

# Fuzzy Multiple Domination

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**Abstract-** The fuzzy domination number  $\gamma(G)$  of the fuzzy graph  $G$  is the minimum cardinality taken over all fuzzy minimal dominating set of  $G$ . The minimum cardinality of a fuzzy  $k$ -dominating set is called the fuzzy  $k$ -dominating number  $\gamma_k(G)$ . The maximum incident degree of a fuzzy graph is  $\Delta(G)$ . In this paper we prove some theorems that relate the parameters  $\gamma(G)$ ,  $\gamma_k(G)$ ,  $\Delta(G)$ .

**Index Terms-** Fuzzy graph, fuzzy dominating set, fuzzy  $k$ -dominating set, fuzzy  $k$ -domination number.

## I. INTRODUCTION

The study of dominating sets in graphs was begun by ore and berge, the domination number is introduced by cockayne and Hedetniemi. Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A Somasundram and S. Somasundram discussed domination in fuzzy graph. They defined domination using effective edges in fuzzy graph. Nagoor Gani and Chandrasekaran discussed domination in fuzzy graph using strong arc. we also discuss the domination number of the fuzzy digraph , fuzzy  $k$ -dominating set,fuzzy  $k$ -domination number,fuzzy multiple domination

### Preliminaries1.1:

- A fuzzy subset of a nonempty set  $V$  is a mapping  $\sigma: V \rightarrow [0,1]$
- A fuzzy relation on  $V$  is a fuzzy subset of  $V \times V$ .
- A fuzzy graph  $G = (\sigma, \mu)$  is a pair of function  $\sigma: V \rightarrow [0,1]$  and  $\mu: V \times V \rightarrow [0,1]$  where  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$  for  $u, v \in V$ .
- The underlying crisp graph of  $G = (\sigma, \mu)$  is denoted by  $G^* = (V, E)$  where  $V = \{u \in V : \sigma(u) > 0\}$  and  $E = \{(u, v) \in V \times V : (\mu(u, v) > 0)\}$ .
- The order  $p$  and size  $q$  of the fuzzy graph  $G = (\sigma, \mu)$  are defined by  $p = \sum_{v \in V} \sigma(v)$  and  $q = \sum_{(u, v) \in E} \mu(u, v)$ .
- Let  $G$  be a fuzzy graph on  $V$  and  $S \subseteq V$ , then the fuzzy cardinality of  $S$  is defined to be  $\sum_{v \in S} \sigma(v)$ .
- The strength of the connectedness between two vertices  $u, v$  in a fuzzy graph  $G$  is  $\mu^\circ(u, v)$

$v) = \sup \{\mu^k(u, v) : k = 1, 2, 3, \dots\}$ , where  $\mu^k(u, v) = \sup \{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge \dots \wedge \mu(u_{k-1}, v)\}$ .

- An arc  $(u, v)$  is said to be a strong arc or strong edge, if  $\mu^\circ(u, v) \geq \mu^\infty(u, v)$  and the vertex  $v$  is said to be a strong neighbor of  $u$ .
- vertex  $u$  is said to be isolated if  $\mu(u, v) = 0$  for all  $u \neq v$ .
- In a fuzzy graph, every arc is a strong arc then the graph is called strong arc fuzzy graph.
- A path in which every arc is a strong arc then the path is called strong path and the path contains  $n$  strong arcs is denoted by  $p_n$ .
- Let  $u$  be a vertex in a fuzzy graph  $G$  then  $N(u) = \{v : (u, v) \text{ is a strong arc}\}$  is called neighborhood of  $u$  and  $N[u] = N(u) \cup \{u\}$  is called closed neighborhood of  $u$ .



**Strong arc fuzzy graph**

## II. FUZZY MULTIPLE DOMINATION

### Definition2.1

$G = (\sigma, \mu)$  be a fuzzy graph. A subset  $D$  of  $V$  is said to be fuzzy dominating set of  $G$  if for every  $v \in V - D$ , there exists  $u \in D$  such that  $(u, v)$  is a strong arc.

### Definition2.2

A fuzzy dominating set  $D$  of a fuzzy graph  $G$  is called fuzzy minimal dominating set of  $G$  , if for every vertex  $v \in D$ ,  $D - \{v\}$  is not a fuzzy dominating set.

### Definition2.3

The fuzzy domination number  $\gamma(G)$  is the minimum cardinality taken over all fuzzy minimal dominating sets of  $G$  .It is also defined by fuzzy  $\gamma$  set of a fuzzy graph  $G$ .

#### Definition2.4

$G = (\sigma, \mu)$  be a fuzzy graph . And let D be a subset of V. A vertex  $v \in V - D$  is said to be fuzzy k-dominated if it is dominated by at least k vertices in D. that is  $|N(v) \cap D| \geq k$ .

#### Definition2.5

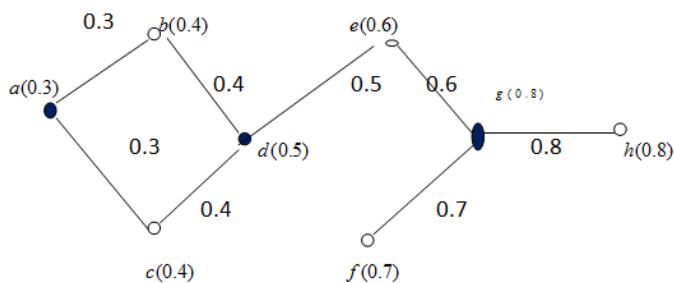
In a fuzzy graph G every vertex in  $V - D$  is fuzzy k-dominated ,then D is called a fuzzy k-dominating set.

