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FuzzyStacked Set and Fuzzy Stacked Semigroup

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Abstract- To developthe concept of stacked set and fuzzy stacked set in the previous twopapers [1] and [2], also generalizethe idea ofstacked systems to becomemore comprehensivein addressingsimilarissues, and the addition ofsomealgebraicrelationsandalgebraicoperations, while maintaining thebasicshape of thestacked systems.

Index Terms- stacked set, fuzzy stacked set , fuzzy set , fuzzy system , stacked

I. INTRODUCTION

This systemthat we have developedin previouspapers, observantandpresence ineverything in thisuniverse. Our worldis full ofoverlappingrelationshipsbetween theelements, andis difficult todetermine the specificarrangementof the importance ofthese elementsto each other, andso it mustgeneralize thissystemto includemany issuesin science.

It is noted that stacked style building close to the idea of fuzzy set , , so you should turn this set related to the ordinal set stacking into obscurity or fuzzy set .
And since the general idea for this study based on the identification of ways to arrange an element within the set, they need to follow this order, to the style of fuzzy set.
Thereforewe needto findotheroperationsandrelationshipsmake ita more positiveidea.

II. PRELIMINARIES

2.1 Definition [1]

let T_α be a finite set , where T_α be a stacked set if and only if $a_\alpha \in T_\alpha$, $\alpha \in N/0$, α is the number of methods stacking elements in the set, and it is called paths ($P_1, P_2, \dots, P_\alpha$).

2.2 Definition [1]

The system (T_α , τ) called stacked- system if and only if $a_\gamma \tau\ b_\beta = \min_0 (a_\gamma , b_\beta)$, and the system looking for (zero convergence) , and The system (T_α , Υ) called stacked- system if and only if $a_\gamma\ \Upsilon\ b_\beta = \max_0 (a_\gamma , b_\beta)$, and the system looking for (zero spacing) . a_γ and $b_\beta \in T_\alpha$.

2.3 Definition [1]

The system (T_α , τ) called stacked- system if and only if $a_\gamma \tau\ b_\beta = \min_t(a_\gamma , b_\beta)$, and the system looking for (convergence oft) , and The system (T_α , Υ) called stacked- system if and only if $a_\gamma\ \Upsilon\ b_\beta = \max_t(a_\gamma , b_\beta)$, and the system looking for (spacing oft) . a_γ and $b_\beta \in T_\alpha$, $t \in \mathcal{R}$.

2.4 Definition [1]

The order element on stacked system T_α ,where the system looking for zero convergence or zero spacing, is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ , ($O_0(a_\gamma)$) :

$$O_0(a_\gamma) = \lceil a_\gamma \rfloor_0 = \left[\frac{|a_{\gamma 1}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \frac{|a_{\gamma 2}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \dots + \frac{|a_{\gamma \alpha}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha =$$

$$\left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha .$$

2.5 Definition [1]

The order element on stacked system T_α ,where the system looking for convergence oft (or spacing of t), is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ ,($O_t(a_\gamma)$) :

$$O_t(a_\gamma) = \lceil a_\gamma \rfloor_t =$$

$$\left[\frac{|a_{\gamma 1} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \frac{|a_{\gamma 2} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \dots + \frac{|a_{\gamma \alpha} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha =$$

$$\left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha .$$

2.6 Definition[1]

The order stacked set T_α in zero convergence system is set $O_0(T_\alpha) = \{x_1, x_2 , \dots ,x_n\}$ if $\lceil x_1 \rfloor_0 < \lceil x_2 \rfloor_0 < \dots < \lceil x_n \rfloor_0$ .And

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the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1}, \dots, x_1\}$ if $\lceil x_1 \rfloor_0 > \lceil x_2 \rfloor_0 > \dots > \lceil x_n \rfloor_0$.
The order stacked set T_α , where the system looking for convergence oft (or spacing of t), is set $O_0(T_\alpha) = \{x_1, x_2, \dots, x_n\}$ if $\lceil x_1 \rfloor_t < \lceil x_2 \rfloor_t < \dots < \lceil x_n \rfloor_t$ .And the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1}, \dots, x_1\}$ if $\lceil x_1 \rfloor_t > \lceil x_2 \rfloor_t > \dots > \lceil x_n \rfloor_t$.

2.7 Definition [1]

- If (T_α , τ) or (T_α , Ʈ) is stacked– system then $\forall\, a_\alpha$, $b_\beta \in T_\alpha$:
  $Max_t(a_\alpha, b_\beta) = a_\alpha$ Ʈ b_β , and $Min_t(a_\alpha, b_\beta) = a_\alpha \tau b_\beta$

- If (T_α , τ) or (T_α , Ʈ) is stacked– system then $\forall\, a_\alpha$, $b_\beta \in T_\alpha$:

$$\max_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha : if\ \lceil a_\alpha \rfloor_t > \lceil b_\beta \rfloor_t . \\ b_\beta : if\ \lceil b_\beta \rfloor_t > \lceil a_\alpha \rfloor_t . \end{cases}$$

$$\min_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha : if\ \lceil a_\alpha \rfloor_t < \lceil b_\beta \rfloor_t . \\ b_\beta : if\ \lceil b_\beta \rfloor_t < \lceil a_\alpha \rfloor_t . \end{cases}$$

- If $a_\alpha = t$,(in $\lceil a_\alpha \rfloor_t$) then we suppose that $| a_\alpha - t | = \Delta t$, and where $\sum_i^{\alpha} | a_i - t |$, $\alpha \in \{1, ..i\}$ then, we compensate $| a_\alpha - t | = 0$.

- If $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil a_\alpha \rfloor_t \neq \lceil b_\beta \rfloor_t$ the system is type-1 .

2.8 Definition[1]

A stacked-semigroup is a stacked-system T_α , with associative binary operation .

2.9 Theorem [1]

(i) If the systems (T_α , τ) is a stacked-system (type – 1), then (T_α , τ) is a semigroup and called a stacked-semigroup .
(ii) If the systems (T_α , Ʈ)is a stacked-system (type – 1), then (T_α , Ʈ) is a semigroup and called a stacked-semigroup.

Prove this theorem earlier in paper [1]

2.10 Definition[2]

Let $T_\alpha = \{ x_{11}, x_{12}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{nn} \}$ be called a stacked set if theelements in $T_\alpha$arestackedin terms oftheplace(horizontally and vertically).

2.11 Definition[2]

If T_α is a stacked system of element denoted generically by x then a fuzzy stacked system T_μ in T_α is a system of ordered pairs:

$$T_\mu = \{(x , \mu_T(x)) \mid x \in T_\alpha \} .$$

$\mu_T(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in T_μ which maps T_α to the membership space M . (When M contains only the two points 0 and 1, T_μ is nonfuzzy and $\mu_T(x)$ is identical to the characteristic function of a nonfuzzy stacked set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite . Elements with a zero degree of membership are normally not listed .

$$T_\mu = \{(x , \mu_T(x)) \mid x \in T \} .$$

2.12 Theorem[2]

If T_α is a stacked system, $\forall$ x , a $\in T_\alpha$:

$$\mu_{T1}(x) = (1 + | a - x |)^{-1} .$$
$$\mu_{T2}(x) = (1 + (a - x)^2)^{-1} .$$
$$\mu_{Tn}(x) = (1 + (|a - x|)^n)^{-1}$$

are types of a function such that $\mu_{Ti}(x) \in [0, 1]$,i $\in \{ 1 , 2 , \dots , n \}$.

proof :let T_α is a stacked system, $\forall$ x , a $\in T_\alpha$ **:**
$| a - x | \geq 0 \Rightarrow (1+| a - x |) \geq 1 \Rightarrow 0 \leq (1 + | a - x |)^{-1} \leq 1$
$\Rightarrow \mu_{T1}(x) = (1 + | a - x |)^{-1} \in [0, 1]$.
$(a - x)^2 \geq 0 \Rightarrow (1+(a - x)^2) \geq 1 \Rightarrow 0 \leq (1 + (a - x)^2)^{-1} \leq 1$
$\Rightarrow \mu_{T1}(x) = (1 + (a - x)^2)^{-1} \in [0, 1]$, so $\mu_{Tn}(x) = (1 + (|a - x|)^n)^{-1}) \in [0, 1]$, i $\in \{ 1 , 2 , \dots , n \}$.

2.13 Theorem[2]

If T_α is a stacked system, $\forall$ x $\in T_\alpha$, $\mu_T(a) \geq \mu_T(x)$ then :
$\mu_T(x) = (1 + (a - x)^n)^{-1}$, n $\in$ N/0 ,is convex function .

