Transcendental equation calculated analytically obeyed by quasi-bound energy levels of quantum well and resonant transmission peaks

M. A. Samad[1], S. Chowdhury[2]

[1] Department of Physics, Jessore University of Science and Technology, Jessore-7408, Bangladesh
[2] Department of Physics, Shahjalal University of Science and Technology, Sylhet-3114, Bangladesh

DOI: 10.29322/IJSRP.8.11.2018.p8386
http://dx.doi.org/10.29322/IJSRP.8.11.2018.p8386

Abstract- We present the analytical calculation of transcendental equation obeyed by quasi-bound energy levels of electron for motion perpendicular to interfaces of the Quantum Well and Nanostructure Physics of non-isolated Quantum Well of symmetric rectangular double barrier. In this investigation we need to calculate expressions of elements of transfer matrix of the left hand rectangular tunnel barrier (T_L matrix) and expressions of elements of inverse transfer matrix of the right hand rectangular tunnel barrier (IT_R matrix) to get the desired calculation. In particular, we shall carry out thorough and complete analytical calculation leading to the transcendental equation obeyed by allowed values of kinetic or total energy of an electron in the non-isolated QW for motion along x direction only. As is evident, we shall deal with a 1D problem or calculation. It is also observed the resonant transmission peaks of symmetric rectangular double barrier obey the same condition. The Quantum Well structure can be realized with GaAs as wells and AlGaAs as barriers for wavelength about 0.8 µm. We anticipate that the application of Quantum Well might be worthwhile for potential applications in novel multifunctional devices. These devices are monolithically integrated with various optoelectronic devices to provide photonic integrated circuit with increased functionality. In addition, quantum well devices find their applications in quantum well lasers or improved lasers, photo detectors, modulators and switches and operate much faster, more economically and have led to a million increases in speed.

Index Terms- Quantum Well (QW), quasi-bound energy levels, resonant transmission peaks, transcendental equation

I. Introduction

The study of quantum mechanics is now a great interesting subject area to understand the physical characteristics of microscopic particles and to describe the behavior of matter and energy at the sub atomic scale. In nanometer scale (nm, 10^{-9} m) which is comparable to the electron coherence length and wavelength, quantum mechanical effects originated from undulatory properties of electrons become more tangible and essential, so that the conventional semi classical theory becomes inapplicable. Hence researchers are mostly attracted in this field for its unprecedented features. It is after 1970's that the quantum-well (QW) is experimentally observed in potential barriers made in semiconductor-on-semiconductor systems or semiconductor super lattices. This research evaluates the nanostructure physics of non-isolated quantum well of symmetric rectangular double barrier. Quantum wells are formed in semiconductors by having a material, like gallium arsenide (GaAs), sandwiched between two layers of a material with a wider band gap, like aluminium gallium arsenide (AlGaAs). These structures can be grown by molecular beam epitaxy or chemical vapor deposition with control of the layer thickness down to monolayers. A thin layer of GaAs sandwiched between two thick layers of AlGaAs can provide us a QW. Since the width of the semiconductor material is comparable to the de Broglie wavelength, size quantization will result and electrons and holes will be confined in the small band gap semiconductors, where their potential energies are lowest. Because of the confinement of electron or hole by two sides of the QW, energy spectrum of an electron or hole in QW for motion perpendicular to the GaAs-AlGaAs interfaces becomes quantized or discrete. The states associated with these confined levels are called bound states, because an electron in such a state essentially cannot leak out of the well and remains essentially confined within the QW, because the barriers on either side are thick. If we impose periodic boundary condition on wave function associated with motion of an electron parallel to interfaces of QW, the continuous energy spectrum in fact becomes quasi-continuous. In our investigation, We shall obtain a condition that is obeyed by allowed values of kinetic or total energy of an electron in the QW for motion along x direction only i.e. for motion perpendicular to interfaces of the QW. As we shall see, the condition will come up in the form of a transcendental equation and the resonant transmission peaks of symmetric rectangular double barrier also obey the same condition. We shall carry out thorough and complete analytical calculation leading to the transcendental equation and finding the similarities that the values of energy at T versus E peaks of symmetric rectangular double barrier are the same as allowed values of energy of quasi-bound levels in the non- isolated QW.

II. Details of the Problem

In this investigation, a thin layer of GaAs sandwiched between two thick layers of AlGaAs to form quantum well. Figure 1 (a) shows structure of symmetric rectangular double barrier. The two AlGaAs layers are identical in width and in Al content. Figure 1 (b) shows band model of symmetric rectangular double barrier.

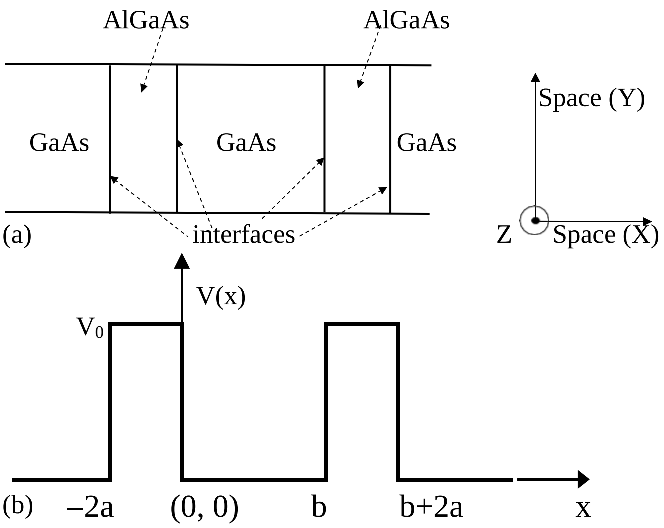

Figure 1: (a) Structure, (b) band model of symmetric rectangular double barrier showing lower edge of conduction band Ec as a function of x.

In the band model we have potential profile of a Quantum Well (QW) between two identical rectangular tunnel barriers. An electron in the QW can feel the Quantum Well confinement for its motion along x direction only, i.e. for motion perpendicular to interfaces of the QW. There are four GaAs-AlGaAs interfaces, which are all parallel to YZ plane. The QW is a non-isolated QW because an electron residing in the QW can tunnel through either barrier and leak out of the QW never to return. Because of small (Nano scale) width b, energy of an electron inside the QW for motion along x direction can have some discrete allowed values only. Potential energy inside the QW has been taken as zero by choice of origin in Figure 1. As such, kinetic energy is the total energy of an electron in the QW.

We shall obtain a condition that is obeyed by allowed values of kinetic or total energy of an electron in the QW for motion along x direction only i.e. for motion perpendicular to interfaces of the QW. It also be seen that the resonant transmission peaks of symmetric rectangular double barrier obey the same condition.

III. Methodology and Involved Physics

Quantum wells can be grown in semiconductors by molecular beam epitaxy or chemical vapor deposition with control of the layer thickness down to monolayers. A thin layer of semiconductor sandwiched between another two thick layers of semiconductor can provide us a QW. Here, a thin layer of GaAs sandwiched between two thick layers of AlGaAs and provided us a Quantum Well. The methodology and involved physics for finding out the transcendental equation is depicted in Figure 2.

