# Simulation of a Cascade Decay of $\psi^{'}$

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**Abstract**

In this paper, we use Monte Carlo Method to simulate the cascade decay process of $\psi^{'}$ in MATLAB. We record the direction, energy and momentum of the production particles (two photons and two muons) above the background of a three-body decay and a $\eta$ decay and estimate the efficiency of the detector model.

**Key words**: Monte Carlo Method; Decay; $\psi^{'}$

## Contents

# 1 Introduction

## 1.1 The decay sequence

Quarkonium refers to a flavorless meson whose constituents are a quark and its own antiquark. Charmonium, a member of the quarkonium system, is the bound state of a charm quark and an anti-charm quark and has different states with different quantum numbers. Known charmonium states and candidates are shown in Figure 1 [1]. The overall decay sequence we focus on is

$$e^+e^- \to \psi^{'} \to \gamma\chi_c \to \gamma_1\gamma_2 J/\psi \to \gamma_1\gamma_2\mu^+\mu^-$$

We will also consider

$$\psi^{'} \to \pi^0\pi^0 J/\psi$$

and

$$\psi^{'} \to \eta J/\psi \to \gamma_1\gamma_2\mu^+\mu^-$$

as the background decays.

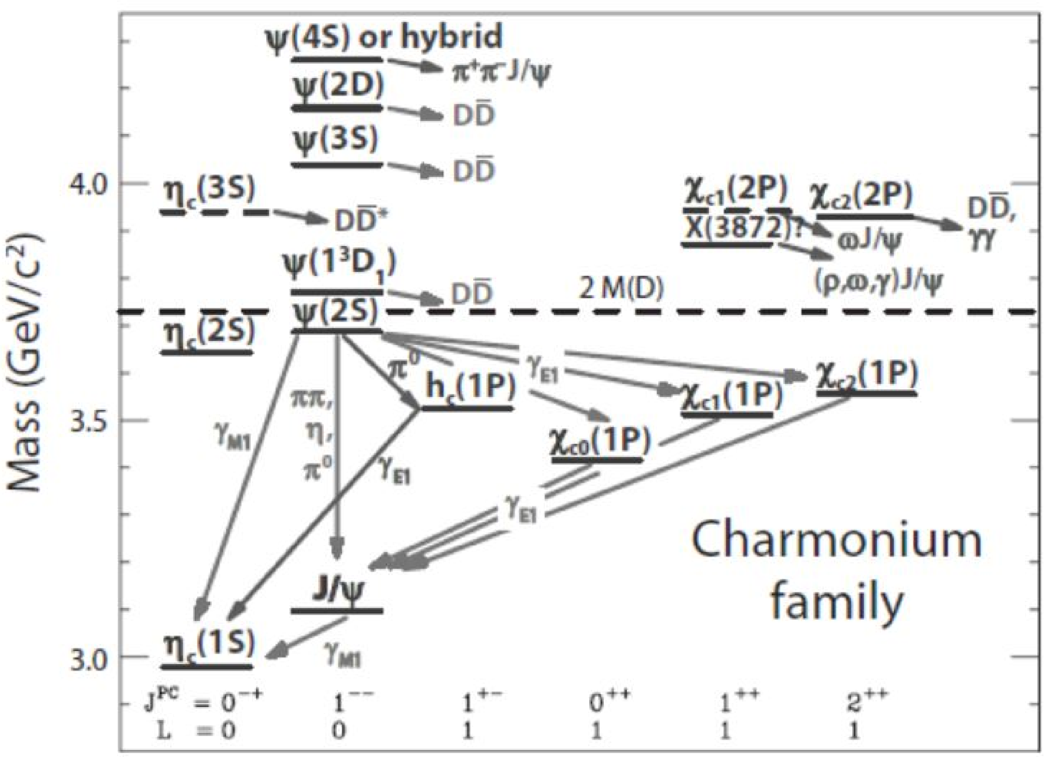



Figure 1: Known charmonium states and candidates

### 1.2 The Crystal Ball Model

The Crystal Ball was a hermetic particle detector used initially with the SPEAR particle accelerator at the Stanford Linear Accelerator Center. In the model, the detector is a spherical shell with holes having $30^\circ$ openings at the ends. Beams of electrons and positrons of the same intensity and energy enter from opposite ends of the detector. They collide to produce $\psi'$ at rest in the middle of the detector and experience the cascade decay described above. For engineering reasons, the holes are large enough to allow space for the beam pipe to pass through the apparatus.
The number of trials in which all of the particles of interest that are produced are detected divided by the total number of trials is called the efficiency of the Crystal Ball. In our model, we neglect the distances traveled by the particles during the decay process since their lifetimes are very short and we assume that the decays are dominated by E1 transitions or M1 transitions by which we can derive the expected angular correlation between the two photons [3].

## 2 Simulation

### 2.1 Relativistic kinematics

The relationship between energy, mass and momentum is given by

$$E^2 = m^2c^4 + p^2c^2 \tag{1}$$

If we choose natural units where $c = 1$, the equation becomes

$$E^2 = m^2 + p^2 \tag{2}$$

where the unit of energy is usually MeV, the unit of mass is $\mathrm{MeV/c^2}$ and the unit of momentum is MeV/$c$, which are adopted throughout the paper. Assume that a rest particle of mass $M$ decays into two lighter particles of masses $m_1$ and $m_2$. Through the conservation of energy, we arrive at

$$M = E_1 + E_2 \tag{3}$$

Then square both sides of equation (3)

$$M^2 = E_1^2 + E_2^2 + 2E_1E_2 \tag{4}$$

Using

$$p^2 = E_1^2 - m_1^2 = E_2^2 - m_2^2 \tag{5}$$

and equation (3) we obtain

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \tag{6}$$

The expression for the energy of the other particle can be obtained by interchanging indices in equation (6). In the simulation, we use this formula to calculate the energy and momentum of the decay particles produced by an initial state at rest in its reference frame.

### 2.2 Lorentz boost

Assume that an inertial frame $\mathbf{S}'$ moves at $\mathbf{v}$ with respect to another inertial frame $\mathbf{S}$. Let $\mathbf{v}/\mathbf{c}$ be a new parameter $\beta$. Let $\gamma = 1/\sqrt{1-\beta^2}$. We can write the Lorentz Transformation as a matrix operator that takes an energy-momentum four-vector from $\mathbf{S}'$ to $\mathbf{S}$ as below [5]

$$\begin{bmatrix} E \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta_x & \gamma\beta_y & \gamma\beta_z \\ \gamma\beta_x & 1+\frac{(\gamma-1)\beta_x^2}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} \\ \gamma\beta_y & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & 1+\frac{(\gamma-1)\beta_y^2}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} \\ \gamma\beta_z & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} & 1+\frac{(\gamma-1)\beta_z^2}{\beta^2} \end{bmatrix} \begin{bmatrix} E' \\ p_x' \\ p_y' \\ p_z' \end{bmatrix} \tag{7}$$

which only depends on the relative velocity $\mathbf{v}$. We use this matrix to transform the energy and momentum of the particles in a moving reference frame to their energy and momentum in the reference frame of the Crystal Ball.

### 2.3 Branching ratios

For the case of interest, $\psi'$ has three main decay modes as described above. And in the $\chi$ decay there are three $\chi$ states with J (the total angular momentum)=0,1,2 which can be produced by $\psi'$ decay. Thus we scale all the branching ratios according to [2] so that the sum of the ratios is 100%. In each trial, we select a $\psi'$ decay mode at random.

