashion. The service time, life time of items in stock and the lead time are all assumed to be independently distributed as exponential.

The concept of retrial demands in inventory was introduced by Artalejo et al. [3]. They have assumed Poisson demand, exponential lead time and exponential retrial time. In that work, the authors proceeded with an algorithmic analysis of the system. Jeganathan et al. [5] studied a retrial inventory system with non-preemptive priority service. Narayanan et al. [6] studied on an (s,S) inventory policy with service time, vacation to server and correlated lead time.

In this present paper, we address a continuous review inventory system with Poisson demand. The server served at different service rates (working vacation and regular), which are exponentially distributed. When the server is busy or the inventory level is zero, any arriving primary demands enter into orbit. When the server is in a working vacation or the inventory level is zero, the orbiting customer either may retry or may leave, which is exponentially distributed. The joint probability distribution of the inventory level, the number of customers in the orbit and the server status is obtained in the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated and some numerical examples.

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model. In Section 3 and 4, we discuss the steady-state analysis of the model and some key system performance measures respectively. In Section 5, calculate the long-run total expected cost rate and the final section 6, a cost function is also studied numerically.

II. Model Description

We consider a continuous review stochastic inventory system on the replenishment policy of (s,Q). The basic assumptions of this inventory model are described as follows: we assume that the inter-arrival times of demands to a single server station according to a Poisson process with rate $\lambda(>0)$ and demands only single unit at a time. It is assumed that there is no waiting space in the system. The items are issued by a server to the customer after some service time due to the service performed on the items. When the server is busy (regular or working vacation) or the inventory level is zero, any arriving primary demands enter into an orbit of infinite size. The service rates are μ_b and μ_v ($< \mu_b$), when the system is in regular and working vacation, respectively, which are exponentially distributed. The server takes a working vacation at times when no customers in the system or the inventory level is zero or both. Working vacation durations are exponentially distributed with parameter θ. At the completion time of the working vacation, the server switches a regular busy ((ie) service rate from μ_v to μ_b) if there are customers (primary or retrial) in the system. Otherwise, the server continues the working vacation. The orbiting customers may either retry or may leave the orbit. The leaving orbiting customers are described as impatient(reneging) customers. If the server is in working vacation or the inventory level is zero, the impatience occurs in orbiting customers. An impatient customer leaves the orbit independenly after a random time which is distributed exponentially with parameter $\eta(> 0)$.

We assume the constant retrial policy for these orbit demands, that is probability of a repeated attempt of an orbiting demand is independent of the number of demands in the orbit. Retrial requests from the orbit follow an exponential distribution with parameter $\alpha(> 0)$. As the (s,Q) replenishment policy, when the on-hand inventory level drops to a prefixed level, say $s(> 0)$ an order for $Q(= S - s > s + 1)$ units is placed. The positive lead time is exponentially distributed with parameter $\beta(> 0)$. We assume that the inter-demand times between the primary demands, the lead times, retrial demand times, server regular periods and server working vacation periods are mutually independent random variables.

Notations

$[A]_{ij}$: The element / submatrix at (i, j) the position of A.

0 : Zero matrix.

I : Identity matrix.

e : A column vector of 1's of appropriate dimension.

δ_{ij} : $\begin{cases} 1, & if\ i = j, \\ 0, & otherwise. \end{cases}$

$\bar{\delta}_{ij}$: $1 - \delta_{ij}$

$Y(t)$: $\begin{cases} 0, & \text{the server is idle in working vacation period at time t.} \\ 1, & \text{the server is busy in working vacation period at time t.} \\ 2, & \text{the server is idle in regular period at time t.} \\ 3, & \text{the server is busy in regular period at time t.} \end{cases}$

E : $\{(i,0,0)$: $i = 0,1,2,\ldots\} \cup \{(i,k,m)$: $i = 0,1,2,\ldots,\quad k = 1,2,\ldots,S,\ m = 0,1,2,3\}$

III. Analysis

Let $X(t)$, $L(t)$ and $Y(t)$ denote the number of demands in the orbit, inventory position of the commodity and the server status at time $t+$. From the assumptions made on the input and output processes, it can be shown that the triplet $\{(X(t),\ L(t),\ Y(t)),\ t \geq 0\}$ with the state space E is a Markov process.

To determine the infinitesimal generator

$$P = (p((i,k,m),(j,l,n))),\quad (i,k,m),(j,l,n) \in E$$

of this process we use the following arguments:

Let $i = 0$, $k = 1,2,\ldots,S$

— any arriving primary demand takes the state of the process from $(i,k,0)$ to $(i,k,1)$ with the intensity λ.

— any arriving primary demand takes the state of the process from (i,k,m) to $(i+1,k,m)$ with the intensity λ, $m=1,3$.

— The server changes from working vacation to regular takes the state of the process from $(i,k,1)$ to $(i,k,3)$ with the intensity θ.

— The completion of service from primary demand makes a transition from $(i,k,1)$ to $(i,k-1,0)$ with the intensity μ_v.

— The completion of service from primary demand makes a transition from $(i,k,3)$ to $(i,k-1,0)$ with the intensity μ_b.

Let $i \geq 0$, $k=0$

— any arriving primary demand takes the state of the process from $(i,k,0)$ to $(i+1,k,0)$ with the intensity λ.

Let $i \geq 1$, $k=1,2,\ldots,S$

— any arriving primary demand takes the state of the process from (i,k,m) to $(i,k,m+1)$ with the intensity λ, m=0, 2.

— any arriving primary demand takes the state of the process from (i,k,m) to $(i+1,k,m)$ with the intensity λ, m=1, 3.

— The server changes from working vacation to regular takes the state of the process from (i,k,m) to $(i,k,m+2)$ with the intensity θ, m=0, 1.

— a retrial requests takes the state of the process from (i,k,m) to $(i-1,k,m+1)$ with the intensity α, m=0, 2.

— an impatient customer takes the state of the process from (i,k,m) to $(i-1,k,m)$ with the intensity η, m=0, 1.

— an impatient customer takes the state of the process from $(i,0,0)$ to $(i-1,0,0)$ with the intensity η.

