

Double Diffusive Heat and Mass Transfer over a Vertical Plate in the Presence of Wall Suction and Chemical Reaction

Mangwiro Magodora¹, Kisswell Basira², Precious Sibanda³

^{1,2} Department of Mathematics & Physics, Bindura University, P. Bag 1020, Bindura, Zimbabwe

³ School of Mathematical Sciences, University of KwaZulu-Natal, P. Bag X01, Pietermaritzburg, Scottsville 3209, South Africa

Abstract- A steady incompressible boundary layer flow over a permeable vertical plate in the presence of a chemical reaction and wall suction is investigated. The governing fluid flow equations are transformed into a set of coupled ordinary differential equations with the help of similarity transformations and solved using asymptotic approximations in the presence of large buoyancy to obtain closed form solutions of the skin friction, Nusselt and Sherwood numbers. The effects of varying the buoyancy parameter on the velocity, concentration, temperature, skin friction and the rates of heat and mass transfer are determined and presented graphically, using MATLAB. Results indicate that an increase in buoyancy is accompanied by an increase in fluid velocity and a decrease in

the fluid temperature and fluid concentration. Results also show that an increase in buoyancy is accompanied by an increase in skin friction, while the rates of heat and mass transfer fall rapidly from very large values close to the wall down to a minimum value and then start to increase as the buoyancy parameter becomes larger. It is also noticed that the increase of the rate of heat transfer is more pronounced than the rate of mass transfer as the buoyancy parameter is increased.

Index Terms- Double Diffusive Convection, Mixed Convection, Boundary Layer, Buoyancy, Wall Suction, Skin Friction.

Nomenclature

a	constant	T_w	temperature at the plate surface
C	concentration of chemical species	T_∞	free stream temperature
C_w	concentration at the plate surface	u	velocity component along x-direction
C_∞	free stream concentration	U_∞	free stream velocity
D	diffusion coefficient	v	velocity component along y-direction
f_w	transpiration rate	$V_0(x)$	wall suction
g	acceleration due to gravity	x	coordinate directed upward along the plate
$Gr_{x,c}$	Grashof number due to concentration	y	coordinate directed normal to the plate
$Gr_{x,t}$	Grashof number due to temperature	α	thermal diffusivity
kr	chemical reaction rate constant	β_c	volumetric-expansion coefficient due to concentration
N	buoyancy ratio	β_t	volumetric-expansion coefficient due to temperature
Pr	Prandtl number	δ	boundary layer thickness
Re _x	Reynolds number	ξ	dimensionless buoyancy parameter
Sc	Schmidt number	γ_v	dynamic viscosity
T	fluid temperature	ψ	stream function

I. INTRODUCTION

The phenomenon of heat and mass transfer, also referred to as double diffusive convection, has attracted extensive research interest due to its many applications in science, engineering and technology. Heat and mass transfer involve buoyancy driven flows induced by a combination of temperature and concentration gradients. Many transport processes occur in nature and industrial applications in which combined heat and mass transfer takes place simultaneously due to combined effects of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is encountered in

chemical process industries such as polymer production and food processing as well as in other fields such as oceanography, geology, biology, astrophysics. Heat and mass transfer processes are also observed in buoyancy induced motions in the atmosphere and in bodies of water. Atmospheric flows are driven appreciably by both temperature and concentration gradients while flows in bodies of water are driven by equally important effects of temperature, concentration of dissolved materials and concentration of suspended particulate matter.

The problem being investigated is a case of mixed convection, in which both free convection and forced convection are significantly present. Convective heat transfer is one of the

major modes of heat and mass transfer in fluids. Mixed convection flow finds application in several industrial and technological processes such as cooling of nuclear reactors, thermal pollution, dispersion of pollutants, cooling of electronic devices by electric fans and the use of heat exchange devices.

