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An Efficient algorithm for Fuzzy based Metagraph ADT

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Abstract- This paper proposes an efficient algorithm for Fuzzy based Metagraph ADT (Abstract Data Type). It is a hierarchical data structure where every node has one or more data. A mathematical model which encompasses both data type and the functions that operate on the data type is called as an ADT. Fuzzy set ADT integrated with Fuzzy Metagraph can perform different operations such as Union, Intersection, Disjoint Subset Union, Clustering and Searching. The proposed method considerably reduces the time complexity which can be applied to solve a number of problems. Fuzzy metagraph is an emerging technique used in the design of many information processing systems like transaction processing systems, decision support systems, and workflow Systems.

Index Terms- ADT, Adjacency Matrix, Adjacency List, Fuzzy Metagraph, Fuzzy set, T-norm.

I. INTRODUCTION

Graphs play an important role in the design of most of the information processing systems. A graph may be a simple graph or an ordered graph. It is defined by a pair G={X, E}, where X ={x_1, x_2, x_3…x_n} is a finite set of vertices and 'E' a collection of edges that happen to connect vertices.

It is a hierarchical data structure where every node has one or more data which exhibit all the properties of a graph and as in a conventional graph structure, there exist a set-to-set mapping. Metagraphs have lot of applications in the field of information processing systems, decision support systems, models management and the rule based management systems in which a single work consists of many information processing tasks to be performed by the humans or the machines. Metagraphs under such situations can provide a useful and comprehensive function for modeling by extending the features offered by the traditional graph structures i.e., digraphs, hypergraphs. Metagraph allows different components of the process to be represented both graphically and analytically [1].

Fuzzy logic is based on fuzzy set theory. In contrast to standard set theory in which each element is either completely in or not in a set, fuzzy set theory allows partial members in sets. This provides a powerful mechanism for representing vague concepts. The world of information is surrounded by uncertainty and imprecision. The human reasoning process can handle inexact, uncertain, and vague concepts in an appropriate manner. Usually, the human thinking, reasoning, and perception process cannot be expressed precisely. Fuzzy systems were first introduced by Zadeh (1965) [3].

In a graph ADT, What is to be done is mentioned but how is to be done is not mentioned. It consists of function name, types of function and behavior of the functions, which includes set of operations like intersection, union, set difference and set complement [5]. According to Fuzzy Metagraph based ADT, we may perform different operation like fuzzy set union, fuzzy set intersection, fuzzy disjoint subset union, Searching, Sorting and clustering.

Rest of the paper is organized as follows. Section 2 gives the related work. Section 3 points out various method of the Metagraph. Section 4 deals with fuzzy based metagraph ADT and the time complexity have been discussed. Section 5 concludes the paper.

II. RELATED WORKS

Deepti Gaur, and Aditya Shastri have proposed a model for metagraph data structure. They have used to store data inside the computer memory either in the form of Adjacency matrix or in Adjacency list so it has been used efficiently [6], they have proposed metagraph based substructure pattern mining technique. They have developed an algorithm which adapts the depth-first search strategy to mine frequent connected sub metagraph efficiently [7]. They have proposed fuzzy metagraph method of clustering to find the similar fuzzy nodes in a fuzzy metagraph. They have used T-norms (Triangular Norms) functions and join two or more T norms to cluster the fuzzy nodes [8]. They have proposed vague metagraph method, which is a graphical model that not only visualized the process of any system but also their formal analysis where the analysis will be accomplished by means of an algebraic representation of the graphical structure. The graphical structure has been represented by the adjacency and incidence matrix of a vague metagraph [9].

Pankaj Dashore, and Suresh Jain have used the rule based system and fuzzy metagraphs for real world applications (Online Transaction such as banking, E commerce and share market,) to make correct decisions which were achieved using a fuzzy metagraph to provide a high level view reducing the unnecessary details [13, 14, 15].

III. BACKGROUND AND METHODOLOGY

This section briefly reviews fuzzy metagraph techniques. Basu and Blanning introduced the concept of metagraph [1].

3.1 Fuzzy Set Declaration and Definition

Let *X* be a space of objects and *x* be a generic element of *X*. A classical set *A*, $A \subseteq X$, is defined as a collection of elements or objects $x \in X$, such that *x* can either belong or not belong to the set *A*. A fuzzy set *A* in *X* is defined as a set of ordered pairs [3].in fuzzy set there is no repeated elements. All Data elements to be unique [3, 11].

