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Bulk Queueing System with Multiple Vacations Set Up Times with N-Policy and Delayed Service

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Abstract- In this paper $M^{(X)}$ / G(a,b) /1 queueing system with multiple vacations ,set up time with N-policy and delayed service. If the queue size is <a after a service completion, the server goes for multiple vacations also after serving M batches continously even if sufficient numbers are there in there queue the server goes for a vacation with probability α_j or resume service with probility (1-α_j). when the server returns from a vacation and if the queue length is still < N he leaves for another vacation and so on, until he find N (N>b) customers in the queue .(ie) the server finds at least N customers waiting for service , then he requires a setup time R to start the service. After the setup he servers a batch of b customers where b$\geq$a. various Characteristics of the queueing system are presented in this paper.

Index Terms- Bulk arrival, delayed service, multiple vacations, setup time, steady state probability.

I. Introduction

In the area of optimal design and control of queues, the N-policy has received great attentation. According to this policy , the server ideals until a fixed number N of customers arrives in the queue at the moment the server is ' switched on ' and servers exhaustively the queue until it empties. The server is then "switched off " and remains idle until N customers accumulate again in the queue .Given the costs of turning the server on and having customers a waiting in the queue, an optimal value of N can be determined that minimizes the expected cost of operating the queue. This model is found to be applicable in analyzing numerous real world queueing situations such as flexible manufacturing systems, service systems, computer and telecommunication system. In many production systems it is assumed that when all the jobs are served, the machine stays idle until next job arrives .If there is a cost associated with operating the machine, It is plausible that a rational way to operate the system is to shut down the machine When the queue length is zero and bring it up again as the queue length grows to a predetermined level of , Say N(≥1), Jobs. Such a control mechanism is usually good when the machine start-up and shut- down costs are high.

The objective of this paper is to analysis a situation that exists in a pump manufacturing industry. A pump manufacturing industry manufactures different types of pumps, which require shafts of various dimensions. The partially finished pump shafts arrive at the copy turning center from the turning centre. The operator starts the copy turning process only if required batch quantity of shafts is available because the operating cost may increase otherwise. After processing, if the number of available shafts is less than the minimum batch quantity, then the operator will start doing other work such as making the templates for copy turning, checking the components. Hence, the operator always shuts down the machine and removes the templates before taking up other works. When the operator returns from other work and finds that the shafts available are more than the maximum batch quantity, the server resumes the copy turning process, for which some amount of time is required to setup the template in the machine. Otherwise the operator will continue with other work until he finds required number of shafts.

The above process can be modeled as $M^X/G\,(a,b)/1$ queueing system with multiple vacations, setup times with *N*-policy and delayed service. Various authors have analysed queueing problems of server vacations with several combinations. A literature survey on queueing systems with server vacations can be found in Doshi . Lee has developed a procedure to calculate the system size probabilities for a bulk queueing model. Several authors have analysed the *N*-policy on queueing systems with vacation. Kella provided detail discussions concerning *N*-policy queueing systems with vacations. Lee et al. have analysed a batch arrival queue with *N*-policy, but considered single service with single vacation and multiple vacations. Chae and Lee , studied a $M^X/G/1$ vacation model with *N*-policy and discussed heuristic interpretation of mean waiting time. Reddy et al. have analysed a bulk queueing model and multiple vacations with setup time. They derived the expected number of customers in the queue at an arbitrary time epoch and obtained other measures. The queue is analysed by Takagi , considering closedown time and setup time. The performance measures are also obtained

II. Mathematical Model

* Let λ be the poisson arrival rate ,X be the group size random variable of the arrival, g_k be the probability that ‘k’ customers arrive in a batch and X(z) be its probability generating function. Let S(.), V(.), and R(.) be the cumulative distributions of

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the service time, vacation time and setup time respectively. Further s(x), v(x) and r(x) are their respective probability density functions.

* $S^0(t)$ denotes the remaining service time at time 't' and $V^0(t)$, and $R^0(t)$ denote the remaining vacation time, and setup time at time 't' respectively. Let us denote the Laplace transforms (LT) of s(x), v(x),and r(x) as $\widetilde{S}$, $\widetilde{V}$, and $\widetilde{R}$ respectively.