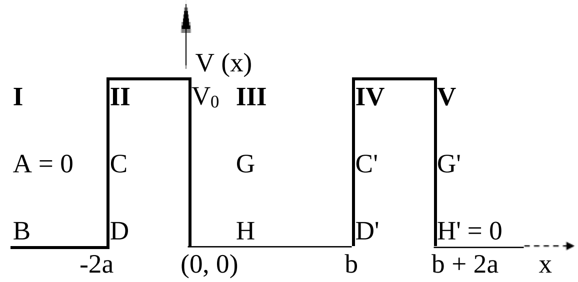

Figure 2: Figure to help describe methodology of the calculation

If we have variation of potential V(x) as shown in the figure 2, we have a one-dimensional, double, rectangular potential barrier. If the width and height of the barrier are finite and small, we have tunnel barrier of width 2a and height V_0.
There are five regions as shown. According to the choice of origin in Figure 2, V(x) = 0 in region I, III and V and V(x) = V_0 in region II and IV. Solutions of time-independent Schrödinger equation $[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)]u(x)=Eu(x)$

$$or,\ [\frac{d^2u}{dx^2}+\frac{2m}{\hbar^2}(E-V(x))u(x)]=0$$

in the five regions are given by

$$u_1(x)=Ae^{ikx}+Be^{-ikx}$$

$$u_2(x)=Ce^{\beta x}+De^{-\beta x}$$

$$u_3(x)=Ge^{ikx}+He^{-ikx}$$

$$u_4(x)=C'e^{\beta x}+D'e^{-\beta x}$$

$u_5(x)=G'e^{ikx}+H'e^{-ikx}$ Where $k^2=\frac{2mE}{\hbar^2}$ and $\beta^2=\frac{2m(V_0-E)}{\hbar^2}$

And u_i's are space part of solutions of time independent Schrödinger equation which is

$$\psi(\vec{r},t)=u(\vec{r})f(t)=u(\vec{r})e^{-\frac{i}{\hbar}Et}=u(\vec{r})e^{-i\omega t}$$

We first recognize that G and H are amplitude of plane waves travelling along + x and - x directions inside the non-isolated Quantum Well between the two tunnel barriers. An electron residing in the QW is reflecting back and forth between the two barriers with these two plane travelling waves associated with it. Every time the electron is incident on the right hand tunnel barrier, there is a probability that it tunnels the barrier and leaks out of the QW, with the plane wave of amplitude G′ with it, never to return. As such H′ = 0. H′ is the amplitude of plane wave with which the electron would attempt to return to the QW if it would return at all, which it does not.

Again, every time the electron is incident on the left hand tunnel barrier, there is a probability that it tunnels the barrier and leaks out of the QW, with the plane wave of amplitude B with it, never to return. As such A = 0. A is the amplitude of plane wave with which the electron would attempt to return to the QW if it would return at all, which it does not.

We shall use boundary conditions like $u_1 = u_2$ and $\frac{du_1}{dx} = \frac{du_2}{dx}$ at the finite potential discontinuities of Figure 2 and obtain the relations

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} T_{L11} & T_{L12} \\ T_{L21} & T_{L22} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \quad \text{............... (1)}$$

and

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} IT_{R11} & IT_{R12} \\ IT_{R21} & IT_{R22} \end{pmatrix} \begin{pmatrix} G' \\ H' \end{pmatrix} \quad \text{............... (2)}$$

where T_L matrix is transfer matrix of the left hand rectangular tunnel barrier and IT_R matrix is inverse transfer matrix of the right hand rectangular tunnel barrier. Equation (1) and (2) give

$$\begin{pmatrix} T_{L11} & T_{L12} \\ T_{L21} & T_{L22} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} IT_{R11} & IT_{R12} \\ IT_{R21} & IT_{R22} \end{pmatrix} \begin{pmatrix} G' \\ H' \end{pmatrix} \quad \text{...... (3)}$$

As already explained, we know from physical ground that A=0 and H′= 0. We now use these in equation (3) and get

$$\begin{pmatrix} T_{L11} & T_{L12} \\ T_{L21} & T_{L22} \end{pmatrix} \begin{pmatrix} 0 \\ B \end{pmatrix} = \begin{pmatrix} IT_{R11} & IT_{R12} \\ IT_{R21} & IT_{R22} \end{pmatrix} \begin{pmatrix} G' \\ 0 \end{pmatrix} \quad \text{..... (4)}$$

Equation (4) yields

$$T_{L12} B = IT_{R11} G' \quad \text{...... (5)}$$

and

$$T_{L12} B = IT_{R11} G' \quad \text{.............(6)}$$

Dividing equation (5) by (6), we get

$$\frac{T_{L12}}{T_{L22}} = \frac{IT_{R11}}{IT_{R21}}$$

or, $$T_{L12} IT_{R21} = T_{L22} IT_{R11} \quad (7)$$

which is the condition obeyed by allowed values of energy E of an electron in the non-isolated QW for motion along x direction only. We need to obtain analytical expressions of elements of transfer matrix of left hand tunnel barrier (T_L matrix) and we also need to obtain analytical expressions of elements of inverse transfer matrix of right hand tunnel barrier (IT_R matrix) and use them in equation (7) to get the transcendental equation (condition) obeyed by allowed values of energy E of an electron in the non-isolated QW for motion along x direction only. The allowed values of energy E are kinetic as well as total energy of an electron for motion along x direction inside the non-isolated QW between the two tunnel barriers; this is because potential energy of electron inside the QW is zero by choice of origin. In the following sections, we calculate expressions of elements of T_L matrix and of IT_R matrix and then obtain the transcendental equation.

IV. CALCULATION OF TRANSFER MATRIX OF THE LEFT HAND RECTANGULAR TUNNEL BARRIER

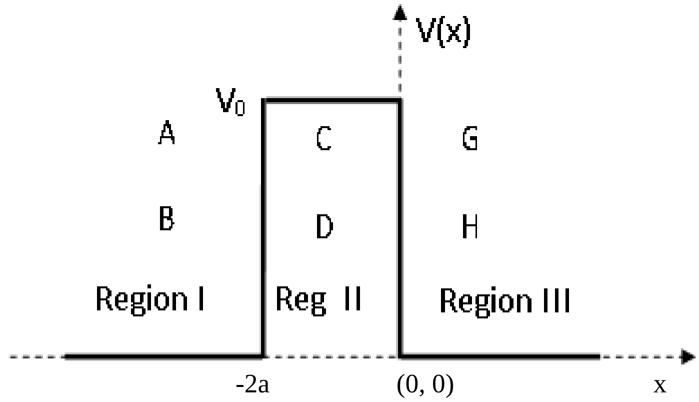

Figure 3: Left hand rectangular tunnel barrier

According to the choice of origin, V(x) is zero in region I and III and is V_0 in region II. Solutions of time-independent Schrödinger equation in the three regions are given by

$$u_1(x) = Ae^{ikx} + Be^{-ikx} \quad \text{where} \quad k^2 = \frac{2mE}{\hbar^2}$$

$$u_2(x) = Ce^{\beta x} + De^{-\beta x} \quad \text{and} \quad \beta^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

$$u_3(x) = Ge^{ikx} + He^{-ikx}$$

The expressions for u_2 and β^2 imply that we are considering free electrons of kinetic energy less than V_0 impinging on the barrier from the left.