Proof : Let $x_1 < a < x_2$,and $\forall$ x $\in T_\alpha$, $\mu_T(a) \geq \mu_T(x)$ then :
$\mu_T(a) \geq \mu_T(x_1)$, and $\mu_T(a) \geq \mu_T(x_2)$
So :$[\mu_T(a) - \mu_T(x_1)] / [a - x_1] < 0 < [\mu_T(x_2) - \mu_T(a)] / [x_2 - a]$.
From definition 2.6 :$[\mu_T(a) - \mu_T(x_1)] / [a - x_1] \geq [\mu_T(x_2) - \mu_T(a)] / [x_2 - a]$,then $\mu_T(x) = (1 + (|a - x|)^n)^{-1}$ is convex function .

2.14 Definition[2]

If T_α is a stacked system, then the fuzzy stacked system T_μ in T_α is a system of ordered pairs:

$$T_\mu = \{(x , \mu_T(x)) \mid x \in T \} .$$

Such that :

$$\mu_T(x) = (1 + (| x - a |)^n)^{-1} , n \in N/0 \text{ and } x \in T_\alpha$$

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2.15 Definition[2]

For a finite fuzzy stacked set T_μ the column(row) cardinality $\backslash T_{\mu\,(C \text{ or } R)}\backslash$ is defined as :

$$\backslash T_{\mu\,(C \text{ or } R)}\backslash = \sum_{x \in (CorR)} \mu_{T(C \text{ or } R)}(x).$$

2.16 Definition[2]

Let (x) $\in T_\mu$, γ is a row and β is column then the order fuzzy stacked of $\mu(x_{\gamma\beta})$ is :

$$O(x) = \| \mu(x_{\gamma\beta}) \| =[\, \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\gamma)}\backslash + \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\beta)}\backslash \,]/2 \quad .$$

2.17 Theorem[2]

If T is a stacked system, $\forall$ x , a $\in$ T :
$O(x) = \| \mu(x_{\gamma\beta}) \| =[\, \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\gamma)}\backslash + \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\beta)}\backslash \,]/2$
is a type of a function such that O(x)$\in$ [0, 1],

proof : from definitions (3.5) , (3.7) , (3.8).
$0 \le \mu(x_{\gamma\beta}) \le \backslash T_{\mu\,(\gamma)}\backslash \Rightarrow 0 / \backslash T_{\mu\,(\gamma)}\backslash \le \mu(x_{\gamma\beta}) / \backslash T_{\mu\,(\gamma)}\backslash \le \backslash T_{\mu\,(\gamma)}\backslash / \backslash T_{\mu\,(\gamma)}\backslash \Longrightarrow 0 \le \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\gamma)}\backslash \le 1$ (A) . And so $0 \le \mu(x_{\gamma\beta}) \le \backslash T_{\mu\,(\beta)}\backslash \Rightarrow 0 / \backslash T_{\mu\,(\beta)}\backslash \le \mu(x_{\gamma\beta}) / \backslash T_{\mu\,(\beta)}\backslash \le \backslash T_{\mu\,(\beta)}\backslash / \backslash T_{\mu\,(\beta)}\backslash \Longrightarrow 0 \le \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\beta)}\backslash \le 1$(B).
Then (A) +(B) $\Rightarrow 0+0 = 0 \le \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\gamma)}\backslash + \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\beta)}\backslash \le 1 + 1 =2 \Longrightarrow 0 \le [\mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\gamma)}\backslash + \mu(x_{\gamma\beta})/ \backslash T_{\mu\,(\beta)}\backslash]/ 2 \le 1 \Longrightarrow O(x)$)$\in$ [0, 1]

2.18 Definition [2]

If T_α is a stacked system, then the fuzzy level stacked system $l(T_\mu)$ in T_α is a system :

$$l(T_\mu) = \{(x , \mu_T(x), O(x)) \mid x \in T_\alpha \} .$$

2.19 Definition[2]

Let $l(T_\mu)$ in T_α is a fuzzy level stacked system $[l(T_\mu)=\{(x, \mu_T(x), O(x))|x \in T_\alpha \}]$, then :
$\max_t 1[O(T)] = \{ x_1 , x_2 ,\dots , x_n \}$ .That's where :
$\max[O(T)] = x_1$, $\max[O(T)/\{R_{x_1},C_{x_1}\}] = x_2$,… , $\max[O(T)/\{R_{x_1},C_{x_1},R_{x_2},C_{x_2},\cdots,R_{x_{n-1}},C_{x_{n-1}}\}]= x_n$, C mean column and R mean row ,and if $x_\beta \in \max_{t1}(O(T))$, then $R_\beta \cap \max_{t1}(O(T))= \{x_\beta\}$, and $C_\beta \cap \max_{t1}(O(T))= \{x_\beta\}$,$|C_i| = |R_i| = | \mathrm{Max}_{t1}(O(T))| = n$.

2.20 Definition[3]

If Ã is a collection of objects denoted generically by x then a fuzzy set A in Ã is a set of ordered pairs:

$$\tilde{A} = \{(x , \mu_{\tilde{A}}(x)) \mid x \in T \} .$$

$\mu_{\tilde{A}}(x)$is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in A which maps X to the membership space M. (When M contains only the two points 0 and 1, Ã is nonfuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set.) The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

2.21 Definition[3]

The support of a fuzzy set Ã, S(Ã), is the crisp set of all x ∈ X such that :

$$\mu_{\tilde{A}}(x) > 0 .$$

2.22 Definition[3]

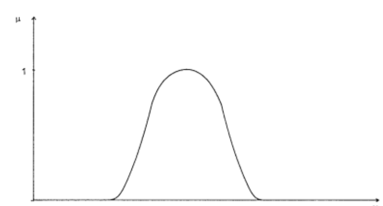
The (crisp) set of elements that belong to the fuzzy set A at least to the degree a is called the α - level set :

$$A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \ge \ \}$$

$\tilde{A}= \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha \}$ is called "strong α - level set" or "strong α -cut."

2.23 Definition[3]

A fuzzy set Ã is convex if :
$\mu_{\tilde{A}}\ (\lambda x_1 + (1 - \lambda)x_2)\ge \min(\mu_{\tilde{A}}(x_1) , \mu_{\tilde{A}}(x_2)), x_1 , x_2 \in X , \lambda \in [0 , 1]$



Figures 1
Convex fuzzy set

2.24 Definition[4]

By groupoid (S, *) we shall mean a non-empty set S on which a binary operation * is defined. That is to say, we have a mapping *: S × S → S.
We shall say that (S, *) is a semigroup if * is associative, i.e. if

$$(\forall\ x, y, z \in S), ((x, y)*, z)* = (x, (y, z)*)*$$

2.25 Definition[4]

S is a finite semigroup if it has only a finitely many elements.

2.26 Definition[4]

A commutative semigroupis a semigroup S with property :

$$(\forall\, x, y \in S) \quad (xy = yx).$$

2.27 Definition[4]

If there exists an element 1 of S such that $(\forall\, x \in S)$ $x1 = 1x = x$ .We say that 1 is an identity (element) of S and that S is a semigroup with identity.

2.28 Definition[5]

Let S be a set and $\sigma : S \times S \to S$ a binary operation that maps each ordered pair (x , y) of S to an element σ(x , y) of S. The pair (S, σ) (or just S, if there is no fear of confusion) is called a groupoid .

2.29 Definition[6]

We denote by L[a] (R[a], J[a], I[a], Q[a], B[a]) the principal left (right, two-sided, interior, quasi-, bi-) ideal of a semigroup S generated by the element a ∈ S, that is,

$L[a] = \{a\} \cup Sa,$
$R[a] = \{a\} \cup aS,$
$J[a] = \{a\} \cup Sa \cup aS \cup SaS,$
$I[a] = \{a\} \cup \{a^2\} \cup SaS,$
$Q[a] = \{a\} \cup (aS \cap Sa),$
$B[a] = \{a\} \cup \{a^2\} \cup aSa$. [10]

2.30 Definition[7]

A real function f defined on a real interval I is convex on I iff:

$\forall x_1,x_2,x_3 \in I: x_1<x_2<x_3 : [f(x_2)-f(x_1)] / [x_2-x_1] \le [f(x_3)-f(x_2)] / [x_3-x_2].$

or:

$\forall x_1,x_2,x_3 \in I: x_1<x_2<x_3 : [f(x_2)-f(x_1)] / [x_2-x_1] \le [f(x_3)-f(x_1)] / [x_3-x_1].$

The function f is strictly convex on I if, in the above inequalities, equality cannot hold.

2.31 Definition[8]

For a set A ,we define a membership function μ_A such as

$$\mu_A(x) = \begin{cases} 1 & iff \quad x \in A \\ 0 & iff \quad x \notin A \end{cases}$$

(" iff " is short for "if and only if ").

X be a classical set of objects, called the universe, whose generic elements are denoted x . Membership in a classical subset A of X is often viewed as a characteristic function , μ_A from X to {0,1} such that :

$$\mu_A : X \to \{ 0 , 1 \} .$$

2.32 Axioms for Union Function[8]

In general sense, union of A and B is specified by a function of the form.

$$\mu_{A\cup B}(x) = [0,1] \times [0,1] \to [0,1]$$

This union function calculates the membership degree of union A∪ B from those of A and B.

$$\mu_{A\cup B}(x) = \cup[\mu_A(x) , \mu_B(x)].$$

this union function should obey next axioms.