Several researchers have carried out studies on mixed convection boundary layer flow. Alam, Rahman and Samad [1] carried a numerical investigation of mixed convection boundary layer flow over a vertical plate in a porous medium with heat generation and thermal diffusion. Chamkha [6] carried out a study of the mixed convective flow of a Non-Newtonian power law fluid over a permeable wedge embedded in a porous media with variable wall temperature and concentration. They conducted a parametric study to illustrate the influence of the various physical parameters on temperature and concentration profiles as well as the local Nusselt and Sherwood numbers.

The effect of a chemical reaction on a moving vertical surface was investigated by Muthucurumaraswamy [20] while Muthucumaraswamy, Chandrakala and Raj [21] looked at effects of radiation on convective flow over a moving isothermal vertical plate in the presence of a chemical reaction.

Considerable research has been carried out to investigate the transfer of heat and mass in the last three decades. Makakula, Sibanda, Motsa and Shateyi [13] looked at new numerical techniques for the magnetohydrodynamic flow past a shrinking sheet with heat and mass transfer in the presence of a chemical reaction.

The phenomenon of combined heat and mass transfer was also studied by Hossain & Rees [11] when they considered natural convection flow over a vertical wavy surface. They used the implicit finite difference method with the Keller box approach to solve the transformed boundary layer equations. Other researches tackling mixed convection include those of

Bachok, Ishak and Pop [3], as well as that of Bachok and Ishak [4] and Gorla, Chamkha and Rashad [9].

Another notable contribution, which tackles similar problems, was made by Guria & Jana [10] when they studied the hydrodynamic effect on three-dimensional flow past a vertical porous plate. Approximate solutions were obtained by using perturbation techniques. They found out that fluid velocity increased with increase in Brandt number. They also observed that fluid velocity also increased with increase in suction parameter. They also found out that decreases with increase in either suction parameter or Prandtl number or frequency parameter.

More recent studies have seen contributions from eminent researchers who have also published widely in the area of boundary flow past a vertical plate. These researches include the work of Makinde and Sibanda [14], in which they investigated the effect of chemical reaction on the boundary layer flow past a vertical stretching surface in the presence of internal heat generation.

II. PROBLEM FORMULATION

The equations governing the heat and mass transfer over a vertical plate in the presence of wall suction and diffusion of chemical species emanate from the basic principles of mass conservation, momentum conservation, energy conservation and mass diffusion. We employ the Boussinesq and boundary layer approximations to obtain the following partial differential equations which, when taken together model the heat and mass transfer of the system under investigation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} + g\beta_t(T - T_\infty) + g\beta_c(C - C_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_r(C - C_\infty) \quad (4)$$

The boundary conditions for the system are:

$$u(x,0) = 0, \quad v(x,0) = -V(x,0), \quad T(x,0) = T_w, \quad T(x, \infty) = T_\infty \quad (5)$$

$$C(x,0) = C_w, \quad C(x, \infty) = C_\infty, \quad U(x,0) = U_\infty = ax \quad (6)$$

The governing equations (1) to (4) are transformed into dimensionless form by making use of the following similarity transformations:

$$\eta = \sqrt{\frac{a}{2\gamma}} y, \quad \xi(x, y) = xf(\eta)\sqrt{2a\gamma}, \quad \theta(x, y) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(x, y) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \xi = \frac{Gr_{x,t}}{Re_x^2} \quad (7)$$

$$Gr_{x,t} = \frac{g\beta_t x^3 (T_w - T_\infty)}{\gamma^2}, \quad Pr = \frac{\gamma}{\alpha}, \quad Gr_{x,c} = \frac{g\beta_c x^3 (C_w - C_\infty)}{\gamma^2}, \quad Sc = \frac{\gamma}{D}, \quad N = \frac{Gr_{x,t}}{Gr_{x,c}} \quad (8)$$

The model equations (1) to (4) reduce to the following set of three non-dimensional nonlinear ordinary differential equations (9) to (11) and their associated boundary conditions (12) to (13) as given below:

$$f'''' + 2ff'' - 2(f')^2 + 2\eta(\theta + N\phi) = 0, \quad (9)$$

$$\phi'' + 2fSc\phi' - 2\lambda Sc\phi = 0, \quad (10)$$

$$\theta'' + 2fPr\theta' = 0, \quad (11)$$

$$f(\xi, 0) = f_w \xi = \xi, \quad f'(\xi, 0) = 0, \quad f'(\xi, \infty) = 1, \quad (12)$$

$$\theta(\xi, 0) = \phi(\xi, 0) = 1, \quad (13)$$

where $f_w = \frac{V_0 x}{U_\infty \sqrt{2}} \frac{\sqrt{Re_x}}{\xi}$.