According to boundary condition $u_1 = u_2$ at x = -2a, we get

$$Ae^{-ik2a} + Be^{ik2a} = Ce^{-2\beta a} + De^{2\beta a} \quad \text{........ (8)}$$

$\frac{du_1}{dx} = \frac{du_2}{dx}$ at x = -2a gives

$$ik\,Ae^{ikx} - ikBe^{-ikx} = \beta Ce^{\beta x} - \beta De^{-\beta x} \text{ at x =-2a}$$

$$ikAe^{-ik2a} - ikBe^{ik2a} = \beta Ce^{-2\beta a} - \beta De^{2\beta a}$$

$$\Rightarrow \frac{ik}{\beta} Ae^{-ik2a} - \frac{ik}{\beta} Be^{ik2a} = Ce^{-2\beta a} - De^{2\beta a} \quad \text{...... (9)}$$

Again, $u_2 = u_3$ at x = 0 gives

$$C + D = G + H \quad \text{........ (10)}$$

$\frac{du_2}{dx} = \frac{du_3}{dx}$ at x = 0 gives

$$\beta Ce^{\beta x} - \beta De^{-\beta x} = ik\,Ge^{ikx} - ikHe^{-ikx} \text{ at x = 0}$$

$$\Rightarrow \beta C - \beta D = ikG - ikH$$

$$\Rightarrow \frac{\beta C}{ik} - \frac{\beta D}{ik} = G - H \quad \text{........ (11)}$$

Adding equation (8) and (9), we get

$$2Ce^{-2\beta a} = \left(1 + \frac{ik}{\beta}\right) Ae^{-ik2a} + \left(1 - \frac{ik}{\beta}\right) \beta e^{ik2a}$$

$$\Rightarrow C = \frac{1}{2}\left(1 + \frac{ik}{\beta}\right) Ae^{-ik2a+2\beta a} + \frac{1}{2}\left(1 - \frac{ik}{\beta}\right) \beta e^{ik2a+2\beta a}$$

Subtracting equation (9) from equation (8) we get

$$2De^{2\beta a}=\left(1-\frac{ik}{\beta}\right)Ae^{-ik2a}+\left(1+\frac{ik}{\beta}\right)\beta e^{ik2a}$$

$$\Rightarrow D=\frac{1}{2}\left(1-\frac{ik}{\beta}\right)Ae^{-ik2a-2\beta a}+\frac{1}{2}\left(1+\frac{ik}{\beta}\right)\beta e^{ik2a-2\beta a}$$

In matrix form these can be written as

$$\begin{pmatrix}C\\D\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{-i2ka+2\beta a} & \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{i2ka+2\beta a}\\ \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{-i2ka-2\beta a} & \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{i2ka-2\beta a}\end{pmatrix}\begin{pmatrix}A\\B\end{pmatrix} \quad \ldots (12)$$

Adding equation (10) and (11), we get

$$G=\frac{1}{2}\left(1+\frac{\beta}{ik}\right)C+\frac{1}{2}\left(1-\frac{\beta}{ik}\right)D$$

Subtracting equation (11) from equation (10) we get

$$H=\frac{1}{2}\left(1-\frac{\beta}{ik}\right)C+\frac{1}{2}\left(1+\frac{\beta}{ik}\right)D$$

In matrix form these can be written as

$$\begin{pmatrix}G\\H\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\left(1+\frac{\beta}{ik}\right) & \frac{1}{2}\left(1-\frac{\beta}{ik}\right)\\ \frac{1}{2}\left(1-\frac{\beta}{ik}\right) & \frac{1}{2}\left(1+\frac{\beta}{ik}\right)\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix} \quad \ldots\ldots.. (13)$$

Using equation (12) in equation (13), we get

$$\begin{pmatrix}G\\H\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\left(1+\frac{\beta}{ik}\right) & \frac{1}{2}\left(1-\frac{\beta}{ik}\right)\\ \frac{1}{2}\left(1-\frac{\beta}{ik}\right) & \frac{1}{2}\left(1+\frac{\beta}{ik}\right)\end{pmatrix}\begin{pmatrix}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{-i2ka+2\beta a} & \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{i2ka+2\beta a}\\ \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{-i2ka-2\beta a} & \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{i2ka-2\beta a}\end{pmatrix}\begin{pmatrix}A\\B\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}G\\H\end{pmatrix}=\begin{pmatrix}T_{L11} & T_{L12}\\T_{L21} & T_{L22}\end{pmatrix}\begin{pmatrix}A\\B\end{pmatrix} \quad \ldots\ldots.. (14)$$

We get

$$T_{L11}=\frac{1}{2}\left(1+\frac{\beta}{ik}\right)\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{-i2ka+2\beta a}+\frac{1}{2}\left(1-\frac{\beta}{ik}\right)\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{-i2ka-2\beta a}$$

$$=\frac{1}{4}[\left(1+\frac{\beta}{ik}\right)\left(1+\frac{ik}{\beta}\right)e^{2\beta a}+\left(1-\frac{\beta}{ik}\right)\left(1-\frac{ik}{\beta}\right)e^{2\beta a}]e^{-i2ka}$$

$$=\frac{1}{4}[(1+\frac{\beta}{ik}+\frac{ik}{\beta}+1)e^{2\beta a}+(1-\frac{\beta}{ik}-\frac{ik}{\beta}+1)e^{-2\beta a}]e^{-i2ka}$$

$$=[\frac{1}{2}(e^{2\beta a}+e^{-2\beta a})+\frac{1}{2}(\frac{\beta}{ik}+\frac{ik}{\beta})(\frac{e^{2\beta a}-e^{-2\beta a}}{2})]e^{-i2ka}$$

$$=[\cosh 2\beta a+\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{-i2ka} \quad \ldots (15)$$

$$T_{L22}=\frac{1}{2}\left(1-\frac{\beta}{ik}\right)\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{i2ka+2\beta a}+\frac{1}{2}\left(1+\frac{\beta}{ik}\right)\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{i2ka-2\beta a}$$

$$=\frac{1}{4}[\left(1-\frac{\beta}{ik}\right)\left(1-\frac{ik}{\beta}\right)e^{2\beta a}+\left(1+\frac{\beta}{ik}\right)\left(1+\frac{ik}{\beta}\right)e^{-2\beta a}]e^{i2ka}$$

$$=\frac{1}{4}[(1-\frac{\beta}{ik}-\frac{ik}{\beta}+1)e^{2\beta a}+(1+\frac{\beta}{ik}+\frac{ik}{\beta}+1)e^{-2\beta a}]e^{i2ka}$$

$$=[\frac{1}{2}(e^{2\beta a}+e^{-2\beta a})-\frac{1}{2}(\frac{\beta}{ik}+\frac{ik}{\beta})(\frac{e^{2\beta a}-e^{-2\beta a}}{2})]e^{i2ka}$$

$$=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka} \quad \ldots (16)$$

$$T_{L12}=\frac{1}{2}\left(1+\frac{\beta}{ik}\right)\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{i2ka+2\beta a}+\frac{1}{2}\left(1-\frac{\beta}{ik}\right)\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{i2ka-2\beta a}$$