### 2.4 Angular correlation

The angular distribution between the two photons is [3]

$$^3P_0: \quad N(\theta_{\gamma\gamma}) = \text{constant},$$

$$^3P_1: \quad N(\theta_{\gamma\gamma}) \propto 5 + \cos^2\theta_{\gamma\gamma},$$

$$^3P_2: \quad N(\theta_{\gamma\gamma}) \propto 73 + 21\cos^2\theta_{\gamma\gamma}$$

For simplicity we'll use the rejection-acceptance method to generate the required distribution. When $J = 0$, there is no correlation between the two photons. When $J = 1$, we first assume the directions of all the production particles are isotropic. The spherical element is $\sin\theta \mathrm{d}\theta \mathrm{d}\phi$ which equals $-\mathrm{d}\cos\theta \mathrm{d}\phi$, so we expect $\cos\theta$ and $\phi$ to be uniformly distributed over the intervals

$$-1 < \cos\theta < 1 \qquad \text{and} \qquad 0 < \phi < 2\pi$$

Then we calculate the opening angle between the two photons and find the maximum value of $5 + \cos^2\theta_{\gamma\gamma}$ among all the trials. For each trial, if the value of $5 + \cos^2\theta_{\gamma\gamma}$ is less than the maximum value multiplied by a random number between 0 and 1, we throw away this trial. In this way we generate a distribution of the shape we want. A similar method is used for the $J = 2$ case.

### 2.5 Energy resolution

Even when a photon is emitted with a perfectly defined energy, its measured energy will be smeared by some amount, as a result of measurement error. To smear the energy, we use a Gaussian distribution whose centered value is $E$ and standard deviation $\sigma$ is $0.028E^{3/4}$ GeV [4].

### 2.6 Breit-Wigner distribution

Since the $\chi_0$ state has such a short lifetime, according to the uncertainty principle of energy and lifetime there will be a significant uncertainty in the measured energy of the $\chi_0$. The measured mass of the $\chi_0$ will follow the relativistic Breit-Wigner distribution with the following probability density function

$$f(E) = \frac{k}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

where

$$k = \frac{2\sqrt{2}M\Gamma\gamma}{\pi\sqrt{M^2 + \gamma}}$$

with

$$\gamma = \sqrt{M^2(M^2 + \Gamma^2)}$$

where $E$ is the mass of the resonance for a particular event, $M$ is the nominal mass of the resonance, and $\Gamma$ is the resonance width, which can be found in [2]. We can write the probability density function as

$$f(x) = \frac{N_0}{1 + x^2}$$

where

$$x = \frac{E^2 - M^2}{M\Gamma}$$

and $N_0$ is the normalization. Thus the integral of this probability density function would be $p(x) = \tan^{-1} x$ and if we randomly choose $-\pi/2 < p(x) < \pi/2$, then $x = \tan y$. We work out $E$ from $x$ and check whether $M_{J/\psi} < E < M_{\psi'}$. If $E$ doesn't fall in this interval, we generate another $p(x)$ to try again until a qualified $E$ appears.

### 2.7 Invariant mass

The invariant mass is a characteristic of the total energy and momentum of an object or a system of objects that is the same in all frames of reference related by Lorentz transformations. It follows that

$$W^2 = (\sum E)^2 - (\sum \mathbf{P})^2$$

where $W$ is the invariant mass of the system of particles, $\sum E$ is the sum of the energies of the particles and $\sum \mathbf{p}$ is the vector sum of the momentum of the particles. We use this property of the overall decay $\psi^{'} \to \gamma_1\gamma_2\mu^+\mu^-$ to check our code and find that the error is about $10^{-12}$ MeV/$c^2$.

### 2.8 Three-body decay

Next we will discuss the background decay

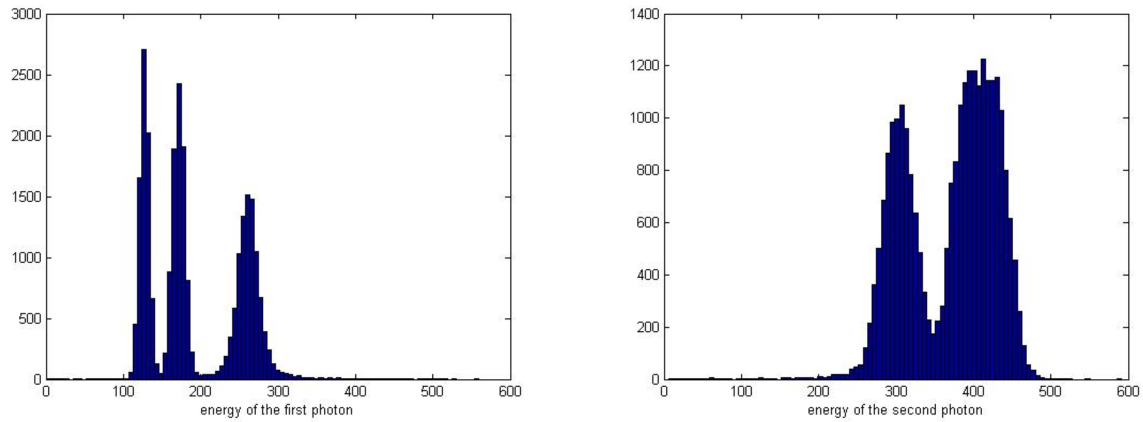
$$\psi^{'} \to \pi^0\pi^0 J/\psi$$

The problem is that the three production particles all have indeterminate energy and momentum. Since we are familiar with the two-body decay, we may assume that $\psi^{'}$ first decays to a $X$ particle and the $\psi$ and then the $X$ particle and the $\psi$ decay to two pions and two muons respectively. Since the mass of the decaying particle must be larger than the sum of the masses of the production particles, one reasonable assumption is that the mass of the $X$ particle is uniformly distributed between the lower limit of $2m_{\pi^0}$ and the upper limit of $m_{\psi'} - m_{\psi}$.

# 3 Results

We do 50,000 trials and find the following results.

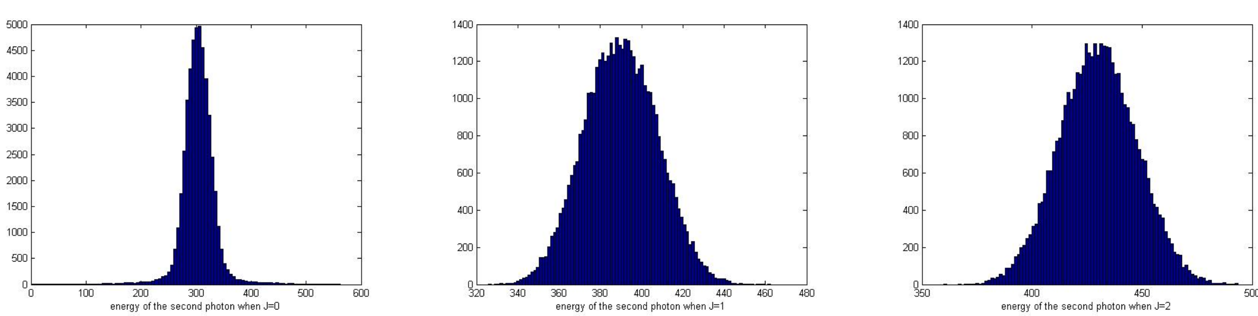
## 3.1 Energy and momentum of the two photons and two muons




(a) (b)

Figure 2: Energy and momentum of the two photons

The energy of photons is the same as their momentum. We see that the energy and momentum of the first photon are smeared instead of the originally definite values and depend on the $J$ value of the $\chi$ state. The heavier the $\chi$ is (the larger the $J$ value is), the less energy the first photon can take away from it. Notice that when $J = 0$ the energy of the first photon is boardened more due to the Breit-Wigner distribution. The transition lines for the second photon are further broadened by the motion of the decaying chi state. This is because the randomness in the direction of the $\chi$ leads to different Lorentz boost matrix and different energy and momentum of the second photon consequently.