— The completion of service in the system makes a transition from $(i,k,1)$ to $(i,k-1,0)$ with the intensity μ_v.

Let $i \geq 1$, $k=2,3,\ldots,S$

— The completion of service in the system makes a transition from $(i,k,3)$ to $(i,k-1,2)$ with the intensity μ_b.

— The completion of service in the system makes a transition from $(i,1,3)$ to $(i,0,0)$ with the intensity

μ_b.

Let $i \geq 0$, $k = 1,2,\ldots s$

— a transition from (i,k,m) to $(i,k+Q,m)$ for $m = 0,1,2,3$ takes place with the intensity β when a replenishment occurs.

— a transition from $(i,0,0)$ to $(i,Q,0)$ takes place with the intensity β when a replenishment occurs.

We observe that no transition other than the above is possible.

Finally, the value of $p((i,k,m),\,(i,k,m))$ is obtained by

$$p((i,k,m),\,(i,k,m)) = -\sum_{\substack{j \\ (i,k,m)\neq(j,l,n)}}\sum_{l}\sum_{n} p((i,k,m),\,(j,l,n)).$$

Hence we have, $p((i,k,m),\,(j,l,n)) =$

$$\begin{cases} \lambda, & j = i, \quad l = k, \quad n = m+1 \\ & i = 0, \quad k = 1,2,\ldots,S, \quad m = 0 \\ & \quad or \\ & j = i+1, \quad l = k, \quad n = m \\ & i \geq 0, \quad k = 1,2,\ldots,S, \quad m = 1,3 \\ & \quad or \\ & j = i+1, \quad l = k, \quad n = m \\ & i \geq 0, \quad k = 0, \quad m = 0 \\ & \quad or \\ & j = i, \quad l = k, \quad n = m+1 \\ & i \geq 1, \quad k = 1,2,\ldots,S, \quad m = 0,2 \end{cases}$$

$$
\begin{cases}
\mu_v, & j=i, \quad l=k-1, \quad n=0 \\
 & i\geq 0, \quad k=1,2,\ldots,S, \quad m=1 \\
\mu_b, & j=i, \quad l=k-1, \quad n=0 \\
 & i=0, \quad k=1,2,\ldots,S, \quad m=3 \\
 & \quad or \\
 & j=i, \quad l=k-1, \quad n=m-1 \\
 & i\geq 1, \quad k=2,\ldots,S, \quad m=3 \\
 & \quad or \\
 & j=i, \quad l=k-1, \quad n=0 \\
 & i\geq 1, \quad k=1, \quad m=3 \\
\theta, & j=i, \quad l=k, \quad n=m+2 \\
 & i=0, \quad k=1,2,\ldots,S, \quad m=1 \\
 & \quad or \\
 & j=i, \quad l=k, \quad n=m+2 \\
 & i\geq 1, \quad k=1,2,\ldots,S, \quad m=0,1 \\
\alpha, & j=i-1, \quad l=k, \quad n=m+1 \\
 & i\geq 1, \quad k=1,2,\ldots,S, \quad m=0,2 \\
\eta, & j=i-1, \quad l=k, \quad n=m \\
 & \quad m=0,1 \\
 & i\geq 1, \quad k=1,2,\ldots,S, \\
 & \quad or \\
 & j=i-1, \quad l=k, \quad n=m \\
 & i\geq 1, \quad k=0 \quad m=0
\end{cases}
$$

$$
\begin{cases}
\beta, & j=i, \quad l=k+Q, \quad n=m \\
 & i\geq 0, \quad k=1,2,\ldots,s, \quad m=0,1,2,3 \\
 & \quad or \\
 & j=i, \quad l=k+Q, \quad n=m \\
 & i\geq 0, \quad k=0, \quad m=0 \\
-\lambda, & j=i, \quad l=k, \quad n=m \\
 & i=0, \quad k=s+1,s+2,\ldots,S, \quad m=0 \\
-(\lambda+\mu_v+\theta), & j=i, \quad l=k, \quad n=m \\
 & i=0, \quad k=s+1,s+2,\ldots,S, \quad m=1 \\
-(\lambda+\mu_b), & j=i, \quad l=k, \quad n=m \\
 & i\geq 0, \quad k=s+1,s+2,\ldots,S, \quad m=3 \\
-(\lambda+\beta), & j=i, \quad l=k, \quad n=m \\
 & i=0, \quad k=0,1,2,\ldots,s, \quad m=0 \\
-(\lambda+\mu_v+\theta+\beta), & j=i, \quad l=k, \quad n=m \\
 & i=0, \quad k=1,2,\ldots,s, \quad m=1 \\
-(\lambda+\mu_b+\beta), & j=i, \quad l=k, \quad n=m \\
 & i\geq 0, \quad k=1,2,\ldots,s, \quad m=3 \\
-\beta, & j=i, \quad l=k, \quad n=m \\
 & i=0, \quad k=1,2,\ldots,s, \quad m=2 \\
-(\lambda+\theta+\eta+\alpha), & j=i, \quad l=k, \quad n=m \\
 & i\geq 1, \quad k=s+1,s+2,\ldots,S, \quad m=0
\end{cases}
$$

$$\begin{cases} -(\lambda+\mu_v+\theta+\beta), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=s+1,s+2,\ldots,S, \quad m=1 \\ -(\lambda+\alpha), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=s+1,s+2,\ldots,S, \quad m=2 \\ -(\lambda+\theta+\eta+\alpha+\beta), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=1,2,\ldots,s, \quad m=0 \\ -(\lambda+\mu_v+\theta+\eta+\beta), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=1,2,\ldots,s, \quad m=1 \\ -(\lambda+\alpha+\beta), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=1,2,\ldots,s, \quad m=2 \\ -(\lambda+\eta+\beta), & j=i, \quad l=k, \quad n=m \\ & i\geq 1, \quad k=0, \quad m=0 \\ 0, & \text{otherwise} \end{cases}$$