The physical quantities of interest in the study are the skin friction coefficient C_f , the Nusselt number Nu_x and the Sherwood number Sh_x which are respectively defined as:

$$C_f = \frac{\tau_w}{(\frac{1}{2})\rho U_\infty^2}, \quad Nu_x = \frac{q_w x}{k_t (T_w - T_\infty)}, \quad Sh_x = \frac{J_w x}{D(C_w - C_\infty)} \quad (14)$$

where ρ is the fluid density, k_t is the fluid thermal conductivity and τ_w , q_w and J_w are defined as

$$\tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0}, \quad q_w = -k_t \frac{\partial T}{\partial y} \Big|_{y=0} \text{ and } J_w = -D \frac{\partial C}{\partial y} \Big|_{y=0}. \quad (15)$$

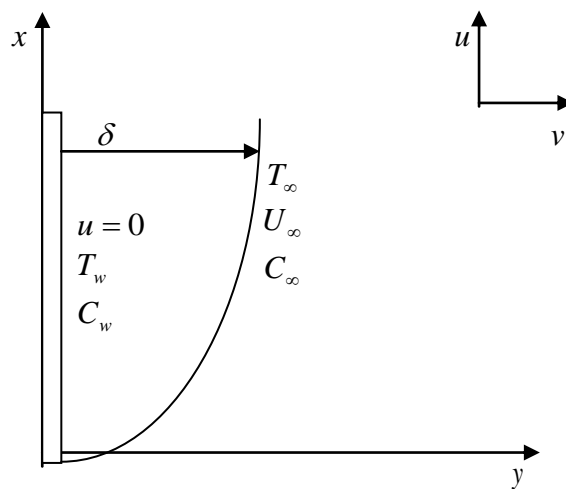


Figure 1: The coordinate system and flow configuration

III. PROBLEM SOLUTION

Asymptotic expansions in the limit where the buoyancy parameter ξ tends to infinity were used to find solutions of the governing equations (9) to (11). Analysis the orders of magnitude of the continuity and momentum equations and letting $u \sim O(U_\infty)$

and $y \sim O(\delta)$ yields $f \sim \frac{1}{\eta}$.

The boundary conditions (18) show that $f \sim O(\xi)$ which in turn implies that $\eta \sim \xi^{-1}$. Assuming that both θ and ϕ remain $O(1)$ as ξ becomes large, we define a function $Y \sim O(1)$ and let

$$\eta = \xi^{-1} Y, \quad f(\eta, \xi) = \xi F(Y), \quad \theta(\eta, \xi) = G(Y), \quad \phi(\eta, \xi) = H(Y) \tag{16}$$

where $F(Y) = 1 + \xi^{-2} F_1(Y) + \xi^{-3} F_2(Y) + \dots$ (17)

$$G(Y) = G_1(Y) + \xi^{-2} G_2(Y) + \xi^{-3} G_3(Y) + \dots \tag{18}$$

$$H(Y) = H_1(Y) + \xi^{-2} H_2(Y) + \xi^{-3} H_3(Y) + \dots \tag{19}$$

Now $f'(\eta, \xi) = \frac{df}{d\eta} = \xi^2 F'(Y), \quad f''(\eta, \xi) = \xi^3 F''(Y), \quad f'''(\eta, \xi) = \xi^4 F'''(Y).$ (20)