$$=\frac{1}{4}[\left(1+\frac{\beta}{ik}\right)\left(1-\frac{ik}{\beta}\right)e^{2\beta a}+\left(1-\frac{\beta}{ik}\right)\left(1+\frac{ik}{\beta}\right)e^{-2\beta a}]e^{i2ka}$$

$$=\frac{1}{4}[(1+\frac{\beta}{ik}-\frac{ik}{\beta}-1)e^{2\beta a}+(1-\frac{\beta}{ik}+\frac{ik}{\beta}-1)e^{-2\beta a}]e^{i2ka}$$

$$=\frac{1}{2}(\frac{\beta}{ik}-\frac{ik}{\beta})(\frac{e^{2\beta a}-e^{-2\beta a}}{2})]e^{i2ka}$$

$$=[\frac{1}{2}(\frac{\beta}{ik}-\frac{ik}{\beta})\sinh 2\beta a]e^{i2ka}$$

$$=[-\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a]e^{i2ka} \quad \ldots (17)$$

$$T_{L21}=\frac{1}{2}\left(1-\frac{\beta}{ik}\right)\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{-i2ka+2\beta a}+\frac{1}{2}\left(1+\frac{\beta}{ik}\right)\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{-i2ka-2\beta a}$$

$$=\frac{1}{4}[\left(1-\frac{\beta}{ik}\right)\left(1+\frac{ik}{\beta}\right)e^{2\beta a}+\left(1+\frac{\beta}{ik}\right)\left(1-\frac{ik}{\beta}\right)e^{-2\beta a}]e^{-i2ka}$$

$$=\frac{1}{4}[(1-\frac{\beta}{ik}+\frac{ik}{\beta}-1)e^{2\beta a}+(1+\frac{\beta}{ik}-\frac{ik}{\beta}-1)e^{-2\beta a}]e^{-i2ka}$$

$$=\frac{1}{2}(\frac{ik}{\beta}-\frac{\beta}{ik})(\frac{e^{2\beta a}-e^{-2\beta a}}{2})e^{-i2ka}$$

$$=[\frac{1}{2}(\frac{ik}{\beta}-\frac{\beta}{ik})\sinh 2\beta a]e^{-i2ka}$$

$$=\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a]e^{-i2ka} \quad \ldots (18)$$

Comparison of equation (13) to (16) shows that the elements of transfer matrix T_L obey the following properties.

$$T^{*}_{L11}=T_{L22} \quad \text{or, } T^{*}_{L22}=T_{L11} \quad \ldots\ldots.. (19)$$

$$T_{L12}=T^{*}_{L21} \quad \text{or, } T_{L21}=T^{*}_{L12} \quad \ldots\ldots.. (20)$$

V. CALCULATION OF INVERSE TRANSFER MATRIX OF THE RIGHT HAND RECTANGULAR TUNNEL BARRIER

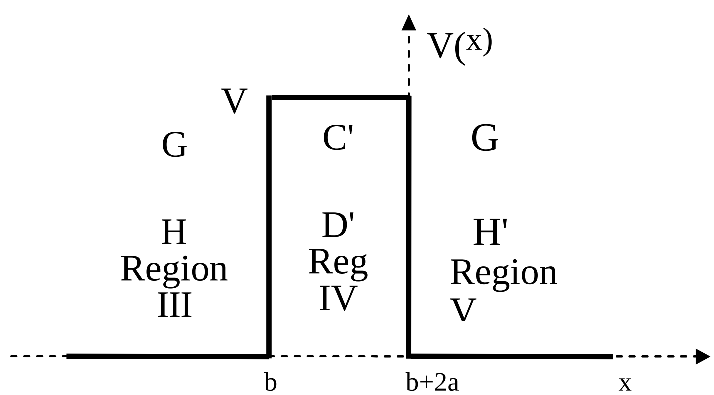

Figure 4: Right hand rectangular tunnel barrier

Solutions of time-independent Schrödinger equation in the three regions are as follows.

$$u_3(x)=Ge^{ikx}+He^{-ikx} \quad \text{where } k^2=\frac{2mE}{\hbar^2}$$

$$u_4(x)=C'e^{\beta x}+D'e^{-\beta x} \quad \text{and } \beta^2=\frac{2m(V_0-E)}{\hbar^2}$$

$$u_5(x)=G'e^{ikx}+H'e^{-ikx}$$

We are considering the case with E < V_0. Using the boundary condition $u_3 = u_4$ at x = b, we get

$$Ge^{ikb} + He^{-ikb} = C'e^{\beta b} + D'e^{-\beta b} \quad \text{........ (21)}$$

and $\frac{du_3}{dx} = \frac{du_4}{dx}$ at x = b gives

$$ik\,Ge^{ikx} - ikHe^{-ikx} = \beta C'e^{\beta x} - \beta D'e^{-\beta x} \text{ at x = b}$$

or, $$Ge^{ikb} - He^{-ikb} = \frac{\beta}{ik}C'e^{\beta b} - \frac{\beta}{ik}D'e^{-\beta b} \quad \text{...... (22)}$$

Again, $u_4 = u_5$ at x =b+2a gives

$$C'e^{\beta(b+2a)} + D'e^{-\beta(b+2a)} = G'e^{ik(b+2a)} + H'e^{-ik(b+2a)} \quad \text{.... (23)}$$

and $\frac{du_4}{dx} = \frac{du_5}{dx}$ at x =b+2a gives

$$\beta C'e^{\beta x} - \beta D'e^{-\beta x} = ik\,G'e^{ikx} - ikH'e^{-ikx} \text{ at x =b+2a}$$

$$\Rightarrow \beta C'e^{\beta(b+2a)} - \beta D'e^{-\beta(b+2a)} = ikG'e^{ik(b+2a)} - ikH'e^{-ik(b+2a)}$$

$$\Rightarrow C'e^{\beta(b+2a)} - D'e^{-\beta(b+2a)} = \frac{ik}{\beta}e^{ik(b+2a)}G' - \frac{ik}{\beta}e^{-ik(b+2a)}H' \quad \text{... (24)}$$

Adding equation (21) and (22), we get

$$2Ge^{ikb} = \left(1+\frac{\beta}{ik}\right)e^{\beta b}C' + \left(1-\frac{\beta}{ik}\right)e^{-\beta b}D'$$

$$\Rightarrow G = \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{\beta b-ikb}C' + \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{-\beta b-ikb}D' \quad \text{... (25)}$$

Subtracting equation (22) from equation (21) we get

$$2He^{-ikb} = \left(1-\frac{\beta}{ik}\right)e^{\beta b}C' + \left(1+\frac{\beta}{ik}\right)e^{-\beta b}D'$$

$$\Rightarrow H = \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{\beta b+ikb}C' + \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{-\beta b+ikb}D' \quad \text{... (26)}$$

In matrix form these can be written as

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{\beta b-ikb} & \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{-\beta b-ikb} \\ \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{\beta b+ikb} & \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{-\beta b+ikb} \end{pmatrix} \begin{pmatrix} C' \\ D' \end{pmatrix} \quad \text{... (27)}$$

Adding equation (23) and (24), we get

$$2C'e^{\beta(b+2a)} = \left(1+\frac{ik}{\beta}\right)e^{ik(b+2a)}G' + \left(1-\frac{ik}{\beta}\right)e^{-ik(b+2a)}H'$$

$$\Rightarrow C' = \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(ik-\beta)(b+2a)}G' + \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(-ik-\beta)(b+2a)}H' \quad \text{... (28)}$$

