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# Fuzzy Regular Generalized Super Closed Set

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**Abstract:** In 1968 Chang [3] introduce the concepts of fuzzy topological spaces. Since then many topologists have contributed to theory of fuzzy topological spaces .The concepts of generalized super closed sets in topology was invented by Levene [4] . In the present paper we study the concepts of Regular generalized super closed sets and Regular generalized super open sets in fuzzy topology and obtained some of their basics properties.

**Index Terms:** Fuzzy topology, fuzzy open set, fuzzy closed set, fuzzy super interior, fuzzy super closure, fuzzy super open set, fuzzy super closed set, fuzzy regular super closed set, fuzzy regular super open set

## I. PRELIMINARIES:

Let X be a non empty set I=[0,1]. A fuzzy set on X is a mapping from X in to 1.The null fuzzy set 0 is the mapping from X in to 1 which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to 1 which takes the values 1 only. The union (resp. intersection) of a family $\{A_\propto:\alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup $A_\propto$ (resp. inf $A_\propto$) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x)\leq B(x)$ for each $x\in X$. A fuzzy point $x_\beta$ in X is a fuzzy set defined by $x_\beta$ (y)= β for y=x and x(y) =0 for $y\neq x,\beta\in[0,1]$ and $y\in X$ a fuzzy point $x_\beta$ is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta qA$ if and only if β+A(x)>1. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by AqB if and only if there exists a point $x\in X$ such that A(x)+B(x)>1.A≤if and only if $\rceil(AqB^c)$[6].

A family τ of fuzzy sets of X is called a fuzzy topology [4] on X if 0,1 belongs to τ and τ is closed with respect to arbitory union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets .For a fuzzy set A of X the closure of A(denoted by cl(A)is the intersection of all the fuzzy closed r sets of A and the interior of A(denoted by int(A) is the union of all fuzzy open subsets of A.

## II. FUZZY SUPER CLOSED SETS

**Defination2.1:** Let (X,τ) fuzzy topological space and $A\subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\leq A\neq\phi\}$

**Definition 2.2:** A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. Fuzzy super closed if scl(A ) ≤ A.
2. Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

**Definition (2.1):**A fuzzy sets A of a fuzzy topological spaces (X,τ) is called

(a) Fuzzy regular super open if A=int{cl(A)}
(b) Fuzzy regular super closed if A=cl{int(A)}

The family of all fuzzy regular super open(resp. fuzzy regular super closed) sets of a fuzzy topological $(X,\tau)$ will be denoted by FRSO(x) (resp FRSC(X)).

**Remark2.1:**Every fuzzy regular super open (resp. fuzzy regular super closed) sets of a fuzzy super open (resp. fuzzy super closed) but the converse may not be true

**Leema2.1:** A fuzzy set $A\in$FRSC(X) if and only if $1-A\in$FRSO(X)

**Definition 2.2:** A fuzzy set A of a fuzzy topological space $(X,\tau)$ is called

(a) Fuzzy g- super closed if $cl(A)\leq G$ whenever $A\leq G$ and $G\in\tau$
(b) Fuzzy g- super open if 1-A is fuzzy g- super closed

**Remark2.2:**Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) but it converse may not be true.

**Defination2.3:** A mapping f: $(X,\tau)\rightarrow(Y,\Gamma)$ is said to be fuzzy almost super continuous if $f^{-1}(G)\in\tau$ for each fuzzy set of $G\in$FRSO(Y).

## III. Fuzzy Regular Generalized - Super Closed Sets

**Defination3.1:**A fuzzy set A of a topological spaces $(X,\tau)$ is called fuzzy regular generalized super closed(written as fuzzy rg-super closed) if $cl(A)\leq U$ whenever $A\leq U$ and $U\in$FRSO(X).

**Remark3.1:** Every fuzzy g-super closed set is rg-super closed but its converse may not be true for,

**Example3.1:** Let X={a,b} and the fuzzy sets A and U be defined as follows A(a)=0.5 , A(b)=0.3

U(a)=0.5 ,U(b)=0.4 ,Let $\tau=\{0,U,1\}$ be a fuzzy topology on X .Then A is fuzzy rg-super closed but not fuzzy g-super closed.

