Root Mean Square Derivative - Based Closed Newton Cotes Quadrature

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Abstract- In this paper, a set of Root mean square derivative - based closed Newton Cotes quadrature formula (RMSDCNC) is introduced in which the derivative value is included in addition to the existing closed Newton Cotes quadrature (CNC) formula for the calculation of a definite integral in the inetrval [a, b]. These derivative value is measured by using the root mean square value. The proposed formula yields improved precision than the existing formula. The error terms are likewise obtained by utilizing the method based on the precision of a Quadrature formula. Lastly, numerical examples are discussed to show the accuracy of the proposed rule.

Index Terms- Closed Newton-Cotes formula,Definite integral, Error terms, Root mean square, Numerical examples

I. Introduction

There are several new quadrature rules were found to improve the accuracy of the existing newton cotes quadrature rule. Dehghan and his companions concentrate to increase the order of accuracy of the closed, open and semi-open [4,5,6] Newton Cotes formula by order two by letting in the location of the boundaries of the interval as two additional parameters and returning the original integral to fit the optimal boundary locations. These authors used the same technique to Gauss Legendre, Gauss Lobatto and Gauss Chebyshev quadrature rules [1,7,8]. Burg et al., suggested derivative based - closed ,open and Midpoint quadrature rules[2,3,9]by admitting the first and higher order derivatives. Also, Weijing Zhao and Hongxing Li [16] have improved the closed newton cotes quadrature formula by putting in the Midpoint derivative. Lately, we proposed Geometric mean [11], Harmonic mean [12], Heronian mean [14] and centroidal mean [15] derivative - based closed Newton cotes quadrature rule and comparison of the arithmetic mean, geometric mean and harmonic mean derivative - based closed Newton Cotes quadrature rules [13].

The general shape of the quadrature formula for the valuation of a definite integral over [a, b] is

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n} w_i f(x_i) \qquad (1)$$

where there are (n+1) weights wi , i=0,1,,2,...,n and (n+1) intermediate points a= x0 , x1 , … , xn=b,so xi=x0+ih, i=0,1,2,..n, where $h = \frac{b-a}{n}$ and the closed Newton Cotes formula for the evaluation of the definite integral over [a,b] is

$$\int_a^b f(x)dx = \int_{x_0}^{x_n} f(x)dx \approx \sum_{i=0}^{n} w_i f(x_i) \qquad (2)$$

These weights can be derived by applying the method based on the precision of the quadrature formula. Pick out the values for w_i, i=0, 1,... n. so that the error of approximation in the quadrature formula is zero,i.e

$$E_n[f] = \int_a^b f(x)dx - \sum_{i=0}^{n} w_i f(x_i) = 0,$$

$$\text{for } f(x) = x^j \quad j = 0,1,\dots n \qquad (3)$$

Definition 1.1.[10] An integration method of the form (1) is said to be of order P, if it produces accurate results (En[f] = 0) for all polynomials of degree less than or equal to P.

We list some of the closed newton cotes quadrature formula obtained by giving various values of n in a general quadrature formula,

When n=1 ⟹ Trapezoidal rule

$$\int_a^b f(x)dx = \frac{b-a}{2}\big(f(a)+f(b)\big) - \frac{(b-a)^3}{12} f''(\xi), \quad \text{where } \xi \in (a,b) \qquad (4)$$

When n =2 ⟹ Simpson's 1/3rd rule

$$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] - \frac{(b-a)^5}{2880} f^{(4)}(\xi), \quad \text{where } \xi \in (a,b) \qquad (5)$$

When n =3 ⟹ Simpson's 3/8th rule

$$\int_a^b f(x)dx = \frac{b-a}{8}\left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right] - \frac{(b-a)^5}{6480} f^{(4)}(\xi), \quad \text{where } \xi \in (a,b) \qquad (6)$$

When n=4 ⟹ Boole's rule

$$\int_a^b f(x)dx = \frac{b-a}{90}\left[7f(a) + 32f\left(\frac{3a+b}{4}\right) + 12f\left(\frac{a+b}{2}\right) + 32f\left(\frac{a+3b}{4}\right) + 7f(b)\right] - \frac{(b-a)^7}{1935360} f^{(6)}(\xi), Where\ \xi \in (a,b) \quad (7)$$

It is recognized that the degree of precision is (n+1) for even values of n and n for odd values of n.

In this paper, the role of derivative at the terminalpoints is evaluated by using the root mean square.These Root mean square derivative-based closed Newton cotes quadrature formula gives more accurate solution than the existing closed newton cotes quadrature formula.The error terms are also derived by utilizing the concept of precision.Lastly, numerical examples exhibit the order of accuracy the closed Newton cotes quadrature and Root mean square derivative-based closed Newton cotes quadrature and are discussed about in particular. The degree of precision of the proposed formula is (n+2) for even n and is (n+1) for odd n.

