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Transient Analysis of Electrical Circuits Using Runge-Kutta Method and its Application

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Abstract- An RLC circuit (or LCR circuit) is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively. The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will.

Index Terms- Damping in, Impedance, Kirchhoff Current Law, Kirchhoff Voltage Law, Runge Kutta Method, Second Order Equation

I. Introduction

The time varying currents and voltages resulting from the sudden application of sources, usually due to switching are called TRANSIENTS. In transient analysis we start by writing the circuit equations using basic concepts of KCL, KVL, node-voltage analysis and mesh-current analysis. Due to the involvement of integrals and derivatives in current-voltage relationships for inductances and capacitances, we obtain integro-differential equations which are converted to pure differential equations by differentiating with respect to time. Thus the study of transients requires solving of differential equations. The order of the differential equation depends on the number of energy storage elements present in the circuit.

Some Basic Concepts:-

1. Kirchhoff's Current Law – The sum of currents flowing in and out of a node is zero.
   For a node, $\sum I = 0$.
2. Kirchhoff's Voltage Law –The sum of voltages across various circuit elements in a mesh is zero.
   In a mesh, $\sum v = 0$.
3. Voltage across a capacitor– V = (∫ i dt) /C i.e integral of current flowing through it over a period of time divided by capacitance
4. Current supplied by capacitor- I = C (dv/dt) i.e product of capacitance and derivative of voltage across capacitor with respect to time.
5. Voltage across an inductor- V = L (di/dt) i.e product of inductance and derivative of current through inductor with respect to time
6. Current stored in an inductor- I=(∫ v dt) /L i.e integral of voltage across it over a period of time divided by inductance.

II. Analysis of RLC Circuit

An RLC circuit (or LCR circuit) is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively. The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will. The main difference that the presence of the resistor makes is that any oscillation induced in the circuit will die away over time if it is not kept going by a source. This effect of the resistor is called damping. The presence of the resistance also reduces the peak resonant frequency somewhat. Some resistance is unavoidable in real circuits, even if a resistor is not specifically included as a component.

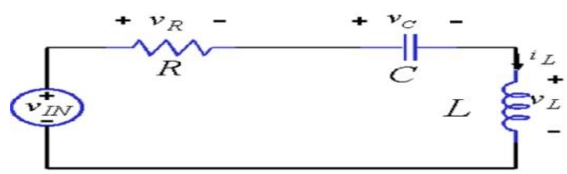

Figure 1.RLC Circuit diagram

The RLC filter is described as a second-order circuit, meaning that any voltage or current in the circuit can be described by a second-order differential equation in circuit analysis.

$I(t)=c.\partial V_c(t)/ \partial t$

Where,

C=capacitance

$V_c(t)$=voltage across capacitance

Then we write KVL equation for the circuit as:

$L\partial I(t)/ \partial t+RI(t)+Vc(t)=Vin$

Substituting for I(t),we get:

$LC\, \partial^2 V_c(t)/ \partial^2(t)+RC\, \partial V_c(t)/ \partial(t)+Vc(t)=Vin$

For the case of the series RLC circuit these two parameters are given by:

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Where ω_0 = natural frequency.

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A useful parameter is the damping factor, ζ, which is defined as the ratio of these two,

$$\zeta = \frac{\alpha}{\omega_0}$$

In the case of the series RLC circuit, the damping factor is given by,

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The value of the damping factor determines the type of transient that the circuit will exhibit.
Some authors do not use ζ and call α the damping factor.
Different conditions for damping factors,
If,

ζ> 1, the system is called over damped.
ζ= 1, the system is called critically damped.
ζ< 1, the system is called under damped.

III. Formulation of RK Method

Runge–Kutta method is an effective method of solving ordinary differential equations of 1storder. If the given ordinary differential equation is of higher order say 'n' then it can be converted to a set of n 1storder differential equations by substitution.

The Runge-Kutta method uses the formulas:

$t_{k+1}=t_{k+h}$

$Y_{j+1}=Y_j+(k1+2k2+2k3+k4)/6$ where K=0,1,2….m-1

Where:

$k_1=hf(tj, Y_j)$

$k_2=hf(t_{j+h}/2,Y_j+k_1/2\ )$

$k_3=hf(t_{j+h}/2,Y_j+k_2/2\ )$

$k_4=hf(t_{j+h},Y_j+k3)$

k_1 is the increment based on the slope at the beginning of the interval, using y_n;

$k_2$is the increment based on the slope at the midpoint of the interval, using $y_n+k_1/2$;

$k_3$is again the increment based on the slope at the midpoint, but now using $y_n+k_2/2$;

$k_4$is the increment based on the slope at the end of the interval, using y_n+k_3.

