Modeling and Optimization of Odometry Error in a Two Wheeled Differential Drive Robot

T.Mathavaraj Ravikumar*, R.Saravanan and N.Nirmal***

* Associate Professor, Department of Mechanical Engineering, United Institute of Technology, Coimbatore-641020, India
** Professor, Department of Mechanical Engineering, Sri Krishna College of Technology, Coimbatore-641105, India
*Assistant Professor, Department of Mechanical Engineering, United Institute of Technology, Coimbatore-641020, India

***Abstract*:** Positioning of mobile robots basically calculated using odometry information. Odometry from the wheel's encoder is mostly used for simple and inexpensive implementation for determining the relative localization of a mobile robot. This paper deals with the estimation of better relative localization of a two wheeled differential drive robot by means of odometry by considering the influence of parameters namely payload, speed, diameter of wheel and thickness of wheel. Experiments have been conducted based on central composite rotatable design matrix. A mathematical model has been developed for the robot using Response Surface Methodology (RSM) with the help of MINITAB software. An optimum relative positioning was obtained by using Excel Solver.

***Index Terms*:** Mobile Robot, Odometry Error, Relative Localization, Response Surface Methodology, Excel solver

1. INTRODUCTION

Positioning in a mobile robot is an important task for many applications while navigating the robots. The objective of localization is to determine the position of a mobile robot in its environment, given a map of the environment and local sensorial data. Pose estimation of robot has been known as one of the most fundamental problems in mobile robotics (Julian Lategahn et al., 2010; Byrne et al., 1992). Absolute and relative positioning are widely employed methods in mobile robots (Hollingum, 1991; Chenavier and Crowley, 1992; Evans, 1994). Odometry is the mostly used navigation method for relative positioning of mobile robot because it provides good accuracy, is inexpensive, and allows very high sampling rates (Borenstein, 1998). Odometry is a useful method for predicting the position of a robot after it has moved. The prediction is accomplished by counting the number of wheel revolutions that each wheel rotated.

Absolute positioning methods usually rely on navigation beacons, active or passive landmarks, map matching and satellite based navigation signals but none of these existing methods are well designed. Another approach to the pose determination of mobile robots is based on inertial navigation with gyros and accelerometers. The experimental results acquired by the researchers (Borenstein and Feng, 1996; Barshan and Durrant-Whyte, 1994) indicate that this approach is not advantageous. Gyros can be more accurate and costly but they provide information only on the rate of rotation of a robot (Komoriya et al., 1994). This kind of problem does not exist with electronic compasses that measure the orientation of the robot relative to magnetic field of earth. But, electronic compasses are not recommended for indoor applications due to the large distortions of the magnetic field near power lines or steel structures (Byrne et al., 1992).

Most of the researchers have concentrated upon the accurate calibration of odometry (Krantz, 1996; Kooktae Lee et al., 2011), measuring errors in odometry and development of models for minimizing the odometry errors (Lauro Ojeda and Borenstein, 2004; Korayem et al., 2006) in wheeled robots. In addition, researchers have focused on the optimum path planning in mobile robots (ShenZhi-cua et al.,2006; Gonzalez-Gomez et al., 2011) and stability analyses in mobile robots (Eghtesad and Necsulescu, 2004; Chaoli Wanget al.,2010; Jianxian Cai and Xiaogang Ruan, 2011;YaoCai et al., 2012).

The motion of the robot in any particular terrain is affected by many factors like geometry and type of locomotion system (wheeled, tracked, hybrid, legged, jumping), properties of effectors (e.g. tyre type for wheeled robots), mass properties of a robot and constraints resulting from characteristics of drives (Maciej Trojnacki, 2012).

From the literature, it is noticed that the major research have been focused on the development of odometry error models, stability analyses and path planning for mobile robots. Very few researchers have considered the effect of parameters like payload, velocity and geometry of wheels for mobile robot in the determination of relative localization using odometry. In this paper, a mathematical model for odometry error of a two wheeled differential drive robot has been developed using RSM and an optimum condition was obtained through XL Solver.