$A = \{(x, \mu A(x)) \mid x \in X\}$

Where $\mu A(x)$ is called the membership function (MF) for the fuzzy set A. The MF maps each element of X to a membership grade between zero and one.

3.1.1 Basic operation of Fuzzy Set ADT

The two fuzzy sets *A* and *B* is specified in general by a function $T : [0,1] \times [0,1] \rightarrow [0,1]$, which aggregates two membership grades as follows: This class of fuzzy set operators is usually referred to as *T*-norm (Triangular Norm) .the most frequently used T-norm operators are

Minimum: $T_{min}(a, b) = min(a, b) = a \wedge b$

Maximum: $T_{max}(a, b) = max(a, b) = a \vee b$

$\mu_{A\cap B}(x) = T(\mu_A(x), \mu_B(x)) = \mu_A(x) \cap \mu_B(x) = \min(\mu_A, \mu_B)$

$\mu_{A\cup B}(x) = T(\mu_A(x), \mu_B(x)) = \mu_A(x) \cup \mu_B(x) = \max(\mu_A, \mu_B)$

$\mu_{\bar{A}} = 1 - \mu A$

Algebraic product: $T_{ap}(a, b) = ab$

Drastic product:

$$T_{dp}(a, b) = \begin{cases} a, & \text{if } b=0 \\ b, & \text{if } a=0 \\ 1 & \text{if } a, b > 0 \end{cases}$$

Minimum $(A, B) = A \wedge B$ and Maximum $(A, B) = A \vee B$. where || Denotes logical OR operation, initially 1st element is minimum and maximum.

3.2 Fuzzy Metagraph

A metagraph S = {X, E} is a graphical representation consisting of two tuples X and E. Here X is its generating set and E is the set of edges defined on generating sets. The generating set X of the metagraph S. The set of elements X = {x1, x2, x3,…,xn} represents variables and occurs in the edges of the metagraph.

The concept of a fuzzy graph is the "fuzzification" of the crisp graphs using fuzzy sets. A fuzzy graph $\tilde{G}$ can be defined as a triple $\{X, \tilde{X}, \tilde{E}\}$, where $\tilde{X}$ is a fuzzy set on X and $\tilde{E}$ is a fuzzy relation on X× X. A fuzzy set $\tilde{X}$ on X is completely characterized by its membership function $\mu: X \rightarrow [0, 1]$ for each $x \in X$, $\mu(x)$ illustrates the truth value of the statement of $x \in \tilde{X}$. The fuzzy metagraph is the concept of Fuzzification of the crisp Metagraph using fuzzy generating set. Fuzzy generating set is the node set of all the elements of fuzzy metagraph [8, 20]. Consider a finite set $X=\{x1, x2, x3,\ldots, x_n\}$. It is defined as a triple $\tilde{S} = \{X, \tilde{X}, \tilde{E}\}$ in which $\tilde{X}$ is a fuzzy set on X and $\tilde{E}$ is a fuzzy edge set $\{\tilde{e}_m, m=1, 2, 3,.. m\}$. Each component $\tilde{e}$ in $\tilde{E}$ is characterized by an ordered pair $<\tilde{V}_m, \tilde{W}_m>$. In the pair $\tilde{V}_m \subseteq \tilde{X}$ is the in-vertex of $\tilde{e}_m$ and $\tilde{W}_m \subseteq \tilde{W}$ is the out-vertex.

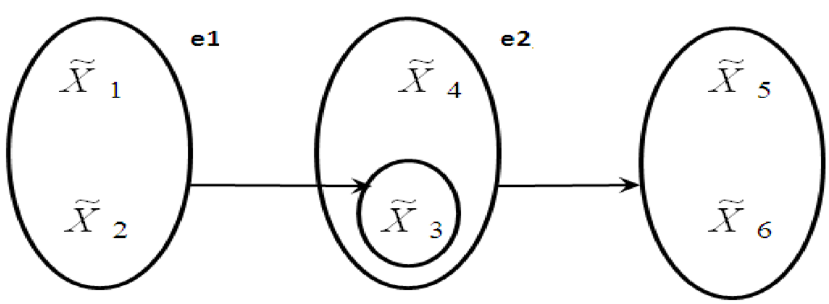

Fig .1 Fuzzy Metagraph

Figure 1 shows fuzzy metagraph whose element set is $X = \{\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_6\}$ is known as fuzzy meta node and whose edge set consists of: $\tilde{e}_1 = <\{\tilde{X}_1, \tilde{X}_2\}, \{\tilde{X}_3\}>$ and $\tilde{e}_2 = <\{\tilde{X}_3, \tilde{X}_4\}, \{\tilde{X}_5, \tilde{X}_6\}>$. The in-vertex and out-vertex of $\tilde{e}_1$ are $\{\tilde{X}_1, \tilde{X}_2\}$ and $\{\tilde{X}_3\}$.