The supplementary variable technique was introduced by Cox was followed by Lee introduced an effective technique for solving queueing models using supplementary variables and this technique is used for solving the proposed model.

We define,

Y(t) = 0; if the server is on vacation or doing setup,
j; where j is the number of batches served up to t, since the commencement of a current busy period, if a service is going on at t ($1 \leq j \leq M$)

Z(t) = j ; if the server is on jth vacation at time t.
$N_s(t)$ = number of customers in the service.
$N_q(t)$ = number of customers in the queue.

The probabilities for the number of customers in the queue and service are defined as follows:

$$P_{ij}(x,t)dt = P\{N_s(t) = i, N_q(t) = j, x \leq S^0(t) \leq x + dt, \ Y(t) = 0\}, \quad a \leq i \leq b, j \geq M,$$

$$Q_{jn}(x,t)dt = P\{N_q(t) = j, x \leq V^0(t) \leq x + dt, Y(t) = 1, Z(t) = j\}, \ n\geq 0,\ j\geq 1,$$

$$R_n(x,t)dt = P\{N_q(t) = n, x \leq R^0(t) \leq x + dt, Y(t) = 2\}, \ n\geq N.$$

Analysis

The steady state queue size equations are obtained as

$$-P_{01}^1(x) = -\lambda P_{01}(x) + \sum_{i=a}^{b} R_i(0)\, s(x) \quad (1)$$

$$-P_{i1}^1(x) = -\lambda P_{i1}(x) + \sum_{k=1}^{i} P_{i-k}(x)\, \lambda g_k + R_{b+i}(0)\, s(x) \quad , i \geq 1 \quad (2)$$

$$-P_{0j}^1(x) = -\lambda P_{0j}(x) + \sum_{i=a}^{b} P_{i,j-1}(0)\, (1-\alpha_{j-1})\, s(x) \quad 2 \leq j \leq M \quad (3)$$

$$-P_{ij}^1(x) = -\lambda P_{ij}(x) + \sum_{k=1}^{i} P_{i-k,j}(x)\, \lambda g_k + (1-\alpha_{j-1})\, P_{i+b,j-1}(0)\, s(x), \quad 2 \leq j \leq M, i \geq 1 \quad (4)$$

$$-p_{bj}^1(x) = -\lambda p_{bj}(x) + \sum_{i=a}^{b} p_{i,b+j}(0)\, s(x) + \sum_{k=1}^{i} p_{i,j-1}(x)\, \lambda g_k \quad 1 \leq i \leq N-b-1 \quad (5)$$

$$-P_{bj}^1(x) = -\lambda P_{bj}(x) + \sum_{i=a}^{b} P_{i,b+j}(0)\, s(x) + \sum_{k=1}^{i} P_{i,j-1}(x)\, \lambda g_k + R_{b+i}(0)\, s(x) \quad i \geq N-b \quad (6)$$

$$-Q_{01}^1(x) = -\lambda Q_{01}(x) + \sum_{j=j}^{M} P_{0j}(0)\, v(x) \quad (7)$$

$$-Q_{i1}^1(x) = -\lambda Q_{i1}(x) + \sum_{k=1}^{i} Q_{i-k\,1}(0)\, \lambda g_k + \sum_{j=1}^{M} P_{i,j}(0)\, v(x) \quad ,1 \leq i \leq a-1 \quad (8)$$

$$-Q_{i1}^1(x) = -\lambda Q_{i1}(x) + \sum_{k=1}^{i} Q_{i-k\,1}(x)\, \lambda g_k + \sum_{j=1}^{M} P_{i,j}(0)\alpha_j\, v(x) \quad , i \geq a \quad (9)$$

$$-Q_{0n}^1(x) = -\lambda Q_{0n}(x) + Q_{0n-1}(0)v(x) \quad ,n \geq 2 \quad (10)$$

