Numerical Study on Heat Transfer of non-Newtonian Fluid Flow over Stretching Surface with Variable Viscosity in Uniform Magnetic Field

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Abstract- An investigation has been carried out to obtain the flow and heat transfer of two dimensional electrically conducting second grade fluids over stretching surface in the presence of uniform magnetic field. The viscosity is assumed to vary as inverse linear function of temperature. The non-linear boundary layer equations together with the boundary conditions are reduced to a system of non-linear ordinary differential equations by using similarity transformations. The resulting equations are solved numerically by Runge-Kutta shooting method. The velocity and temperature profiles, the skin-friction and the rate of heat transfer are computed and discussed for different values of parameters. Some numerical results are presented graphically and discussed.

Index Terms- Heat Transfer**,** Non-Newtonian fluid, Stretching sheet, Thermal Conductivity, Variable Viscosity.

I. Introduction

The study of the flow and heat transfer created by a moving surface is relevant to several applications in the fields of metallurgy and chemical engineering, polymer processing, electro-chemistry, MHD power generators, flight magneto hydro dynamics as well as in the field of planetary magneto spheres, aeronautics and chemical engineering. Sakiadis [1] was the first to study the boundary layer flow due to a moving wall in fluid at rest. The study of flow over a stretching surface has generated much interest in recent years in view of its numerous industrial applications such as extension of polymer sheets, glass blowing, rolling and manufacturing plastic films and artificial fibers. The pioneer work on the boundary layer flows over stationary and continuously moving surfaces was initially done by Blasius [2] and Crane [3]. Ali [4] carried out a study for a stretching surface subject to suction or injection for uniform and variable surface temperatures. Rajgopal et al [5], Dandapat and Gupta [6], Shit [7] and Reddaiah and Rao [8] extensively studied on various aspects of boundary layer flow problems over a stretching sheet.

In most of the studies of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However, it is known that the physical properties can changed sufficiently with temperature .and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are significantly changed compared to the constant property. Hassanien et al [9] revealed that the fluid viscosity and thermal conductivity might function of temperatures as well as the fluid is considering.

Recently Sharma and Hazarika [10] studies the effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field.

Also, most of the practical situations demand for fluids that are non-Newtonian in nature which are mainly used in many industrial and engineering applications. It is well known that a number of fluids such as molten plastic, polymeric liquid, food stuffs etc exhibit non-Newtonian character. The boundary layer flow of non-Newtonian fluids over a starching sheet has been studied extensively in the recent years. Ericksen et al [11] discussed the heat and mass transfer over a stretching sheet with suction or injection. The flow was assumed to be an incompressible viscous fluid and the surface speed was assumed to be constant. Later, Gupta and Gupta [12] extended this work by assuming that the surface speed varies with the coordinate along the flow. Dutta et al [13] analyzed the case where a uniform heat flux is prescribed at the surface and the surface is non-porous. Hassanien et al [14] presented a work on flow and heat transfer in power law fluid over a stretching porous surface with variable surface temperature. Very recently, Abel and Mahesha [15] considered the effects of buoyancy and variable thermal conductivity in a power law fluid past on a vertical stretching sheet in the presence of non-uniform heat source.

In the present work, the effect of variable viscosity of viscous incompressible second grade fluid over a stretching surface in uniform magnetic field is investigated. Numerical results are presented for velocity and temperature profiles for different parameters of the problem.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the steady two dimensional boundary layer flow of an incompressible and electrically conducting second grade fluid past over an isothermal stretching sheet coinciding with the plane $y = 0$ and in the presence of a uniform transverse magnetic field. The flow is assumed to be in the *x*-direction. Two equal and opposite forces are introduced along the x- axis, so that the sheet is stretched with a velocity, which varies linearly with the distance from the fixed origin point on the sheet. The fluid is permeated by an externally applied uniform magnetic field of strength B_0 , which acts in the direction perpendicular to the sheet. The magnetic Reynolds number is assumed to be small and the effect of induced magnetic fluid is neglected.