(a) (b) (c)

Figure 3: Energy and momentum of the second photon

If we look at the energy of the second photon for the three $J$ values, we find that when $J = 0$ the energy is clustered at 300 MeV and significantly broadened, when $J = 1$ the energy is centered at 390 MeV, and when $J = 2$ the energy is clustered at 430 MeV. Since the last two energies are close to each other, they appear as a single peak in figure 2.

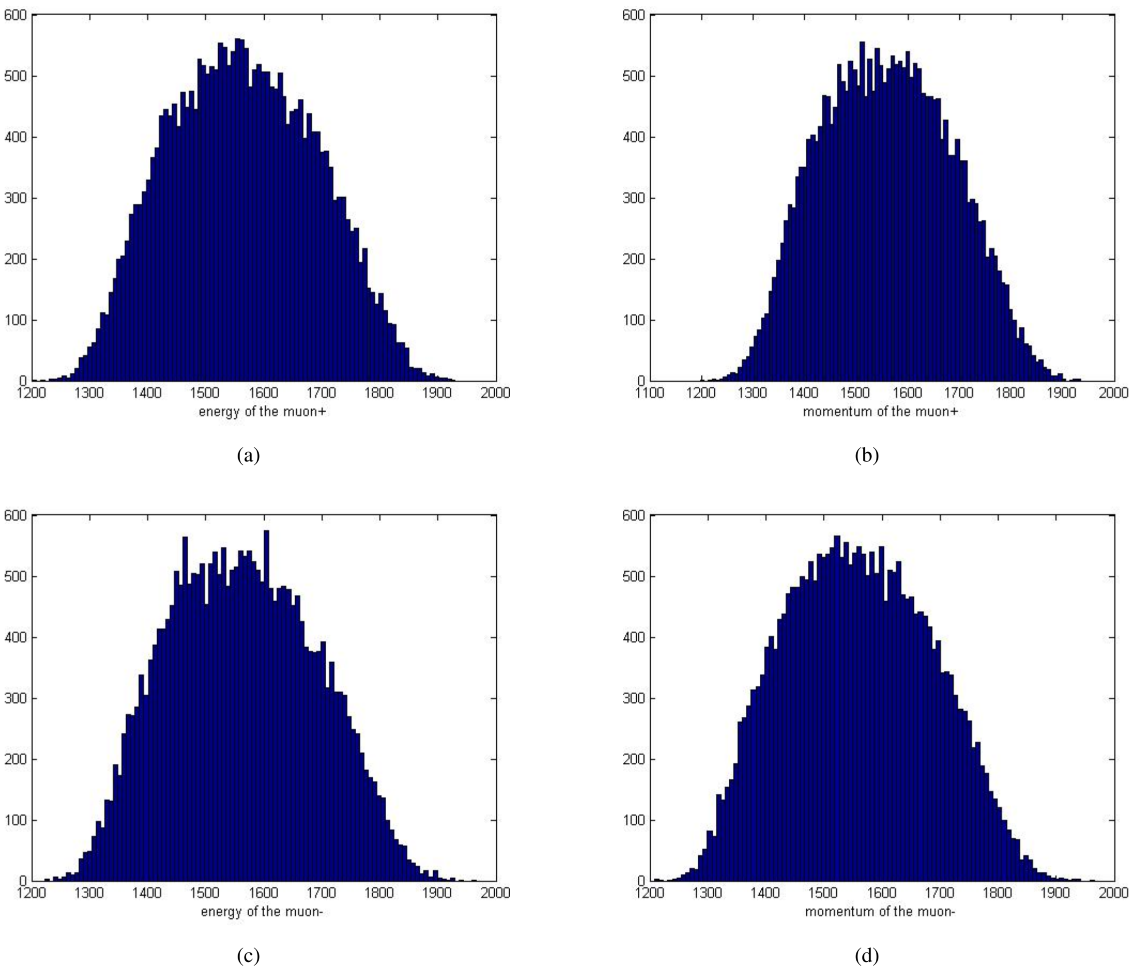
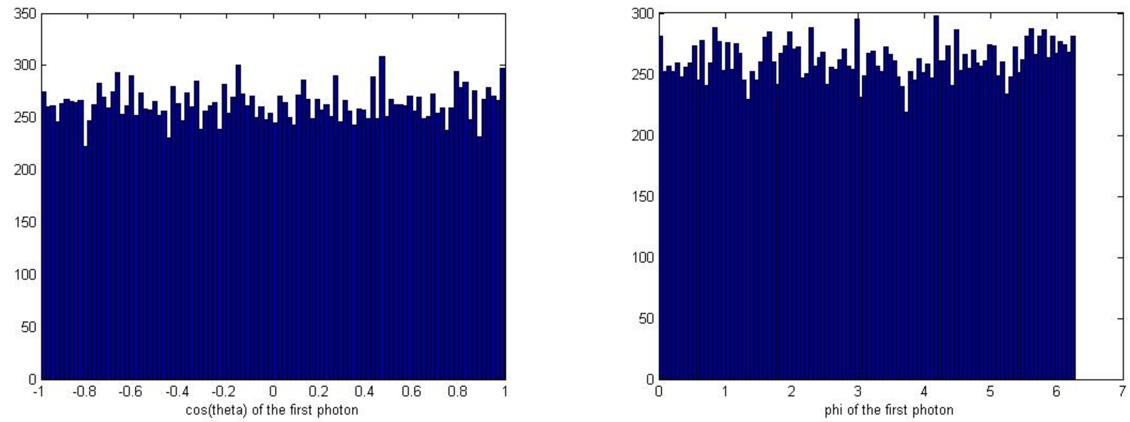



Figure 4: Energy and momentum of the two muons

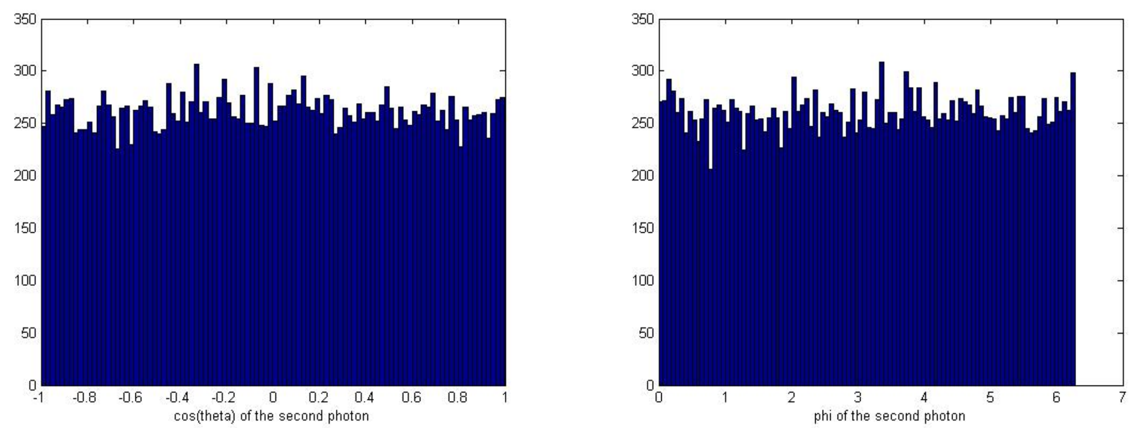
As for the two muons, the energy and momentum are roughly the same since muon has a very small mass. Note that the energies of the two muons aren't exactly the same because the $J/\psi$ is moving.