(Axiom U1)

∪(0,0) = 0, ∪(0,1) = 1, ∪(1,0) = 1, ∪(1,1) = 1

so this union function follows properties of union operation of crisp sets (boundary condition).

(Axiom U2)

∪(a , b) = ∪(b , a) Commutativity holds.

(Axiom U3)

If a ≤ a' and b ≤ b', ∪(a, b) ≤ ∪(a', b') Function ∪ is a monotonic function.

(Axiom U4)

∪(∪(a, b), c) = ∪(a, ∪(b, c)) Associativity holds.

The above four statements are called as "axiomatic skeleton". It is often to restrict the class of fuzzy unions by adding the following axioms.

(Axiom U5)

Function ∪ is continuous.

(Axiom U6)

∪(a, a) = a (idempotency).

2.35 Some Algebraic Operations [8]

(1) Probabilistic sum $A \hat{+} B$ (Algebraic sum)

Fuzzy union $A \hat{+} B$ is defined as,

$$\forall\, x \in X , \mu_{A \hat{+} B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\, \mu_B(x)$$

It follows commutativity, associativity, identity, and De Morgan's law.

This operator holds also the following :

$$A \hat{+} X = X$$

(2) Bounded sum A ⊕ B (Bold union)

$$\forall\, x \in X , \mu_{A\oplus B}(x) = \text{Min}[1, \mu_A(x) + \mu_B(x)]$$

This operator is identical to Yager function at w = 1. Commutativity, associativity, identity, and De Morgan's Law are perfected, and it has relations,

$$A \oplus X = X$$
$$A \oplus A = X$$

but it does not idempotency and distributivity at absorption.

(3) Drastic sum $A \dot{\cup} B$

Drastic sum is defined as follows:

$$\forall x \in X , \quad \mu_{A \dot{\cup} B}(x) = \begin{cases} \mu_A(x), & when \quad \mu_B(x) = 0 \\ \mu_B(x), & when \quad \mu_A(x) = 0 \\ 1, & for \quad others \end{cases}$$

(4) Hamacher's sum A ∪ B

$$\forall x \in X, \quad \mu_{A\cup B}(x)=\frac{\mu_A(x)+\mu_B(x)-(2-\gamma)\mu_A(x)\mu_B(x)}{1-(1-\gamma)\mu_A(x)\mu_B(x)}, \gamma \geq 0 \ .$$

(5) Algebraic product A ● B (probabilistic product)

$$\forall x\in X, \quad \mu_{A\bullet B}(x) = \mu_A(x) \bullet \mu_B(x).$$

Operator ● is obedient to rules of commutativity, associativity, identity, and De Morgan's law.
(6) Bounded product A⊙B (Bold intersection)
This operator is defined as,

$$\forall x\in X, \quad \mu_{A\odot B}(x) = \mathrm{Max}[\, 0 \,, \ \mu_A(x) + \mu_B(x) - 1 \,]$$

and is identical to Yager intersection function with w = 1,

$$I_1(a, b) = 1 - \mathrm{Min}[1, 2 - a - b]$$

commutativity, associativity, identity, and De Morgan's law hold in this
operator . The following relations

$$A \odot \emptyset = \emptyset$$

$$A \odot \bar{A} = \emptyset \ .$$

are also satisfied, but not idempotency, distributability, and absorption.

2.36 Definition[9]

The mathematical systems is a set of interacting or interdependent components forming an integrated whole or a set of elements (often called 'components') and relationships which are different from relationships of the set or its elements to other elements or sets .

Example: (R , +) is a system , R is a set of all the reel numbers, and (+) the relation between the elements .

2.37 Definition[7]

A binary operation on a set S is a mapping of the Cartesian product S × S into S.

2.38 Definition[10]

Let X be a space of points (objects),with a generic element of X denoted by x . Thus, X = {x}. A fuzzy set (class) A in X is characterized by a membership (characteristic) function $f_A(x)$ which associates with each point in X a real number in the interval [0, 1] , with the value of $f_A(x)$ at x representing the "grade of membership" of : x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of 0 in A. When A is a set in the ordinary sense of the term, its membership function can take on only two values 0 and 1, with $f_A(x)$ = 1 or 0 according as x does or does not belong to A . Thus, in this case $f_A(x)$ reduces to the familiar characteristic function of a set A . (When there is a need to differentiate between such sets and fuzzy sets, the sets with two-valued characteristic functions will be referred to as ordinary sets or simply sets).

III. BASICCONCEPTSIN THE STACKED FUZZY SET

3.1 Theorem

If T_α be a stacked set then the set of ordered pairs :

$$T_{\mu(t)} = \{(\, x_\gamma \,,\, \mu_t(x_\gamma)) \mid x_\gamma\in T_\alpha \,,\, \mu_t(x_\gamma) = \lceil x_\gamma \rfloor_t \,\}$$

is a fuzzy stacked set.

Proof :

We only need to prove that $\mu_t(x_\gamma)\in[\,0\,,\,1\,]$,and that's enough to prove $T_{\mu(t)}$ is a fuzzy stacked set .
From the definition of the stacked set, $x_\gamma\in$ T ,

$$0 \leq |\, x_\gamma - t \,| \leq \sum_{i=1}^{\alpha} |x_{\gamma i} - t\,| \Longrightarrow$$

$$0 \leq \frac{|\, x_\gamma - t \,|}{\sum_{i=1}^{\alpha} |\, x_{\gamma i} - t \,|} \leq 1 \Longrightarrow$$

$$0 \leq \frac{|\, x_{\gamma 1} - t \,|}{\sum_{i=1}^{\alpha} |\, x_{\gamma i} - t \,|} + \frac{|\, x_{\gamma 2} - t \,|}{\sum_{i=1}^{\alpha} |\, x_{\gamma i} - t \,|} + \ldots + \frac{|\, x_{\gamma \alpha} - t \,|}{\sum_{i=1}^{\alpha} |\, x_{\gamma i} - t \,|} \leq 1+1+$$

$$\ldots \ (\text{ to } \alpha \text{ times }) = \alpha \Longrightarrow$$

$$0 \leq [\sum_{i=1}^{\alpha} \frac{|\, x_{\gamma i} - t \,|}{\sum_{i=1}^{\alpha} |\, x_{\gamma i} - t \,|}] / \alpha \leq 1 \Longrightarrow$$

$$0 \leq \lceil x_\gamma \rfloor_t \leq 1$$

$$\text{Then } \lceil x_\gamma \rfloor_t = \mu_t(x_\gamma) \in [\, 0 \,,\, 1 \,] \ .$$

3.2 Example

If there are three distribution centers,consumer productsofthetype(A,B, C), wheretheyaretransportedtosales centers(X, Y, Z)atacost, asinthefollowingtable:

	X	Y	Z
A	1	3	2
B	4	5	1
C	7	3	1

Table 1

So thereisaprocess of transferringbetween(A, B, C) and (X, Y, Z).
Transportation betweenAandXcost1, so

$$\text{cost}\ (\, A,X) = 1 \equiv 1_{11}$$

$$\text{cost}\ (\, A,Y) = 3 \equiv 3_{12}$$

$$\text{cost}\ (\, A,Z) = 2 \equiv 2_{13}$$

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cost (B,X) = 4 ≡ 4_{21}
cost (B,Y) = 5 ≡ 5_{22}
cost (B,Z) = 1 ≡ 1_{23}
cost (C,X) = 7 ≡ 7_{31}
cost (C,Y) = 3 ≡ 3_{32}
cost (C,Z) = 1 ≡ 1_{33} .

And $T_{2,3}$ (Non-Order)= { 1_{11} , 3_{12} , 2_{13} , 4_{21} , 5_{22} , 1_{23} , 7_{31} , 3_{32} , 1_{33} } .

In this example, if requiredthetransferofconsumer products atthelowestcost, so that thetransferofa single productfrom each single distributioncenterto the single center ofthesale,we godirectlyto :

$$Min_0\{cost[(A,B,C),(X,Y,Z)]\} = \{ 1_{11} , 1_{23} , 3_{32} \}$$

But whentheaveragetransfer requestarelimitedtoaspecificvalue, we identifythrough :

$$T_{(t)} = \{(x , \mu_t(x)) \mid x \in T_\alpha \} .$$

Now supposethat the averagetransportationintendedfordistribution3 unitsof products, fromeachdistribution centertoeach centersale .
Then :

$$\mu_t(x_\gamma) = \lceil a_\gamma \rfloor_3 = [\sum_{i=1}^{3} \frac{|x_{\gamma i} - 3|}{\sum_{i=1}^{3} |x_{\gamma i} - 3|}] / \alpha$$

If x_a = t, then we suppose that | x_a – t | = Δt , and where $\sum_{i}^{\alpha} | x_i - t |$, a∈ {1 , ..i} then, we compensate | x_a – t | = 0 .