The sheet is kept at a constant temperature T_w. Under these assumptions, the boundary layer equations governing the flow and heat transfer can be written as:

The equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The equation of momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{k_0}{\rho_\infty}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] - \frac{\sigma B^2}{\rho_\infty}u \tag{2}$$

The equation of energy:

$$\rho_\infty c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + k_0\left[u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right] \tag{3}$$

Along with the boundary conditions,

$$\begin{aligned} &u = U_w = cx, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \\ &u = 0, \quad v = 0 \quad T = T_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{4}$$

Where u and v are the flow velocity components along x and y directions respectively, B_0 is the applied magnetic field, μ_∞ and k_∞ are the constant viscosity and constant thermal conductivity of the free stream of the fluid respectively. μ and k are the coefficient of variable viscosity and variable thermal conductivity respectively of the fluid which are considered to vary as a function of temperature. C_p is the specific heat at constant pressure. κ is the thermal conductivity and k_0 is the coefficient of visco-elasticity. σ is the electrical conductivity. c is the constant stretching rate. T_∞ and ρ_∞ are the free stream temperature and density.

Flowing Lai and Kulacki [16] , We assume:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma\left(T - T_\infty\right)\right] \quad or \quad \frac{1}{\mu} = a\left(T - T_r\right) \tag{5}$$

$$where \quad a = \frac{\gamma}{\mu_\infty} \quad and \quad T_r = T_\infty - \frac{1}{\gamma}$$

and

$$\frac{1}{k}=\frac{1}{k_\infty}\left[1+\kappa\left(T-T_\infty\right)\right] \quad or \quad \frac{1}{k}=\varepsilon\left(T-T_\varepsilon\right)$$

Where $\varepsilon=\frac{\kappa}{k_\infty}$ and $T_e=T_\infty-\frac{1}{\kappa}$ (6)

Where a, ε, T_r, T_e are constants and their values depend on the reference state and thermal properties of the fluid i.e γ and κ. In general a , ε f 0 for liquids and a , ε p 0 for gases (the viscosity and thermal conductivity of liquid/gas usually decrease/increase with increasing temperature).

The mathematical analysis of the problem is simplified by introducing the following similarity variables:

$$\psi(x,y)=\sqrt{c\upsilon}xf(\eta), \qquad \eta=\sqrt{\frac{c}{\upsilon}}y, \qquad \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{7}$$

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x}, \tag{8}$$

On substitution of the transformation given in Eqs. (7) and (8) into (2) - (3), the following equations are obtained:

$$\left(\frac{\theta-\theta_r}{\theta_r}\right)\left[(f')^2-ff''\right]+f'''-\frac{\theta'}{\theta-\theta_r}f''-K\left(\frac{\theta-\theta_r}{\theta_r}\right)\left[2ff'''-ff^{IV}-(f'')^2\right]+M\left(\frac{\theta-\theta_r}{\theta_r}\right)f'=0 \tag{9}$$

and

$$\theta''-Ec\Pr\left(\frac{\theta_r}{\theta-\theta_r}\right)(f'')^2+\Pr f\theta'-\frac{\theta'^2}{\theta-\theta_e}+KEc\Pr\left[f'(f'')^2-ff''f'''\right]=0 \tag{10}$$

The transformed boundary conditions are reduce to

$$f'(\eta)=1, \qquad f(\eta)=0, \qquad \theta(\eta)=1, \qquad at \qquad \eta=0, \tag{11}$$

$$f'(\eta)\to 0, \qquad f''(\eta)\to 0, \qquad \theta(\eta)\to 0, \qquad as \qquad \eta\to\infty, \tag{12}$$

Where prime denotes differentiation with respect to η only and

$K=\frac{k_0 c}{\rho_\infty \upsilon}$ is the viscoelastic parameter, $M=\frac{\sigma B_0^{\ 2}}{\rho_\infty c}$ is the magnetic parameter,

$\Pr=\frac{\mu c_p}{k}$ is the Prandtle number, $Ec=\frac{U_w^{\ 2}}{c_p\left(T_w-T_\infty\right)}$ is the Eckert number.