## 3.2 Distribution of directions of the two photons and two muons



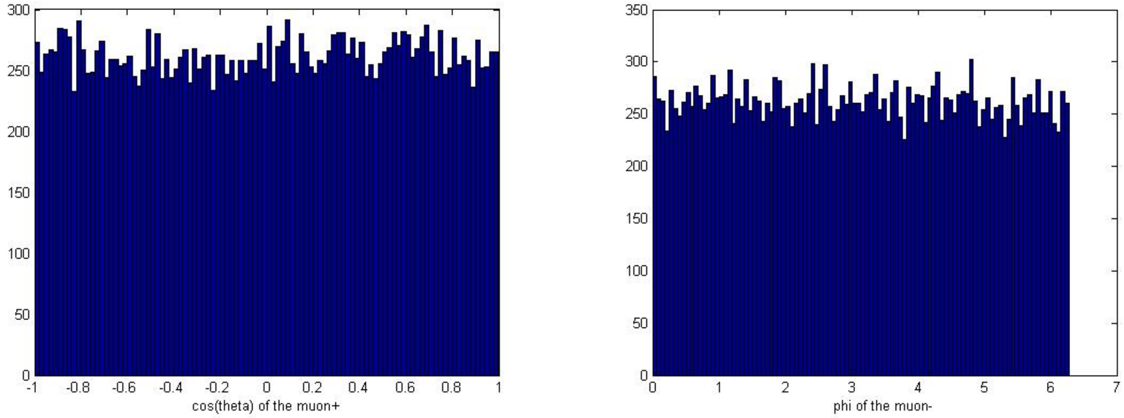

(a) (b)

Figure 5: Directions of the first photon



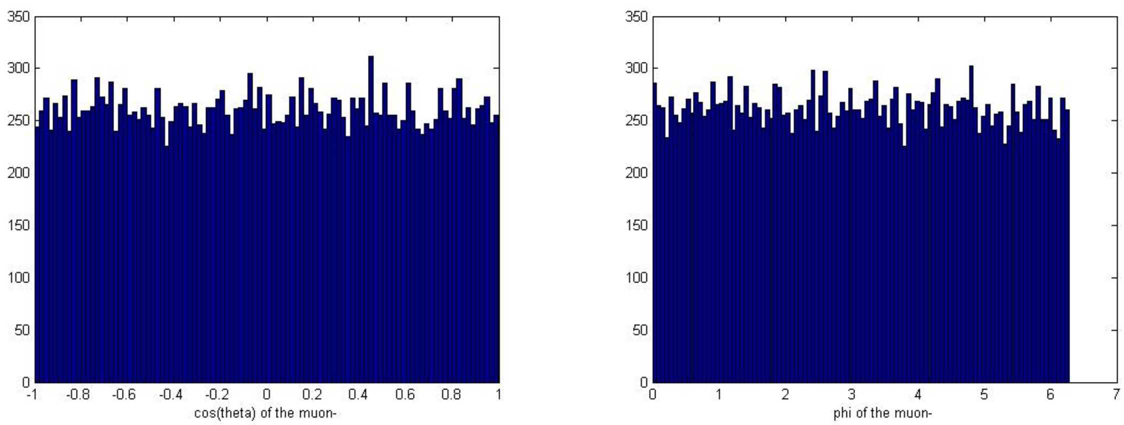

(a) (b)

Figure 6: Directions of the second photon




(a) (b)

Figure 7: Directions of the $\mu^+$

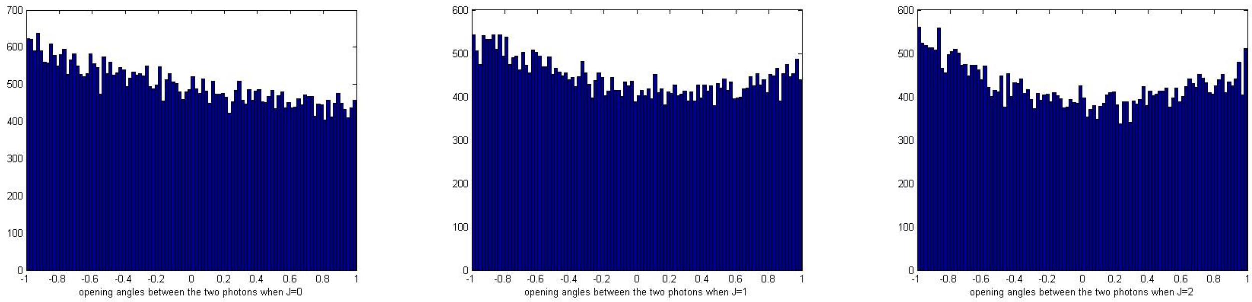



(a) (b)

Figure 8: Directions of the $\mu^-$

Before we throw away the events that are not qualified for the angular correlation between the two photons, the distribution of the directions should be isotropic in the rest frame of the decaying particle. If that particle is moving (as in the cases of the second photon and the two muons), the decay products will not quite be isotropic since the direction of the moving particle represents a preferred direction in space. However, if we do 50,000 trials the randomness in the direction of the decaying particle and the randomness in the direction of the production particle together won't make any overall effect on the directions of all four particles which look uniformly distributed.

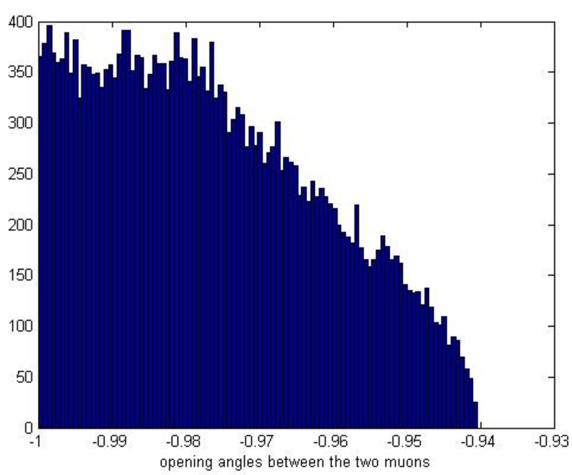
### 3.3 Opening angles between the two photons and two muons




(a) (b) (c)

Figure 9: Opening angles between the two photons

When $J = 0$, though the opening angles between the two photons are roughly uniform, a tendency to be opposite to each other is shown in the histogram since the second photon prefers the direction of the $\chi$ particle, which is opposite to the direction of the first photon. Though their directions remain seemingly random, their relative directions aren't isotropic. When $J = 1$ and $J = 2$, the opening angles follow the angular correlation of the form $N(\theta) = 1 + \cos^2\theta$.