Subtracting equation (24) from equation (23) we get

$$2D'e^{-\beta(b+2a)} = \left(1-\frac{ik}{\beta}\right)e^{ik(b+2a)}G' + \left(1+\frac{ik}{\beta}\right)e^{-ik(b+2a)}H'$$

$$\Rightarrow D' = \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(ik+\beta)(b+2a)}G' + \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(-ik+\beta)(b+2a)}H' \quad \text{... (29)}$$

In matrix form equation (28) and (29) can be written as

$$\begin{pmatrix} C' \\ D' \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(ik-\beta)} & \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(-ik-\beta)} \\ \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(ik+\beta)} & \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(-ik+\beta)} \end{pmatrix} \begin{pmatrix} G' \\ H' \end{pmatrix} \quad \text{... (30)}$$

Equation (27) and (30), give

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{\beta b-ikb} & \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{-\beta b-ikb} \\ \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{\beta b+ikb} & \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{-\beta b+ikb} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(ik-\beta)} & \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(-ik-\beta)} \\ \frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(ik+\beta)} & \frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(-ik+\beta)} \end{pmatrix} \begin{pmatrix} G' \\ H' \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} IT_{R11} & IT_{R12} \\ IT_{R21} & IT_{R22} \end{pmatrix} \begin{pmatrix} G' \\ H' \end{pmatrix} \quad \text{........ (31)}$$

We now obtain expressions of elements of the inverse transfer matrix IT_R.

$$IT_{R11} = \frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{(\beta b-ikb)}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(ik-\beta)}$$

$$+\frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{(-\beta b-ikb)}\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(ik+\beta)}$$

$$=\frac{1}{4}[(1+\frac{\beta}{ik}+\frac{ik}{\beta}+1)e^{(\beta-ik)(b-b-2a)} + \frac{1}{4}(1-\frac{\beta}{ik}-\frac{ik}{\beta}+1)e^{(b+2a-b)(ik+\beta)}$$

$$=\frac{1}{4}[(2+\frac{\beta}{ik}+\frac{ik}{\beta})e^{-2\beta a+i2ka} + \frac{1}{4}(2-\frac{\beta}{ik}-\frac{ik}{\beta})e^{2\beta a+i2ka}$$

$$=\frac{1}{2}(e^{-2\beta a}+e^{2\beta a})e^{i2ka} + \frac{1}{2}(\frac{\beta}{ik}+\frac{ik}{\beta})(\frac{e^{-2\beta a}-e^{2\beta a}}{2})e^{i2ka}$$

$$=[\cosh 2\beta a - \frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka} \quad \text{... (32)}$$

$$IT_{R22} = \frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{(\beta b+ikb)}\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(-ik-\beta)}$$

$$+\frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{(-\beta b+ikb)}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(-ik+\beta)}$$

$$=\frac{1}{4}[(1-\frac{\beta}{ik}-\frac{ik}{\beta}+1)e^{(\beta+ik)(b-b-2a)} + \frac{1}{4}(1+\frac{\beta}{ik}+\frac{ik}{\beta}+1)e^{(b-b-2a)(ik-\beta)}$$

$$=\frac{1}{4}[(2-\frac{\beta}{ik}-\frac{ik}{\beta})e^{-2\beta a-i2ka} + \frac{1}{4}(2+\frac{\beta}{ik}+\frac{ik}{\beta})e^{2\beta a-i2ka}$$

$$=\frac{1}{2}(e^{-2\beta a}+e^{2\beta a})e^{-i2ka} + \frac{1}{2}(\frac{\beta}{ik}+\frac{ik}{\beta})(\frac{e^{2\beta a}-e^{-2\beta a}}{2})e^{-i2ka}$$

$$=[\cosh 2\beta a + \frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{-i2ka} \quad \text{... (33)}$$

Comparison of equation (32) and (33) shows that

$$IT^{*}_{R11} = IT_{R22} \quad \text{........ (34)}$$

$$IT_{R12}=\frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{(\beta b-ikb)}\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(-ik-\beta)}$$

$$+\frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{(-\beta b-ikb)}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(-ik+\beta)}$$

$$=\frac{1}{4}[(1+\frac{\beta}{ik}-\frac{ik}{\beta}-1)e^{\beta b-ikb-ikb-ik2a-\beta b-2\beta a}$$

$$+\frac{1}{4}(1-\frac{\beta}{ik}+\frac{ik}{\beta}-1)e^{-\beta b-ikb-ikb-i2ka+\beta b+2\beta a}$$

$$=\frac{1}{4}(\frac{\beta}{ik}-\frac{ik}{\beta})e^{-i2kb-i2ka-2\beta a}+\frac{1}{4}(-\frac{\beta}{ik}+\frac{ik}{\beta})e^{-i2kb-i2ka+2\beta a}$$

$$=\frac{1}{2}(\frac{\beta}{ik}-\frac{ik}{\beta})(\frac{e^{-2\beta a}-e^{2\beta a}}{2})e^{-i2ka-i2kb}$$

$$=[\frac{i}{2}(\frac{k}{\beta}+\frac{\beta}{k})\sinh 2\beta a]e^{-i2k(a+b)} \quad \ldots (35)$$

$$IT_{R21}=\frac{1}{2}\left(1-\frac{\beta}{ik}\right)e^{(\beta b+ikb)}\frac{1}{2}\left(1+\frac{ik}{\beta}\right)e^{(b+2a)(ik-\beta)}$$

$$+\frac{1}{2}\left(1+\frac{\beta}{ik}\right)e^{(-\beta b+ikb)}\frac{1}{2}\left(1-\frac{ik}{\beta}\right)e^{(b+2a)(ik+\beta)}$$

$$=\frac{1}{4}[(1-\frac{\beta}{ik}+\frac{ik}{\beta}-1)e^{\beta b+ikb+ikb+i2ka-\beta b-2\beta a}$$

$$+\frac{1}{4}(1+\frac{\beta}{ik}-\frac{ik}{\beta}-1)e^{-\beta b+ikb+ikb+i2ka+\beta b+2\beta a}$$

$$=\frac{1}{4}(-\frac{\beta}{ik}+\frac{ik}{\beta})e^{-2\beta a+i2ka+i2kb}-\frac{1}{4}(-\frac{\beta}{ik}+\frac{ik}{\beta})e^{i2\beta a+i2ka+i2kb}$$

$$=\frac{1}{2}(-\frac{\beta}{ik}+\frac{ik}{\beta})(\frac{e^{-2\beta a}-e^{2\beta a}}{2})e^{i2ka+i2kb}$$

$$=[-\frac{i}{2}(\frac{k}{\beta}+\frac{\beta}{k})\sinh 2\beta a]e^{i2k(a+b)} \quad \ldots (36)$$

From equation (35) and (36), we find that

$$IT^{*}_{R12}=IT_{R21} \quad \ldots\ldots\ldots (37)$$

V. CALCULATION OF TRANSCENDENTAL EQUATION OBEYED BY QUASI-BOUND ENERGY LEVELS

We now gather

equation (17): $T_{L12}=[-\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a]e^{i2ka}$

equation(16): $T_{L22}=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka}$

equation (32): $IT_{R11}=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka}$

equation (36): $IT_{R21}=[-\frac{i}{2}(\frac{k}{\beta}+\frac{\beta}{k})\sinh 2\beta a]e^{i2k(a+b)}$

to use in equation (7): $T_{L12}IT_{R21}=T_{L22}IT_{R11}$ to get the transcendental equation obeyed by allowed values of energy E of an electron in the non-isolated Quantum Well of the symmetric rectangular double barrier for motion perpendicular to interfaces of the Quantum Well.