Voltage equation across 2nd Order RLC circuit is given by,

$$\mathbf{LC}\ \frac{\partial^2 Vc(t)}{\partial t^2}+\mathbf{RC}\frac{\partial Vc(t)}{\partial t}+\mathbf{Vc(t)=Vin}$$

$$\mathbf{I(t)=c}\frac{\partial Vc(t)}{\partial t}$$

So,

$$\frac{\partial I(t)}{\partial t}=\mathbf{Vin\text{-}RI(t)\text{-}Vc(t\ )/L}$$

Now let,

$I(t)=x_1$

$Vc(t)=x_2$

$$\frac{\partial I(t)}{\partial t}=\text{Vin-R*x1-x2/L=g(t,x1,x2)}$$

$$\text{x1/c=}\frac{\partial Vc(t)}{\partial t}\text{=f(t,x1,x2)}$$

Let,

$X(i)=x_1$

$Y(i)=x_2$

Solving the above equation using 4^{th} order R-K method:

f1=h*f(t,x1,x2)

g1=h*g(t,x1,x2)

f2=h*f((t+h/2),(x1+f1/2),(x2+g1/2))

g2=h*g((t+h/2),(x1+f1/2),(x2+g1/2))

f3=h*f((t+h/2),(x2+f2/2),(x2+g2/2))

g3=h*g((t+h/2),(x1+f2/2),(x2+g2/2))

f4=h*f((t+h),(x1+f3),(x2+g3)
g4=h*g((t+h),(x1+f3),(x2+g3))

IV. Solution of the RLC Circuit

x1=x1+((f1+f4)+2*(f2+f3))/6.0

x2=x2+((g1+g4)+2*(g2+g3))/6.0

Where,

$h=(T_f-T_0)/n$

here:

h=step size

T_f=final time

T_o=initial time

Example: Let's take an example to get the transient analysis of circuit for an over damped system.

Let:

R=300, L= 10mH,C=1uF and Vin=10V

Here,

I(0)=0,Vc(0)=0

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

SO, W_0 =1000

$$\alpha = \frac{R}{2L}$$

This gives a result with magnitude 15000

Now, after dividing the two values we get 1.5, therefore the condition is over damped

V. GRAPHS FOR DIFFERENT CONDITIONS OF DAMPING

A. Over Damped Condition

A plot for voltage developed across capacitor and time for a time interval of 1ms. This graph shows that as the time increases, the voltage also increases parabolic ally. The voltage reaches steady state at or after 1 millisecond

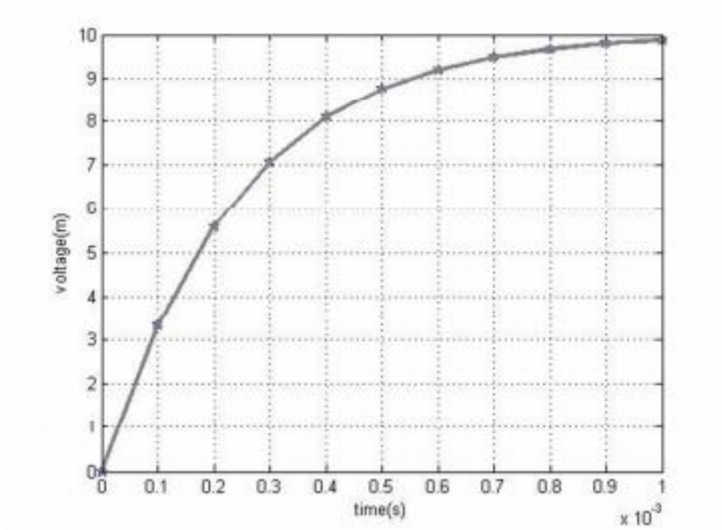

Figure 2. Voltage Vs time graph for over damped conditions

B. Critically Damped Conditions

let us take R=200Ω

then,

α=10000 and ζ=1.5

A plot for voltage developed across capacitor and time for a time interval of 1ms