2. TWO WHEELED DIFFERENTIAL DRIVE ROBOT

A two wheeled differential drive robot is a mobile robot whose movement is based on two separately driven wheels placed on either side of the robot body. It can thus change its direction by varying the relative rate of rotation of its wheels and hence does not require an additional steering motion. The robot "VENTRA" shown in Figure 1 was employed for the conduct of experiments in this study. The robot was driven in an indoor environment for a distance of 2 m in a straight line path. An evenly paved cement concrete floor was used as terrain which normally minimizes the chance of non-systematic errors such as wheel slippage, interaction with external bodies and travel over unexpected objects on the floor.

The self weight of the robot is 1.2 kg and the distance between two wheels (W) is 120 mm. The maximum speed is upto 200 mm/s. Two encoder wheels with encoders are used to calculate the linear displacement of each wheel. The new orientation of the robot can be estimated from difference in encoder counts, diameter of the wheels and distance between the wheels.

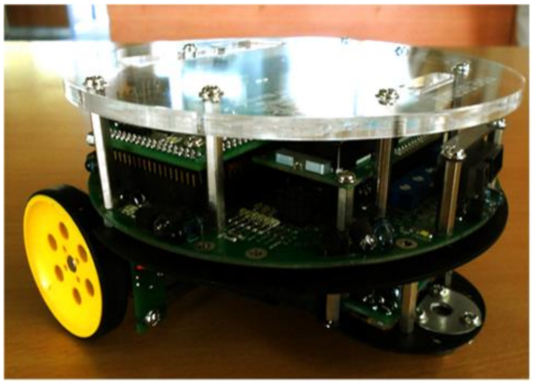


Figure 1: VENTRA Robot

A. *Calculation of odometry error*

Odometry is a measuring method of wheel rotation as a function of time. If the two wheels of the robot are joined to a common axle, orientation of the centre of the axle relative to the previous orientation can be determined from odometry measurements on both wheels. In actual practice, optical encoders mounted on both wheels feed discretised wheel increment information to the controller, which in turn used to calculate the robot's state using geometric equations.

The wheel base (W) of the robot is the space between the contact points of two rear wheels. The center of the robot with respect to odometry is the midpoint between these two contacts. To calculate the variation in position and orientation of the robot with respect to starting point (P) across a given span of time, linear distance D_R and D_L of each wheel traveled (computed from number of ticks of the encoders and diameter of the wheels) and wheel base (W) are substituted in the following equation 1. The new orientation (R) in radians shown in Figure 2 is calculated by

$$R = P + (D_R - D_L) / W \qquad (1)$$

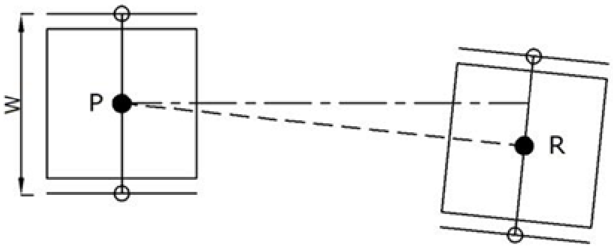


Figure 2: Orientation of robot

3. RESPONSE SURFACE METHODOLOGY

Response surface methodology (RSM) is one of the useful methods of analyzing the result of a factorial experiment. Odometry error (Oe) can be treated as output response and expressed as a function of parameters namely payload (L), speed (S), diameter of wheel (D) and thickness of wheel (T) as indicated in equation (2).