Substituting for $F(Y), F'(Y), F''(Y)$ and $F'''(Y)$ in the equation $f(\eta, \xi) = \xi F(Y)$ and equations (20) results in the equations:

$$f(\eta, \xi) = \xi + \xi^{-1} F_1(Y) + \xi^{-2} F_2(Y) + \dots \tag{21}$$

$$f'(\eta, \xi) = F_1'(Y) + \xi^{-1} F_2'(Y) + \dots \tag{22}$$

$$f''(\eta, \xi) = F_1''(Y) + F_2''(Y) + \dots \tag{23}$$

$$f'''(\eta, \xi) = \xi^4 F_1'''(Y) + \xi F_2'''(Y) \dots \tag{24}$$

Similarly for θ and ϕ we obtain the following systems of equations:

$$\theta(\eta, \xi) = G_1(Y) + \xi^{-2} G_2(Y) + \dots \tag{25}$$

$$\theta'(\eta, \xi) = \xi G_1'(Y) + \xi^{-1} G_2'(Y) + \dots \tag{26}$$

$$\theta''(\eta, \xi) = \xi^2 G_1''(Y) + G_2''(Y) + \dots \tag{27}$$

$$\phi(\eta, \xi) = H_1(Y) + \xi^{-2} H_2(Y) + \dots \tag{28}$$

$$\phi'(\eta, \xi) = \xi H_1'(Y) + \xi^{-1} H_2'(Y) + \dots \tag{29}$$

$$\phi''(\eta, \xi) = \xi^2 H_1''(Y) + H_2''(Y) + \dots \tag{30}$$

We substitute equations (21) – (30) into equations (9) - (11) and compare coefficients to get the following system of ordinary differential equations:

$$F_1''' + 2F_1'' = 0, \tag{31}$$

$$F_2''' + 2F_2'' + 2G_1 + 2NH_1 = 0, \tag{32}$$

$$\text{Pr}^{-1} G_1'' + 2G_1' = 0, \tag{33}$$

$$\text{Pr}^{-1} G_2'' + 2G_2' + 2F_1 G_1' = 0, \tag{34}$$

$$Sc^{-1} H_1'' + 2H_1' = 0, \tag{35}$$

$$Sc^{-1} H_2'' + 2H_2' + 2F_1 H_1' - \lambda H_1 = 0. \tag{36}$$

The boundary conditions (12) and (13) imply that the associated transformed boundary conditions for the ordinary differential equations (31) to (36) are:

$$F(0) = 1, F'(0) = 0, F'(\infty) = \xi^{-2}, G(0) = 1, G(\infty) = 0, H(0) = 1, H(\infty) = 0. \tag{37}$$

Solving equations (31), (33) and (35) gives the solutions:

$$F_1(Y) = \delta_1 e^{-2Y} + \delta_2 Y + \delta_3, \tag{38}$$

$$G_1(Y) = \alpha_1 e^{-2\text{Pr}Y} + \alpha_2, \tag{39}$$

$$H_1(Y) = \beta_1 e^{-2ScY} + \beta_2, \tag{40}$$

where $\delta_1, \delta_2, \delta_3, \alpha_1, \alpha_2, \beta_1$ and β_2 are constants to be determined.

Now substituting for H_1 and G_1 in equations (32), (34) and (36) yields:

$$F_2'''' + 2F_2'' = -2\alpha_1 e^{-2\text{Pr}Y} - 2N\beta_1 e^{-2ScY} - 2\alpha_2 - 2N\beta_2, \tag{41}$$

$$\text{Pr}^{-1} G_2'' + 2G_2' = 4\text{Pr} \alpha_1 e^{-2\text{Pr}Y} (\delta_1 e^{-2Y} + \delta_2 Y + \delta_3), \tag{42}$$

$$Sc^{-1} H_2'' + 2H_2' = 4Sc\beta_1 e^{-2ScY} (\delta_1 e^{-2Y} + \delta_2 Y + \delta_3) + \lambda\beta_1 e^{-2ScY} + \lambda\beta_2. \tag{43}$$