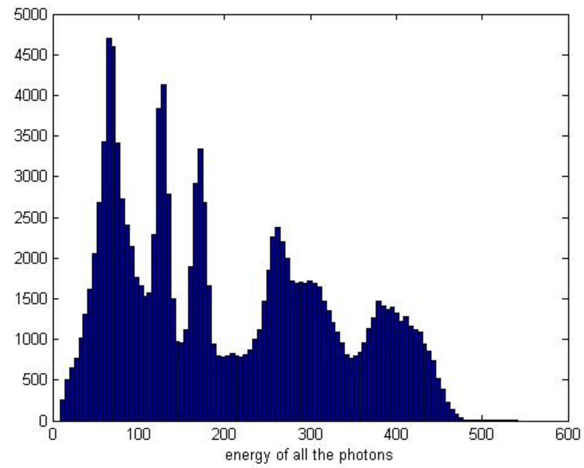



(a)

Figure 10: Opening angles between the two muons

The two muons are virtually opposite to each other since the velocity of the $J/\psi$ particle is so small (in my simulation, $\beta_{J/\psi} = 0.1431$) compared with the velocities of the muons ($\beta_{\mu^+} = 0.9976, \beta_{\mu^-} = 0.9978$) that the muons have an extremely weak preference for deviating from the opposite directions.

### 3.4 Energy of all the photons




(a)

Figure 11: Energy of all the photons

In real experiments, it is impossible to differentiate the photons generated by the $\chi$ decay and other decays which produce photons (especially the decays that produce two photons, both of which are detected). In the simulation we use $\eta$ decay and $\pi^0\pi^0$ decay as background decays. Comparing this figure to the figures above, we recognize that the three peaks around 130 MeV, 170 MeV and 260 MeV are energies of the first photon and the two peaks around 300 MeV and 400 MeV are energies of the second photon. The background pattern with a peak at 90 MeV which looks like a chi-square distribution is due to the background decays (mainly the $\pi^0\pi^0$ decay).

### 3.5 Efficiency of the detector

The efficiency of the detector for the $\chi_0$, $\chi_1$ and $\chi_2$ are 62.89%, 62.08% and 61.92% respectively. The standard error of the efficiency, $\epsilon$, is approximately

$$\sigma_\epsilon = \sqrt{\frac{\epsilon(1-\epsilon)}{N_{\text{trials}}}} \approx 0.0048(\chi_0), 0.0049(\chi_1), 0.0052(\chi_2).$$

## 4 Conclusion

In this paper, we focus on three main decay modes of $\psi'$ and evaluate the behaviors of the production particles (the photons and the muons) under some basic assumptions.

We also simulate a plot of detected energy of all the photons during real experiments and approximate the efficiency of the Crystal Ball Detector.

## Acknowledgement

Thank Prof. Daniel Marlow at Princeton for supporting my research.