$$\text{Odometry error (Oe)} = \phi(L_{iu}, S_{iu}, D_{iu}, T_{iu}) + e_u \qquad (2)$$

where, Φ = response surface, e_u = residual, u = number of observations in the factorial experiment and iu represents level of the i^{th} factor in the u^{th} observation. When the mathematical form of Φ is unknown, it can be approximated by polynomials satisfactorily within the experimental region in terms of parameter variables. The ranges of all the parameters were fixed by conducting trial runs. This was performed by varying one of the parameters while keeping the rest of them as constant values. The upper limit of a given parameter was coded as (+2) and the lower limit was coded as (-2). The coded values for intermediate values were calculated using the equation (3).

$$X_i = \frac{2(2X - (X_{max} + X_{min}))}{(X_{max} - X_{min})} \qquad (3)$$

where

X_i : required coded value of a variable X

X : any value of the variable from X_{min} to X_{max}

X_{min} : lower limit of the variable

X_{max} : upper limit of the variable

The intermediate values were coded as -1, 0, and 1. The parameters with their limits and notations are given in Table 1. The design matrix chosen to conduct the experiments was a five level, four factor central composite rotatable design consisting of 31 sets of coded conditions and comprising a half replication $2^4 = 16$ factorial design plus 8 star points and 7 centre points as given in Table 2. All parameters at the intermediate level (0) constitute the centre points while the combination of each parameter at either its lower level (-2) or its higher level (+2) with the other two parameters at the intermediate level constitute the star points. Thus the 31 experimental runs allow the estimation of linear, quadratic, and two way interactive effects of the parameters on odometry error (Montgomery, 2000).

Table 1: Parameters and their levels

Parameters & Notations	**Units**	**Levels**				
		-2	**-1**	**0**	**1**	**2**
Payload (L)	kg	0.2	0.4	0.6	0.8	1
Speed (S)	mm/sec	100	120	140	160	180
Diameter of wheel (D)	mm	40	50	60	70	80
Thickness of wheel (T)	mm	5	10	15	20	25

Table 2: Experimental design – Central composite rotatable design matrix

Exp.No.	**Control parameters**				**Odometry error (rad)**		**% error**
	L	**S**	**D**	**T**	**Measured**	**Predicted**	
1	-1	-1	-1	-1	0.03565	0.03590	-0.69
2	1	-1	-1	-1	0.01785	0.01756	1.64
3	-1	1	-1	-1	0.03425	0.03399	0.75
4	1	1	-1	-1	0.01485	0.01465	1.34
5	-1	-1	1	-1	0.03205	0.03106	3.08
6	1	-1	1	-1	0.01765	0.01722	2.43
7	-1	1	1	-1	0.03425	0.03405	0.59
8	1	1	1	-1	0.01895	0.01921	-1.35
9	-1	-1	-1	1	0.03235	0.03174	1.89
10	1	-1	-1	1	0.02185	0.02215	-1.36
11	-1	1	-1	1	0.02955	0.03009	-1.81
12	1	1	-1	1	0.01885	0.01949	-3.41
13	-1	-1	1	1	0.02625	0.02655	-1.15
14	1	-1	1	1	0.02155	0.02146	0.42
15	-1	1	1	1	0.02985	0.02979	0.21
16	1	1	1	1	0.02385	0.02370	0.64
17	-2	0	0	0	0.03755	0.03794	-1.04

18	2	0	0	0	0.01365	0.01351	1.03
19	0	-2	0	0	0.02485	0.02551	-2.64
20	0	2	0	0	0.02625	0.02584	1.57
21	0	0	-2	0	0.03025	0.02994	1.02
22	0	0	2	0	0.02875	0.02931	-1.94
23	0	0	0	-2	0.02015	0.02096	-4.01
24	0	0	0	2	0.02185	0.02129	2.56
25	0	0	0	0	0.02085	0.02092	-0.34
26	0	0	0	0	0.02125	0.02092	1.55
27	0	0	0	0	0.02065	0.02092	-1.31
28	0	0	0	0	0.02185	0.02092	4.25
29	0	0	0	0	0.02015	0.02092	-3.83
30	0	0	0	0	0.02085	0.02092	-0.34
31	0	0	0	0	0.02085	0.02092	-0.34