$$-Q^{1}_{in}(x) = -\lambda Q_{in}(x) + \sum_{k=1}^{i} Q_{i-k\,n}(x)\lambda g_k + Q_{0n-1}(0)v(x) \quad ,n \geq 2,\ 1 \leq i \leq a-1 \tag{11}$$

$$-Q^{1}_{in}(x) = -\lambda Q_{in}(x) + \sum_{k=1}^{i} Q_{i-k\,n}(x)\lambda g_k,\ n \geq 2,\ n \geq N \tag{12}$$

$$-R^{1}_{i}(x) = -\lambda R_{i}(x) + \sum_{k=1}^{n-N} R_{i-k}(x)\lambda g_k + \sum_{l=1}^{\infty} Q_{i1}(0)\, r(x) \quad i \geq a,\ n \geq N \tag{13}$$

III. Queue Size Distribution

The Laplace -Stieljes transform of transform of $P_{ij}(x)$, $Q_{in}(x)$ and $R_i(x)$ are defined respectively as follows:

$$\tilde{P}_{ij}(\theta) = \int_0^\infty e^{-\theta x} P_{ij}(x)\,dx,\quad \tilde{Q}_{in}(\theta) = \int_0^\infty e^{-\theta x} Q_{in}(x)\,dx,\quad \tilde{R}_n(\theta) = \int_0^\infty e^{-\theta x} R_n(x)\,dx \tag{14}$$

Then the Laplace-Stieltjes transform of $P'_{ij}(x), Q'_{in}(x)$ and $R'_i(x)$ are defined

respectively as follows:

$$\int_0^\infty e^{-\theta x}\, p^{1}_{ij}(x)\,dx, = \left[e^{-\theta x} P_{ij}(x)\right]_0^\infty - \int_0^\infty (-\theta e^{-\theta x}) P_{ij}(x)\,dx = \theta \tilde{P}_{ij}(x) - P_{ij}(0)$$

$$\int_0^\infty e^{-\theta x}\, q^{1}_{ij}(x)\,dx, = \left[e^{-\theta x} q_{in}(x)\right]_0^\infty - \int_0^\infty (-\theta e^{-\theta x}) q_{in}(x)\,dx = \theta \tilde{q}_{in}(\theta) - q_{in}(0)$$

$$\int_0^\infty e^{-\theta x}\, R^{1}_{ij}(x)\,dx, = \left[e^{-\theta x} R_{in}(x)\right]_0^\infty - \int_0^\infty (-\theta e^{-\theta x}) R_{in}(x)\,dx = \theta \tilde{R}_{in}(\theta) - Q_{in}(0)$$

Taking Laplace - Stiltje's transform on both sides of the above equations we get,

$$\theta\tilde{P}_{0,1}(\theta) - P_{0,1}(0) = \lambda\tilde{P}_{0,1}(\theta) - \sum_{i=a}^{b} R_i(0)\tilde{S}(\theta) \tag{15}$$

$$\theta\tilde{P}_{i,1}(\theta) - P_{i,1}(0) = \lambda\tilde{P}_{i,1}(\theta) - \lambda\sum_{k=1}^{i} \tilde{P}_{i-k,1}(\theta)g_k - R_{b+i}(0)\tilde{S}(\theta);\ i \geq 1 \tag{16}$$

$$\theta\tilde{P}_{0,j}(\theta) - P_{0,j}(0) = \lambda\tilde{P}_{0,j}(\theta) - \lambda\sum_{i=a}^{b} \tilde{P}_{i,j-1}(0)(1-\alpha_{j-1})\tilde{S}(\theta);\ 2 \leq j \leq M \tag{17}$$

$$\theta\tilde{P}_{i,j}(\theta) - P_{i,j}(0) = \lambda\tilde{P}_{i,j}(\theta) - \lambda\sum_{k=1}^{i} \tilde{P}_{i-k,j}(\theta)g_k - P_{i+bj-i}(0)\left(1-\alpha_{j-i}\right)\tilde{S}(\theta);\ 2 \leq j \leq M;\ i \geq 1 \tag{18}$$