# A Simulation Code

```
clear
trial=50000;% number of trials
trial0=0;% number of chi0 decay
trial1=0;% number of chi1 decay
trial2=0;% number of chi2 decay
num0=0;% number of chi0 decay successfully detected
num1=0;% number of chi1 decay successfully detected
num2=0;% number of chi2 decay successfully detected
Mpsiprime=3686.109;
Mpsi=3096.916;
width0=10.5; % full width of chi0
index1=0; % index of opening angles between two photons for chi1
index2=0; % index of opening angles between two photons for chi2
for n=1:trial
r=rand;
if r<0.199242
    Mchi=3414.75;% mass of chi0
    trial0=trial0+1;
elseif r<0.389709
    Mchi=3510.66;% mass of chi1
    trial1=trial1+1;
elseif r<0.5714
    Mchi=3556.20;% mass of chi2
    trial2=trial2+1;
elseif r<0.638412 % psi' --> psi + eta
Calpha=(rand-0.5)*2; % direction of eta particle
Salpha=sqrt(1-Calpha.^2);
beta=rand*2*pi;
Meta=547.862;
Epsi=(Mpsiprime^2-Meta^2+Mpsi^2)/2/Mpsiprime;
Ppsi=sqrt(Epsi^2-Mpsi^2);
Vpsi=Ppsi/Epsi;
Vpsix=-Vpsi*Salpha*cos(beta);
Vpsiy=-Vpsi*Salpha*sin(beta);
Vpsiz=-Vpsi*Calpha;
Peta=Ppsi;
Eeta=sqrt(Peta^2+Meta^2);
Veta=Peta/Eeta;
Vex=Veta*Salpha*cos(beta);
Vey=Veta*Salpha*sin(beta);
Vez=Veta*Calpha;
%% eta --> 2 photons
Ephotona=Meta/2;
Pphotona=Ephotona;
Ephotonb=Meta/2;
Pphotonb=Ephotonb;
gamma=1/sqrt(1-Veta.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vex;
boost(1,3)=gamma*Vey;
boost(1,4)=gamma*Vez;
boost(2,1)=gamma*Vex;
boost(2,2)=1+(gamma-1)*Vex.^2/Veta.^2;
boost(2,3)=(gamma-1)*Vex*Vey/Veta.^2;
boost(2,4)=(gamma-1)*Vex*Vez/Veta.^2;
boost(3,1)=gamma*Vey;
```

```
boost(3,2)=(gamma-1)*Vex*Vey/Veta.^2;
boost(3,3)=1+(gamma-1)*Vey.^2/Veta.^2;
boost(3,4)=(gamma-1)*Vey*Vez/Veta.^2;
boost(4,1)=gamma*Vez;
boost(4,2)=(gamma-1)*Vex*Vez/Veta.^2;
boost(4,3)=(gamma-1)*Vey*Vez/Veta.^2;
boost(4,4)=1+(gamma-1)*Vez.^2/Veta.^2;
Calpha=(rand-0.5)*2;
Salpha=sqrt(1-Calpha.^2);
beta=rand*2*pi;
etaphotona=boost*[Ephotona;Pphotona*Salpha*cos(beta);Pphotona*Salpha*sin(beta);Pphotona*Calpha];
Pphotona=sqrt(etaphotona(2,1).^2+etaphotona(3,1).^2+etaphotona(4,1).^2);
Ephotona=etaphotona(1,1);
if n==1
    Allphoton=Ephotona;
else
    Allphoton=[Allphoton;Ephotona];
end
etaphotonb=boost*[Ephotonb;-Pphotonb*Salpha*cos(beta);-Pphotonb*Salpha*sin(beta);-Pphotonb*Calpha];
Pphotonb=sqrt(etaphotonb(2,1).^2+etaphotonb(3,1).^2+etaphotonb(4,1).^2);
Ephotonb=etaphotonb(1,1);
Allphoton=[Allphoton;Ephotonb];

% %% psi--> muon pair
% Mmuon=105.658;
% Emuon2=Mpsi/2;% in psi reference frame
% Pmuon2=sqrt(Emuon2^2-Mmuon^2);% in psi reference frame
% Calpha=(rand-0.5)*2;
% Salpha=sqrt(1-Calpha.^2);
% beta=rand*2*pi;
% gamma=1/sqrt(1-Vpsi.^2);
% boost=zeros(4,4);% boost matrix
% boost(1,1)=gamma;
% boost(1,2)=gamma*Vpsix;
% boost(1,3)=gamma*Vpsiy;
% boost(1,4)=gamma*Vpsiz;
% boost(2,1)=gamma*Vpsix;
% boost(2,2)=1+(gamma-1)*Vpsix.^2/Vpsi.^2;
% boost(2,3)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
% boost(2,4)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
% boost(3,1)=gamma*Vpsiy;
% boost(3,2)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
% boost(3,3)=1+(gamma-1)*Vpsiy.^2/Vpsi.^2;
% boost(3,4)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
% boost(4,1)=gamma*Vpsiz;
% boost(4,2)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
% boost(4,3)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
% boost(4,4)=1+(gamma-1)*Vpsiz.^2/Vpsi.^2;
% etamuon1=boost*[Emuon2;Pmuon2*Salpha*cos(beta);Pmuon2*Salpha*sin(beta);Pmuon2*Calpha];% transform of
% Pmuon1=sqrt(etamuon1(2,1).^2+etamuon1(3,1).^2+etamuon1(4,1).^2);
% Emuon1=etamuon1(1,1);
% Vmuon1=Pmuon1/Emuon1;
% etamuon2=boost*[Emuon2;-Pmuon2*Salpha*cos(beta);-Pmuon2*Salpha*sin(beta);-Pmuon2*Calpha];% transform
% Pmuon2=sqrt(etamuon2(2,1).^2+etamuon2(3,1).^2+etamuon2(4,1).^2);
% Emuon2=etamuon2(1,1);
% Vmuon2=Pmuon2/Emuon2;

continue
else % three-body decay
```

```
%% psi'--> x + psi
Mpi0=134.9766;
Max=Mpsiprime-Mpsi;
Min=Mpi0*2;
Mx=Min+rand*(Max-Min);
Epsi=(Mpsiprime^2-Mx^2+Mpsi^2)/2/Mpsiprime;
Ppsi=sqrt(Epsi^2-Mpsi^2);
Vpsi=Ppsi/Epsi;
Px=Ppsi;
Ex=sqrt(Px^2+Mx^2);
Calpha=(rand-0.5)*2; % direction of x particle
Salpha=sqrt(1-Calpha.^2);
beta=rand*2*pi;
%% x--> pi0 + pi0
Epi02=Mx/2;
Ppi02=sqrt(Epi02^2-Mpi0^2);
Ctheta=(rand-0.5)*2;
Stheta=sqrt(1-Ctheta.^2);
phi=rand*2*pi;
Vx=Px/Ex;
Vxx=Vx*Stheta*cos(phi);
Vxy=Vx*Stheta*sin(phi);
Vxz=Vx*Ctheta;
gamma=1/sqrt(1-Vx.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vxx;
boost(1,3)=gamma*Vxy;
boost(1,4)=gamma*Vxz;
boost(2,1)=gamma*Vxx;
boost(2,2)=1+(gamma-1)*Vxx.^2/Vx.^2;
boost(2,3)=(gamma-1)*Vxx*Vxy/Vx.^2;
boost(2,4)=(gamma-1)*Vxx*Vxz/Vx.^2;
boost(3,1)=gamma*Vxy;
boost(3,2)=(gamma-1)*Vxx*Vxy/Vx.^2;
boost(3,3)=1+(gamma-1)*Vxy.^2/Vx.^2;
boost(3,4)=(gamma-1)*Vxy*Vxz/Vx.^2;
boost(4,1)=gamma*Vxz;
boost(4,2)=(gamma-1)*Vxx*Vxz/Vx.^2;
boost(4,3)=(gamma-1)*Vxy*Vxz/Vx.^2;
boost(4,4)=1+(gamma-1)*Vxz.^2/Vx.^2;
result=boost*[Epi02;Ppi02*Stheta*cos(phi);Ppi02*Stheta*sin(phi);Ppi02*Ctheta];
Ppi0=sqrt(result(2,1).^2+result(3,1).^2+result(4,1).^2);
Epi0=result(1,1);
Vpi0=Ppi0/Epi0;
resultb=boost*[Epi02;-Ppi02*Stheta*cos(phi);-Ppi02*Stheta*sin(phi);-Ppi02*Ctheta];
Ppi0b=sqrt(resultb(2,1).^2+resultb(3,1).^2+resultb(4,1).^2);
Epi0b=resultb(1,1);
Vpi0b=Ppi0b/Epi0b;
%% 2 pi0 --> 4 photons
Vpi0x=result(2,1)/Epi0;
Vpi0y=result(3,1)/Epi0;
Vpi0z=result(4,1)/Epi0;
Ctheta=(rand-0.5)*2;
Stheta=sqrt(1-Ctheta.^2);
phi=rand*2*pi;
Ephoton1=Mpi0/2;