4. EXCEL (XL) SOLVER

Solver is a powerful optimization tool, bundled with Microsoft Excel and is widely used for optimizing nonlinear engineering models. Optimization in Microsoft Excel begins with an ordinary spreadsheet. The solver finds a maximum or minimum or specified value of a target cell by varying the values in one or several changing cells. It accomplishes this by means of an iterative process, beginning with trial values of the coefficients. The value of the each coefficient is changed by a suitable increment, the new value of the function is calculated and the change in the value of the function is used to calculate improved values for each of the coefficients. The process is repeated until the desired result is obtained. The solver uses gradient methods to find the optimum set of coefficients.

5. RESULTS AND DISCUSSION

A. Development of mathematical model

The general form of a quadratic polynomial which gives the relation between response surface ‘Y’ and the process variable ‘X’ is given in equation (4).

$$Y = a_0 + \sum_{i=1}^{4} a_i X_i + \sum_{i=1}^{4} a_{ii} X_i^2 + \sum_{i<j}^{4} a_{ij} X_i X_j \qquad (4)$$

where a_o = constant, a_i = linear term coefficient, a_{ii}= quadratic term coefficient and a_{ij} = interaction term coefficient. The values of the coefficients of the polynomials were calculated using the multiple regression method. A statistical analysis software MINITAB was used to calculate the values of these coefficients. The second order mathematical model was developed for Odometry error (Oe) as given in equation (5).

$$Oe = 0.020921 - 0.006108L + 0.000083S - 0.0000158D + 0.000083T + 0.001201L^2 + 0.001188S^2 + 0.002176D^2 + 0.000051T^2 - 0.00025LS + 0.001125LD + 0.002187LT + 0.0012225SD + 0.000063ST - 0.000088DT \quad (5)$$

where

L : Payload in kg
S : Speed of robot in mm/sec
D : Diameter of wheel in mm
T : Thickness of wheel in mm

B. Adequacy of the model

The adequacy of the model is tested using the analysis of variance (ANOVA). As per the ANOVA technique (Box and Hunter, 1978), the model can be considered to be adequate if the calculated value of F-ratio of the model should not exceed the standard tabulated value of F-ratio for a desired level of confidence (95%). From the values in Table 3, it can be deduced that the current model is adequate. It is evident from the Table 2 that the error between the experimental value and predicted value is less than 5%.

Table 3: Adequacy of the model

Response	**DOF**		**F – ratio**		**Remarks**
	Lack of fit	**Error Term**	**Model**	**Standard**	
Odometry error	10	6	1.8	4.06	Model is adequate