$$\theta\tilde{P}_{bj}(\theta) - P_{bj}(0) = \lambda\tilde{P}_{bj}(\theta) - \sum_{m=a}^{b} P_{mb+j}(0)\tilde{S}(\theta) - \sum_{k=1}^{j} \tilde{P}_{bj-k}(\theta)\lambda g_k \quad 1 \leq j \leq N-b-1, \tag{19}$$

$$\theta\tilde{P}_{bj}(\theta) - P_{bj}(0)\lambda\tilde{P}_{bj}(\theta) - \sum_{m=a}^{b} P_{mb+j}(0)\tilde{S}(\theta) - \sum_{k=1}^{j} \tilde{P}_{bj-k}(\theta)\lambda g_k + R_{b+j}(0)\tilde{S}(\theta) \quad j \geq N-b \tag{20}$$

$$\theta\tilde{q}_{0,1}(\theta) - q_{0,1}(0) = \lambda\tilde{q}_{0,1}(\theta) - \sum_{j=1}^{M} P_{0j}(0)\tilde{v}(\theta) \tag{21}$$

$$\theta\tilde{q}_{i,1}(\theta) - q_{i,1}(0) = \lambda\tilde{q}_{i,1}(\theta) - \lambda\sum_{k=1}^{i} \tilde{q}_{i-k,1}g_k - \sum_{j=1}^{M} p_{ij}(\theta)\tilde{v}(\theta) \quad ; 1 \leq i \leq a-1 \tag{22}$$

$$\theta\tilde{q}_{i,1}(\theta) - q_{i,1}(0) = \lambda\tilde{q}_{i,1}(\theta) - \lambda\sum_{k=1}^{i} \tilde{q}_{i-k,}(\theta)g_k - \sum_{j=1}^{M} p_{ij}(\theta)\alpha_j\tilde{v}(\theta) \quad ; i \geq a \tag{23}$$

$$\theta\tilde{q}_{0,n}(\theta) - q_{0,n}(0) = \lambda\tilde{q}_{0,n}(\theta) - q_{0,n-1}(0)\tilde{v}(\theta) \ ; n \geq 2 \tag{24}$$

$$\theta\tilde{q}_{i,n}(\theta) - q_{i,n}(0) = \lambda\tilde{q}_{i,n}(\theta) - \lambda\sum_{k=1}^{i} \tilde{q}_{i-k,n}(\theta)g_k - q_{i,n-1}(0)\tilde{v}(\theta) \ ; n \geq 2 \quad ; 1 \leq i \leq a-1$$

(25)

$$\theta\tilde{q}_{i,n}(\theta) - q_{i,n}(0) = \lambda\tilde{q}_{i,n}(\theta) - \lambda\sum_{k=1}^{i}\tilde{q}_{i-k,n}(\theta)g_k \quad ; n \geq 2, i \geq a \tag{26}$$

$$\theta\tilde{R}_i(\theta) - R_i(0) = \lambda\tilde{q}_i(\theta) - \lambda\sum_{k=1}^{i-a}\tilde{R}_{i-k}(\theta)g_k - \sum_{j=1}^{\infty}q_{i,1}(0)\tilde{r}(\theta) \quad ; i \geq a \tag{27}$$

Queue Size Distribution

Lee H. S has developed a new technique to find the steady state probability generating function of the number of customers in the system at an arbitrary time.

To apply this technique, the following probability generating function are defined.

$$\tilde{P}_j(z,\theta) = \sum_{i=0}^{\infty}\tilde{P}_{ij}(\theta)z^i,\ P_j(z,0) = \sum_{i=0}^{\infty}\tilde{P}_{ij}(0)z^i;\ \ a \leq j \leq M$$

$$\tilde{Q}_n(z,\theta) = \sum_{i=0}^{\infty}\tilde{q}_{in}(\theta)z^i,\ Q_n(z,0) = \sum_{i=0}^{\infty}\tilde{q}_{in}(0)z^i;\ \ n \geq 1 \tag{28}$$

$$\tilde{R}_n(z,\theta) = \sum_{i=a}^{\infty}\tilde{R}_i(\theta)z^i,\ R(z,0) = \sum_{i=a}^{\infty}\tilde{R}_i(0)z^i$$