3.2.1 Adjacency Matrix of Fuzzy Metagraph

An example is the adjacency matrix, a square matrix with one row and one column for each element in the generating set. Each member of the matrix is a set of triples, one for each edge connecting the row element to the column element. The triples define the invertex, outvertex, and the edge [20].

Table I. Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_3$	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$<\tilde{X}_2, \emptyset, \tilde{e}_1>$	$\emptyset$	$\emptyset$
$\tilde{X}_2$	$<\tilde{X}_1, \emptyset, \tilde{e}_1>$	$\emptyset$	$\emptyset$
$\tilde{X}_3$	$\emptyset$	$<\tilde{X}_4, \tilde{X}_6, \tilde{e}_2>$	$<\tilde{X}_4, \tilde{X}_5, \tilde{e}_2>$
$\tilde{X}_4$	$\emptyset$	$<\tilde{X}_3, \tilde{X}_6, \tilde{e}_2>$	$<\tilde{X}_3, \tilde{X}_5, \tilde{e}_2>$

3.2.2 Adjacency List of a Fuzzy Metagraph

An adjacency list basically has *V* linked lists, with each corresponding linked list containing the elements that are adjacent to a particular vertex.

$\tilde{X}_1 \rightarrow <\tilde{X}_2, \emptyset, \tilde{e}_1>$, $\tilde{X}_2 \rightarrow <\tilde{X}_1, \emptyset, \tilde{e}_1>$, $\tilde{X}_3 \rightarrow <\tilde{X}_4, \tilde{X}_6, \tilde{e}_2> \rightarrow <\tilde{X}_4, \tilde{X}_5, \tilde{e}_2>$,
$\tilde{X}_4 \rightarrow <\tilde{X}_3, \tilde{X}_6, \tilde{e}_2> \rightarrow <\tilde{X}_3, \tilde{X}_5, \tilde{e}_2>$.

IV. FUZZY BASED METAGRAPH ADT

An ADT is a set of operations such as intersection, union, set difference and set complement. What is to be done is mentioned but how is to be done is not mentioned. A Fuzzy Metagraph ADT involves the following operations.

- A single node is transformed to a new node after processing or transformation
- Two or more kinds of nodes are assembled to form a new node.
- Merging and clustering
- Search - BFS(Breadth First Search) and DFS (Depth First Search)
- Sorting - Topological Sort

```
Algorithm Fuzzy Metagraph based Set ADT Declaration (A [1 ….m], B [1…n])
{
 // input: A is having first fuzzy set and B is having second fuzzy set, both A and B are input sets, m is the total number of elements
in first set and n is the total number of elements in second set
   for i = 1 to m do
    {
       write A[ i];
     }
    for j = 1 to n do
    {
         write B[j];
     }
}
Algorithm Fuzzy set ADT (A [1 ….m], B [1…n])
{
//input: A is having first Fuzzy set and B is having second Fuzzy set. m is the total number of elements in first set and n is the total
number of elements in second set.
//output: The result set may be minimum or maximum.
min=max= A [1];
min1=max1= B [1];
for i=2 to n||m do
{
   if((B[i]<min1)||(A[i]<min))then
   {
```

```
    min1= B [i];
    min= A [i];
   }
   if(B[i]>max1||(A[i]>max))then
   {
   max1= B [i];
   max= A [i];
   }
}
if((min<min1)||(max<max1))  then
{
 Write (" intersection ",min);
 Write (" union ",max1);
}
else
{
Write (" intersection ",min1);
 Write (" union ",max);
}
}
}
```

The Fuzzy ADT algorithm can be applied to a graph structure to find out the minimum and the maximum values[10].