C. Validation of the model

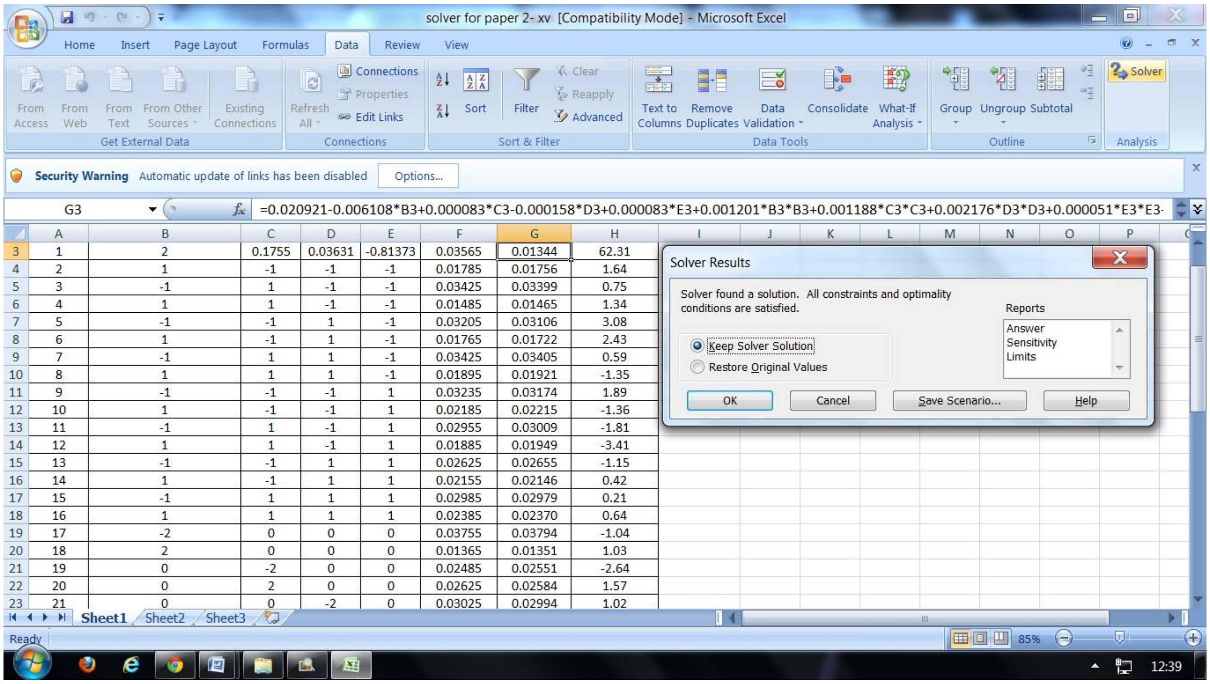
The validation of the model is checked for certain levels of the parameters, which have not been included in the experimental design. The validations of the experimental data are shown in Table 4. From this table, it is observed that the error between the measured value and predicted value is less than 5%, which confirms the validity of the model.

Table 4: Result of conformity tests for validating the model

Exp. No.	**Control factors**				**Odometry error (rad)**		**% error**
	L	**S**	**D**	**T**	**Measured**	**Predicted**	
1	-2	2	-2	2	0.04385	0.04472	-1.97
2	2	-2	2	-2	0.01955	0.01899	2.88

D. Optimum condition through XL Solver

In order to obtain optimum parametric condition for minimal odometry error, XL solver tool was used in this study. The mathematical model given in equation (5) was used as the target function with a condition of minimization. The constraints were set for all control parameters an optimized odometry error (0.01344 rad) was found at optimum parametric condition (Coded values:: L: 2, S: 0.1755, D: 0.03631, T: - 0.81373 and corresponding decoded values :: L: 1 kg, S: 143.5 mm/sec, D: 60.36 mm, T: 10.93 mm). XL solver results are shown in Figure 3.



Security Warning Automatic update of links has been disabled

G3 =0.020921-0.006108*B3+0.000083*C3-0.000158*D3+0.000083*E3+0.001201*B3*B3+0.001188*C3*C3+0.002176*D3*D3+0.000051*E3*E3-

	A	B	C	D	E	F	G	H
3	1	2	0.1755	0.03631	-0.81373	0.03565	0.01344	62.31
4	2	1	-1	-1	-1	0.01785	0.01756	1.64
5	3	-1	1	-1	-1	0.03425	0.03399	0.75
6	4	1	1	-1	-1	0.01485	0.01465	1.34
7	5	-1	-1	1	-1	0.03205	0.03106	3.08
8	6	1	-1	1	-1	0.01765	0.01722	2.43
9	7	-1	1	1	-1	0.03425	0.03405	0.59
10	8	1	1	1	-1	0.01895	0.01921	-1.35
11	9	-1	-1	-1	1	0.03235	0.03174	1.89
12	10	1	-1	-1	1	0.02185	0.02215	-1.36
13	11	-1	1	-1	1	0.02955	0.03009	-1.81
14	12	1	1	-1	1	0.01885	0.01949	-3.41
15	13	-1	-1	1	1	0.02625	0.02655	-1.15
16	14	1	-1	1	1	0.02155	0.02146	0.42
17	15	-1	1	1	1	0.02985	0.02979	0.21
18	16	1	1	1	1	0.02385	0.02370	0.64
19	17	-2	0	0	0	0.03755	0.03794	-1.04
20	18	2	0	0	0	0.01365	0.01351	1.03
21	19	0	-2	0	0	0.02485	0.02551	-2.64
22	20	0	2	0	0	0.02625	0.02584	1.57
23	21	0	0	-2	0	0.03025	0.02994	1.02