Denoting $q=((q,0,0),\ (q,1,0)\ ,(q,1,1),\ (q,1,2),\ (q,1,3),\ldots,\ (q,S,0),\ (q,S,1),\ (q,S,2),\ (q,S,3))$ for $q=0,1,\ldots$. By ordering states lexicographically, the infinitesimal generator P can be conveniently expressed in a block partitioned matrix with entries

$$[P]_{ij}=\begin{cases} A_1, & j=i, \quad i=0 \\ A, & j=i, \quad i=1,2,\ldots \\ L, & j=i+1, \quad i=0,1,2,\ldots \\ M, & j=i-1, \quad i=1,2,\ldots \\ 0, & \text{otherwise} \end{cases}$$

More explicitly,

$$P = \begin{pmatrix} A_1 & L & 0 & 0 & 0 & \cdots \\ M & A & L & 0 & 0 & \cdots \\ 0 & M & A & L & 0 & \cdots \\ 0 & 0 & M & A & L & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where

$$[A_1] = \begin{cases} J, & j = i, \quad i = s+1, s+2, \ldots, S \\ J_1, & j = i, \quad i = 1, 2, \ldots, s \\ -(\lambda + \beta), & j = i, \quad i = 0 \\ J_3, & j = i-1, \quad i = 2, \ldots, S \\ J_4 & j = i-1 \quad i = 1 \\ C, & j = i+Q, \quad i = 1, 2, \ldots, s \\ C_1, & j = i+Q, \quad i = 0 \\ 0, & \text{otherwise} \end{cases}$$

with

$$[J]_{kl} = \begin{cases} \lambda, & l = k+1, \quad k = 0 \\ \theta, & l = k+2, \quad k = 1 \\ -\lambda, & l = k, \quad k = 0 \\ -(\lambda + \mu_v + \theta), & l = k, \quad k = 1 \\ -(\lambda + \mu_b), & l = k, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$[J_1]_{kl} = \begin{cases} \lambda, & l = k+1, \quad k = 0 \\ \theta, & l = k+2, \quad k = 1 \\ -(\lambda+\beta), & l = k, \quad k = 0 \\ -(\lambda+\mu_v+\theta+\beta), & l = k, \quad k = 1 \\ -(\lambda+\mu_b+\beta), & l = k, \quad k = 3 \\ -\beta, & l = k, \quad k = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$[J_3]_{kl} = \begin{cases} \mu_v, & l = k-1, \quad k = 1 \\ \mu_b, & l = k-3, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$[J_4]_{kl} = \begin{cases} \mu_v, & l = k-1, \quad k = 1 \\ \mu_b, & l = k-3, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$[C]_{kl} = \begin{cases} \beta, & l = k, \quad k = 0,1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

$$[C_1]_{kl} = \begin{cases} \beta, & l = k, \quad k = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[L]_{ij} = \begin{cases} F, & j = i, \quad i = 1,2,\ldots,S \\ \lambda, & j = i, \quad i = 0 \\ 0, & \text{otherwise} \end{cases}$$

with

$$[F]_{kl} = \begin{cases} \lambda, & l = k, \quad k = 1,3 \\ 0, & \text{otherwise} \end{cases}$$

$$[M]_{ij} = \begin{cases} K, & j = i, \quad i = 1,2,\ldots,S \\ \eta, & j = i, \quad i = 0 \\ 0, & \text{otherwise} \end{cases}$$

with

$$[K]_{kl} = \begin{cases} \alpha, & l = k+1, \quad k = 0,2 \\ \eta, & l = k, \quad k = 0,1 \\ 0, & \text{otherwise} \end{cases}$$

$$[A]_{ij} = \begin{cases} G, & j = i, \quad i = s+1, s+2, \ldots, S \\ G_1, & j = i, \quad i = 1, 2, \ldots, s \\ -(\lambda + \eta + \beta), & j = i, \quad i = 0 \\ G_3, & j = i-1, \quad i = 2, 3, \ldots, S \\ J_4, & j = i-1, \quad i = 1 \\ C, & j = i+Q, \quad i = 1, 2, \ldots, s \\ C_1, & j = i+Q, \quad i = 0 \\ 0, & \text{otherwise} \end{cases}$$

with

$$[G]_{kl} = \begin{cases} \lambda, & l = k+1, \quad k = 0, 2 \\ \theta, & l = k+2, \quad k = 0, 1 \\ -(\lambda + \theta + \eta + \alpha), & l = k, \quad k = 0 \\ -(\lambda + \mu_v + \theta + \eta), & l = k, \quad k = 1 \\ -(\lambda + \alpha), & l = k, \quad k = 2 \\ -(\lambda + \mu_b), & l = k, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$[G_1]_{kl} = \begin{cases} \lambda, & l = k+1, \quad k = 0,2 \\ \theta, & l = k+2, \quad k = 0,1 \\ -(\lambda+\theta+\eta+\alpha+\beta), & l = k, \quad k = 0 \\ -(\lambda+\mu_v+\theta+\eta+\beta), & l = k, \quad k = 1 \\ -(\lambda+\alpha+\beta), & l = k, \quad k = 2 \\ -(\lambda+\mu_b+\beta), & l = k, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$[G_3]_{kl} = \begin{cases} \mu_v, & l = k-1, \quad k = 1 \\ \mu_b, & l = k-1, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

It may be noted that the matrices $A_1,\ A,\ M$ and L are square matrices of order $(4S+1)$ and $J,\ J_1,\ J_3,\ F,\ K,\ G,\ G_1,\ G_3$ and C are square matrices of order 4. The matrix J_4 is of size 4×1 and the matrix C_1 is of size 1×4.