1- $T_{3,2}$ =

1	**3**	**2**
4	**5**	**1**
7	**3**	**1**

Table2

2- $T_{3,2}$ [| $x_{\gamma i}$ – 3 |] =

2	Δt	1
1	2	2
4	Δt	2

Table3

3- $T_{3,2}$ [$\mu_T(x_\gamma)$] = $T_{3,2}$ [$\lceil a_\gamma \rfloor_3$] =

$$T_{3,2} [[\sum_{i=1}^{3} \frac{|x_{\gamma i} - 3|}{\sum_{i=1}^{3} |x_{\gamma i} - 3|}] / \alpha] =$$

0.2	1	0.5
0.5	0.2	0.2
0	1	0.2

Table4

when O_3($T_{3,2}$) = { 3_{32}, 3_{12}, 4_{21}, 2_{13}, 1_{33}, 1_{23}, 1_{11}, 7_{31} , 5_{22} } .
so

$T_{\mu(3)}$ = { (3_{32} ,$0_{\mathbf{0.333}}$) , (3_{12} , $0_{\mathbf{0.417}}$) , (4_{21} , 0.171) , (2_{13} , 0.267) , (1_{33} , 0.367) , (1_{23} , 0.400) , (1_{11} , 0.476) , (7_{31} , 0.619) , (5_{22} , 0.700) } .

Then if we have identifiedaveragetransportationextent3 units , most suitableconductfor transportare:

Min_3\{cost[(A,B,C),(X,Y,Z)]\} = {(3_{32} ,$0_{\mathbf{0.333}}$), (4_{21} , 0.171) , (2_{13} , 0.267)}

3.3 Example

In example 3.2 , if requiredthetransferofconsumer products atthelowestcost, so that thetransferofa single productfrom each single distributioncenterto the single center ofthesale,we godirectlyto :

$$Min_0\{cost[(A,B,C),(X,Y,Z)]\} = \{ 1_{11} , 1_{23} , 3_{32} \}$$

But whentheaveragetransfer requestislimitedtoaspecificvalue, we identifythrough:

$$T_\mu = \{(x, \mu_T(x)) \mid x \in T\}.$$

Now supposethat the averagetransportationintendedfordistribution3 unitsof products, fromeachdistribution centertoeach centersale. And that does not exceed6 units fortransportationproducts

So:

$$\mu_T(x) = \begin{cases} 0 & : x > 6 \\ (1 + (x - 3)^2)^{-1} & : 0 \le x \le 6 \end{cases}$$

Then
$\mu_T(1_{11})$ = (1 + (1 – 3 $)^2$ $)^{-1}$ = 0.2 , $\mu_T(3_{12})$= 1 , … , $\mu_T(7_{31})$= 0 ,….

when $T_{3,2}$ = { 1_{11} , 3_{12} , 2_{13} , 4_{21} , 5_{22} , 1_{23} , 7_{31} , 3_{32} , 1_{32} } (non-order).
So:

T_μ = { (1_{11} ,0.2) , (3_{12} , 1) , (2_{13} , 0.5) , (4_{21} , 0.5) , (5_{22} , 0.2) , (1_{23} , 0.2) , (7_{31} , 0) , (3_{32} , 1) , (1_{32} , 0.2) } .
Or

T_μ =

0.476	0.417Δt	0.267
0.171	0.700	0.400
0.619	0.333Δt	0.367

Table5

Then if we have identifiedaveragetransportationextent3 units,most suitableconductfor transportare:

$\text{Max}_3\{\text{cost}[(A,B,C),(X,Y,Z)]\} = \{(3_{32}, 1), (4_{21}, 0.5), (2_{13}, 0.5)\}$

It has already been mentioned that the membership function is not limited to values between 0 and 1. lf $\sup_x \mu_T(x) = 1$ the fuzzy stacked set T_μ is called normal . A nonempty fuzzy stacked set T_μ can always be normalized by dividing $\mu_T(x)$ by $\sup_x \mu_T(x)$: As a matter of convenience we will generally assume that fuzzy stacked sets are normalized. For the representation of fuzzy stacked sets we will use the notation illustrated in example above respectively. A fuzzy stacked set is obviously a generalization of a classical set and the membership function a generalization of the characteristic function. Since we are generally referring to a universal (crisp) set T some elements of a fuzzy set may have the degree of membership zero. Often it is appropriate to consider that element of the universe which has a nonzero degree of membership in fuzzy stacked set.

3.4 Definition

The support of a fuzzy stacked set $T_{\mu(t)}$, $S(T_{\mu(t)})$, is the crisp set of all $x \in T$ such that $\mu_t(x) > 0$.

3.5 Example

In example above $S(T_\mu) = \{1_{11}, 3_{12}, 2_{13}, 4_{21}, 5_{22}, 1_{23}, 3_{32}, 1_{32}\}$. The element $\{7_{31}\}$ is not part of the support of T_μ

3.6 Definition

The (crisp) set of elements that belong to the fuzzy stacked set T_μ at least to the degree α – level stacked set:

$$T_\alpha = \{x \in T \mid \mu_T(x) \geq \alpha\}$$

$T_{\underline{\alpha}} = \{x \in T \mid \mu_T(x) > \alpha\}$ is called (strong α – level stacked set or strong α– cut).

3.7 Example:

The list possible α – level stacked set in the above example:

$T_{0.2} = \{1_{11}, 3_{12}, 2_{13}, 4_{21}, 5_{22}, 1_{23}, 3_{32}, 1_{32}\}$,

$T_{0.5} = \{3_{12}, 2_{13}, 4_{21}, 1_{23}, 3_{32}\}$,

$T_{1.0} = \{3_{12}, 3_{32}\}$.

The strong α – level stacked set for $\alpha = 0.5$ is $T_{\underline{0.5}} = \{3_{12}, 3_{32}\}$.

Convexity also plays a role in fuzzy set theory. By contrast to classical set theory, however, convexity conditions are defined with reference to then membership function rather than the support of a fuzzy set.

3.8 Example

Supposethat the averagetransportationintendedfordistribution3 unitsof products, fromeachdistribution centertoeach centersale. And that does not exceed6 units fortransportationproducts

So:

$$\mu_T(x) = \begin{cases} 0 : x > 6 \\ (1 + (x-3)^2)^{-1} : 0 \leq x \leq 6 \end{cases}$$

when $T_{3,2} = \{1_{11}, 3_{12}, 2_{13}, 4_{21}, 5_{22}, 1_{23}, 7_{31}, 3_{32}, 1_{32}\}$.

So:

$T_\mu = \{(1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{32}, 0.2)\}$.

The cardinality is:

$$|T_\mu| = \sum_{x \in X} \mu_T(x) = 0.2 + 1 + 0.5 + 0.5 + 0.2 + 0.2 + 0.2 + 0 + 1 + 0.2 = 3.8$$

Its relative cardinality is:

$$\|T_\mu\| = \frac{3.8}{9} = 0.422$$

IV. THEORETIC OPERATIONS FOR FUZZY STACKED SETS

The membership function is obviously the crucial component of a fuzzy set. It is therefore not surprising that operations with fuzzy sets are defined via their membership functions.

4.1 Definition

The membership function $\mu_{CT}(x)$ of the intersection $C_T = A_T \cap B_T$ is point wise defined by:

$$\mu_{CT}(x) = \min_t\{\mu_{AT}(x), \mu_{BT}(x)\}$$

Such that: if $a_\alpha \in \mu_{AT}(x)$ and $b_\beta \in \mu_{BT}(x)$

$$\text{Min}_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha : \text{i} \ldots \rfloor_t \\ b_\beta : \text{if } \lceil b_\beta \rfloor_t < \lceil a_\alpha \rfloor_t \end{cases}$$

If $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil a_\alpha \rfloor_t \neq \lceil b_\beta \rfloor_t$ the system is type-1 .

4.2 Example

Supposethat the averagetransportationintendedfordistribution3 unitsof products, fromeachdistribution centertoeach centersale . And that does not exceed6 units fortransportationproducts:

$$\mu_{T1}(x) = \begin{cases} 0 : x > 6 \\ (1 + (x-3)^2)^{-1} : 0 \leq x \leq \end{cases}$$

when $T_{3,2} = \{1_{11}, 3_{12}, 2_{13}, 4_{21}, 5_{22}, 1_{23}, 7_{31}, 3_{32}, 1_{32}\}$.

so

$T_{\mu 1} = \{(1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{32}, 0.2)\}$.