Multiply equation (15) by z^0 and (16) by z^i (i≥1) summing up from i=0 to ∞ and using (28)

we have

$$Z^b\big(\theta - \lambda + \lambda x(z)\big)\tilde{P}_1(z,\theta) = Z^b P_1(z,0) - \left[R(z,0) - \sum_{i=a}^{b-1}R_i(0)\left(z^b - z^i\right)\right]\tilde{S}(\theta) \tag{29}$$

Multiply equation (17) by Z^0 and (18) by $Z^i(i \geq 1)$, Summing up from i=0 to ∞, we have

$$Z^b[\theta - \lambda + \lambda x(z)]\tilde{P}_j(z,\theta) = Z^b P_j(z,0) - \left[\sum_{i=a}^{b-1}P_{i\,j-1}(0)\left(z^b - z^1\right) + P_{j-1}(z,0)\right](1 - \alpha_{j-1})\tilde{S}(\theta)$$

$$,2 \leq j \leq M \tag{30}$$

Multiply (17) by Z^0, (19)by $Z^i(i \leq i \leq N - b - 1)$, (20)by $Z^i(i \geq N - b)$ Summing up from

$i = 0$ to ∞ and by using (28), We have

$$(\theta - \lambda + \lambda x(z))\tilde{P}_b(z,\theta) = P_b(z,0) - \frac{\tilde{S}(\theta)}{z^b}\left[\sum_{m=a}^{b}\left(P_m(z,0) - \sum_{i=a}^{b-1}P_{i\,j-1}(0)z^1\right) + R(z,0)\right] \tag{31}$$

Multiply equation (21) by z^0 and (22) by z^i (i≥1) summing up from i=0 to ∞ and using (28) we have

$$(\theta-\lambda+\lambda x(z))\,\tilde{Q}_1(z,\theta) = \tilde{Q}_1(z,0) - \tilde{V}(\theta)\left[\sum_{i=0}^{a-1}\sum_{i=1}^{M}P_{ij}(0)z^i - \sum_{i=0}^{\infty}\sum_{i=1}^{M}P_{ij}(0)\alpha_j z^i\right] \tag{32}$$

Multiply equation (24) by Z^0, (25) by z^i (1≤n≤N-b-1)(1≤i≤a-1) and (26) by z^i (n≥N-b),(i≥a) summing up from i=0 to ∞ using (28) we have.

$$\big(\theta - \lambda + \lambda x(z)\big)\tilde{Q}_n(z,\theta) = Q_n(z,0) - \left[\sum_{i=0}^{N-1}Q_{i\,n-i}(0)Z^i\right]\tilde{v}(\theta) \qquad n \geq 2 \tag{33}$$

Multiply equation (27) by $z^i(i \geq a)$ Summing up from i=0 to ∞ and using (28) we have

$$\big(\theta - \lambda + \lambda x(z)\big)\tilde{R}(z,\theta) = R(z,0) - \left[\sum_{l=1}^{\infty}Q_l(z,0) - \sum_{i=0}^{N-1}Q_{li}(0)Z^i\right]\tilde{R}(\theta) \tag{34}$$

Let $C_i = \sum_{i=j}^{M}P_{ij}(0)\left(1 - \alpha_j\right),\ d_i = \sum_{n=1}^{\infty}q_{in}(0),$ and

Let $\widetilde{P}(z,0)=\sum_{j=1}^{M}\widetilde{P}_j(z,\theta), P(z,0)=\sum_{j=1}^{M}P_j(z,0)$

$\widetilde{Q}(z,\theta)=\sum_{n=1}^{\infty}\widetilde{Q}_n(z,\theta),\quad Q(z,0)=\sum_{n=1}^{\infty}Q_n(z,0)$