**Theorem3.1:**Let $(X,\tau)$ be a fuzzy topological spaces and A is fuzzy regular super closed subset of X .Then A is fuzzy rg-super closed if and only if $\rceil(AqF)\Rightarrow\rceil(cl(A)qF)$ for every fuzzy regular super closed set of X.

**Proof: Necessity**: Let F be a fuzzy regular super closed subsets of X and $\rceil(AqF)$.Then $A\leq1-F$ and 1-F is fuzzy regular super open in X. Therefore $cl(A)\leq1-F$ because A is fuzzy rg-super closed. Hence $\rceil(cl(A)qF)$.

**Sufficiency:** Let $U\in$FRSO(X) such that $A\leq U$ then $\rceil(Aq(1-U))$ and 1-U is fuzzy regular super closed in X. Hence by hypothesis $\rceil(cl(A)q(1-U)$. Therefore $cl(A)\leq U$, Hence A is fuzzy rg-super closed in X.

**Theorem3.2:**Let A be a fuzzy rg-super closed set in a fuzzy topological space$(X,\tau)$ and $x_\beta$ be a fuzzy point of X such that $x_\beta q(cl(int(A)))$ then $cl(int(x_\beta))qA$.

**Proof:** If $\rceil(cl(int(x_\beta)qA)$ then $A\leq1-cl(int(x_\beta))$ and so $cl(A)\leq1-cl(int(x_\beta))\leq1-x_\beta$ because A is fuzzy rg-super closed in X. Hence $\rceil(x_\beta qcl(int(A)))$, a contradiction.

**Theorem3.3:**If A and B are fuzzy rg-super closed sets in a fuzzy topological space $(X,\tau)$ then $A\cup B$ is fuzzy rg-super closed.

**Proof:** Let U∈FRSO(X) such that A∪B≤U. Then A≤U and B≤U , so cl(A)≤U and cl(B)≤U. Therefore cl(A) ∪cl(B)≤ cl(A∪B)≤U. Hence A∪B is fuzzy rg-super closed.

**Remark3.2:** The intersection of any two fuzzy rg-super closed sets in a fuzzy topological space(X,τ) may not be fuzzy rg-super closed for,

**Example3.2:** Let X={a,b} and the fuzzy sets U,A and B of X are defined as follows

U(a)=0.7, U(b)=0.6,A(a)=0.6,A(b)=0.7,B(a)=0.8,B(b)=0.5,Let τ ={0,U,1} be a fuzzy topology on X,then A and B are fuzzy topological space (X,τ) but A∩B is not rg-super closed.

**Theorem3.4:** Let A≤B≤cl(A) and A is fuzzy rg-super closed set in a fuzzy topological space (X,τ) then B is fuzzy rg-super closed.

**Proof:** Let U∈FRSO(X) such that B≤U. Then A≤U and since A is fuzzy rg-super closed cl(A)≤U. Now B≤ cl(A)⇒cl(B)≤cl(A)≤U. Consequently B is fuzzy rg-super closed.

**Defination3.2:** A fuzzy set A of a fuzzy topological space (X,τ) is called fuzzy regular generalized super open (written as fuzzy rg-super open) if and only if 1-A is fuzzy rg-super closed.

**Remark3.3:** Every fuzzy regular super open set is fuzzy rg-super open. But converse may not be true. For the fuzzy set B defined by B(a)=0.5 and B(b)=0.7 in the fuzzy topological (X,τ) of example (3.1) is fuzzy rg-super open but not fuzzy regular super open.

**Theorem3.5:**A fuzzy set A of a fuzzy rg-super open if and only if F ≤ int(A) whenever F≤A and F is fuzzy regular super closed.

**Proof:** Obvious.

**Theorem3.6:** Let A be a fuzzy rg-super open set in a fuzzy topological spaces (X,τ) and int (A)≤B≤A then B is fuzzy rg-super open.

Proof: Obvious

**Theorem3.7:** Let (X,τ) be a fuzzy topological space and FRC(X) be the family of of all fuzzy regular super closed sets of X. Then FRSO(X)=FRSC(X) if and only if every fuzzy subset of X is fuzzy rg-super closed.

**Proof: Necessity:** Suppose that FRSO(X)=FRSC(X) and that A≤U∈FRSO(X) then cl(A)≤cl(U)=cl(int(U))=U and A is fuzzy rg-super closed.