Or:

$T_{\mu 1}$ =

0.2	1	0.5
0.5	0.2	0.2
0	1	0.2

Table6

In the same set $T_{3,2} = \{ 1_{11} , 3_{12} , 2_{13} , 4_{21} , 5_{22} , 1_{23} , 7_{31} , 3_{32} , 1_{32} \}$.

Assume that thedistributionofproducts fromalldistribution centers,topoints of saleatthe largestcostand takingintoaccountthat the biggestcostfor distributionis7 units

So:

$\mu_{T2}(x) = (1 + (x-7)^2)^{-1} \;:\; 0 \le x \le 7$

$T_{\mu 2} = \{ (1_{11}, 0.027), (3_{12}, 0.0588), (2_{13}, 0.03846), (4_{21}, 0.1), (5_{22}, 0.2), (1_{23}, 0.027), (7_{31}, 1), (3_{32}, 0.0588), (1_{32}, 0.027) \}$.

Or:

$T_{\mu 2}$ =

0.201	0.521	0.425
0.6349	0.1566	0.222
0	0.644	0.194

Table7

the order element on stacked system T_μ ,where the system looking for zero convergence or zero spacing, is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ, ($O(a_\gamma)$) :

$$\lceil a_\gamma \rfloor_0 = [\sum_{i=1}^{\alpha} \frac{a_i}{\sum_{i=1} a_{\gamma i}}] / \alpha$$

Then:

$\lceil (0.027_{11}) \rfloor_0 = [(0.027 / (0.027 + 0.1 + 1)) + (0.027 / (0.027 + 0.0588 + 0.03846))] = 0.241243744 / 2 \approx 0.12$

And so:

$T_{\gamma 2} [\lceil a_\gamma \rfloor_0]$=

0.121	0.329	0.3627
0.197	0.62	0.082
0.904	0.1196	0.158

Table8

And

$T_{\gamma 1} [\lceil a_\gamma \rfloor_0]$ =

0.201	0.521	0.425
0.6349	0.1566	0.222
0	0.644	0.194

Table9

Then :

$$\mu_{T1}(x) \cap \mu_{T2}(x) = \min_t\{ \mu_{T1}(x), \mu_{T2}(x) \}$$

$= \{ (1_{11}, 0.121), (3_{12}, 0.329), (2_{13}, 0.3627), (4_{21}, 0.197), (5_{22}, 0.1566), (1_{23}, 0.082), (7_{31}, 0), (3_{32}, 0.1966), (1_{32}, 0.158) \}$.

4.3 Definition

The membership function $_{DT}(x)$ of the union $D_T = A_T \cup B_T$ is point wise defined by

$$\mu_{DT}(x) = \max_t\{ \mu_{AT}(x), \mu_{BT}(x) \}$$

Such that: if $a_\alpha \in \mu_{AT}(x)$ and $b_\beta \in \mu_{BT}(x)$

$$Max_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha & \rfloor_t . \\ b_\beta & \rfloor_t . \end{cases}$$

if $\lceil \mathbf{a_\alpha} \rfloor_t = \lceil \mathbf{b_\beta} \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil \mathbf{a_\alpha} \rfloor_t \neq \lceil \mathbf{b_\beta} \rfloor_t$ the system is type-1 .

4.4 Example

From example above

$T\mu_{1(3,2)} [\lceil a\gamma \rfloor_0]$ =

0.027	0.0588	0.03846
0.1	0.2	0.027
1	0.0588	0.027

Table10

$T\mu_{2(3,2)} [\lceil a\gamma \rfloor_0]$ =

0.121	0.329	0.3627
0.197	0.62	0.082
0.904	0.1196	0.158

Table11

$\mu_{T1}(x) \cup \mu_{T2}(x) = \max_t\{ \mu_{T1}(x), \mu_{T2}(x) \}$

$= \{ (1_{11}, 0.201), (3_{12}, 0.521), (2_{13}, 0.425), (4_{21}, 0.6349), (5_{22}, 0.62), (1_{23}, 0.222), (7_{31}, 0.904), (3_{32}, 0.644), (1_{32}, 0.194) \}$.

4.5 Definition

The membership function of the complement of a fuzzy stacked set $T\mu$, $\mu^c_T(x)$ is defined by

$\mu^c_T(x) = 1 - \mu_T(x)$, $x \in T$

4.6 Example

Let : $T\mu_1 = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{32}, 0.2) \}$.

Then : $T^c\mu_1 = \{ (1_{11}, 0.8), (3_{12}, 0), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.8), (1_{23}, 0.8), (7_{31}, 1), (3_{32}, 0), (1_{32}, 0.8) \}$.

4.7 Additional operations on fuzzy stacked set :

It has already been mentioned that min and max are not the only operators that could have been chosen to model the intersection or union of fuzzy stacked sets respectively .

The question arises, why those and not others‘!

from a logical point of view, interpreting the intersection as “logical and," the union as “logical or," and the fuzzy stacked set Z as the statement “The element x belongs to set Z“ can be accepted as more or less true. lt is very instructive to follow their line of argument, which is an excellent example for an axiomatic justification of specific mathematical models. We shall therefore sketch their reasoning: Consider two statements, S and T, for which the truth values are μ_S and μ_T , respectively, μ_S , $\mu_T \in [0, 1]$.

The truth value of the “and” and "or" combination of these statements, μ(S and T) and t μ(S or T), both from the interval [0, 1] are interpreted as the values of the membership functions of the intersection and union, respectively , of S and T. We are now looking for two real-valued functions f and g such that

$$\mu_{S \text{ and } T} = f(\mu_S , \mu_T)$$

$$\mu_{S \text{ or } T} = g(\mu_S, \mu_T)$$

That the following restrictions are reasonably imposed on f and g:

- i- f and g are non decreasing and continuous in μ_S and μ_T .
- ii- f and g are symmetric, that is,
  $f(\mu_S, \mu_T) = f(\mu_T, \mu_S)$
- iii- $f(\mu_S, \mu_S)$ and $g(\mu_S, \mu_S)$ are strictly increasing in μ_S .
- iv- $f(\mu_S, \mu_T) \leq \min (\mu_S, \mu_T)$ and $g(\mu_S, \mu_T) \geq \max(\mu_S, \mu_T)$. That implies that accepting the truth of the statements “S and T" requires more , and accepting the truth of the statement “S or T" less than accepting S or T alone as true.
- v- $f(1, 1)= 1$ and $g(0, 0)= 0$.
- vi- Logically equivalent statements must have equal truth values and fuzzy sets with the same contents must have the same membership functions, that is,
  S_1 and (S_2 or S_3)
  is equivalent to (S_1 and S_2) or (S_1 and S_3)
  and therefore must be equally true.

Using the symbols ∧ for “and” (= intersection) and ∨ for “or” (= union), this amounts to the following 7 restrictions, to be imposed on the two commutative (see (ii)) and associative (see (vi)) binary compositions ∧ and ∨ on the closed interval [0, 1] which are mutually distributive (see (vi)) with respect to one another.

1- $\mu_S \wedge \mu_T = \mu_T \wedge \mu_S$
   $\mu_S \vee \mu_T = \mu_T \vee \mu_S$
2- $(\mu_S \wedge \mu_T) \wedge \mu_U = \mu_S \wedge (\mu_T \wedge \mu_U)$
   $(_S \vee \mu_T) \vee \mu_U = \mu_S \vee (\mu_T \vee \mu_U)$
3- $\mu_S \wedge (\mu_T \wedge \mu_U) = (\mu_S \wedge \mu_T) \vee (\mu_S \wedge \mu_U)$
   $\mu_S \vee (_T \wedge \mu_U) = (\mu_S \vee \mu_T) \wedge (\mu_S \vee \mu_U)$
4- $\mu_S \wedge \mu_T$ and $\mu_S \vee \mu_T$ are continuous and non decreasing in each component
5- $\mu_S \wedge \mu_T$ and $\mu_S \vee \mu_T$ are strictly increasing in μ_S (see (iii))
6- $\mu_S \wedge \mu_T \leq \min(\mu_S , \mu_T)$
   $\mu_S \vee \mu_T \geq \max(\mu_S , \mu_T)$ (see (iv))
7- $1 \wedge 1 = 1$
   $0 \vee 0 = 0$ (see (v))

$$\mu_{S \wedge T} = \min(\mu_S , \mu_T) \text{ and } \mu_{S \vee T} = \max(\mu_S , \mu_T)$$

For the complement it would be reasonable to assume that if statement
“S“ is true, its complement “non S" is false, or if $\mu_S = 1$ then $\mu_{nonS} = 0$ and vice versa . The function h (as complement in analogy to f and g for intersection and union) should also be continuous and monotonically decreasing and we would like the complement of the complement to be the
original statement (in order to be in line with traditional logic and set
theory).