3.1 Stability Analysis

To discuss the stability condition of the process, we consider the matrix $Q = M + A + L$ which is given by

$$[Q]_{ij} = \begin{cases} H_0, & j = i, \quad i = s+1, s+2, \ldots, S \\ H_1, & j = i, \quad i = 1,2,\ldots, s \\ H_2, & j = i, \quad i = 0 \\ H_3, & j = i-1, \quad i = 2,3,\ldots, S \\ H_4, & j = i-1, \quad i = 1 \\ C, & j = i+Q, \quad i = 1,2,\ldots, s \\ C_1, & j = i+Q, \quad i = 0 \\ 0, & \text{otherwise} \end{cases}$$

Where

$$[H_0]_{kl} = \begin{cases} (\lambda+\alpha), & l = k+1, \quad k = 0,2 \\ \theta, & l = k+2, \quad k = 0,1 \\ -(\lambda+\theta+\alpha), & l = k, \quad k = 0 \\ -(\mu_v+\theta), & l = k, \quad k = 1 \\ -(\lambda+\alpha), & l = k, \quad k = 2 \\ -\mu_b, & l = k, \quad k = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$
[H_1]_{kl} = \begin{cases} (\lambda+\alpha), & l=k+1, \quad k=0,2 \\ \theta, & l=k+2, \quad k=0,1 \\ -(\lambda+\theta+\alpha+\beta), & l=k, \quad k=0 \\ -(\mu_v+\theta+\beta), & l=k, \quad k=1 \\ -(\lambda+\alpha+\beta), & l=k, \quad k=2 \\ -(\mu_b+\beta), & l=k, \quad k=3 \\ 0, & \text{otherwise} \end{cases}
$$

$$
[H_2]_{kl} = \begin{cases} -\beta, & l=k, \quad k=0 \end{cases}
$$

$$
[H_3]_{kl} = \begin{cases} \mu_v, & l=k-1, \quad k=1 \\ \mu_b, & l=k-1, \quad k=3 \\ 0, & \text{otherwise} \end{cases}
$$

$$
[H_4]_{kl} = \begin{cases} \mu_v, & l=k-1, \quad k=1 \\ \mu_b, & l=k-3, \quad k=3 \\ 0, & \text{otherwise} \end{cases}
$$

$$
[C]_{kl} = \begin{cases} \beta, & l=k, \quad k=0,1,2,3 \\ 0, & \text{otherwise} \end{cases}
$$

$$[C_1]_{kl} = \begin{cases} \beta, & l = k, \quad k = 0 \\ 0, & \text{otherwise} \end{cases}$$

Clearly H_0, H_1, H_3, H_4 and C are square matrices of order 4. The matrix H_4 is of size 4 $\times$ 1 and the matrix C_1 is of size 1 $\times$ 4. The matrix H_2 is of size 1 $\times$ 1.

Let $\mathbf{\Pi}$ be the steady-state probability vector of Q. That is, $\mathbf{\Pi}$ satisfies

$$\mathbf{\Pi}Q = 0, \quad \mathbf{\Pi e} = 1.$$

The vector $\mathbf{\Pi}$ can be represented by

$\mathbf{\Pi} = (\mathbf{\Pi}^{(0)}, \mathbf{\Pi}^{(1)}, \ldots, \mathbf{\Pi}^{(S)})$

where

$\mathbf{\Pi}^{(0)} = (\pi^{(0,0)})$

$\mathbf{\Pi}^{(i)} = (\pi^{(i,0)}, \pi^{(i,1)}, \pi^{(i,2)}, \pi^{(i,3)})$, $i = 1, 2, \ldots, S$

Theorem 1 *The steady-state probability vector* $\mathbf{\Pi}$ *coresponding to the generator* Q *is given by*

$$\Pi^{(i)} H_4 + \Pi^{(i-1)} H_2 = 0, \qquad i = 1,$$

$$\Pi^{(i)} H_3 + \Pi^{(i-1)} H_1 = 0, \qquad i = 2, 3, \ldots, s+1,$$

$$\Pi^{(i)} H_3 + \Pi^{(i-1)} H_0 = 0, \qquad i = s+2, s+3, \ldots, Q,$$

$$\Pi^{(i)} H_3 + \Pi^{(i-1)} H_0 + \Pi^{(i-Q-1)} C_1 = 0, \qquad i = Q+1$$

$$\Pi^{(i)} H_3 + \Pi^{(i-1)} H_0 + \Pi^{(i-Q-1)} C = 0, \qquad i = Q+2, Q+3, \ldots, S,$$

$$\Pi^{(S)} H_0 + \Pi^{(s)} C = 0.$$

Proof: We have

$\mathbf{\Pi}Q = 0$, $\mathbf{\Pi e} = 1$.

After long simplications, the above equations, except the last one, yields

$$
\Pi^{(i)} = \begin{cases}
(-1)^i \Pi^{(0)} \left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{i-1}, & i = 1, 2, \ldots, s+1 \\
(-1)^i \Pi^{(0)} \left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{i-s-1}, & i = s+2, s+3, \ldots, Q \\
(-1)^i \Pi^{(0)} \left\{\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{i-s-1} + \left(\frac{C_1}{H_3}\right)\right\}, & i = Q+1 \\
(-1)^i \Pi^{(0)} \left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{i-s-1} + \left\{\left(\frac{H_0}{H_3}\right)^{i-Q-1} + \sum_{k=1}^{i-Q-1}\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{k-1}\left(\frac{H_0}{H_3}\right)^{i-Q-k-1}\right\}\Pi^{(0)}\left(\frac{-C}{H_3}\right)^{i-Q}, & i = Q+2, Q+3, \ldots, S
\end{cases}
$$

where $\Pi^{(0)}$ can be obtained by solving,

$\Pi^{(S)} H_0 + \Pi^{(s)} C = 0$ and $\sum_{i=0}^{S} \Pi^{(i)} e = 1$,

that is

$$
\mathbf{\Pi^{(0)}}\left[\left(\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{S-s-1}(-1)^S + \left\{\left(\frac{H_0}{H_3}\right)^{S-Q-1} + \sum_{k=1}^{S-Q-1}\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{k-1}\left(\frac{H_0}{H_3}\right)^{S-Q-k-1}\right\}\left(\frac{-C}{H_3}\right)^{S-Q}\right)H_0 + \left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s-1} C(-1)^{(s)}\right] = 0
$$

and

$$
\mathbf{\Pi^{(0)}}\left[I + \sum_{i=1}^{s+1}(-1)^i\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{i-1} + \sum_{i=s+2}^{Q}(-1)^i\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{i-s-1} + (-1)^{Q+1}\left\{\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{Q-s} + \left(\frac{C_1}{H_3}\right)\right\} + \sum_{i=Q+2}^{S}\left((-1)^i\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{s}\left(\frac{H_0}{H_3}\right)^{i-s-1} + \left\{\left(\frac{H_0}{H_3}\right)^{i-Q-1} + \sum_{k=1}^{i-Q-1}\left(\frac{H_2}{H_4}\right)\left(\frac{H_1}{H_3}\right)^{k-1}\left(\frac{H_0}{H_3}\right)^{i-Q-k-1}\right\}\left(\frac{-C}{H_3}\right)^{i-Q}\right)\right]e = 1
$$

Next, we derive the condition under which the system is stable.