Algorithm **Metagraph based set intersection** (A [1 ….m], B [1…n], result [1….count])

```
{
//input: A is having first set and B is having second set. m is the total number of elements in first set and n is the total number of elements in second set
//output: A∩B result will be stored in result set; count is total number of elements in result set
Count ⟵ 0
          for i = 1 to m do
          {
            for j = 1 to n do
            {
              if( A[i] = B[j] ) then
                    {
                              count++;
                              result [count ]⟵ A[i];
                    }
            }
      }
   for i = 1 to count do
    {
           Write result[i];
    }
}
```

Algorithm **Metagraph based set union** (A [1 ….m], B [1…n], result [1….count])

```
{
//input: A is having first set and B is having second set, m is the total number of elements in first set, n is the total number of elements in second set
//output: A ∪ B result will be stored in result set; count is total number of elements in result set
          Count ⟵0, flag ⟵1;
           for i = 1 to m do
          {
                    Count++;
                     result[count ]⟵ A[i];
          }
          for i = 1 to n do
          {
              flag⟵ 1;
              for j = 1 to m do
```

```
        {
            if( B[i] = A[j] ) then
            {
            flag ← 0;
            break;
            }
            if (flag) then
          result[count ++ ] ← B[ i];

        }
      }
      for i = 1 to count do
      return result[i]
}
```

Time complexity analysis of Metagraph based set ADT
Input size: n and m
Basic operations: union, comparison and copy
Number of instructions to be executed per iteration: 1
To setup the mathematical equations:

$$= \sum_{i=1}^{m} 1 + \sum_{i=1}^{count} 1 + \sum_{i=1}^{n} \sum_{j=1}^{m} 1$$

$$= (m-1+1)+(count-1+1)+n(m-1+1) = m+ count + n.m \approx o(n.m)$$

If the number of elements of sets A and B are equal than m=n, the time complexity of set ADT is o (n^2)

Metagraph based Disjoint Sub Set and Union

Let x be the total number of elements in the set. It may have 2^x subsets. If A and B are Disjoint set then $A \cap B = \Phi$. So we propose an algorithm for metagraph based disjoint subset and union find algorithm. Figure 2, shows an example of Fuzzy based Metagraph ADT. Let us consider the Sub set elements A={ $\tilde{X}$ 1, $\tilde{X}$ 2},B={ $\tilde{X}$ 3, $\tilde{X}$ 4}),C={ $\tilde{X}$ 5},D={ $\tilde{X}$ 6, $\tilde{X}$ 7}, F={ $\tilde{X}$ 8},G={ $\tilde{X}$ 9, $\tilde{X}$ 10}, and H={ $\tilde{X}$ 11}. A, B, C $\subseteq V_1$, D,F$\subseteq V_2$. G, H $\subseteq V_3$, $A \cap B \cap C = \Phi$, $D \cap F = \Phi$, $G \cap H = \Phi$. Where A, B, C, D, F, G and H are collection of disjoint subset elements. From the above sets we develop an algorithm for Fuzzy based Metagraph disjoint subset and union find algorithm. It has been based on number of elements in the fuzzy set. It also obeys the principal of union and subset [3, 5, 10]. The time complexity of fuzzy based metagraph disjoint subset algorithm is o (n.m) where n is total number of cluster node which consists of number of vertices and m is total number of data elements in the cluster. This algorithm will help for making clustering in real world problem like medical, social science, wireless sensor network, MANET and Bioinformatics.

Given a Fuzzy Metagraph $\tilde{S}$ ={X, $\tilde{X}$, $\tilde{E}$ } and its adjacency matrix A is defined as an infinite sum, namely
$A^* = A + A^2 + A^3 + A^4 + \ldots + A^n,\ n \to \infty$

$$A^* = \sum_{n=1}^{\infty} A^n$$

The closure matrix A* for the Fuzzy Metagraph is formed by adding the successive powers of the adjacency matrices. Warshall algorithm is used for computing the transitive closure of the fuzzy metagraph. Reducing the edges by merging the vertices in the fuzzy metagraph based data structures. To identify the loops which are connected in graph, combine those vertices in to a single node by using union find algorithm. Strongly component Fuzzy Metagraph is DAG (Direct Acyclic Graph). If G has a cycle G=s1, s2, s3….sn then s=s1 U s2 U s3…U s_n will be strongly connected component.