4.8 Definition

The Cartesian product of fuzzy stacked sets is defined if $T_{\mu 1}$, $T_{\mu 2}$, … , $T_{\mu n}$ be fuzzy stacked sets in T_1 , T_2 , …, T_n . Then the Cartesian product of fuzzy stacked sets in the product space $T_1 \times T_2 \times \ldots \times T_n$, and the membership function if $\delta = T_{\mu 1} \times T_{\mu 2} \times \ldots \times T_{\mu n}$ is :

$$\mu_\delta(x) = \min_t[\mu T_{\mu i}(x_i) \mid x = (x_1 , x_2 , \ldots , x_n) , x_i \in T_i]$$

4.9 Example

In example above :

$T_{\mu 1} = \{ (1_{11} ,0.2) , (3_{12} , 1) , (2_{13} , 0.5) , (4_{21} , 0.5) , (5_{22} , 0.2) , (1_{23} , 0.2) , (7_{31} , 0) , (3_{32} , 1) , (1_{33} , 0.2) \}$.

$T_{\mu 2} = \{ (1_{11} ,0.027) , (3_{12} , 0.0588) , (2_{13} , 0.03846) , (4_{21} , 0.1) , (5_{22} , 0.2) , (1_{23} , 0.027) , (7_{31} ,1) , (3_{32} , 0.0588) , (1_{33} , 0.027) \}$.

Then :

$T_{\mu 1} \times T_{\mu 2} = \{ [(1_{11} ;1_{11}) ,0.027] , ([(1_{11} ;3_{12}), 0.0588] , [(1_{11} ;2_{13}) , 0.03846] , [(1_{11} ;4_{21}) , 0.1] , [(1_{11} ;5_{22}) , 0.2] , [(1_{11} ;1_{23}) , 0.027] , \underline{[(1_{11} ;7_{31}) ,0.2}] ,[(1_{11} ;3_{32}) , 0.0588] , [(1_{11} ;1_{33}) , 0.027], [(3_{12} ;1_{11}) ,0.027] , ([(3_{12} ;3_{12}), 0.0588] , [(3_{12} ;2_{13}) , 0.03846] , [(3_{12} ;4_{21}) , 0.1] , [(3_{12} ;5_{22}) , 0.2] , [(3_{12} ;1_{23}) , 0.027] , [(3_{12} ;7_{31}) ,1] ,[(3_{12} ;3_{32}) , 0.0588] , [(3_{12} ;1_{33}) , 0.027], [(2_{13} ;1_{11}) ,0.027] , ([(2_{13} ;3_{12}), 0.0588] , [(2_{13} ;2_{13}) , 0.03846] , [(2_{13} ;4_{21}) , 0.1] , [(2_{13} ;5_{22}) , 0.2] , [(2_{13} ;1_{23}) , 0.027] , \underline{[(2_{13} ;7_{31}) ,0.5]} ,[(2_{13} ;3_{32}) , 0.0588] , [(2_{13} ;1_{33}) , 0.027], [(4_{21};1_{11}) ,0.027] , ([(4_{21} ;3_{12}), 0.0588] , [(4_{21} ;2_{13}) , 0.03846] , [(4_{21} ;4_{21}) , 0.1] , [(4_{21} ;5_{22}) , 0.2] , [(4_{21};1_{23}) , 0.027] , \underline{[(4_{21} ;7_{31}) ,0.5]} ,[(4_{21};3_{32}) , 0.0588] , [(4_{21};1_{33}) , 0.027], [(5_{22} ;1_{11}) ,0.027] , ([(5_{22} ;3_{12}), 0.0588] , [(5_{22} ;2_{13}) , 0.03846] , [(5_{22};4_{21}) , 0.1] , [(5_{22} ;5_{22}) , 0.2] , [(5_{22};1_{23}) , 0.027] , \underline{[(5_{22} ;7_{31}) ,0.2}$

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$]$,$[(5_{22};3_{32}), 0.0588]$, $[(5_{22};1_{33}), 0.027]$, $[(1_{23};1_{11}), 0.027]$, $([(1_{23};3_{12}), 0.0588]$, $[(1_{23};2_{13}), 0.03846]$, $[(1_{23};4_{21}), 0.1]$, $[(1_{23};5_{22}), 0.2]$, $[(1_{23};1_{23}), 0.027]$, $[(1_{23};7_{31}), 0.2]$,$[(1_{23};3_{32}), 0.0588]$, $[(1_{23};1_{33}), 0.027]$, $[(7_{31};1_{11}), 0]$, $([(7_{31};3_{12}), 0]$, $[(7_{31};2_{13}), 0]$, $[(7_{31};4_{21}), 0]$, $[(7_{31};5_{22}), 0]$, $[(7_{31};1_{23}), 0]$, $[(7_{31};7_{31}), 0]$,$[(7_{31};3_{32}), 0]$, $[(7_{31};1_{33}), 0]$, $[(3_{32};1_{11}), 0.027]$, $([(3_{32};3_{12}), 0.0588]$, $[(3_{32};2_{13}), 0.03846]$, $[(3_{32};4_{21}), 0.1]$, $[(3_{32};5_{22}), 0.2]$, $[(3_{32};1_{23}), 0.027]$, $[(3_{32};7_{31}), 1]$,$[(3_{32};3_{32}), 0.0588]$, $[(3_{32};1_{33}), 0.027]$, $[(1_{33};1_{11}), 0.027]$, $([(1_{33};3_{12}), 0.0588]$, $[(1_{33};2_{13}), 0.03846]$, $[(1_{33};4_{21}), 0.1]$, $[(1_{33};5_{22}), 0.2]$, $[(1_{33};1_{23}), 0.027]$, $[(1_{33};7_{31}), 0.2]$,$[(1_{33};3_{32}), 0.0588]$, $[(1_{33};1_{33}), 0.027]\}$.

4.10Definition

The nth power of a fuzzy stacked set T is a fuzzy stacked set with the membership function

$$_{(T\mu)}{}^{n}(x) = [\ \mu T_i(x)\ ]^{n}\ ,\ x \in T_{\mu}.$$

4.11Example

In example above :

$T_{\mu 1} = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.2) \}$.

Then :

$[T_{\mu 1}]^2 = \{ (1_{11}, 0.04), (3_{12}, 1), (2_{13}, 0.25), (4_{21}, 0.25), (5_{22}, 0.04), (1_{23}, 0.04), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.04) \}$.

4.12Definition

The algebraic sum (probabilistic sum) $T_{\mu a} + T_{\mu b}$ is defined as :

$T_{\mu a} + T_{\mu b} = \{ (x, \mu_{(T\mu a + T\mu b)}(x)) \mid x \in T_{\mu} \}$ .Where :

$\mu_{(T\mu a+T\mu b)}(x) = \mu_{(T\mu a)}(x) + \mu_{(T\mu b)}(x) - \mu_{(T\mu a)}(x) \cdot \mu_{(T\mu b)}(x)$

4.13Example

In example above :

$T_{\mu 1} = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.2) \}$.

$T_{\mu 2} = \{ (1_{11}, 0.027), (3_{12}, 0.0588), (2_{13}, 0.03846), (4_{21}, 0.1), (5_{22}, 0.2), (1_{23}, 0.027), (7_{31}, 1), (3_{32}, 0.0588), (1_{33}, 0.027) \}$.

Then :

$T_{\mu 1} + T_{\mu 2} = \{ (1_{11}, 0.2216), (3_{12}, 1), (2_{13}, 0.51923), (4_{21}, 0.55), (5_{22}, 0.36), (1_{23}, 0.2216), (7_{31}, 1), (3_{32}, 1), (1_{33}, 0.2216) \}$.

4.14Definition

The bounded sum $T_{\mu a} \oplus T_{\mu b}$ is defined as

$$T_{\mu a} \oplus T_{\mu b} = \{ (x, \mu_{(T\mu a \oplus T\mu b)}(x)) \mid x \in T_{\mu} \}$$

Where

$$\mu_{(T\mu a \oplus T\mu b)}(x) = \min\{ 1, \mu_{(T\mu a)}(x) + \mu_{(T\mu b)}(x) \}$$

4.15Example

In example above :

$T_{\mu 1} = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.2) \}$.

$T_{\mu 2} = \{ (1_{11}, 0.027), (3_{12}, 0.0588), (2_{13}, 0.03846), (4_{21}, 0.1), (5_{22}, 0.2), (1_{23}, 0.027), (7_{31}, 1), (3_{32}, 0.0588), (1_{33}, 0.027) \}$.

Then :

$T_{\mu 1} \oplus T_{\mu 2} = \{ (1_{11}, 0.227), (3_{12}, 1), (2_{13}, 0.53846), (4_{21}, 0.6), (5_{22}, 0.4), (1_{23}, 0.227), (7_{31}, 1), (3_{32}, 1), (1_{33}, 0.227) \}$.