By substituting $\theta=\lambda-\lambda x(z)$ in (29)-(34)

$$Z^bP_1(z,0)=\left[R(z,0)-\sum_{i=a}^{b-1}R_i(0)(z^b-z^i)\right]\tilde{S}(\lambda-\lambda x(z)) \quad (35)$$

$$Z^bP_j(z,0)=\left[\sum_{i=a}^{b-1}\widetilde{P}_{i\,j-1}(0)(z^b-z^i)+P_{j-1}(z,0)\right](1-\alpha_{j-1})\,\tilde{S}(\lambda-\lambda x(z)) \quad (36)$$

$$Z^bP_b(z,0)=\left[\sum_{i=a}^{b-1}P_i(z,0)-\sum_{m=a}^{b}\sum_{j=0}^{b-1}P_{mj}(0)z^j+R(z,0)\right]\tilde{S}(\lambda-\lambda x(z)) \quad (37)$$

$$Q_1(z,0)=\left[\sum_{i=0}^{a-1}\sum_{j=1}^{M}P_{ij}(0)x^i-\sum_{i=a}^{\infty}\sum_{j=1}^{M}P_{ij}(0)\alpha_jz^i\right]\widetilde{V}(\lambda-\lambda x(z)) \quad (38)$$

$$Q_n(z,0)=\left[\sum_{i=0}^{N-1}Q_{i\,n-1}(0)\,z^i\widetilde{V}(\lambda-\lambda x(z)\right] \quad (39)$$

$$R(z,0)=\left[\sum_{l=1}^{\infty}Q_i(z,0)-\sum_{i=0}^{N-1}Q_{li}z^i\right]\widetilde{R}(\lambda-\lambda x(z)) \quad (40)$$

Using the expressions of $P_1(z,0),P_j(z,0),P_b(z,0),Q_1(z,0)Q_n(z,0)$ and R(z,0) from (35)- (40)

using (35) in (29) we have

$$z^b\widetilde{P}_1(z,\theta)=\left[R(z,0)-\sum_{i=a}^{b-1}R_i(0)(z^b-z^i\right]\left(\frac{\tilde{S}(\lambda-\lambda x(z))-\tilde{S}(\theta)}{\theta-\lambda+\lambda x(z)}\right) \quad (41)$$

$$z^b\widetilde{P}_j(z,\theta)=\left[\sum_{i=a}^{b-1}\widetilde{P}_{ij-1}(0)(z^b-z^i)+P_{j-i}(z,0)\right](1-\alpha_{j-1})\left(\frac{\tilde{S}(\lambda-\lambda x(z))-\tilde{s}(\theta)}{\theta-\lambda+\lambda x(z)}\right) \quad (42)$$

Using (37) in (31) we have

$$z^b\widetilde{P}_b(z,\theta)=\left[\frac{\tilde{S}-\lambda-\lambda x(z)-\tilde{S}(\theta)}{\theta-\lambda+\lambda x(z)}\right]f(z) \quad (43)$$

Where

$$f(x)=\sum_{m=a}^{b-1}P_m(z,0)-\sum_{m=0}^{b}\sum_{j=0}^{b-1}P_{mj}(0)z^i+R(z,0)$$

$$=\sum_{m=a}^{b-1}\sum_{j=0}^{b}P_{mj}(0)z^j+\widetilde{R}(\lambda-\lambda x(z))\left[\sum_{l=1}^{\infty}Q_l(z,0)-\sum_{i=0}^{N-1}Q_{1i}(0)Z^i\right]$$

Using (38) in (32) we have

$$\widetilde{Q}_1(z,\theta)=\left(\sum_{i=0}^{a-1}\sum_{j=1}^{M}P_{ij}(0)z^i-\sum_{i=a}^{\infty}\sum_{j=1}^{M}P_{ij}(0)\alpha_jz^i\right)\left(\frac{\tilde{V}(\lambda-\lambda x(z))-\tilde{v}(\theta)}{\theta-\lambda+\lambda x(z)}\right) \quad (44)$$