Algorithm **Transitive closure of the Fuzzy Metagraph** (A[1…n])

```
{
 //Input: The adjacency matrix A of a fuzzy metagraph with n generating set.
 //Output: the transitive closure of the fuzzy metagraph
        R(0) = A
        for k=1 to n do
        for i=1 to n do
        for j=1 to n do
        R(k)[i ,j] = R(k-1)[i ,j] or ( R(k)[i ,k] and R(k)[k ,j])
        return R(n)
}
```

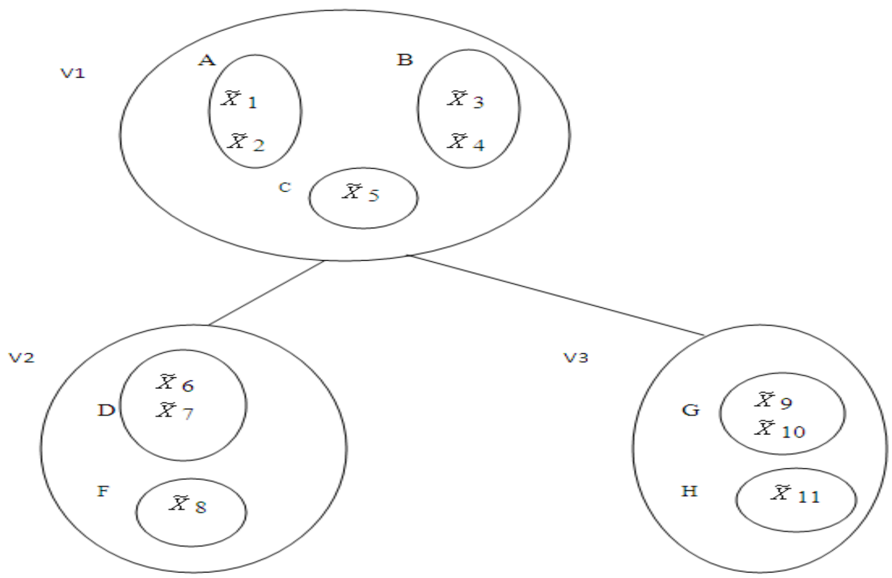

Fig.2. Example of Fuzzy based Metagraph ADT

Algorithm **Fuzzy Based Metagraph ADT** ()

{

// **input:** V= $\{V_1,V_2,V_3,\ldots V_n\}$ is finite non empty set of nodes, where $V_1,V_2,..V_n$ is collection of disjoint subset elements (or) collection of fuzzy metagraph vertices. For each node may have one or more elements and these nodes are disjoint element.

$\tilde{X} = \{ \tilde{X}1, \tilde{X}2, \tilde{X}3 \ldots\ldots \tilde{X}m\}$ where $\tilde{X}$ is collection of data elements.

// **output:** $\tilde{X}1 \cup \tilde{X}2 \cup \tilde{X}3 \cup \tilde{X}4 \ldots\ldots \cup \tilde{X}_m$ or $V= V_1 \cup V_2 \cup V_3 \cup V_{4,\ldots} V_{n-1} \cup V_n$

// $\tilde{X}1 \cap \tilde{X}2 \cap \tilde{X}3 \cap \tilde{X}4 \ldots\ldots \cap \tilde{X}_m = \Phi$.

for j=1 **to** n **do**

{

for i = 2 **to** m **do**

{

if (($\tilde{X}$ i or V_j)= threshold value) then

{

Union ($\tilde{X}_i$, $\tilde{X}_{i+1,\ldots\ldots} \tilde{X}_m$);

break;

}

}

Assign edge for ($V_{j-1}, V_j, V_{j+1\ldots\ldots} V_n$);

break;

}

}

V. CONCLUSION

Fuzzy metagraph is best suited for a number of applications including biometrics, transaction processing systems, workflow systems and so on. Distinct algorithms have been proposed for Fuzzy based Metagraph ADT like union, intersection, disjoint subset union and searching. The algorithm has shown an excellent performance in terms of reduction in time complexity and space complexity. The graphical model not only visualized the process of any system but also their formal analysis where the analysis will be accomplished by means of an algebraic representation of the graphical structure. Data will be stored inside the computer memory either in the form of Adjacency matrix or in Adjacency list so that it can be used efficiently. Future works can be concentrated on applying Fuzzy Metagraph ADT to clustering techniques to enhance the performance of the system.

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