4.16Definition

The bounded difference $T_{\mu a} \ominus T_{\mu b}$ is defined as

$T_{\mu a} \ominus T_{\mu b} = \{ (x, \mu_{(T\mu a \ominus T\mu b)}(x)) \mid x \in T_{\mu} \}$

Where

$\mu_{(T\mu a \ominus T\mu b)}(x) = \max\{0, \mu_{(T\mu a)}(x) + \mu_{(T\mu b)}(x) - 1\}$

4.17Example

In example above :

$T_{\mu 1} = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.2) \}$.

$T_{\mu 2} = \{ (1_{11}, 0.027), (3_{12}, 0.0588), (2_{13}, 0.03846), (4_{21}, 0.1), (5_{22}, 0.2), (1_{23}, 0.027), (7_{31}, 1), (3_{32}, 0.0588), (1_{33}, 0.027) \}$.

Then :

$T_{\mu 1} \ominus T_{\mu 2} = \{ (1_{11}, 0), (3_{12}, 0.0588), (2_{13}, 0), (4_{21}, 0), (5_{22}, 0), (1_{23}, 0.227), (7_{31}, 0), (3_{32}, 0.0588), (1_{33}, 0) \}$

$= \{ (3_{12}, 0.0588), (3_{32}, 0.0588) \}$.

4.18Definition

The algebraic product of tow fuzzy stacked sets $T_{\mu a} \bullet T_{\mu b}$ is defined as

$T_{\mu a} \bullet T_{\mu b} = \{ (x, \mu_{(T\mu a)}(x) \bullet \mu_{(T\mu b)}(x)) \mid x \in T_{\mu} \}$

4.19Example

In example above :

$T_{\mu 1} = \{ (1_{11}, 0.2), (3_{12}, 1), (2_{13}, 0.5), (4_{21}, 0.5), (5_{22}, 0.2), (1_{23}, 0.2), (7_{31}, 0), (3_{32}, 1), (1_{33}, 0.2) \}$.

$T_{\mu 2} = \{ (1_{11}, 0.027), (3_{12}, 0.0588), (2_{13}, 0.03846), (4_{21}, 0.1), (5_{22}, 0.2), (1_{23}, 0.027), (7_{31}, 1), (3_{32}, 0.0588), (1_{33}, 0.027) \}$.

Then :

$T_{\mu 1} \bullet T_{\mu 2} = \{ (1_{11}, 0.0054), (3_{12}, 0.0588), (2_{13}, 0.01923), (4_{21}, 0.05), (5_{22}, 0.04), (1_{23}, 0.0054), (7_{31}, 0), (3_{32}, 0.0588), (1_{33}, 0.0054) \}$.

V. FUZZY STACKED SEMIGROUPS

5.1 Definition

Let $T_{\alpha,n}$ be staked set ,and $(T_{\alpha,n}, \tau)$ be a stacked semigroup , then a map $\mu: T_{\alpha,n} \to [0, 1]$ is called a fuzzy stacked semigroup if $\mu(x\ \tau\ y) = \min_t \{\mu(x), \mu(y)\}$ for all $x, y \in T_{\alpha,n}$.

5.2 Example

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From example above , suppose that the distribution ofproducts from alldistribution centers,to thepoint of saleatlesscostand taking into accountthatthebiggestcostofdistributionis7units .
And :
$\mu_T(x) = 1 - \quad (1 + (x - 7)^2)^{-1} \;:\; 0 \le x \le 7$.
when

$T_{3,2} = \{ 1_{11}, 3_{12}, 2_{13}, 4_{21}, 5_{22}, 1_{23}, 7_{31}, 3_{32}, 1_{32} \}$.

or $T_{3,2}$ =

1	3	2
4	5	1
7	3	1

Table12

$lT_{3,2}(1_{11}) = 1$, $lT_{3,2}(3_{12}) = 6$, $lT_{3,2}(4_{21}) = 5$,
$lT_{3,2}(5_{22}) = 8$, $lT_{3,2}(7_{31}) = 9$, $lT_{3,2}(2_{13}) = 7$.

$lT_{3,2}$ =

1	6	7
5	8	3
9	4	2

Table13
and

$\min_t(4_{21}, 3_{12}) = 4_{21}$, $\min_t(1_{11}, 7_{31}) = 1_{11}$, $\min_t(5_{22}, 2_{13}) = 5_{22}$

from ,
$\mu_T(x) = 1 - \quad (1 + (x - 7)^2)^{-1} \;:\; 0 \le x \le 7$
$T_\mu = \{ (1_{11}, 0.97), (3_{12}, 0.94), (2_{13}, 0.96), (4_{21}, 0.9), (5_{22}, 0.8), (1_{23}, 0.97), (7_{31}, 0), (3_{32}, 0.94), (1_{32}, 0.97) \}$.

Or

T_μ=

0.97	0.94	0.96
0.9	0.8	0.97
0	0.94	0.97

Table14

$$\lceil a_\gamma \rfloor_t = [\frac{|a_{\gamma 1} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \frac{|a_{\gamma 2} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \ldots + \frac{|a_{\gamma \alpha} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|}]/\alpha$$

$$= [\sum_{i=1}^{\alpha} \frac{|a_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|}]/\alpha .$$

Then :

$\lceil \mathbf{0.97}_{11} \rfloor_0 = [(\mathbf{0.97} / (\mathbf{0.97} + \mathbf{0.9} + \mathbf{0})) + (\mathbf{0.97} / (\mathbf{0.97} + \mathbf{0.94} + \mathbf{0.96}))] \approx 0.428$

and so :

$T_{\mu(3,2)}[\lceil a_\gamma \rfloor_0]$ =

0.428	0.339	0.3329
0.409	0.299	0.34889
0	0.4214	0.4211

Table15

and so :

$lT_{\mu(3,2)}[\lceil a_\gamma \rfloor_0]$ =

1	6	7
4	8	5
9	2	3

Table16

Then :
$lT_{\mu(3,2)}[\lceil a_\gamma \rfloor_0] = lT_{3,2}$
When
$\mu_T(4_{21} \tau\ 3_{12}) = \mu_T[\min_t(4_{21}, 3_{12})] = \mu_T(4_{21}) = 0.9$
And, $\min_t\{\mu_T(4_{21}), \mu_T(3_{12})\} = \min_t\{0.9, 0.94\} = 0.9$
Then: $\mu_T(4_{21} \tau\ 3_{12}) = \min_t\{\mu_T(4_{21}), \mu_T(3_{12})\}$
And $\forall$ x , y $\in T_{3,2}$, $\mu(x \tau y) = \min_t\{\mu(x), \mu(y)\}$, then $(T_{3.2}, \tau)$ is fuzzy stacked semigroup .

5.3 Theorem

Let $(T_{\alpha,n}, \tau)$ be a stacked semigroup , then a map $\mu: T_{\alpha,n} \to [0, 1]$ is called a fuzzy stacked semigroup if $lT_\alpha = l(T_\mu)$ or $lT = l(1\text{-}T_\mu)$.

Proof:

Let a , b $\in T_{\alpha,n}$, where $(T_{\alpha,n}, \tau)$ be a stacked semigroup , and $\mu: T_{\alpha,n} \to [0, 1]$
Then
$\min_t(a, b) = a$ if $\lceil a \rfloor_t < \lceil b \rfloor_t$ or b if $\lceil a \rfloor_t < \lceil b \rfloor_t$
So,
$\min_t(a, b)$ corresponds $\min[\lceil a \rfloor, \lceil b \rfloor]$
then there is corresponds between $\min_t[\mu_T(a), \mu_T(b)]$ and $\min[\lceil a \rfloor, \lceil b \rfloor]$ from:
$\min_t[a_\mu, b_\mu] = a_\mu$ if $\lceil a_\mu \rfloor_t < \lceil b_\mu \rfloor_t$ or b_μ if $\lceil a_\mu \rfloor_t < \lceil b_\mu \rfloor_t$.

When $|T_{\alpha,n}| = |T_\mu| \Longrightarrow |lT_{\alpha,n}| = |l(T_\mu)| = r$ (from the corresponds)
so $lT_{\alpha,n} = \{1, 2, \ldots, r\}$ and $lT_\mu = \{1, 2, \ldots, r\} \Longrightarrow lT_{\alpha,n} = l(T_\mu)$.
But sometimes $\mu_T(x)$ is define a $\max_t$ operations so it is corresponds $l(1\text{-}T_\mu)$ then $lT_{\alpha,n} = l(1\text{-}T_\mu)$ or $lT_{\alpha,n} = l(T_\mu)$.

Let $T_{\alpha,n}$ be a stacked semigroup. A function f from $T_{\alpha,n}$ to the unit interval [0, 1] is a fuzzy stacked subset of T_α . A stacked semigroup T_α itself is a fuzzy stacked subset of T_α such that $T_\alpha(x) = 1$ for all $x \in T_\alpha$ denoted by T_α . Let μ and δ be any two fuzzy stacked subsets of T . Then the inclusion relation $\mu \subseteq \delta$ is defined by $\mu(x) \le \delta(x)$ for all x ∈ T . (1 − μ) is a fuzzy stacked subset of T_α defined for all $x \in T_\alpha$.