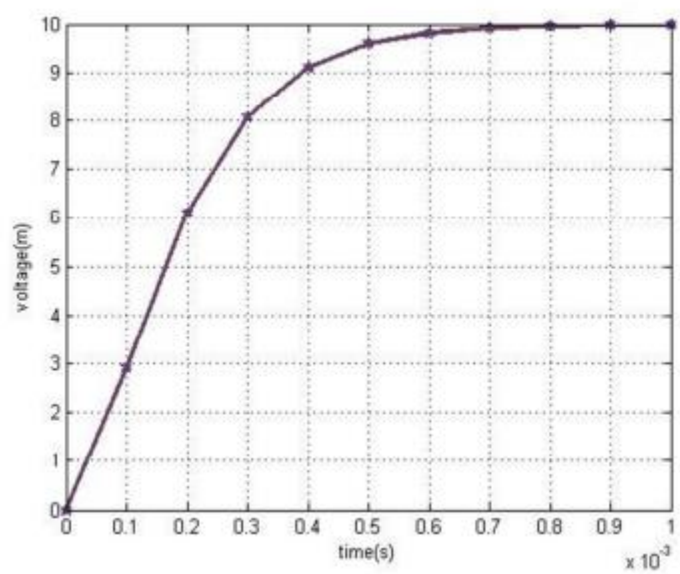

Figure 3. Voltage Vs time graph for critically damped

The voltage here varies parabolic ally with time but it reaches steady state much before 1 millisecond.

C. Under Damped Conditions

Let us take R=100Ω

Then,

α=5000 and, ζ=0.5

A plot for voltage developed across capacitor and time for a time interval of 1ms

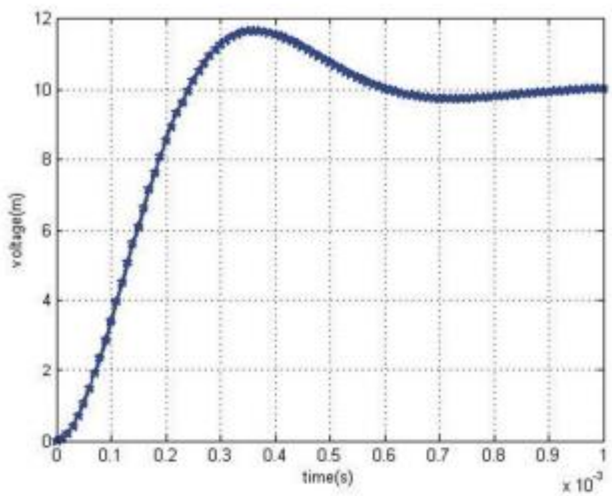

Figure 4. Voltage Vs time for under damped conditions

VI. MATLAB PROGRAMMING

```
clear all;
clc
t=0;
T=0.001;
h=0.0001;
x1=0;
x2=0;
i=1;
c=0.000001;
l= 0.01; r=300; Vin=10 ;
f=@(t,x1,x2) x2/c;
g= @(t,x1,x2)((Vin-x1-r*x2)/l);
fprintf('time\t\tvoltage\tcurrent\n');
for t=0:h:T
fprintf('%f\t%f\t%f\n',t,x1,x2);
X(i)=x1;
grid off Y(i)=x2;
f1=h*f(t,x1,x2);
g1=h*g(t,x1,x2);
f2=h*f((t+h/2),(x1+f1/2),(x2+g1/2));
g2=h*g((t+h/2),(x1+f1/2),(x2+g1/2));
f3=h*f((t+h/2),(x2+f2/2),(x2+g2/2));
g3=h*g((t+h/2),(x1+f2/2),(x2+g2/2));
f4=h*f((t+h),(x1+f3),(x2+g3));
g4=h*g((t+h),(x1+f3),(x2+g3));
x1=x1+((f1+f4)+2*(f2+f3))/6.0;
x2=x2+((g1+g4)+2*(g2+g3))/6.0;
i=i+1;
end
time=[0:h:T];
plot(time,X,'-p');
grid on;
xlabel('time(s)');
ylabel('voltage(m)');
hold on
```

VII. Result and Discussion

From the experiment conducted above, we obtained the following results for transient analysis:

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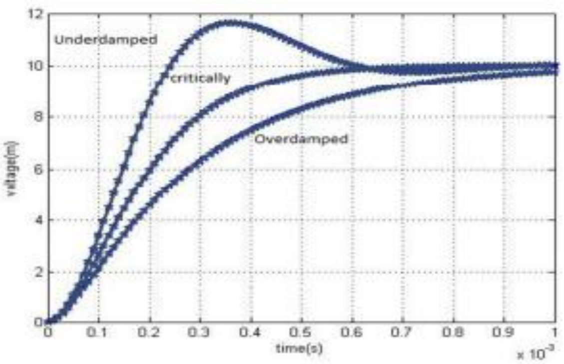

Figure 5. MATLAB Generated final result

This shows that the RK method is very efficient in solving second order differential equations. Thus, we can conclude that by carrying out the transient analysis of a system, we can find out the response of the system by changing the conditions from one steady state value to another. This response helps in designing a system which meets our requirements, and we can further optimize the time domain parameters of the system

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