Using (39) in (33) we have

$$\widetilde{Q}_n(z,\theta)=\sum_{i=0}^{N-1}Q_{i\,n-i}(0)z^i\left(\frac{\tilde{V}(\lambda-\lambda x(z))-\tilde{v}(\theta)}{\theta-\lambda+\lambda x(z)}\right) \quad (45)$$

$$\widetilde{R}(z,0)=\left[\sum_{l=1}^{\infty}Q_l(z,0)-\sum_{i=0}^{a-1}Q_{li}(0)z^i\right]\left(\frac{\tilde{R}(\lambda-\lambda x(z))-\tilde{R}(\theta)}{\theta-\lambda+\lambda x(z)}\right) \quad (46)$$

Let P(z) of the queue size at an arbitrary time epoch is the sum of PGF of queue size at service completion epoch and vacation completion epoch then

$$P(z) = \tilde{P}(z,0) + \tilde{Q}(z,0) + \tilde{R}(z,0) \quad (47)$$

Substituting θ=0 in equations (41)-(46).

$$P(z)=\frac{\left[\begin{array}{c}\left(\tilde{s}(\lambda-\lambda(x)-1)\sum_{i=a}^{b-1}(z^b-z^i)\right)(c_i-R_i)+(z^b-1)\tilde{R}(\lambda-\lambda x(z)) \\ (\tilde{V}(\lambda-\lambda x(z))-1)\alpha_j\sum_{i=0}^{M}p_i z^i+(z^b-1)\tilde{R}(\lambda-\lambda x(z))(\tilde{V}(\lambda-\lambda x(z))-1) \\ \sum_{i=0}^{N-1}Q_i z^i\end{array}\right]}{(z^b - \tilde{s}(\lambda-\lambda x(z))(-\lambda+\lambda x(z))} \quad (48)$$

Equation (48) has N+b unknowns $p_0, p_1, p_2, p_3, \ldots, p_{b-1}, q_0, q_1, \ldots, q_{N-1}$. Thefollowing theorms are proven to express q_i in terms p_i in such a way that the numerators has only b constants. By Rouche's theorm of complex variables, it can be proved that $(z^b-1)\tilde{s}(\lambda-\lambda x(z))$ has b-1 zeros inside and one of the unit circle $|z|$=1. since P(z) is analytic within and on the unit circle , the numerator must vanish at these points, which gives b equations with b unknowns.

Steady state Condition

The probability generating function has to satisfy P(1)=1. In order to satisfy this condition, applying L' Hospital's rule and evaluating limit z→1 of P(z) and equating the expression to 1, we have

$$E(S)\left(\sum_a^{b-1}(Ci\_R_i)(b-i)\right)+b\,E(V)\sum_{i=0}^{a-1}\left(C_i+\sum_{j=1}^{M}P_j(z,0)\alpha_j\right)=b-\lambda E(X)E(s) \quad (50)$$

Since left hand side of (50) are the probabilities of 'i' customers being in the queue it follows that left hand side must be positive. Thus P(1)=1 is satisfied iff (b- λE(X)E(s))>0.

If $\rho=\frac{\lambda E(X)E(S)}{b}$ then ρ<1 is the condition to be satisfied for the existence of steady state.

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Particular Case;

When setup time is zero and $\alpha_j = 0$ for j=1,3,...M equation (48) reduces to

$$P(z)=\frac{\left[(\tilde{s}(\lambda-\lambda x(z)-1)\sum_{i=a}^{b-1}(z^b-z^i)(c_i)+(z^b-1)(\tilde{V}(\lambda-\lambda x(z))-1)\sum_{i=0}^{N-1}Q_i z^i\right]}{(z^b - \tilde{s}(\lambda-\lambda x(z))(-\lambda+\lambda x(z))} \quad (51)$$

Equation (48) gives the queue size distribution of $M^X/G(a,b)/1$ queueing system with multiple vacations.

IV. Conclusion

An M x /G(a,b)/1 queue with multiple vacations setup times with N- policy and delayed service has been studied. The PGF of queue size at an arbitrary time epoch is obtained . Some performance measures are also derived As a future work expected queue length, busy period ,Idle period and Cost analysis will be discussed.

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