θ_r and θ_e are the dimensionless parameters characterizing the influence of viscosity and thermal conductivity respectively, can be written as:

$$\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma\left(T_w-T_\infty\right)} \tag{13}$$

$$\theta_e = \frac{T_e - T_\infty}{T_w - T_\infty} = -\frac{1}{\kappa\left(T_w - T_\infty\right)} \tag{14}$$

For engineering purpose, one is usually less interested in the shape of the velocity, temperature or concentration profiles then in the value of the skin-friction, heat transfer. The expression for the local skin-friction coefficient C_f and the local Nusselt number Nu defined by:

$$C_f = \frac{\tau_w}{\mu_\infty (cx)\sqrt{\frac{c}{\upsilon}}} = -\left[\frac{\theta_r}{\theta - \theta_r} - 2K_1\right] f''(0), \tag{15}$$

$$Nu = \frac{q_w}{k\sqrt{\frac{c}{\upsilon}}(T_w - T_\infty)} = -\theta'(0) \tag{16}$$

Where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} + k_0\left[u\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right]_{y=0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\sqrt{\frac{c}{\upsilon}}(T_w - T_\infty)\theta'(0),$$

III. NUMERICAL RESULTS AND DISCUSSION

The system of differential equations (9) and (10) governed by boundary conditions (11) and (12) are solved numerically by applying an efficient numerical technique based on the fourth order Runge-Kutta shooting method and an iterative method. It is experienced that the convergence of the iteration process is quite rapid.

The purpose of this study is to bring out the effects of the variable viscosity on the governing flow with the combinations of the other flow parameters, namely the visco-elastic parameter K, the Eckert number Ec, the Prandtl number $\Pr$, the Magnetic parameter M and the dimensionless viscosity parameter θ_r. An insight into the effects of these parameters of the flow field can be obtained by the study of the temperature distributions.

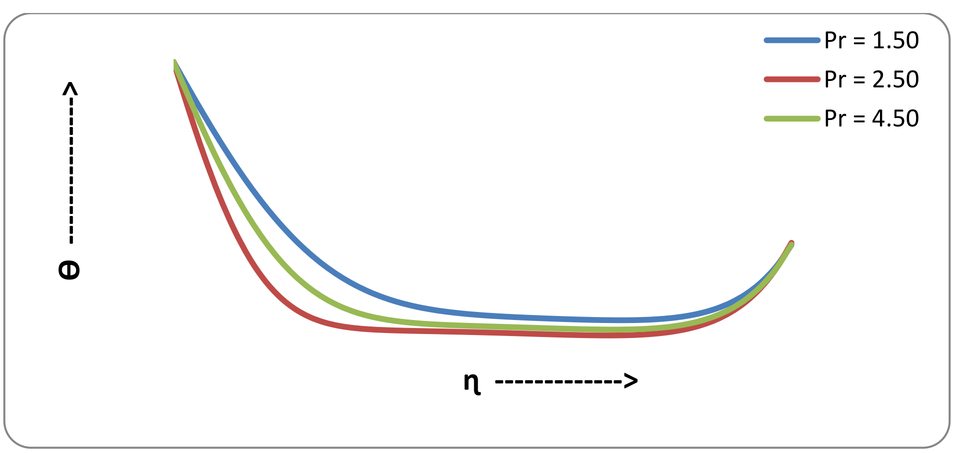

Figure 1: Variation of $\theta(\eta)$ with η for different values of $\Pr$.

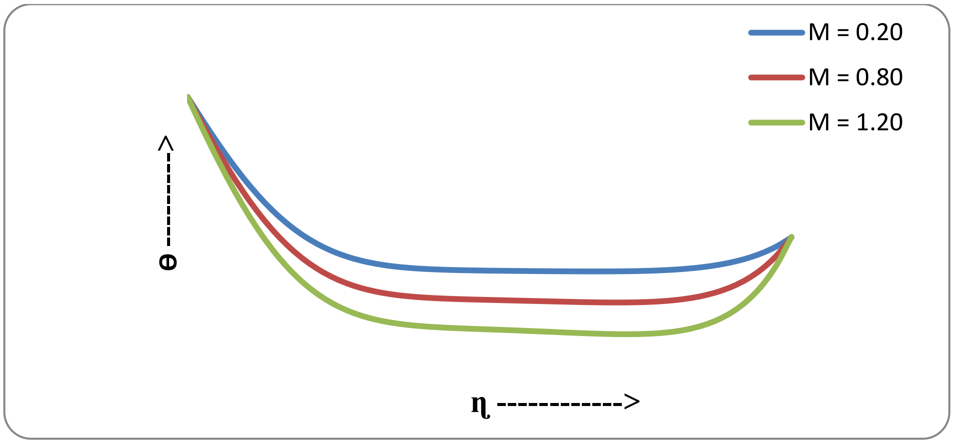

Figure 2: Variation of $\theta(\eta)$ with η for different values of M .