Thus we get

$$[-\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a]e^{i2ka}[-\frac{i}{2}(\frac{k}{\beta}+\frac{\beta}{k})\sinh 2\beta a]e^{i2k(a+b)}$$

$$=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka}[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]e^{i2ka}$$

$$\Rightarrow[\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a e^{ikb}]^2=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a]^2$$

$$\Rightarrow\frac{i}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a\, e^{ikb}=[\cosh 2\beta a-\frac{i}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a \quad \ldots\ldots (38)$$

Equating real parts of equation (38) we get

$$-\frac{1}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a\sin kb=\cosh 2\beta a \quad \ldots\ldots (39)$$

and equating imaginary parts of equation (38) we get

$$\frac{1}{2}(\frac{\beta}{k}+\frac{k}{\beta})\sinh 2\beta a\cos kb=-\frac{1}{2}(\frac{k}{\beta}-\frac{\beta}{k})\sinh 2\beta a \quad \ldots (40)$$

Dividing equation (40) by equation (39) we get

$$\cot kb=\frac{1}{2}(\frac{k}{\beta}-\frac{\beta}{k})\tanh 2\beta a \quad \ldots (41)$$

where $k^2=\frac{2mE}{\hbar^2}$ b = width of QW

and $\beta^2=\frac{2m(V_0-E)}{\hbar^2}$ 2a = width of each barrier.

Equation (41) is the condition or transcendental equation obeyed by quasi-bound energy levels of non-isolated Quantum Well of symmetric rectangular double barrier. In other words, equation (41) is the condition or transcendental equation obeyed by allowed values of kinetic or total energy of an electron in the QW for motion along x direction only i.e. for motion perpendicular to interfaces of the QW.

VI. RESONANT TRANSMISSION PEAKS OBEY THE SAME CONDITION

Resonant transmission peaks of symmetric rectangular double barrier obey the condition

$$\cos[k(2a+b)-\theta]=0 \quad \ldots\ldots\ldots (42)$$

Where $\theta=-\tan^{-1}[(\frac{k^2-\beta^2}{2k\beta})\tanh 2\beta a]+2ka$ .... (43)

Equation (42) becomes

$$\cos[(2ka-\theta)+kb]=0$$
$$\Rightarrow\cos(2ka-\theta)\cos kb-\sin(2ka-\theta)\sin kb=0$$
$$\Rightarrow\cos(2ka-\theta)\cos kb=\sin(2ka-\theta)\sin kb$$
$$\Rightarrow\cot kb=\tan(2ka-\theta) \quad \ldots\ldots\ldots (44)$$

Equation (43) becomes

$$(2ka-\theta)=\tan^{-1}[(\frac{k^2-\beta^2}{2k\beta})\tanh 2\beta a]$$

$$\Rightarrow\tan(2ka-\theta)=[(\frac{k^2-\beta^2}{2k\beta})\tanh 2\beta a] \quad \ldots\ldots\ldots (45)$$

Using equation (44) and equation (45), we get

$$\cot kb=[(\frac{k^2-\beta^2}{2k\beta})\tanh 2\beta a]$$

$$\Rightarrow\cot kb=\frac{1}{2}[(\frac{k}{\beta}-\frac{\beta}{k})\tanh 2\beta a] \quad \ldots\ldots\ldots (46)$$

$$Here\ k^2 = \frac{2mE}{\hbar^2}$$ b = width of QW

$$\beta^2 = \frac{2m(V_0 - E)}{\hbar^2}$$ 2a = width of each barrier.

Equation (46) is the same as equation (41). Thus values of energy at which resonant transmission peaks of symmetric rectangular double barrier occur obey the same condition or transcendental equation as that obeyed by quasi-bound energy levels of non-isolated Quantum Well of symmetric rectangular double barrier. In other words, values of energy at T versus E peaks of symmetric rectangular double barrier are the same as allowed values of energy of quasi-bound levels in the non- isolated QW of symmetric rectangular double barrier.

It is mentionable that resonance occurs if only the condition $\cos[k(2a+b)-\theta]=0$ is satisfied

$$\Rightarrow k(2a+b)-\theta] = (2n+1)\frac{\pi}{2}$$

Where, n = 0, 1, 2, 3,...

Above equation is also the condition for quantized energy levels in the QW between the two barriers. Thus values of resonance energy are equal to bound state energy levels of the quantum well formed between the two barriers. Pronounced transmission of electrons occurs when the energy of electrons is aligned with one of the quantized levels in the quantum well.

Profile of transmission peak

To get an expression for the profile of transmission peak i.e. shape of peak of T versus E curve, let us rewrite equation

$$T(E) = \frac{T_1^2}{T_1^2 + 4(1-T_1)\cos^2[k(2a+b)-\theta]}$$

in the form

$$T(E) = \frac{T_1^2}{T_1^2 + 4(1-T_1)\cos^2\phi} \quad \text{.......... (47)}$$

and recognize that $\phi = k(2a+b)-\theta$ ....(48)

is a function of energy E of tunneling electron. At a resonance peak of T versus E curve, we have

$$\phi = (2n+1)\frac{\pi}{2} \quad \text{.......... (49)}$$

Hence, near a resonance peak of T versus E curve, we have

$$\phi = (2n+1)\frac{\pi}{2} + \delta\phi \quad \text{.......... (50)}$$

where $\delta\phi$ is a small change in ϕ. For the condition given by equation (50), we have

$$\cos\phi = \cos\{(2n+1)\frac{\pi}{2} + \delta\phi\} = \pm\sin\delta\phi \approx \pm\delta\phi \quad \text{.......... (51)}$$

Using equation (51), equation (47) gives

$$T(E) = \frac{T_1^2}{T_1^2 + 4(1-T_1)\cos(\delta\phi)^2}$$

or, $$T(E) = \frac{1}{1 + \frac{4(1-T_1)}{T_1^2}(\delta\phi)^2} \quad \text{.......... (52)}$$

which gives variation of transmission coefficient near a resonance peak.

We have

$$T(E) = \frac{T_{res}}{1 + \left(\frac{\delta\phi}{\frac{T_1}{2\sqrt{1-T_1}}}\right)^2} \quad \text{.......... (53)}$$

Here $T_{res} = 1$ is the peak value of T. Equation (52) shows that T versus E curve near resonance has Lorentzian profile. From equation (52) we find that

at $T(E) = \frac{1}{2}T_{res}$, $\delta\phi = \frac{T_1}{2\sqrt{1-T_1}}$

Since ϕ is a function of energy E, FWHM

$$\Gamma \sim \frac{T_1}{\sqrt{1-T_1}} \quad \text{.......... (54)}$$

We may now write equation (53) in terms of energy as

$$T(E) = \frac{T_{res}}{1 + \left(\frac{E - E_{res}}{\Gamma/2}\right)^2} \quad \text{.......... (55)}$$

This is the shape of T versus E curve near resonance. The shape is Lorentzian.