**Sufficiency:** Suppose that every fuzzy subset of X is fuzzy rg-super closed .If U∈FRSO(X) then since int(U)≤U and int(U) is fuzzy rg-super closed, cl(int(U))≤U≤cl(int(U)) and U∈FRSC(X).Thus FRSO(X)≤FRSC(X).If T∈FRSC(X) then 1-T∈FRSO(X) ≤FRSC(X) and hence T∈FRSO(X).Consequently, FRSC(X) ≤FRSO(X).Hence FRSO(X)=FRSC(X).

**Defination3.3:**A fuzzy topological space (X,τ) is said to be fuzzy almost normal if for every pair of fuzzy regular super closed sets A and B of X such that A≤1-B there exists fuzzy super open sets U and V such that A≤U,B≤V and U≤1-V.

**Theorem3.8:**If F is fuzzy regular super closed sets and A is fuzzy rg-super closed subsets of a almost normal space (X,τ) and ⌉(AqF) .Then there exists fuzzy super open sets U and V such that cl(int(A))≤U,F≤V and ⌉(UqV).

**Proof:** Since $\rceil$(AqF), A≤1-F and hence cl(A)≤1-F because A is fuzzy rg-super closed. And so cl(int(A))≤1-F.Since X is fuzzy almost normal and cl(int(A)) and F are fuzzy regular super closed sets in X ,there exists fuzzy super open sets U and V such that cl(int(A)) ≤U,F and U≤1-V. Hence $\rceil$(UqV).

**Theorem 3.9:**Let A be a fuzzy g-super closed set in a fuzzy topological space (X,τ) and f: (X,τ)→(Y,σ) is a fuzzy almost continuous and fuzzy super closed mappings then f(A) is fuzzy rg-super closed in Y.

**Proof:** If f(A)≤G where G∈FRO(Y).Then A≤$f^{-1}$(G)∈τ and hence cl(A) ≤ $f^{-1}$(G)because A is a fuzzy g-super closed in X. Since f is fuzzy super closed, f(cl(A)) is a fuzzy super closed set in Y. It follows that cl(f(A))≤ cl(f(cl(A)))= f(cl(A))≤G. Thus cl(f(A)≤G and f(A) is a fuzzy rg-super closed set in Y.

**Defination3.4:**A collection $\{G_\alpha:\alpha\in\Lambda\}$ of fuzzy rg-super open sets in a fuzzy topological space (X,τ) is called a fuzzy rg-super open cover of a fuzzy set A of X if $A\leq\cup\{G_\alpha:\alpha\in\Lambda\}$.

**Defination3.5:** A fuzzy set of a topological space (X,τ) is said to be fuzzy rg- super compact if every fuzzy rg-super open cover of X has a finite sub cover.

**Definition 3.6:**A fuzzy topological space (X,τ) is said to be fuzzy rg- super compact relative to X ,if for every collection $\{G_\alpha:\alpha\in\Lambda\}$ of fuzzy rg-super open sub sets of X such that $A\leq\cup\{G_{\alpha j}:\alpha j\in\Lambda_0\}$.

**Defination3.7:** A crisp subset of A of a fuzzy topological space (X,τ) is said to be fuzzy rg- super compact if A is fuzzy rg- super compact as a fuzzy subspace of X.

**Theorem3.10:** A fuzzy rg-super closed crisp subsets of a fuzzy rg- super compact space is fuzzy rg- super compact relative to X.

**Proof:** Let A be a fuzzy rg-super closed crisp set off a fuzzy rg-compact space (X,τ) .Then 1-A is fuzzy rg-super open in X .Let $G=\{G_\alpha:\alpha\in\Lambda\}$. Be a cover of A fuzzy rg-super open sets in X .Then the family{G,1-A} is a fuzzy rg-super open in X is fuzzy rg-compact.it had sub cover $\{ G_{\alpha1}\ G_{\alpha2}\ G_{\alpha3}\ldots\ldots\ G_{\alpha n}\}$. If the sub cover contain 1-A,we discard it otherwise take sub cover as it is, thus we have obtained a finite fuzzy rg-super open sub cover of A is fuzzy rg-compact relative to X.

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