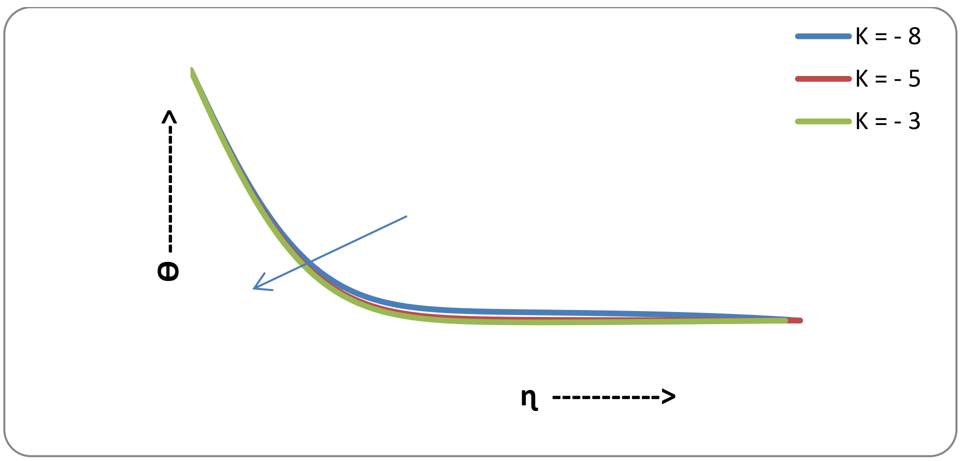

Figure 3: Variation of $\theta(\eta)$ with η for different values of K .

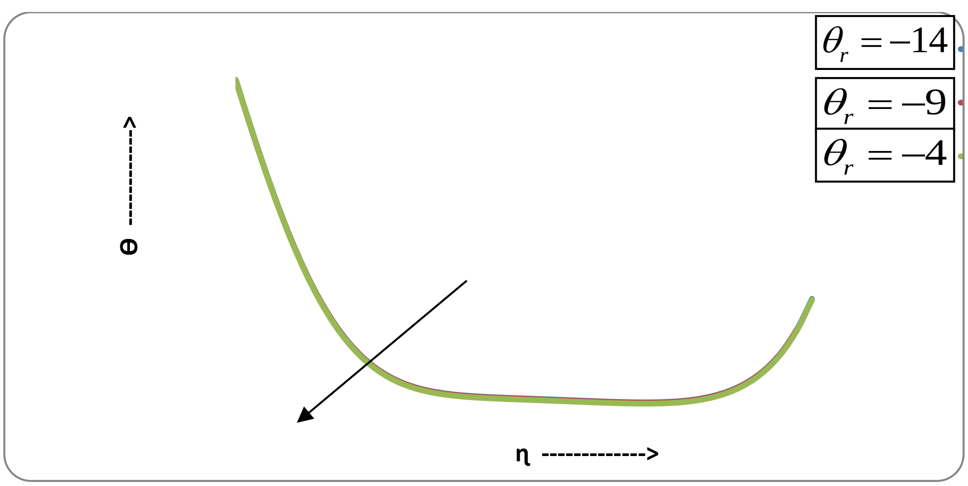

Figure 4: Variation of $\theta(\eta)$ with η for different values of θ_r.