II. ROOT MEAN SQUARE DERIVATIVE - BASED CLOSED NEWTON COTES QUADRATURE RULE

In this section, Root mean square derivative - based closed Newton cotes quadrature formula were derived for the calculation of a definite integral over [a,b] by using the root mean square value at the end points.

Theorem 2.1. Closed Trapezoidal rule (n=1) using Root mean square derivative is

$$\int_a^b f(x)dx \approx \frac{b-a}{2}[f(a) + f(b)] - \frac{(b-a)^3}{12} f''\left(\sqrt{\frac{a^2+b^2}{2}}\right), (8)$$

The precision of this method is 2.

Proof: For f (x) = x2

$$\text{The Exact value of } \int_a^b x^2\,dx = \frac{1}{3}(b^3 - a^3);$$

$$From\ (8) \Rightarrow \frac{b-a}{2}(a^2+b^2) - \frac{2(b-a)^3}{12} = \frac{1}{3}(b^3 - a^3).$$

It indicates that the solution is exact. Thus, the precision of the closed Trapezoidal rule with Root mean square derivative is 2 where as the precision of the existing Trapezoidal rule(4)is 1.

Theorem 2.2**.** Closed Simpson's1/3rd rule with Root mean square derivative (n=2) is

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] - \frac{(b-a)^5}{2880} f^{(4)}\left(\sqrt{\frac{a^2+b^2}{2}}\right), \quad (9)$$

The precision of this method is 4.

Proof: For f (x) = x4.

$$\text{The Exact value of } \int_a^b x^4\,dx = \frac{1}{5}(b^5 - a^5);$$

$$From\ (9) \Rightarrow \left(\frac{b-a}{6}\right)\left[a^4 + 4\left(\frac{a+b}{2}\right)^4 + b^4\right] - \frac{24(b-a)^5}{2880} = \frac{1}{5}(b^5 - a^5).$$

It indicates that the solution is exact. Thus, the precision of the closed Simpson's1/3rd rule with Root mean square derivative is 4 where as the precision of the existing Simpson's1/3rd rule (5) is 3.

Theorem 2.3. Closed Simpson's3/8rd rule with Root mean square derivative (n=3) is

$$\int_a^b f(x)dx \approx \frac{b-a}{8}\left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right] - \frac{(b-a)^5}{6480} f^{(4)}\left(\sqrt{\frac{a^2+b^2}{2}}\right), \quad (10)$$

The precision of this method is 4.

Proof: For f (x) = x4.

$$\text{The Exact value of } \int_a^b x^4\,dx = \frac{1}{5}(b^5 - a^5);$$

$$From\ (10) \Rightarrow \left(\frac{b-a}{8}\right)\left[a^4 + 3\left(\frac{2a+b}{3}\right)^4 + 3\left(\frac{a+2b}{3}\right)^4 + b^4\right] - \frac{24(b-a)^5}{6480} = \frac{1}{5}(b^5 - a^5).$$

It indicates that the solution is exact. Thus, the precision of the closed Simpson's3/8rd rule with Root mean square derivative is 4 where as the precision of the existing Simpson's3/8th rule (6) is 3.

Theorem 2.4. Closed Boole's rule with Root mean square derivative (n=4) is

$$\int_a^b f(x)dx \approx \frac{b-a}{90}\left[7f(a) + 32f\left(\frac{3a+b}{4}\right) + 12f\left(\frac{a+b}{2}\right) + 32f\left(\frac{a+3b}{4}\right) + 7f(b)\right] - \frac{(b-a)^7}{1935360} f^{(6)}\left(\sqrt{\frac{a^2+b^2}{2}}\right) \quad (11)$$

The precision of this method is 6.

Proof: For f (x) = x6.

$$\text{The Exact value of } \int_a^b x^6\,dx = \frac{1}{7}(b^7 - a^7);$$

$$From\ (11) \Rightarrow \left(\frac{b-a}{90}\right)\left[7a^6 + 32\left(\frac{3a+b}{4}\right)^6 + 12\left(\frac{a+b}{2}\right)^6 + 32\left(\frac{a+3b}{4}\right)^6 + 7b^6\right] + \frac{720(b-a)^7}{1935360} = \frac{1}{7}(b^7 - a^7).$$

It indicates that the solution is exact. Thus, the precision of the closed Boole's rule with Root mean square derivative is 6 where as the precision of the existing Boole's rule (7) is 5.

III. THE ERROR TERMS OF ROOT MEAN SQUARE DERIVATIVE - BASED CLOSED NEWTON COTES QUADRATURE RULE

In this section, the error terms for the Root mean square derivative -based closed Newton cotes quadrature formula is derived by using the remainder between the quadrature formula for the monomial $\frac{x^{p+1}}{(p+1)!}$ and the exact result $\frac{1}{(p+1)!}\int_a^b x^{p+1}\,dx$ where p is the precision of the quadrature formula.