5.4 Definition

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$(1 - \mu)(x) = 1 - \mu(x)$, $\mu \cap \delta$ and $\mu \cup \delta$ are fuzzy sacked subsets of T defined by :
$(\mu \cap \delta)(x) = \min_t\{\mu(x), \delta(x)\}$, $(\mu \cup \delta)(x) = \max_t\{\mu(x), \delta(x)\}$ for all $x \in T_\alpha$. The product $\mu \bullet \delta$ is defined as follows:

$$(\mu \bullet \delta)(x) = \begin{cases} \sup\{\min[\{\mu(y), \delta(z)\}]\} \\ 0 \text{ if x is not expressible as x=yz} \end{cases}$$

'●' is an associative operation .

5.5 Example

In example above :

$T_{\mu 1}$ = { (1_{11} ,0.2) , (3_{12} , 1) , (2_{13} , 0.5) , (4_{21} , 0.5) , (5_{22} , 0.2) , (1_{23} , 0.2) , (7_{31} , 0) , (3_{32} , 1) , (1_{33} , 0.2) } .

$T_{\mu 2}$ = { (1_{11} ,0.027) , (3_{12} , 0.0588) , (2_{13} , 0.03846) , (4_{21} , 0.1) , (5_{22} , 0.2) , (1_{23} , 0.027) , (7_{31} ,1) , (3_{32} , 0.0588) , (1_{33} , 0.027) } .

$\lceil 1_{11} \rfloor_0$ =[(1/ (1+3+2))+(1/(1+4+7))]/ 2 = 0.125 .
$\lceil 3_{12} \rfloor_0$ = 0.38636 .
$\lceil 2_{13} \rfloor_0$= 0.41667 .
$\lceil 4_{21} \rfloor_0$ = 0.36667 .
$\lceil 5_{22} \rfloor_0$ = 0.47727 .
$\lceil 1_{23} \rfloor_0$ = 0.175 .
$\lceil 7_{31} \rfloor_0$ = 0.60985 .
$\lceil 3_{32} \rfloor_0$ = 0.27273 .
$\lceil 1_{33} \rfloor_0$ = 0.17045 .

$1_{11}\ T_\alpha = 1_{11} T_\alpha = \text{Min}_0[1_{11}, T_\alpha] = 1_{11}$, so

- $1_{11}1_{33} = 1_{11}$, $1_{11}1_{23} = 1_{11}$, $1_{11}3_{32} = 1_{11}$, $1_{11}4_{21} = 1_{11}$, $1_{11}3_{12} = 1_{11}$, $1_{11}2_{13} = 1_{11}$, $1_{11}5_{22} = 1_{11}$, $1_{11}7_{31} = 1_{11}$.
- $1_{23}1_{23} = 1_{11}$, $1_{23}3_{32} = 1_{23}$, $1_{23}4_{21} = 1_{23}$, $1_{23}3_{12} = 1_{23}$, $1_{23}2_{13} = 1_{23}$, $1_{23}5_{22} = 1_{23}$, $1_{23}7_{31} = 1_{23}$.
- $3_{32}3_{32} = 1_{23}$, $3_{32}4_{21} = 3_{32}$, $3_{32}3_{12} = 3_{32}$, $3_{32}2_{13} = 3_{32}$, $3_{32}5_{22} = 3_{32}$, $3_{32}7_{31} = 3_{32}$.
- $4_{21}4_{21} = 4_{21}$, $4_{21}3_{12} = 4_{21}$, $4_{21}2_{13} = 4_{21}$, $4_{21}5_{22} = 4_{21}$, $4_{21}7_{31} = 4_{21}$.
- $3_{12}3_{12} = 3_{12}$, $3_{12}2_{13} = 3_{12}$, $3_{12}5_{22} = 3_{12}$, $3_{12}7_{31} = 3_{12}$.
- $2_{13}2_{13} = 2_{13}$, $2_{13}5_{22} = 2_{13}$, $2_{13}7_{31} = 2_{13}$.
- $5_{22}5_{22} = 5_{22}$, $5_{22}7_{31} = 5_{22}$.
- $7_{31}7_{31} = 7_{31}$.

$$(\mu_1 \bullet \mu_2)(x) = \begin{cases} \sup\{\min[\{\mu_1(y), \mu_2(z)\}]\} \\ 0 \text{ if x is not expressible as x=yz} \end{cases}$$

Then :
$(\mu_1 \bullet \mu_2)(1_{11}) = \sup\{ \min[\mu_1(1_{11}), \mu_2(1_{33})], \min[\mu_1(1_{11}), \mu_2(1_{23})], \min[\mu_1(1_{11}), \mu_2(3_{32})], \min[\mu_1(1_{11}), \mu_2(4_{21})], \min[\mu_1(1_{11}), \mu_2(3_{12})], \min[\mu_1(1_{11}), \mu_2(2_{13})], \min[\mu_1(1_{11}), \mu_2(5_{22})], \min[\mu_1(1_{11}), \mu_2(7_{31})]\}$
= *sup*{ 0.027 , 0.0588 , 0.03846 , 0.1 , 0.2 } = 0.2 .

A fuzzy stacked subset f of T_α is called a fuzzy stacked subsemigroup of T_α if :

$$f(xy) \geq f(x) \wedge f(y) , \forall x ,y \in T_\alpha.$$

5.7 Definition

for all a, b ∈ T_α , and is called a fuzzy stacked left (right) ideal of S if :

$$f(ab) > f(b) , \quad (f(ab) > f(a))$$

5.8 Definition

- Let T_α be a stacked semigroup. Let A and B be subsets of T_α . Then multiplication of A and B is defined as follows:
  $AB = \{ ab \in T_\alpha \mid a \in A \text{ and } b \in B\}$
- A nonempty subset A of T_α is called a stacked subsemigroup of T_α if $AA \subseteq A$.
- A nonempty stacked subset A of T_α is called a left (right) stacked ideal of T_α if $T_\alpha A \subseteq A$ ($A T_\alpha \subset A$). Further, A is called a two-sided stacked ideal of T_α if it is both a left and a right stacked ideal of T_α.
- A nonempty stacked subset A of T_α is called an interior stacked ideal of T_α if $T_\alpha A T_\alpha \subseteq A$, and a quasi-stacked ideal of T_α if $A T_\alpha \cap T_\alpha A \subseteq A$. A stacked subsemigroup A of T_α is called a stacked bi-ideal of T_α if $A T_\alpha A \subseteq A$. A nonempty subset A is called a generalized stacked bi-ideal of T_α if $A T_\alpha A \subset A$.
- A semigroup T_α is called regular if for each element a of T_α, there exists an element $x \in T_\alpha$ such that a = axa .

5.9 Definition

We denote by $L_t[a]$ ($R_t[a]$, $J_t[a]$, $I_t[a]$, $Q_t[a]$, $B_t[a]$) the principal (left , right, two-sided, interior, quasi- and bi-) ideal of a stacked semigroup S generated by the element $a \in T_\alpha$, that is,

1- $L_t[a] = \{a\} \cup T_\alpha a$,

2- $R_t[a] = \{a\} \cup a T_\alpha$,

3- $J_t[a] = \{a\} \cup T_\alpha a \cup a T_\alpha \cup T_\alpha a T_\alpha$,

4- $I_t[a] = \{a\} \cup \{a^2\} \cup T_\alpha a T_\alpha$,

5- $Q_t[a] = \{a\} \cup (a T_\alpha \cap T_\alpha a)$,

6- $B_t[a] = \{a\} \cup \{a^2\} \cup a T_\alpha a$.

5.10 Definition

A fuzzy stacked subset f of a stacked semigroup T_α is called a fuzzy stacked bi-ideal of T_α if :

$$F(xyz) \geq f(x) \wedge f(z) \quad \text{, for all } x, y, z \text{ of } T_\alpha$$

5.11 Definition

A fuzzy stacked subset f of T_α is called a fuzzy stacked interior ideal of T_α if $f(xay) \geq f(a)$ for all x, a and y of T_α.

5.12 Definition

A fuzzy stacked subset f of a stacked semigroup T_α is called a fuzzy stacked quasi-ideal of T_α if

$$(f \circ T_\alpha) \cap (T_\alpha \circ f) \subseteq f .$$

5.13 Definition

A nonempty stacked subset A of a stacked semigroup T_α is called a generalized stacked bi-ideal of T_α if $A\, T_\alpha\, A \subseteq A$. A fuzzy stacked subset f of T_α is called a fuzzy stacked generalized bi-ideal of T_α if

$$f(xyz) \geq f(x) \wedge f(z)$$

for all x, y and z of T_α .

It is clear that every fuzzy stacked bi-ideal of a stacked semigroup T_α is a fuzzy stacked generalized bi-ideal of T_α, but the converse of this statement does not hold in general.

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