T versus E curve of symmetric rectangular double barrier

To find how the actual T versus E curves of symmetric rectangular double barrier look like, we have used Mathematica to actually plot T as a function of E for E < V_0. The only inputs are a, b and x. We found the result shown in Figure 5.

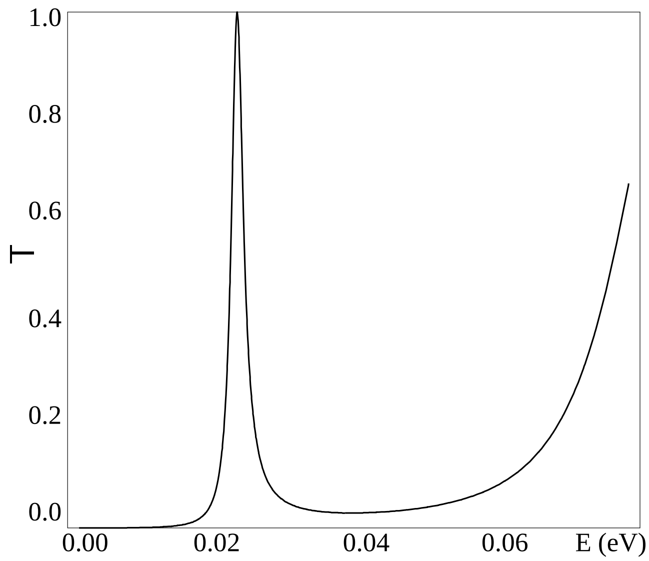

Figure 5: T versus E plot of symmetric rectangular double barrier for the energy range 0 to V_0 for 2a = 5 nm, b = 10 nm and x = 0.1

Notable feature are as follows.

1) Indeed the shape of resonance peak is close to a Lorentzian.

2) The transmission coefficient is unity at resonance and falls off rapidly with energy on either side of the resonance.
3) We have only one peak in the range 0 to V_0 because for x =0.1, the QW is very shallow which contains only one quasi-bound energy level.

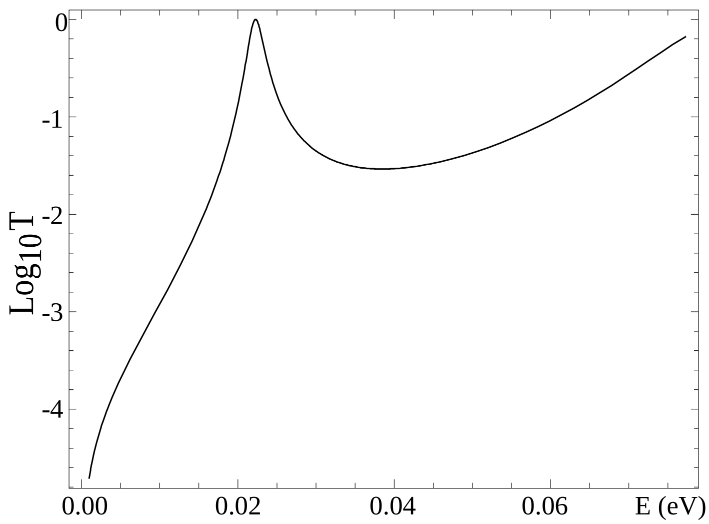

Figure 6: $Log_{10}T$ versus E plot of symmetric rectangular double barrier for the energy range 0 to V_0 for 2a = 5 nm, b = 10 nm and x = 0.1.

For larger value of x = 0.2, the QW is deeper and accommodates two quasi- bound energy levels giving rise to two resonance peaks in the energy range 0 to V_0. This is shown on log scale in figure 7 and on linear scale in figure 8.

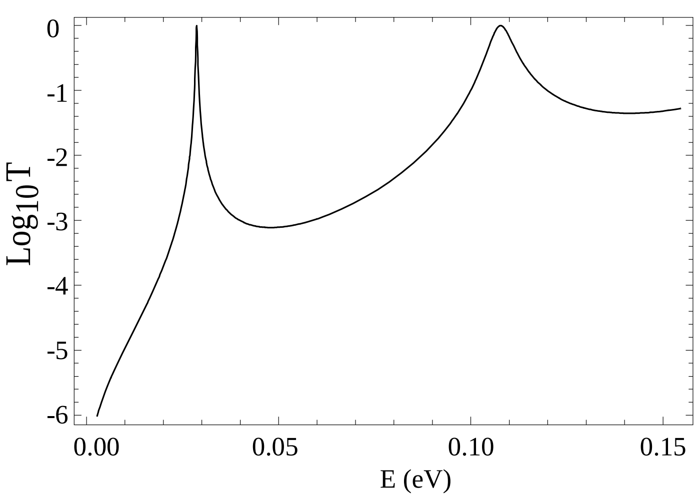

Figure 7: $Log_{10}T$ versus E plot of symmetric rectangular double barrier for the energy range 0 to V_0 for 2a = 5 nm, b = 10 nm and x = 0.2.

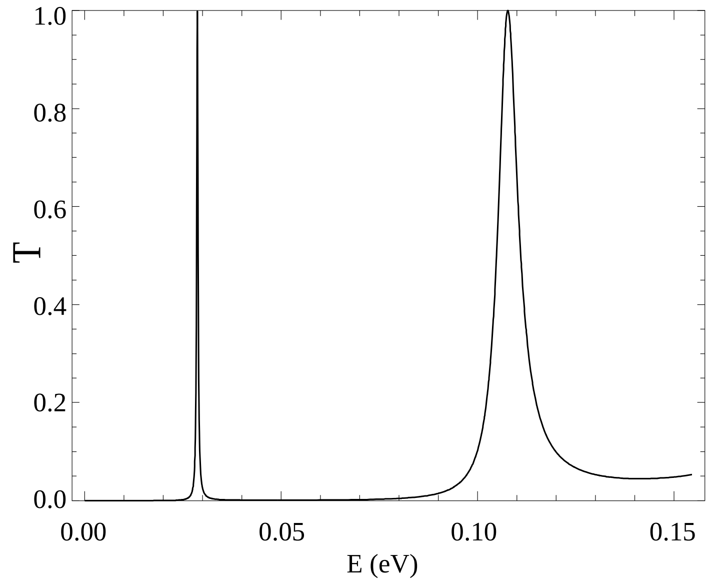

Figure 8: T versus E plot of symmetric rectangular double barrier for the energy range 0 to V_0 for 2a = 5 nm, b = 10 nm and x =0 .2

We can compare transmission coefficient of symmetric rectangular double barrier with transmission coefficient of one of the two single barriers in the tunneling regime ($E < V_0$) depicted in Figure 9.