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

● Keep Solver Solution
○ Restore Original Values

Reports
Answer
Sensitivity
Limits

OK Cancel Save Scenario... Help

Figure 3: Optimum condition through XL solver

E. Confirmatory test for XL solver optimum condition

A confirmatory experiment was conducted for the optimum condition (L: 1 kg, S: 144 mm/sec, D: 60 mm, T: 11 mm) and odometry error (0.01205 rad) was obtained, which is very close to the XL solver result.

6. CONCLUSION

In this work, experiments were conducted based on central composite rotatable design matrix. A mathematical model was developed for the two wheeled differential drive robot using Response Surface Methodology. XL Solver tool was employed to determine the optimum condition for the better relative positioning i.e. minimum odometry error. The optimum level of parameters was found as follows.

Payload, L (kg)	: 1 kg
Speed, S (mm/s)	: 144 mm/sec
Diameter of wheel, D (mm)	: 60 mm
Thickness of wheel, T (mm)	: 11 mm

The optimum condition was checked through the confirmation experiment. From this study, this optimum parametric setting is suggested for achieving optimum relative positioning of two wheeled differential drive robot and the mathematical model developed in this study can be used for optimizing the odometry error using other optimization techniques.

NOMENCLATURE

D_L : Linear distance travelled by left wheel

D_R : Linear distance travelled by right wheel

DOF : Degree of freedom

REFERENCES

1. Barshan, B. &Durrant-White, H. F (1994), "Orientation Estimate for Mobile Robots Using Gyroscopic Information", *Proceedings of International Conference on Intelligent Robots and Systems (IROS '94)*. Munich, Germany, pp. 1867-1874.
2. Borenstein, J. &Feng, L (1996), "Measurement and correction of systematic odometry errors in mobile robots," *IEEE Trans. Robot.Autom.*, Vol. 12, No. 6, pp. 869-880.
3. Borenstein, J (1998), "Experimental results from internal odometry error correction with the omnimate mobile robot", *IEEE Trans. Robot. Autom.*, Vol. 14, No. 6, pp. 963-969.
4. Box Gep& Hunter, W G(1978), *Statistics for experiments: An introduction to design data analysis and model building*, Wiley, New York.
5. Byrne, R.H., kiarer, P.R. &Pletta, J.B (1992), "Techniques for Autonomous Navigation", *Sandia Report SAND92-0457*, Sandia National Laboratories, Albuquerque, NM.
6. Chaoli Wang, Zhenying Liang &QingweiJia (2010), "Dynamic feedback robust stabilization of nonholonomic mobile robots based on visual servoing", *Journal of Control Theory Applications*, Vol.8, No.2, pp. 139-144.
7. Chenavier, F. & Crowley, J (1992), "Position Estimation for a Mobile Robot Using Vision and Odometry", *Proceedings of IEEE International Conference on Robotics and Automation*, Nice, France, pp. 2588-2593.
8. Eghtesad, M. &Necsulescu, D.S(2006), "Study of the internal dynamics of an autonomous mobile robot", *Robotics and Autonomous Systems*, Vol.54, pp. 342-349.
9. Evans, J. M (1994), "HELPMATE: An Autonomous Mobile Robot Courier for Hospitals", *International Conference on Intelligent Robots and Systems (IROS '94)*. Munich, Germany, pp. 1695-1700.