Pphoton1=Ephoton1;
Ephoton2=Mpi0/2;
```

```
Pphoton2=Ephoton2;
gamma=1/sqrt(1-Vpi0.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vpi0x;
boost(1,3)=gamma*Vpi0y;
boost(1,4)=gamma*Vpi0z;
boost(2,1)=gamma*Vpi0x;
boost(2,2)=1+(gamma-1)*Vpi0x.^2/Vpi0.^2;
boost(2,3)=(gamma-1)*Vpi0x*Vpi0y/Vpi0.^2;
boost(2,4)=(gamma-1)*Vpi0x*Vpi0z/Vpi0.^2;
boost(3,1)=gamma*Vpi0y;
boost(3,2)=(gamma-1)*Vpi0x*Vpi0y/Vpi0.^2;
boost(3,3)=1+(gamma-1)*Vpi0y.^2/Vpi0.^2;
boost(3,4)=(gamma-1)*Vpi0y*Vpi0z/Vpi0.^2;
boost(4,1)=gamma*Vpi0z;
boost(4,2)=(gamma-1)*Vpi0x*Vpi0z/Vpi0.^2;
boost(4,3)=(gamma-1)*Vpi0y*Vpi0z/Vpi0.^2;
boost(4,4)=1+(gamma-1)*Vpi0z.^2/Vpi0.^2;
result=boost*[Ephoton1;Pphoton1*Stheta*cos(phi);Pphoton1*Stheta*sin(phi);Pphoton1*Ctheta];
Pphoton1=sqrt(result(2,1).^2+result(3,1).^2+result(4,1).^2);
Ephoton1=result(1,1);
if n==1
    Allphoton=Ephoton1;
else
    Allphoton=[Allphoton;Ephoton1];
end

result2=boost*[Ephoton2;-Pphoton2*Stheta*cos(phi);-Pphoton2*Stheta*sin(phi);-Pphoton2*Ctheta];
Pphoton2=sqrt(result2(2,1).^2+result2(3,1).^2+result2(4,1).^2);
Ephoton2=result2(1,1);
Allphoton=[Allphoton;Ephoton2];

Vpi0bx=resultb(2,1)/Epi0b;
Vpi0by=resultb(3,1)/Epi0b;
Vpi0bz=resultb(4,1)/Epi0b;
Ctheta=(rand-0.5)*2;
Stheta=sqrt(1-Ctheta.^2);
phi=rand*2*pi;
Ephoton3=Mpi0/2;
Pphoton3=Ephoton3;
Ephoton4=Mpi0/2;
Pphoton4=Ephoton4;
gamma=1/sqrt(1-Vpi0b.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vpi0bx;
boost(1,3)=gamma*Vpi0by;
boost(1,4)=gamma*Vpi0bz;
boost(2,1)=gamma*Vpi0bx;
boost(2,2)=1+(gamma-1)*Vpi0bx.^2/Vpi0b.^2;
boost(2,3)=(gamma-1)*Vpi0bx*Vpi0by/Vpi0b.^2;
boost(2,4)=(gamma-1)*Vpi0bx*Vpi0bz/Vpi0b.^2;
boost(3,1)=gamma*Vpi0by;
boost(3,2)=(gamma-1)*Vpi0bx*Vpi0by/Vpi0b.^2;
boost(3,3)=1+(gamma-1)*Vpi0by.^2/Vpi0b.^2;
boost(3,4)=(gamma-1)*Vpi0by*Vpi0bz/Vpi0b.^2;
boost(4,1)=gamma*Vpi0bz;
boost(4,2)=(gamma-1)*Vpi0bx*Vpi0bz/Vpi0b.^2;
```

```
boost(4,3)=(gamma-1)*Vpi0by*Vpi0bz/Vpi0b.^2;
boost(4,4)=1+(gamma-1)*Vpi0bz.^2/Vpi0b.^2;
result3=boost*[Ephoton3;Pphoton3*Stheta*cos(phi);Pphoton3*Stheta*sin(phi);Pphoton3*Ctheta];
Pphoton3=sqrt(result3(2,1).^2+result3(3,1).^2+result3(4,1).^2);
Ephoton3=result3(1,1);
Allphoton=[Allphoton;Ephoton3];

result4=boost*[Ephoton4;-Pphoton4*Stheta*cos(phi);-Pphoton4*Stheta*sin(phi);-Pphoton4*Ctheta];
Pphoton4=sqrt(result4(2,1).^2+result4(3,1).^2+result4(4,1).^2);
Ephoton4=result4(1,1);
Allphoton=[Allphoton;Ephoton4];

% %% psi--> muon pair
% Vpsix=-Vpsi*Salpha*cos(beta);
% Vpsiy=-Vpsi*Salpha*sin(beta);
% Vpsiz=-Vpsi*Calpha;
% Mmuon=105.658;
% Emuon2=Mpsi/2;% in psi reference frame
% Pmuon2=sqrt(Emuon2^2-Mmuon^2);% in psi reference frame
% Calpha=(rand-0.5)*2;
% Salpha=sqrt(1-Calpha.^2);
% beta=rand*2*pi;
% gamma=1/sqrt(1-Vpsi.^2);
% boost=zeros(4,4);% boost matrix
% boost(1,1)=gamma;
% boost(1,2)=gamma*Vpsix;
% boost(1,3)=gamma*Vpsiy;
% boost(1,4)=gamma*Vpsiz;
% boost(2,1)=gamma*Vpsix;
% boost(2,2)=1+(gamma-1)*Vpsix.^2/Vpsi.^2;
% boost(2,3)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
% boost(2,4)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
% boost(3,1)=gamma*Vpsiy;
% boost(3,2)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
% boost(3,3)=1+(gamma-1)*Vpsiy.^2/Vpsi.^2;
% boost(3,4)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
% boost(4,1)=gamma*Vpsiz;
% boost(4,2)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
% boost(4,3)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
% boost(4,4)=1+(gamma-1)*Vpsiz.^2/Vpsi.^2;
% result=boost*[Emuon2;Pmuon2*Salpha*cos(beta);Pmuon2*Salpha*sin(beta);Pmuon2*Calpha];% transform of u+
% Pmuon=sqrt(result(2,1).^2+result(3,1).^2+result(4,1).^2);
% Emuon=result(1,1);
% Vmuon=Pmuon/Emuon;
%
% result2=boost*[Emuon2;-Pmuon2*Salpha*cos(beta);-Pmuon2*Salpha*sin(beta);-Pmuon2*Calpha];% transform o
% Pmuonb=sqrt(result2(2,1).^2+result2(3,1).^2+result2(4,1).^2);
% Emuonb=result2(1,1);
% Vmuonb=Pmuonb/Emuonb;

continue
end

count=0;
%% chi--> photon + psi
Ctheta=(rand-0.5)*2;% spherical coordinates of first photon
Stheta=sqrt(1-Ctheta.^2);
phi=rand*2*pi;
if Mchi==3414.75
```

```
while 1
    r=tan(rand*pi-pi/2);
    xsquare=r*Mchi*width0+Mchi^2;
    if xsquare<=0
        continue
    else
        x=sqrt(xsquare);
    end
    if x>Mpsi && x<Mpsiprime
        break
    end
end
Mchi=x;
end
    Efirstphoton=(Mpsiprime^2-Mchi^2)/2/Mpsiprime;% energy and momentum of first photon
    Pchi=Efirstphoton;% momentum of chi
    Echi=sqrt(Mchi.^2+Pchi.^2);
if n==1
    Allphoton=Efirstphoton;
else
    Allphoton=[Allphoton;Efirstphoton];
end
if abs(Ctheta)<sqrt(3)/2
    count=count+1;
end
Pfphotonx=Efirstphoton*Stheta*cos(phi);
Pfphotony=Efirstphoton*Stheta*sin(phi);
Pfphotonz=Efirstphoton*Ctheta;

% check 1:
if n==1
if abs(Mpsiprime-Echi-Efirstphoton)<0.001 % momentum cancels out, there is round-off
     disp('check 1 passed');
end
end

Vchi=Pchi/Echi;
Vchix=-Vchi*Stheta*cos(phi);
Vchiy=-Vchi*Stheta*sin(phi);
Vchiz=-Vchi*Ctheta;
gamma=1/sqrt(1-Vchi.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vchix;
boost(1,3)=gamma*Vchiy;
boost(1,4)=gamma*Vchiz;
boost(2,1)=gamma*Vchix;
boost(2,2)=1+(gamma-1)*Vchix.^2/Vchi.^2;
boost(2,3)=(gamma-1)*Vchix*Vchiy/Vchi.^2;
boost(2,4)=(gamma-1)*Vchix*Vchiz/Vchi.^2;
boost(3,1)=gamma*Vchiy;
boost(3,2)=(gamma-1)*Vchix*Vchiy/Vchi.^2;
boost(3,3)=1+(gamma-1)*Vchiy.^2/Vchi.^2;
boost(3,4)=(gamma-1)*Vchiy*Vchiz/Vchi.^2;
boost(4,1)=gamma*Vchiz;
boost(4,2)=(gamma-1)*Vchix*Vchiz/Vchi.^2;