Theorem 3.1. Root mean square derivative-based closed Trapezoidal rule (n=1)with the error term is.

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\big(f(a)+f(b)\big) - \frac{(b-a)^3}{12} f''\left(\sqrt{\frac{a^2+b^2}{2}}\right) + \frac{(b-a)^3}{24}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(3)}(\xi), \quad (12)$$

where ξ ∈ (a, b).The order of accuracy is 4 with the error term

$$E_1[f] = \frac{(b-a)^4}{24}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(3)}(\xi).$$

Proof:

$$\text{Let } f(x) = \frac{x^3}{3!},$$

$$The\ Exact\ value\ of\ \frac{1}{3!}\int_a^b x^3\,dx = \frac{1}{24}(b^4 - a^4);$$

$$(8) \Rightarrow \frac{b-a}{3!.2}\left(b^3 + a^3 - (b-a)^2\left(\sqrt{\frac{a^2+b^2}{2}}\right)\right),$$

Hence,

$$\frac{1}{24}(b^4 - a^4) - \frac{b-a}{3!.2}\left(b^3 + a^3 - (b-a)^2\left(\sqrt{\frac{a^2+b^2}{2}}\right)\right) = \frac{(b-a)^3}{24}\left(2\sqrt{a^2+b^2}-b-a\right).$$

Hence the error term is,

$$E_1[f] = \frac{(b-a)^3}{24}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(3)}(\xi).$$

Theorem 3.2. Root mean square derivative-based closed Simpson's1/3rd rule (n=2)with the error term is

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] - \frac{(b-a)^5}{2880} f^{(4)}\left(\sqrt{\frac{a^2+b^2}{2}}\right) + \frac{(b-a)^5}{5760}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(5)}(\xi), \quad (13)$$

where ξ ∈ (a, b). The order of accuracy is 6 with the error term

$$E_2[f] = \frac{(b-a)^5}{5760}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(5)}(\xi).$$

Proof:

$$\text{Let } f(x) = \frac{x^5}{5!},$$

$$The\ Exact\ value\ of\ \frac{1}{5!}\int_a^b x^5\,dx = \frac{1}{720}(b^6 - a^6);$$

$$(9) \Rightarrow \frac{b-a}{5!.48}\left(8a^5 + (a+b)^5 + 8b^5 - 2(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right)\right),$$

Hence,

$$\frac{1}{720}(b^6 - a^6) - \frac{b-a}{5!.48}\left(8a^5 + (a+b)^5 + 8b^5 - 2(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right)\right) = \frac{(b-a)^5}{5760}\left(2\sqrt{a^2+b^2}-b-a\right).$$

Hence the error term is,

$$E_2[f] = \frac{(b-a)^5}{5760}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(5)}(\xi).$$

Theorem 3.3. Root mean square derivative-based closed Simpson's 3/8th rule (n=3) with the error term is

$$\int_a^b f(x)dx \approx \frac{b-a}{8}\left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right] - \frac{(b-a)^5}{6480} f^{(4)}\left(\sqrt{\frac{a^2+b^2}{2}}\right) + \frac{(b-a)^5}{12960}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(5)}(\xi), \quad (14)$$

Where ξ ∈ (a, b). The order of accuracy is 6 with the error term

$$E_3[f] = \frac{(b-a)^5}{12960}\left(2\sqrt{a^2+b^2}-b-a\right)f^{(5)}(\xi).$$

Proof:

$$\text{Let } f(x) = \frac{x^5}{5!},$$

$$The\ Exact\ value\ of\ \frac{1}{5!}\int_a^b x^5\,dx = \frac{1}{720}(b^6 - a^6);$$

$$(10) \Rightarrow \frac{b-a}{5!.648}\begin{pmatrix} 81a^5+(2a+b)^5+(a+2b)^5+81b^5 \\ -12(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right) \end{pmatrix},$$

Hence,

$$\frac{1}{720}(b^6-a^6) - \frac{b-a}{5!.648}\begin{pmatrix} 81a^5+(2a+b)^5+(a+2b)^5+81b^5 \\ -12(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right) \end{pmatrix}$$

$$= \frac{(b-a)^5}{12960}(2\sqrt{a^2+b^2}-b-a).$$

Hence the error term is,

$$E_3[f] = \frac{(b-a)^5}{12960}(2\sqrt{a^2+b^2}-b-a)f^{(5)}(\xi).$$

Theorem 3.4. Root mean square derivative-based closed Boole's rule (n=4) with the error term is