#### Definition2.6

The minimum cardinality of a fuzzy k-dominating set is called the fuzzy k-domination number  $\gamma_k(G)$ .

#### Definition2.7

The fuzzy k-domination number of a fuzzy graph G and the fuzzy domination of a fuzzy graph G are equal when  $k=1$  that is if  $k=1$  then  $\gamma_1(G) = \gamma(G)$ .



$$D = \{a, d, g\} \text{ and } V-D = \{b, c, e, f, h\} \quad \gamma_1(G) = \gamma(G).$$

#### Definition2.8

$G = (\sigma, \mu)$  be a fuzzy graph . And let D be a subset of V. also for  $1 \leq j \leq k$ ,if D is a fuzzy k-dominated set ,it is also a fuzzy j-dominated set and then  $\gamma_j(G) \leq \gamma_k(G)$ .

#### Definition2.9

A fuzzy dominating set D of a fuzzy graph G is called multiple dominating set of G if for each vertex in  $V - D$  be dominated by multiple(more than one vertex) vertices in D.

#### Definition2.10

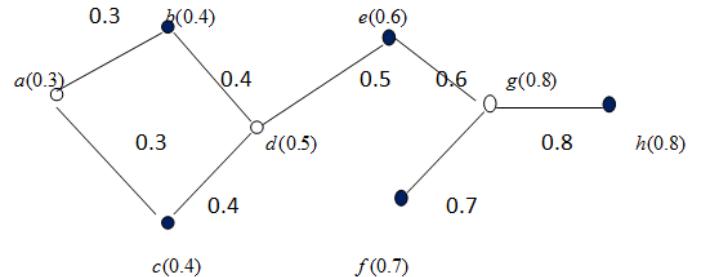
$G = (\sigma, \mu)$  be a fuzzy graph. The incident degree of a fuzzy graph is defined as number of incident arcs on a vertex  $v_i$ . It is denoted as  $d(v_i)$ .

The maximum incident degree of a fuzzy graph G is defined by  $\Delta(G) = \max_{v \in V} \{d(v) : v \in V\}$

#### Theorem2.1

If D is a fuzzy  $\gamma$  set of a of a fuzzy graph G, then at least one vertex in  $V - D$  is not dominated by multiple vertices.

**Proof:**



$$D = \{b, c, e, f, h\} \text{ and } V-D = \{a, d, g\} \quad D \text{ is a fuzzy two dominating set}$$

Let D be a fuzzy minimum dominating set in G and assume that every vertex in  $V - D$  is dominated by multiple vertices .Let  $u \in V - D$  and let v and w be two vertices in D which dominate u. It follows from our assumption that every vertex in  $V - D$  is dominated by at least one vertex in  $D - \{v, w\}$ .Therefore, the set  $D^1 = D - \{v, w\}$  is fuzzy dominating set. But since  $|D^1| < |D|$ , contradict the assumption that D is a fuzzy minimum dominating set.

#### Theorem2.2

If G is a fuzzy graph with  $\Delta(G) \geq k \geq 2$ , then  $\gamma_k(G) \geq \gamma(G) + k - 2$ .

**Proof:**

Let D be a fuzzy minimum k-dominating set in G, let  $u \in V - D$  and let  $v_1, v_2, v_3, \dots, v_k$  be distinct vertices in D which dominate u .and  $\Delta(G) \geq k \geq 2$ , we know that  $V - D = \emptyset$  because there is always a fuzzy k-dominated set, each vertex in  $V - D$  is dominated by at least one vertex in  $D - \{v_2, v_3, \dots, v_k\}$ .

Therefore u dominates each vertex in  $\{v_2, v_3, \dots, v_k\}$ , we know that the set  $D^1 = D - \{v_2, v_3, \dots, v_k\} \cup \{u\}$  is a fuzzy dominating set in G. Therefore  $\gamma(G) \leq |D^1| = \gamma_k(G) - (k-1) + 1 = \gamma_k(G) - k + 2$ .

**Theorem2.3** For any fuzzy graph G,  $\gamma_k(G) \geq kn / (\Delta(G) + k)$ .

**Proof:**

Let D be a fuzzy minimum k-dominating set and let t denote the number of arcs between D and  $V - D$ . since the degree of each vertex in D is at most  $\Delta, t \leq \Delta \gamma_k(G)$ . But since each vertex in  $V - D$  is adjacent with at least k vertices in D, we know  $t \geq k(n - \gamma_k(G))$ .combining these two inequalities produces  $\gamma_k(G) \geq kn / (\Delta(G) + k)$ .

### III. CONCLUSION

In this paper we define the concepts of fuzzy k-dominating set, fuzzy k-domination number .Next we introduce the fuzzy multiple domination and further we proved the theorems based on fuzzy domination number and fuzzy k-domination number of a fuzzy graph.

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