10. Gonzalez-Gomez, J., Victores, G., Valero-Gomez, A. &Abderrahim, M (2011), "Motion Control of Differential Wheeled Robots with Joint Limit Constraints", *Proceedings of 6th International Symposium on Autonomous Mini robots for Research and Edutainment*, pp. 165-172.
11. Hollingum, J (1991), "Caterpillar make the earth move: automatically", *The Industrial Robot*, Vol. 18, No. 2, pp. 15-18.
12. JianxianCai&XiaogangRuan (2011), "Bionic autonomous learning control of a two-wheeled self-balancing flexible robot", *Journal of Control Theory Applications*, Vol.9, No.4, pp. 521-528.
13. Julian Lategahn, Frank Kuenemund&ChristofRoehrig (2010), "Mobile Robot Localization using Wlan,Odometry and Gyroscope data", *International Journal of Computing*, Vol. 9, Issue 1, pp. 22-30.
14. Kleeman, L (1995), "Odometry Error Covariance Estimation for Two Wheel Robot Vehicles", *Technical Report MECSE-95-1*, Intelligent Robotics Research Centre, Department of Electrical and Computer Systems Engineering, Monash University.
15. Komoriya, K. &Oyama, E (1994), "Position estimation of a mobile robot using optical fiber gyroscope (OFG)," *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, Munich, Germany, pp. 143–149.
16. Kooktae Lee, Changbae Jung &Woojin Chung (2011), "Accurate calibration of kinematic parameters for two wheel differential", *Journal of Mechanical Science and Technology*, Vol.25 No.6, pp. 1603-1611.
17. Korayem, M.H., BaniRostam, T. &Nakhai, A (2006), "Design, modelling and errors measurement of wheeled mobile robots", *International Journal of Advanced Manufacturing Technology*, Vol.28, pp.403–416.
18. Krantz, D (1996), "Non-Uniform Dead-Reckoning Position Estimate Updates", *Proceedings of IEEEInternational Conference on Robotics and Automation*, Vol. 2, pp. 2061-2066.
19. Lauro Ojeda & Johann Borenstein (2004), "Methods for the Reduction of Odometry Errors in Over-Constrained Mobile Robots", *Autonomous Robot*, Vol. 16, pp 273–286.
20. MaciejTrojnacki (2012), "Analysis of Influence of Drive System Configurations of a Four Wheeled Robot on its Mobility", *Journal of Automation, Mobile Robotics & Intelligent Systems*, Vol.6, No.4, pp. 65-70.
21. Montgomery, D.C (2000), *Design and analysis of experiments*, John Wiley and sons, New York.
22. ShenZhi-cua, zhoying-kai& Wu Wei-wei (2006), "Niche pseudo-parallel genetic algorithms for path optimization of autonomous mobile robot", Journal of Shanghai University, Vol.10, No.5, pp. 449-453.
23. Yao Cai, Qiang Zhan & Xi Xi (2012), "Path tracking control of a spherical mobile robot", *Mechanism and Machine Theory*, Vol.51, pp. 58-73.

AUTHORS

First Author - T. Mathavaraj Ravikumar, Associate Professor, Department of Mechanical Engineering, United Institute of Technology, Coimbatore – 641022, E-mail: mathurajravi@gmail.com

Second Author - R. Saravanan, Professor, Department of Mechanical Engineering, Sri Krishna College of Technology, Coimbatore – 641105, E-mail: saradharani@hotmail.com

Third Author - N. Nirmal, Assistant Professor, Department of Mechanical Engineering, United Institute of Technology, Coimbatore – 641022, E-mail: nirmalnellaiappan@gmail.com

Corresponding Author: T. Mathavaraj Ravikumar, Associate Professor, Department of Mechanical Engineering, United Institute of Technology, Coimbatore – 641022, E-mail: mathurajravi@gmail.com, Mobile: 9894162425