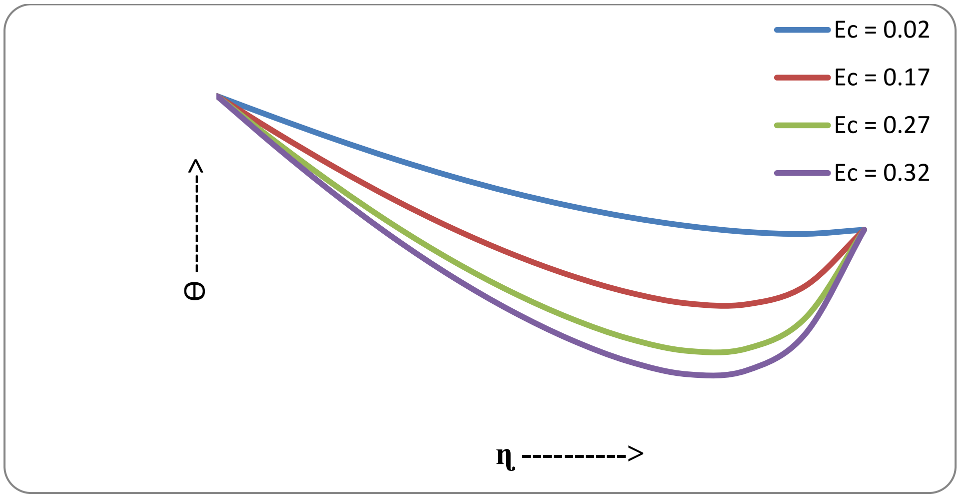

Figure 5: Variation of $\theta(\eta)$ with η for different values of Ec.

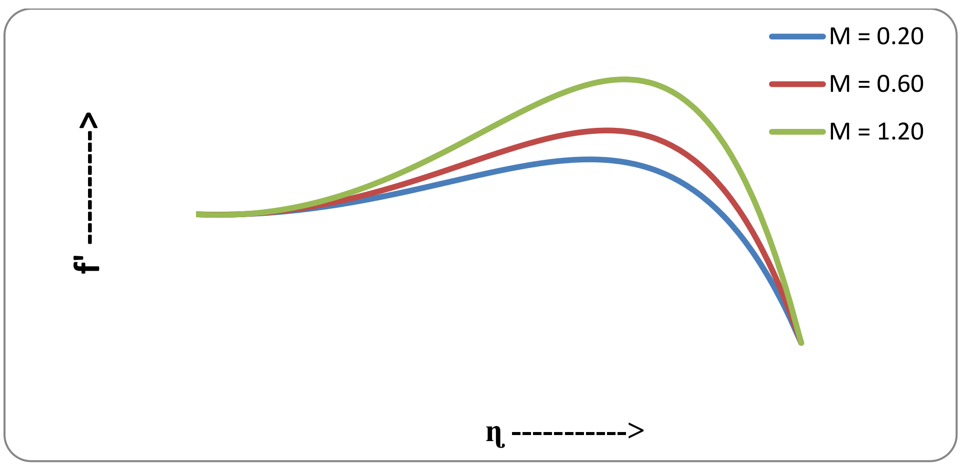

Figure 6: Variation of $f'(\eta)$ with η for different values of M.

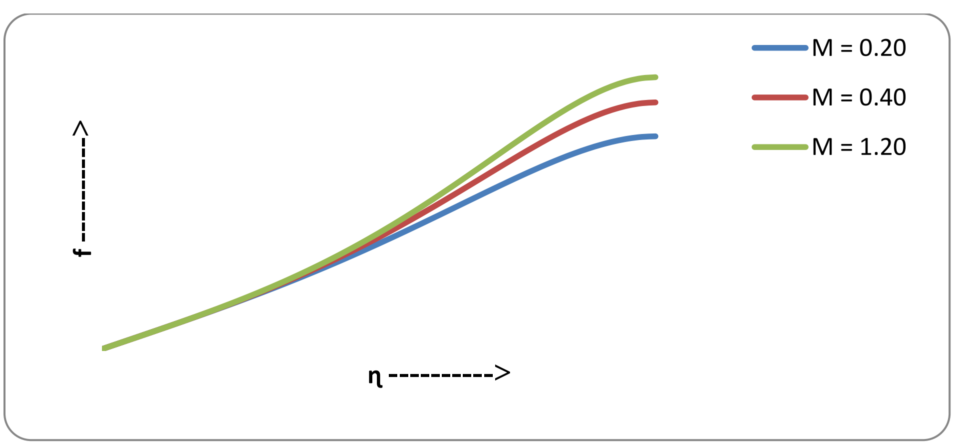

Figure 7: Variation of $f(\eta)$ with η for different values of M.