$$\int_a^b f(x)dx \approx \frac{b-a}{90}\Big[7f(a)+32f\Big(\frac{3a+b}{4}\Big)+12f\Big(\frac{a+b}{2}\Big) + 32f\Big(\frac{a+3b}{4}\Big)+7f(b)\Big] - \frac{(b-a)^7}{1935360}f^{(6)}\left(\sqrt{\frac{a^2+b^2}{2}}\right) + \frac{(b-a)^7}{3870720}(2\sqrt{a^2+b^2}-b-a)f^{(7)}(\xi), \qquad (15)$$

where $\xi \in (a, b)$. The order of accuracy is 8 with the error term

$$E_4[f] = \frac{(b-a)^7}{3870720}(2\sqrt{a^2+b^2}-b-a)f^{(7)}(\xi).$$

Proof:

$$\text{Let } f(x) = \frac{x^7}{7!},$$

$$\textit{The Exact value of } \frac{1}{7!}\int_a^b x^7\,dx = \frac{1}{40320}(b^6-a^6);$$

$$(11) \Rightarrow \frac{b-a}{7!.768}\Big(97a^7+91a^6b+105a^5b^2+91a^4b^3 +91a^3b^4+105a^2b^5+91ab^6+97b^7 -2(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right)\Big),$$

Hence,

$$\frac{1}{720}(b^6-a^6) - \frac{b-a}{7!.768}\Big(97a^7+91a^6b+105a^5b^2 +91a^4b^3+91a^3b^4+105a^2b^5+91ab^6 +97b^7 -2(b-a)^4\left(\sqrt{\frac{a^2+b^2}{2}}\right)\Big),$$

$$= \frac{(b-a)^7}{3870720}(2\sqrt{a^2+b^2}-b-a).$$

Hence the error term is,

$$E_4[f] = \frac{(b-a)^7}{3870720}(2\sqrt{a^2+b^2}-b-a)f^{(7)}(\xi).$$

The following Table I gives a summary of precision, the orders and the error terms for Root mean square derivative based closed Newton- Cotes quadrature.

Table I: Comparison of Error terms

Rules	Precision	Order	Error terms
Trapezoidal rule (n=1)	2	4	$\frac{(b-a)^3}{24}(2\sqrt{a^2+b^2}-b-a)f^{(3)}(\xi)$
Simpson's 1/3rd rule (n=2)	4	6	$\frac{(b-a)^5}{5760}(2\sqrt{a^2+b^2}-b-a)f^{(5)}(\xi)$
Simpson's 3/8th rule (n=3)	4	6	$\frac{(b-a)^5}{12960}(2\sqrt{a^2+b^2}-b-a)f^{(5)}(\xi)$
Boole's rule (n=4)	6	8	$\frac{(b-a)^7}{3870720}(2\sqrt{a^2+b^2}-b-a)f^{(7)}(\xi)$

IV. NUMERICAL EXAMPLES

To prove the accuracy of the proposed formula , the values of the following integrals: $\int_0^2 e^x\,dx$ and $\int_0^2 \frac{dx}{1+x}$ are approximated. The comparative results of Root mean square derivative - based closed Newton- Cotes quadrature formula and the existing closed Newton- Cotes quadrature formula with the error terms are presented in Table II and III.

Example 1**:**solve $\int_0^2 e^x\,dx$ and compare the results with the CNC and RMSDCNC rules.

Solution:

Exact value of $\int_0^2 e^x\,dx$=6.389056099.

Table II: Comparison of CNC and RMSDCNC rules

Example 2: solve $\int_0^2 \frac{dx}{1+x}$ and compare the results with the CNC and RMSDCNC rules.

value of n	CNC		RMSDCNC	
	Approximate value	Error	Approximate value	Error
n = 1	8.389056099	2.000000000	5.646889180	0.742166919
n = 2	6.420727804	0.031671705	6.375019998	0.014036101
n = 3	6.403315477	0.014259378	6.383003129	0.006052970
n = 4	6.389242346	0.000186247	6.388970306	0.000085793

Solution:

Exact value of $\int_0^2 \frac{dx}{1+x}$=1.098612289.

Table III : Comparison of CNC and RMSDCNC rules

Value of n	CNC		RMSDCNC	
	Approximate value	Error	Approximate value	Error
n = 1	1.333333333	0.234721044	1.238576251	0.139963962
n = 2	1.111111111	0.012498822	1.107859562	0.009247273
n = 3	1.104761905	0.006149616	1.103316772	0.004704483
n = 4	1.099259259	0.00064697	1.099159638	0.000547349

V. CONCLUSION

In this paper, Root mean square derivative - based closed Newton - Cotes quadrature formulas are introduced, which uses the root mean square value of terminal points at the derivative. The proposed scheme increase a single order of precision than the existing closed Newton - Cotes quadrature formula. The error terms are also obtained by using the concept of precision. Lastly, comparisons were drawn between the Root mean square derivative - based closed Newton - Cotes quadrature formula and the existing closed Newton - Cotes quadrature formula by using the numerical examples.

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