Lemma 1 *The stability condition of the system under study is given by*

$$(K-F)\mathbf{e} > \pi^{(0)}(K-K_1-F-F_1)\mathbf{e} \quad (1)$$

Proof: From the well known result of Neuts(7) on the positive recurrence of P we have

$$\mathbf{\Pi}M\mathbf{e} > \mathbf{\Pi}L\mathbf{e}$$

and by the exploiting the structure of matrices M and L and $\mathbf{\Pi}$ the stated result follows.

3.2 Steady state analysis

It can be seen from the structure of the rate matrix P and from the lemma (1) that the markov process $\{(X(t),\ L(t),\ Y(t)),\ t \geq 0\}$ with the state space E is regular. Hence the limiting probability distribution $\phi^{(i,k,m)} = \lim_{t\to\infty} pr\{X(t)=i,\ L(t)=k,\ Y(t)=m \mid X(0),\ L(0),\ Y(0)\}$, exists and is independent of the initial state. That is, $\Phi = (\Phi^{(0)},\ \Phi^{(1)},\ \ldots)$ satisfies

$$\mathbf{\Phi}P = 0,\ \mathbf{\Phi e} = 1.$$

We partition the vector $\Phi^{(i)}$, for $i = 0,1,2,\ldots$ as follows

$$\Phi^{(i)} = (\phi^{(i,0)},\ \phi^{(i,1)},\ \ldots,\ \phi^{(i,S)})$$

which is partitioned as follows, for $1 \leq k \leq S$

$$\phi^{(i,0)} = (\phi^{(i,0,0)})$$

$$\phi^{(i,k)} = (\phi^{(i,k,0)},\ \phi^{(i,k,1)},\ \phi^{(i,k,2)},\ \phi^{(i,k,3)})$$

Theorem 2 *When the stability condition (1) holds good, the steady state probability vector* $\mathbf{\Phi}$ *is given by*

$$\Phi^{(i)} = \mathbf{\Phi^{(0)}}R^{(i)}, \qquad i = 0,1,2,\ldots \quad (2)$$

where the matrix R satisfies the matrix quadratic equation

$$R^2M + RA + L = 0 \quad (3)$$

and the vector $\mathbf{\Phi^{(0)}}$ is obtained by solving

$$\mathbf{\Phi^{(0)}}(A_1 + RM) = 0$$

subject to normalizing condition

$$\mathbf{\Phi^{(0)}}(I-R)^{-1}e = 1$$

Proof: The theorem follows from the well known result on matrix-geometric methods (Neuts(7)).

3.3 Computation of R matrix

In this subsection we present an efficient algorithm for computing the rate matrix R which is the main ingredient for discussing qualitative behavior of the model under study. The R matrix is of size (4S + 1) can be computed by using logarithmic reduction algorithm.

Logarithmic reduction algorithm

Logarithmic reduction algorithm is developed by Latouche and Ramaswami [11] which has extremely fast quadratic convergence. Here we discuss only the important steps involved in this algorithm. We refer the reader to Latouche and Ramaswami [11] for more details about this algorithm.

Step 0: $H \leftarrow (-A)^{-1}L$, $F \leftarrow (-A)^{-1}M$, $G = F$, and $T = H$

Step 1:

$$U = HF + FH$$
$$E = H^2$$
$$H \leftarrow (I-U)^{-1}E$$
$$E \leftarrow F^2$$
$$F \leftarrow (I-U)^{-1}E$$
$$G \leftarrow G + TL$$
$$T \leftarrow TH$$

Continue **Step 1:** until $\| e - Ge \|_\infty < \in$.

Step 2: $R = -L(A+LG)^{-1}$.

IV. System Performance Measures

In this section some performance measures of the system under consideration in the steady state are derived.

4.1 Expected inventory level

Let η_i denote the average inventory position in the steady state. Then

$$\eta_i = \sum_{i=0}^{\infty} \sum_{j=1}^{S} \sum_{k=0}^{3} j \left[\pi^{(i,j,k)}\right] \qquad (4)$$

4.2 Expected reorder rate

Let η_r denote the expected reorder rate in the steady state. Then

$$\eta_r = \sum_{i=0}^{\infty} \left[\mu_v \pi^{(i,s+1,1)} + \mu_b \pi^{(i,s+1,3)}\right] \qquad (5)$$

4.3 Expected number of demands in the orbit

Let η_o denote the average inventory position in the steady state. Then

$$\eta_o = \sum_{i=1}^{\infty} \sum_{j=1}^{S} \sum_{k=0}^{3} i \left[\pi^{(i,j,k)}\right] + \sum_{i=1}^{\infty} i \left[\pi^{(i,0,0)}\right] \qquad (6)$$

4.4 Effective reneging rate for an orbiting customer

Let η_{ro} denote the effective reneging rate for an orbiting customer in the steady state. Then

$$\eta_{ro} = \sum_{i=1}^{\infty} \sum_{j=1}^{S} \sum_{k=0}^{1} \eta \left[\pi^{(i,0,0)} + \pi^{(i,j,k)}\right] \qquad (7)$$

4.5 Overall rate of retrials

Let η_{or} denote overall rate of retrials in the steady state. Then

$$\eta_{or} = \sum_{i=1}^{\infty} \sum_{j=1}^{S} \sum_{k=0}^{3} \alpha \left[\pi^{(i,j,k)}\right] + \sum_{i=1}^{\infty} \alpha \left[\pi^{(i,0,0)}\right] \qquad (8)$$

4.6 The successful retrial rate

Let η_{sr} denote successful retrial rate in the steady state. Then

$$\eta_{sr} = \sum_{i=1}^{\infty} \sum_{j=1}^{S} \alpha \left[\pi^{(i,j,0)} + \pi^{(i,j,2)}\right] \qquad (9)$$

4.7 The fraction of successful rate of retrial

Let η_{sr} denote successful retrial rate in the steady state. Then

$$\eta_{fr} = \frac{\eta_{sr}}{\eta_{or}} \qquad (10)$$

V. Total Expected Cost Rate

To compute the total expected cost per unit time, we consider the following costs.

c_s : Setup cost per order.

c_h : The inventory carrying cost per unit item per unit time.

c_w : Waiting cost of a customer in the orbit per unit time.

c_r : reneging cost per customer per unit time.