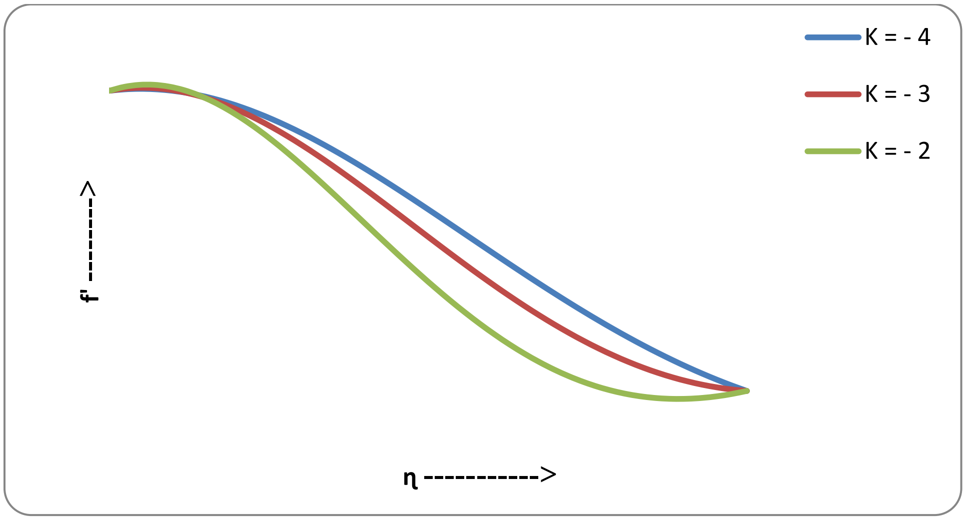

Figure 8: Variation of $f'(\eta)$ with η for different **values** of K .

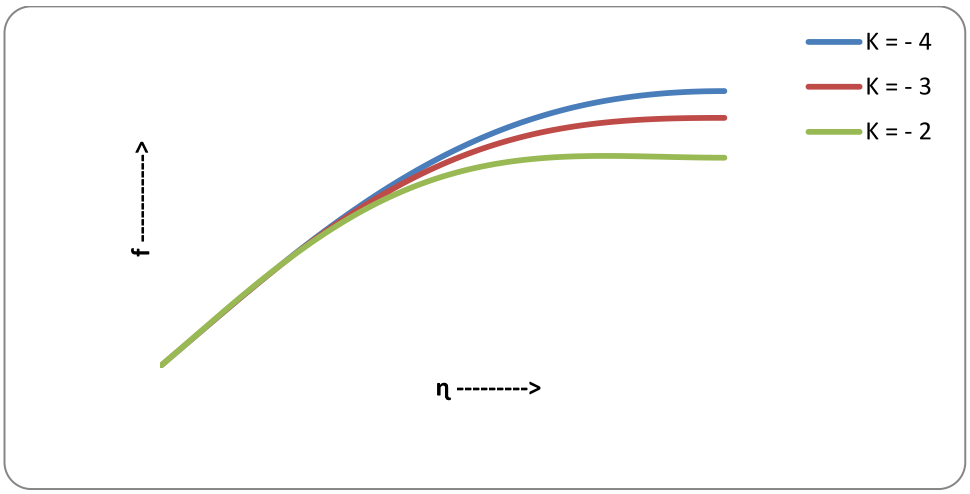

Figure 9: Variation of $f(\eta)$ with η for different values of K .

Figs 1 – 5 give the temperature profile for different values of the Prandtl number Pr , the Magnetic parameter M , the visco-elastic parameter K , the variable viscosity θ_r and the Eckert number Ec respectively. Figs 6 and 8 give the axial velocity profile $f'(\eta)$ for different values the Magnetic parameter M , the visco-elastic parameter K respectively. Figs 7 and 9 show the effect of the Magnetic parameter M and the visco-elastic parameter K on the velocity profile $f(\eta)$. From the Fig 4, it is observed that the effect of the variable viscosity is not prominent in case of temperature profile.