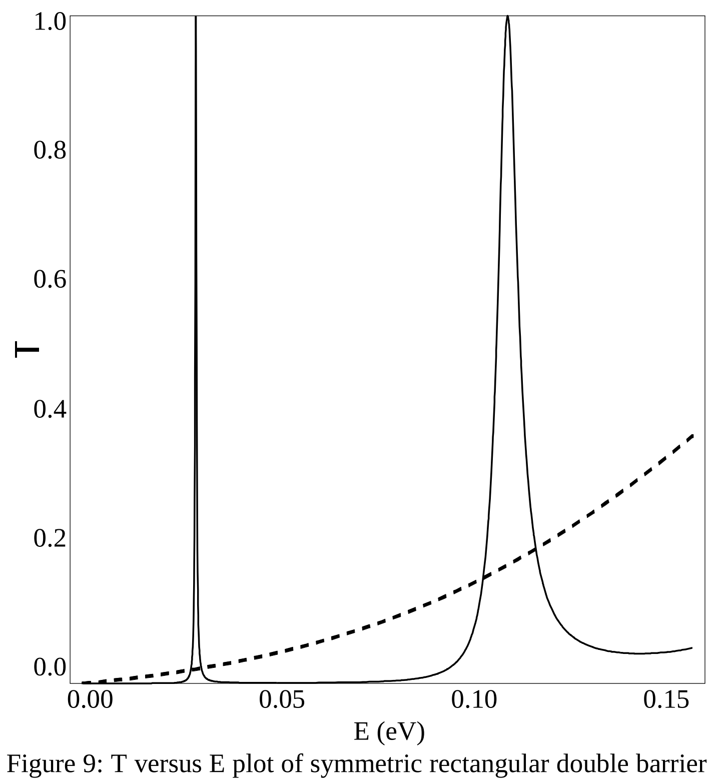

Figure 9: T versus E plot of symmetric rectangular double barrier for the energy range 0 to V_0 for 2a = 5 nm, b = 10 nm and x = 0.2. **Dashed curve** shows T_1 versus E curve in the tunneling regime.

We find that T_1 does not rise to more than 0.4, but T rises up to 1 at resonance. For the resonances at E_1 and E_2, T_1 = 0.025 and 0.2 respectively while T = 1. This is resonant tunneling.

VII. CONCLUSION

We have solved the problem analytically with the method of transfer matrix and found the physics of nanostructures of non-isolated Quantum Well of symmetric rectangular double barrier. It also contains resonant transmission peaks, transmission coefficient of symmetric rectangular and Delta function double potential barriers having a Quantum Well between them and compared the characteristics with those of rectangular and Delta function single potential barrier. Mathematica was used to write programs for calculating and plotting transmission coefficient as a function of energy of electron. This research covers materials selection, processing, involved physics, and technology, thorough calculation and devices aspect of Quantum Well (QW).

REFERENCES

[1] Saxena, D., Jiang, N., Yuan, X., Mokkapati, S., Guo, Y., Tan, H. H., & Jagadish, C. (2016). Design and Room-Temperature Operation of GaAs/AlGaAs Multiple Quantum Well Nanowire Lasers. Nano Letters, 16(8), 5080–5086.

[2] Fu, L., v. d. Heijden, R. W., Tan, H. H., Jagadish, C., Dao, L. V., & Gal, M. (2002). Study of intermixing in a GaAs/AlGaAs quantum-well structure using doped spin-on silica layers. Applied Physics Letters, 80(7), 1171–1173.

[3] Matsuda, I., Tanikawa, T., Hasegawa, S., Yeom, H. W., Tono, K., & Ohta, T. (2004). Quantum-Well States in Ultra-Thin Metal Films on Semiconductor Surfaces. J. Surf. Sci. Nanotech, vol. 2, 169-177.

[4] Odoh, E. O., & Njapba, A. S. (2015). A Review of Semiconductor Quantum Well Devices. Advances in Physics Theories and Applications, vol. 46, 26-32.

[5] Sakaki, H., Tsuchiya, M., & Yoshino, J. (1985). Energy levels and electron wave functions in semiconductor quantum wells having superlattice alloylike material (0.9 nm GaAs/0.9 nm AlGaAs) as barrier layers. Applied Physics Letters, 47(3), 295–297.

[6] Dimastrodonato,V., Mereni, L. O., Young, R. J., & Pelucchi, E. (2011). AlGaAs/GaAs/AlGaAs quantum wells as a sensitive tool for the MOVPE reactor environment.

[7] Ekins-Daukes, N. J., Adams, J., Ballard, I. M., Barnham, K. W. J., Browne, B., Connolly, J. P., Tibbits, T., Hill, G., & Roberts, J. S. (2015). Physics of Quantum Well Solar Cells. Proc. of SPIE, vol. 7211, 72110L-7.

[8] Chowdhury, S., Samad, A., & Dash, D. (2013). Nanostructure Physics of non-isolated Quantum Well of symmetric double barrier. Munich, GRIN Verlag, V211440,1-137.

[9] Chowdhury, S. (2014). Nanostructure Physics and Microelectronics. Alpha science intl. ltd.,1-200.

[10] Saha, S.(2006). Lecture Notes on Quantum Mechanics: Part I, 4th year Honours project, Department of Physics, Shahjalal University of Science and Technology, Sylhet 3114, Bangladesh.

[11] Chowdhury, A. T., (2007). Lecture Notes on Quantum Mechanics: Part II, 4th year Honours project, Department of Physics, Shahjalal University of Science and Technology, Sylhet 3114, Bangladesh.

[12] Ferry, D. K., & Goodnick, S. M. (1997). Transport in Nanostructures. Cambridge University Press.

[13] Ruhel, M. B. H.,& Chowdhury, S. (2010). WKB method applied to Nanostructure Physics: transmission coefficient of single and symmetric double barriers encountered in Nanostructure Physics. Lap Lambert Academic Publishing, Germany.

[14] Acharjee, M. C., & Chowdhury, S. (2010). Nanostructure Physics of symmetric rectangular double barrier: transmission coefficient and parametric dependence of energy of resonant peaks. Lap Lambert Academic Publishing, Germany.

[15] Islam, R., Ullah, F., & Chowdhury, S. (2011). Transverse wavevector in Nanostructure Physics: QW and tunnel barrier: effects on confined energy levels of isolated Quantum Well (QW) and on transmission coefficient of tunnel barrier. VDM Publishing, Verlag Dr. Müller, Germany.

[16] Nagaraja, K. K., Telenkov, M. P., Kazakov, I. P., Savinov, S. A., & Mityagin, Y. A. (2017). Resonant tunneling GaAs/AlGaAs quantum well structures for p-i-n photovoltaic cells. Bulletin of the Lebedev Physics Institute, 44(3), 72–76.

[17] Solovev, I. A., Davydov, V. G., Kapitonov, Y. V., Shapochkin, P. Y., Efimov, Y. P., Lovcjus, V. A., Eliseev, S. A., Petrov, V. V., & Ovsyankin, V. V. (2015). Increasing of AlGaAs/GaAs quantum well robustness to resonant excitation by lowering Al concentration in barriers. Journal of Physics: Conference Series, 643, 012085.

AUTHORS

First Author – M. A. Samad, Department of Physics, Jessore University of Science and Technology, Jessore-7408, Bangladesh

Second Author –S. Chowdhury, (Phd.), Professor, Department of Physics, Shahjalal University of Science and Technology, Sylhet-3114, Bangladesh

Correspondence Author – M. A. Samad, Department of Physics, Jessore University of Science and Technology, Jessore-7408, Bangladesh**.**

msamad25@gmail.com