The long run total expected cost rate is given by

$$TC(s,S) = c_s\eta_i + c_h\eta_r + c_w\eta_o + c_r\eta_{ro}$$

From equations (4),(5),(6) and (7), we obtain

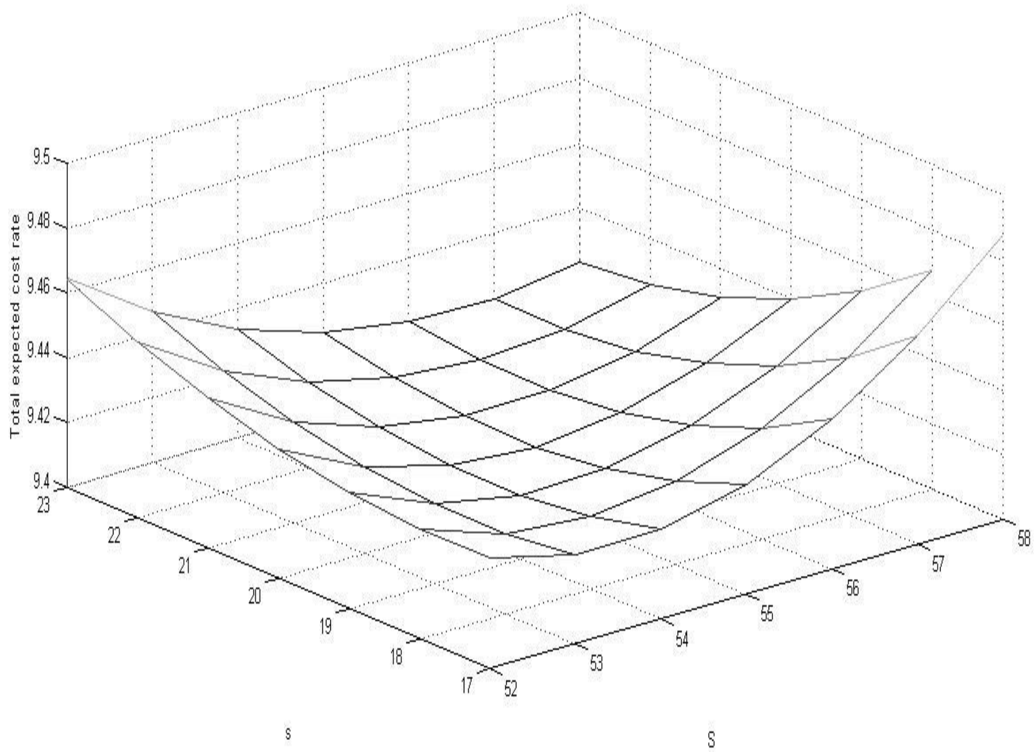
$$TC(s,S) = c_s\left(\sum_{i=0}^{\infty}\sum_{j=1}^{S}\sum_{k=0}^{3} j\ \pi^{(i,j,k)}\right) + c_h\left(\sum_{i=0}^{\infty}\left[\mu_v\ \pi^{(i,s+1,1)} + \mu_b\ \pi^{(i,s+1,3)}\right]\right) +$$
$$c_w\left(\sum_{i=1}^{\infty}\sum_{j=1}^{S}\sum_{k=0}^{3} i\ \pi^{(i,j,k)} + \sum_{i=1}^{\infty} i\ \pi^{(i,0,0)}\right) + c_r\left(\sum_{i=1}^{\infty}\sum_{j=1}^{S}\sum_{k=0}^{1} \eta\ \left[\pi^{(i,0,0)} + \pi^{(i,j,k)}\right]\right).$$

VI. Numerical Illustrations

In this section, we discuss some numerical examples indicates the function $TC(s,S)$ to be convex.Figure 1.refers the changes of s and S are how to affect the total expected cost rate. The table 1 presents the total expected cost rate for various combinations of s and S by fixing the other parameters and costs as $\lambda = 5, \beta = 1.8, \ \eta = 0.4, \ \alpha = 2, \ \mu_v = 17.7, \ \mu_b = 19, \ \theta = 0.2$ and $c_s = 0.09, c_h = 3.8, c_w = 0.48, c_r = 1$. The simple numarical search procedure are used to obtain the optimal values of $TC(s,S)$ (say $TC^*(s^*,S^*)$). The optimal value of the total expected cost rate is $TC(20,55)$ =9.41742. The optimal cost for each *S* is shown in underline and the optimal cost for each *s* is bold. Some of the results are presented in Tables 2 through 13 where the lower entry in each cell gives the optimal total expected cost rate and the upper entries the corresponding S^* and s^*.

Table 1: Total expected cost rate as a function of S and s

s / S	17	18	19	20	21	22	23
52	9.43446	9.42764	9.42770	9.43400	9.44618	9.46413	9.48790
53	**9.43385**	9.42473	9.42213	9.42540	9.43417	9.44829	9.46777
54	9.43574	**9.42450**	9.41946	9.41996	9.42562	9.43626	9.45185
55	9.43990	9.42673	**9.41945**	**9.41742**	9.42023	9.42769	9.43975
56	9.44618	9.43122	9.42189	9.41752	**9.41773**	9.42228	9.43111
57	9.45440	9.43780	9.42657	9.42007	9.41787	**9.41976**	9.42563
58	9.46443	9.44631	9.43333	9.42485	9.42045	9.41988	9.42304



$\lambda = 5, \beta = 1.8, \eta = 0.4, \alpha = 2, \mu_v = 17.7, \mu_b = 19, \theta = 0.2$ $c_s = 0.09, c_h = 3.8, c_w = 0.48, c_r = 1$

Fig. 1: A three dimensional plot of the cost function $TC(s, S)$

Example 1 In this example we study the impact of arrival rate λ, service rates μ_v and μ_b, the lead time parameter β, the retrial rate α, and the renaging rate η, on the optimal values (s^*, S^*) and the corresponding total expected cost rate TC^*. By fixing parameter $\theta = 0.2$ and the costs values as $c_s = 0.09,\ c_h = 3.8,\ c_w = 0.46,\ c_r = 1$. We observe the following from table 2 to 7.