The impacts of the Magnetic parameter M on the velocity and temperature profiles are very significant in practical point of view. In Figure 2 and Figure 6, the variations in temperature distribution and velocity field for several values of M are presented. The dimensionless velocity $f'(\eta)$ increases with increasing values of M . Accordingly, the thickness of the momentum boundary layer decreases. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. To reduce momentum boundary layer thickness the general Lorentz force enhances the fluid motion in the boundary layer region. On the other hand, from Figure 2, it is noticed that the temperature $\theta(\eta)$ decreases with increasing values of M .

The temperature field for various values of the Prandtl number Pr is represented in Figure 1. With increasing Pr, the dimensionless temperature profile as well as thermal boundary layer thickness quickly decreases. An increase in Prandtl number means a decrease of fluid thermal conductivity which causes a decrease in temperature.

In Figure 5, the effect of Eckert number Ec on the temperature is exhibited. It is noticed that the dimensionless temperature $\theta(\eta)$ decreases for increasing values of Ec. So, the thickness of thermal boundary layer reduces.

Table-1: Numerical values of the local skin-friction: $C_f = \dfrac{\tau_w}{\mu_\infty (cx)\sqrt{\dfrac{c}{\upsilon}}} = -\left[\dfrac{\theta_r}{\theta-\theta_r} - 2K_1\right] f''(0),$

Pr	K	θ_r	Ec	$M=0.0$	$M=0.4$	$M=0.8$
2.5	2	-10	.05	-0.136449	-0.198107	-0.26316
5.5				-0.144564	-0.207283	-0.272177
6.5				-0.145579	-0.208532	-0.272743
2.5	-8	-10	.05	-0.107486	-0.202212	-0.301971
	-6			-0.141665	-0.234047	-0.331744
	-4			-0.169960	-0.262464	-0.360796
2.5	2	-10	.05	-0.136449	-0.198107	-0.26316
		-6		-0.133113	-0.193353	-0.256571
		-4		-0.124399	-0.180365	-0.209293
2.5	2	-10	.05	-0.136449	-0.198107	-0.26316
			.15	-0.142268	-0.206547	-0.272880
			.25	-0.145675	-0.210323	-0.277054

Table- 2: Numerical values of local Nusselt number: $Nu = -\theta'(0)$

Pr	K	θ_r	Ec	$M=0.0$	$M=0.4$	$M=0.8$
2.5	2	-10	0.05	1.582884	1.740261	1.955533
5.5				2.384658	2.622688	2.949071
6.5				2.589813	2.835063	3.070676
2.5	-8	-10	0.05	1.234033	1.252961	1.261490
	-6			1.270197	1.286124	1.299361
	-4			1.301762	1.312465	1.320547
2.5	2	-10	0.05	1.582884	1.740261	1.955533
		-6		1.639997	1.811143	2.047467
		-4		1.788585	1.962371	3.181905
2.5	2	-10	0.05	1.582884	1.740261	1.955533
			0.15	2.088649	2.522476	3.100220
			0.25	2.560794	3.196641	4.736850

The important characteristics in the present study are the local skin-friction coefficient C_f and the local rate of heat transfer at the sheet (Nusselt number Nu) defined in equations in (15) and (16) respectively. Tables 1 and Table 2 exhibit the numerical values to the local skin-friction and local Nusselt number respectively. It has been observed empirically that for any particular values of $\Pr$, K, θ_r and Ec the local skin-friction decreases with the increase in the magnetic parameter M. The skin-friction is also decreases with the increase of the Prandtl number $\Pr$ and the visco-elastic parameter K. But the reversal trend is observed in the presence of viscosity parameter θ_r. It is worthwhile to mention here that the rate of heat transfer increases with the increasing values of $\Pr$, K, θ_r and Ec.

IV. CONCLUSIONS

In the present study, the problem of two dimensional boundary layer flow of second order incompressible laminar, electrically conducting fluid past a stretching sheet with the variation in the viscosity in the presence of a uniform transverse magnetic field is investigated. Numerical solutions are presented for the fluid flow and heat transfer characteristics for different values of parameters involved in the problem. The present study will serve as a scientific tool for understanding more complex flow problems concerning with the various physical parameters.

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