boost(4,3)=(gamma-1)*Vchiy*Vchiz/Vchi.^2;
boost(4,4)=1+(gamma-1)*Vchiz.^2/Vchi.^2;
```

```
Calpha=(rand-0.5)*2;
Salpha=sqrt(1-Calpha.^2);
beta=rand*2*pi;
Esecondphoton2=(Mchi^2-Mpsi^2)/2/Mchi;% energy and momentum of second photon
photon=boost*[Esecondphoton2;Esecondphoton2*Salpha*cos(beta);Esecondphoton2*Salpha*sin(beta);Esecondpho
Psphoton=sqrt(photon(2,1).^2+photon(3,1).^2+photon(4,1).^2);
Esecondphoton=photon(1,1);
Allphoton=[Allphoton;Esecondphoton];
Vphoton=Psphoton/Esecondphoton;% always equal to 1
if abs(photon(4,1)/Psphoton)<sqrt(3)/2
    count=count+1;
end
cosphoton12=(Pfphotonx*photon(2,1)+Pfphotony*photon(3,1)+Pfphotonz*photon(4,1))/Efirstphoton/Psphoton;%

Ppsi2=Esecondphoton2;
Epsi2=sqrt(Mpsi^2+Ppsi2^2);
result=boost*[Epsi2;-Ppsi2*Salpha*cos(beta);-Ppsi2*Salpha*sin(beta);-Ppsi2*Calpha];% transform
Ppsi=sqrt(result(2,1).^2+result(3,1).^2+result(4,1).^2);
Epsi=result(1,1);
Vpsi=Ppsi/Epsi;

% check 2:
if n==1
m1=Echi^2-Pchi^2;
m2=(Esecondphoton+Epsi)^2-(photon(2,1)+result(2,1))^2-(photon(3,1)+result(3,1))^2-(photon(4,1)+result(4
if abs(m1-m2)<0.001
    disp('check 2 passed');
end
end

Calpha=photon(4,1)/Psphoton;% cos(theta) of second photon
Salpha=sqrt(1-Calpha.^2);
Cbeta=photon(2,1)/Psphoton/Salpha;
Sbeta=photon(3,1)/Psphoton/Salpha;
if Sbeta>0
    beta=acos(Cbeta);% phi of second photon
else
    beta=2*pi-acos(Cbeta);
end
%% psi--> muon pair
Vpsix=result(2,1)/Epsi;
Vpsiy=result(3,1)/Epsi;
Vpsiz=result(4,1)/Epsi;
Mmuon=105.658;
Emuon2=Mpsi/2;% in psi reference frame
Pmuon2=sqrt(Emuon2^2-Mmuon^2);% in psi reference frame
Calpha=(rand-0.5)*2;
Salpha=sqrt(1-Calpha^2);
beta=rand*2*pi;
gamma=1/sqrt(1-Vpsi.^2);
boost=zeros(4,4);% boost matrix
boost(1,1)=gamma;
boost(1,2)=gamma*Vpsix;
boost(1,3)=gamma*Vpsiy;
boost(1,4)=gamma*Vpsiz;
boost(2,1)=gamma*Vpsix;
boost(2,2)=1+(gamma-1)*Vpsix.^2/Vpsi.^2;
boost(2,3)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
boost(2,4)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
```

```
boost(3,1)=gamma*Vpsiy;
boost(3,2)=(gamma-1)*Vpsix*Vpsiy/Vpsi.^2;
boost(3,3)=1+(gamma-1)*Vpsiy.^2/Vpsi.^2;
boost(3,4)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
boost(4,1)=gamma*Vpsiz;
boost(4,2)=(gamma-1)*Vpsix*Vpsiz/Vpsi.^2;
boost(4,3)=(gamma-1)*Vpsiy*Vpsiz/Vpsi.^2;
boost(4,4)=1+(gamma-1)*Vpsiz.^2/Vpsi.^2;
result=boost*[Emuon2;Pmuon2*Salpha*cos(beta);Pmuon2*Salpha*sin(beta);Pmuon2*Calpha];% transform of u+
Pmuon=sqrt(result(2,1).^2+result(3,1).^2+result(4,1).^2);
Emuon=result(1,1);
Vmuon=Pmuon/Emuon;
if abs(result(4,1)/Pmuon)<sqrt(3)/2
    count=count+1;
end

result2=boost*[Emuon2;-Pmuon2*Salpha*cos(beta);-Pmuon2*Salpha*sin(beta);-Pmuon2*Calpha];% transform of
Pmuonb=sqrt(result2(2,1).^2+result2(3,1).^2+result2(4,1).^2);
Emuonb=result2(1,1);
Vmuonb=Pmuonb/Emuonb;
if abs(result2(4,1)/Pmuonb)<sqrt(3)/2
    count=count+1;
end

cosmuon12=(result(2,1)*result2(2,1)+result(3,1)*result2(3,1)+result(4,1)*result2(4,1))/Pmuon/Pmuonb;% t

Calpha1=result(4,1)/Pmuon;% cos(theta) of muon+
Salpha1=sqrt(1-Calpha1^2);
Cbeta1=result(2,1)/Pmuon/Salpha1;
Sbeta1=result(3,1)/Pmuon/Salpha1;
if Sbeta1>0
    beta1=acos(Cbeta1);% phi of muon+
else
    beta1=2*pi-acos(Cbeta1);
end

Calpha2=result2(4,1)/Pmuonb;% cos(theta) of muon-
Salpha2=sqrt(1-Calpha2^2);
Cbeta2=result2(2,1)/Pmuonb/Salpha2;
Sbeta2=result2(3,1)/Pmuonb/Salpha2;
if Sbeta2>0
    beta2=acos(Cbeta2);% phi of muon-
else
    beta2=2*pi-acos(Cbeta2);
end

% check 3:
if n==1
    m1=Epsi^2-Ppsi^2;
    m2=(Emuon+Emuonb)^2-(result2(2,1)+result(2,1))^2-(result2(3,1)+result(3,1))^2-(result2(4,1)+result(
if abs(m1-m2)<0.001
    disp('check 3 passed');
end
end

% check 4 (overall check):
m1=Mpsiprime^2;
m2=(Efirstphoton+Esecondphoton+Emuon+Emuonb)^2-(Pfphotonx+photon(2,1)+result2(2,1)+result(2,1))^2-(Pfph
```

```
if Mchi==3510.66
   Allphoton=[Allphoton;5+cosphoton12^2];
   index1=[index1;length(Allphoton)];
elseif Mchi==3556.20
   Allphoton=[Allphoton;73+21*cosphoton12^2];
   index2=[index2;length(Allphoton)];
end

%    if n==1
%         check4=sqrt(m1)-sqrt(m2);
%    else
%        check4=[check4;sqrt(m1)-sqrt(m2)];
% end
if count==4
    if Mchi==3556.20;% mass of chi2
        num2=num2+1;
    elseif Mchi==3510.66;% mass of chi1
        num1=num1+1;
    else  % mass of chi0
        num0=num0+1;
    end
end
end

%% non-isotropic distribution
chi1=Allphoton(index1(2:length(index1)));
chi2=Allphoton(index2(2:length(index2)));
max1=max(chi1);
max2=max(chi2);
del=0;
for n=2:length(index1)
    if Allphoton(index1(n))<max1*rand
          del=[del;index1(n)-1];
    end
end
for n=2:length(index2)
    if Allphoton(index2(n))<max2*rand
         del=[del;index2(n)-1];
    end
end
for n=1:length(Allphoton)
    if ismember(n,index1)==0 && ismember(n,index2)==0 && ismember(n,del)==0
        if n==1
         Dphoton=normrnd(Allphoton(n),0.028*(Allphoton(n)/1000).^0.75*1000);

        else
         Dphoton=[Dphoton;normrnd(Allphoton(n),0.028*(Allphoton(n)/1000).^0.75*1000)];
        end
    end
end

efficiency0=num0/trial0;
efficiency1=num1/trial1;
efficiency2=num2/trial2;
SD0=sqrt(efficiency0*(1-efficiency0)/trial0);
SD1=sqrt(efficiency1*(1-efficiency1)/trial1);
SD2=sqrt(efficiency2*(1-efficiency2)/trial2);
figure,hist(Dphoton,100),xlabel('energy of all the photons');
% figure,hist(check4,50),title('Result of overall check');
```

## References

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