1. The total expected cost rate increases when λ increases and the total expected cost rate decreases when $\beta,\ \eta,\ \alpha,\ \mu_v,\ \mu_b$ increase.
2. If λ, μ_v increase, then S^* monotonically increases. If $\beta,\ \eta,\ \alpha,\ \mu_b$ increase, then S^* monotonically decrease.
3. If λ increases, then s^* monotonically increases. If $\beta,\ \eta,\ \alpha,\ \mu_v,\ \mu_b$ increase, then s^* monotonically decreases .

Table 2: Effect of arrival rate λ and sevice rate μ_v on the optimal values

$$\beta = 1.8,\ \ \eta = 0.4,\ \ \alpha = 2,\ \mu_b = 19$$

λ \ μ_v	17.5		17.6		17.7		17.8		17.9	
4.99	55	20	55	20	55	20	56	20	56	20
	8.98862		8.97611		8.96371		8.95137		9.93898	
5.00	55	20	55	20	55	20	55	20	56	20
	9.20023		9.18744		9.17475		9.16217		9.14959	
5.01	56	21	56	21	56	21	56	21	56	21
	9.43021		9.41729		9.40448		9.39176		9.37915	
5.02	56	21	56	21	56	21	56	21	56	21
	9.68171		9.66850		9.65539		9.64239		9.62948	
5.03	57	22	57	22	56	21	56	21	56	21
	9.95759		9.94430		9.93090		9.91759		9.90439	

Table 3: Effect of arrival rate λ and sevice rate μ_b on the optimal values

$$\beta = 1.8,\ \ \eta = 0.4,\ \ \alpha = 2,\ \mu_v = 17.7$$

λ \ μ_b	18.8		18.9		19.0		19.1		19.2	
4.99	56	21	55	20	55	20	56	20	55	19
	9.63374		9.27375		8.96371		8.69475		8.45912	
5.00	56	21	56	21	55	20	55	20	56	20
	9.91251		9.51501		9.17475		8.88071		8.62467	
5.01	57	22	56	21	56	21	55	20	56	20
	10.21998		9.77924		9.40447		9.08232		8.80211	
5.02	57	22	57	22	56	21	56	21	56	20
	10.56019		10.07003		9.65539		9.30142		8.99502	
5.03	57	22	57	22	56	21	56	21	56	21
	10.61398		10.39072		9.93090		9.54012		9.20521	

Table 4: Effect of arrival rate λ and retrial rate α on the optimal values

$$\beta = 1.8,\ \ \eta = 0.4,\ \ \mu_v = 17.7,\ \mu_v = 19$$

λ \ α	1.99		2.00		2.01		2.02		2.03	
4.99	55	20	55	20	55	20	55	19	55	19
	9.18425		8.96371		8.76306		8.57946		8.41008	
5.00	56	21	55	20	55	20	55	20	55	19
	9.41545		9.17475		8.95623		8.75729		8.57523	
5.01	56	21	56	21	55	20	55	20	55	20
	9.66804		9.40447		9.16612		8.94953		8.75221	

5.02	56	21	56	21	56	21	55	20	55	20
	9.94549		9.65539		9.39443		9.15832		8.94356	
5.03	57	22	56	21	56	21	56	21	55	20
	10.25085		9.93090		9.64379		9.38531		9.15135	

Table 5: Effect of lead time β and sevice rate μ_v on the optimal values

$$\lambda = 5,\ \eta = 0.4,\ \ \alpha = 2,\ \mu_b = 19$$

β \ μ_v	17.5		17.6		17.7		17.8		17.9	
1.79	56	21	56	21	55	20	55	20	56	20
	9.20086		9.18820		9.17556		9.16296		9.15033	
1.80	55	20	55	20	55	20	55	20	56	20
	9.20023		9.18744		9.17475		9.16217		9.14959	
1.81	55	20	55	20	55	20	55	20	56	20
	9.19944		9.18666		9.17400		9.16144		9.14889	
1.82	55	20	55	20	55	20	55	20	56	20
	9.19869		9.18594		9.17330		9.16076		9.14824	
1.83	55	20	55	20	55	20	55	20	56	20
	9.19800		9.18527		9.17264		9.16012		9.14764	

Table 6: Effect of lead time β and sevice rate μ_b on the optimal values

$$\lambda = 5,\ \eta = 0.4,\ \ \alpha = 2,\ \mu_v = 17.7$$

β \ μ_b	18.8		18.9		19.0		19.1		19.2	
1.79	56	21	56	21	55	20	55	20	56	19
	9.91335		9.51569		9.17556		8.88137		8.62519	
1.80	56	21	56	21	55	20	55	20	56	20
	9.91251		9.51501		9.17475		8.88071		8.62467	
1.81	56	21	56	21	55	20	55	20	55	19
	9.91172		9.51437		9.17400		8.88009		8.62407	
1.82	56	21	55	20	55	20	55	20	55	19
	9.91098		9.51371		9.17330		8.87953		8.62338	
1.83	56	21	55	20	55	20	55	20	55	19
	9.91028		9.51291		9.17264		8.87901		8.62274	

Table 7: Effect of lead time β and sevice rate η on the optimal values

$$\lambda = 5,\ \ \alpha = 2,\ \mu_v = 17.7,\ \mu_b = 19$$

β \ η	0.39		0.40		0.41		0.42		0.43	
1.79	56	21	55	20	55	20	55	20	55	20
	9.19097		9.17556		9.16013		9.14477		9.12949	
1.80	55	20	55	20	55	20	55	20	55	20
	9.19023		9.17475		9.15935		9.14402		9.12876	
1.81	55	20	55	20	55	20	55	20	55	20
	9.18946		9.17400		9.15862		9.14331		9.12808	
1.82	55	20	55	20	55	20	55	20	55	20
	9.18873		9.17330		9.15794		9.14265		9.12744	
1.83	55	20	55	20	55	20	55	20	55	20

	9.18805	9.17264	9.15730	9.14204	9.12685

Exapmle 2 In this example we study the impact of setup cost c_s, holdiing cost c_h, waiting cost c_w and the reneging cost c_r on the optimal values (s^*, S^*) and the corresponding total expected cost rate TC^*. By fixing the parameter values as $\lambda = 5, \beta = 1.8, \eta = 0.4, \alpha = 2, \mu_v = 17.7, \mu_b = 19, \theta = 0.2$. We observe the following from table 8 to 13.

1. The total expected cost rate increases when c_s, c_h, c_w and c_r increase.
2. If c_h, c_w increase, then S^* monotonically increases. If c_s increases, then S^* monotonically decreases.

Table 8: Sensitivity of c_s and c_h on the optimal values

$c_w = 0.46, c_r = 1$

c_s \ c_h	3.6		3.7		3.8		3.9		4.0	
0.087	55	20	56	20	56	20	56	20	57	20
	9.03707		9.07498		9.11200		9.14902		9.18524	
0.088	55	20	55	20	56	20	56	20	57	20
	9.05758		9.09566		9.13301		9.17003		9.20675	
0.089	55	20	55	20	56	20	56	20	56	20
	9.07809		9.11617		9.15402		9.19104		9.22806	
0.090	55	20	55	20	55	20	56	20	56	20
	9.09860		9.13668		9.17975		9.21205		9.24907	
0.091	54	20	55	20	55	20	56	20	56	20
	9.11873		9.15719		9.19526		9.23306		9.27008	

Table 9: Sensitivity of c_s and c_w on the optimal values

$c_h = 3.8, c_r = 1$

c_s \ c_w	0.43		0.44		0.45		0.46		0.47	
0.087	56	20	56	20	56	20	56	20	56	20
	8.74826		8.86950		8.99075		9.11200		9.23325	
0.088	56	20	56	20	56	20	56	20	56	20
	8.76927		8.89051		9.01176		9.13301		9.25426	
0.089	55	20	56	20	56	20	56	20	56	20
	8.790244		8.91153		9.03277		9.15402		9.27527	
0.090	55	20	55	20	55	20	55	20	55	20
	8.81075		8.93209		9.05342		9.17475		9.29609	
0.091	55	20	55	20	55	20	55	20	55	20
	8.83126		8.95260		9.07393		9.19526		9.31660	

Table 10: Sensitivity of c_s and c_r on the optimal values

$c_h = 3.8, c_w = 0.46$

c_s \ c_r	0.99		1.00		1.01		1.02		1.03	
0.087	56	20	56	20	56	20	56	20	56	20

	9.10900		9.11200		9.11500		9.11800		9.12100	
0.088	56	20	55	20	56	20	56	20	56	20
	9.13001		9.13301		9.13601		9.13901		9.14201	
0.089	56	20	56	20	56	20	56	20	55	20
	9.15102		9.15402		9.15702		9.16002		9.16302	
0.090	55	20	55	20	55	20	55	20	55	20
	9.17175		9.17475		9.17776		918076		9.18377	
0.091	55	20	55	20	55	20	55	20	55	20
	9.19226		9.19526		9.19827		9.20128		9,20428	

Table 11: Sensitivity of c_h and c_w on the optimal values

$c_s = 0.09,\ c_r = 1$

c_w / c_h	0.43		0.44		0.45		0.46		0.47	
3.6	54	20	54	20	55	20	55	20	55	20
	8.73445		8.85588		8.97727		9.09860		9.21993	
3.7	55	20	55	20	55	20	55	20	55	20
	8.77268		8.89401		9.01534		9.13668		9.25801	
3.8	55	20	55	20	55	20	55	20	55	20
	8.81075		8.93209		9.05342		9.17475		9.29609	
3.9	56	20	56	20	56	20	56	20	56	20
	8.84831		8.96955		9.09080		9.21205		9.33330	
4.0	56	20	56	20	56	20	56	20	56	20
	8.88533		9.00657		9.12782		9.24907		9.37032	

Table 12: Sensitivity of c_h and c_r on the optimal values

$c_s = 0.09,\ c_w = 0.46$

c_r / c_h	0.99		1.00		1.01		1.02		1.03	
3.6	55	20	55	20	55	20	55	20	55	20
	9.09559		9.09860		9.10161		9.10461		9.10762	
3.7	55	20	55	20	55	20	55	20	55	20
	9.13367		9.13668		9.13968		9.14269		9.14569	
3.8	55	20	55	20	55	20	55	20	55	20
	9.17175		9.17475		9.17776		9.18076		9.18377	
3.9	56	20	56	20	56	20	56	20	56	20
	9.20905		9.21205		9.21505		9.21805		9.22105	
4.0	56	20	56	20	56	20	56	20	56	20
	9.24607		9.24907		9.25221		9.25507		9.25807	

Table 13: Sensitivity of c_w and c_r on the optimal values

$c_s = 0.09,\ c_h = 3.8$

c_r / c_w	0.99		1.00		1.01		1.02		1.03	
0.43	55	20	55	20	55	20	55	20	55	20
	8.80775		8.81075		8.81376		8.81677		8.81977	
0.44	55	20	55	20	55	20	55	20	55	20
	8.92908		8.93209		8.93509		8.93810		8.94111	

0.45	55	20	55	20	55	20	55	20	55	20
	9.05041		9.05342		9.05643		9.05943		9.06244	
0.46	55	20	55	20	55	20	55	20	55	20
	9.17175		9.17475		9.17776		9.18076		9.18377	
0.47	55	20	55	20	55	20	55	20	55	20
	9.29308		9.29609		9.29909		9.30210		9.30510	

VII. Conclusion

We analysed an (s,Q) inventory system with retrial customers and working vacation. Primary inter arrival times, retrial times, service times and working vacation times are independent exponentially distributed random variables. We have derived the steady state distribution of the system using Matrix analytic methods and several performance measures have also been calculated. Some numerical solutions are presented to illustrate the qualitative behavior of the system.

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