Applying reduction of order to equation (41) – (43) gives the solutions of $F_2(Y)$, $G_2(Y)$ and $H_2(Y)$ as

$$F_2(Y) = \frac{\alpha_1}{4\text{Pr}^2(\text{Pr}-1)} e^{2\text{Pr}Y} + \frac{N\beta_1}{4Sc^2(Sc-1)} e^{-2ScY} - (\alpha_2 + N\beta_2)Y - \frac{\delta_4}{2} e^{-2Y} + \delta_5, \tag{44}$$

$$G_2(Y) = \frac{\alpha_1 \delta_1 \text{Pr}^2}{\text{Pr}+1} e^{-2(\text{Pr}+1)Y} - 2\text{Pr}^2 \alpha_1 \delta_2 \left(\frac{Y^2}{2\text{Pr}} + \frac{Y}{2\text{Pr}^2} + \frac{1}{4\text{Pr}^3} \right) e^{2\text{Pr}Y} - 4\text{Pr}^2 \alpha_1 \delta_3 \left(\frac{Y}{2\text{Pr}} + \frac{1}{4\text{Pr}^2} \right) e^{-2\text{Pr}Y} - \frac{\alpha_3}{2\text{Pr}} e^{-2\text{Pr}Y} + \alpha_4, \tag{45}$$

$$H_2(Y) = \frac{\beta_1 \delta_1 Sc^2}{Sc+1} e^{-2(Sc+1)Y} - 2Sc^2 \beta_1 \delta_2 \left(\frac{Y^2}{2Sc} + \frac{Y}{2Sc^2} + \frac{1}{4Sc^3} \right) e^{2ScY} - 4Sc^2 \beta_1 \delta_3 \left(\frac{Y}{2Sc} + \frac{1}{4Sc^2} \right) e^{-2ScY} - \frac{\beta_3}{2Sc} e^{-2ScY} + \frac{\beta_2}{2} \lambda Y + \beta_4, \tag{46}$$

where $\alpha_3, \alpha_4, \beta_3, \beta_4, \delta_5$ and δ_6 are new constants to be determined.

Considering boundary conditions (37) and comparing coefficients gives the values of all the required constants as:

$$\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = \frac{\text{Pr}^3}{\text{Pr}+1} + \text{Pr}-1, \alpha_4 = 0$$

$$\beta_1 = 1, \beta_2 = 0, \beta_3 = \frac{Sc^3}{Sc+1} + Sc - \frac{\lambda}{2} - 1, \beta_4 = 0$$

$$\delta_1 = \frac{1}{2}, \delta_2 = 1, \delta_3 = -\frac{1}{2}, \delta_4 = \frac{1}{Pr(1-Pr)} + \frac{N}{Sc(1-Sc)}, \delta_5 = 0 \text{ and } \delta_6 = \frac{1}{4Pr^2} + \frac{N}{4Sc^2}.$$

Substituting these values in equations (38) – (40) and (44)-(46) and letting , $N = 1$ and $\lambda = 1$ equations (22), (25) and (28) imply that

$$f'(\eta) = 1 - e^{-2\xi\eta} + \frac{1}{2Pr(1-Pr)} e^{-2Pr\xi\eta} + \frac{1}{2Sc(1-Sc)} e^{-2Sc\xi\eta} - \frac{1}{2Pr(1-Pr)} e^{-2\xi\eta} - \frac{1}{2Sc(1-Sc)} e^{-2\xi\eta},$$

$$\theta(\eta, \xi) = e^{-2Pr\xi\eta} + \frac{Pr^2}{2\xi^2(1+Pr)} e^{-2(1+Pr)\xi\eta} - \left[\eta^2 Pr + \frac{\eta}{\xi} - \frac{\eta}{\xi} Pr + \frac{Pr^2}{2\xi^2(1+Pr)} \right] e^{-2Pr\xi\eta},$$

$$\phi(\eta, \xi) = e^{-2Sc\xi\eta} + \frac{Sc^2}{2\xi^2(1+Sc)} e^{-2(1+Sc)\xi\eta} - \left[\eta^2 Sc + \frac{\eta}{\xi} - \frac{\eta}{\xi} Sc + \frac{Sc^2}{2\xi^2(1+Sc)} + \frac{\eta}{2\xi} \right] e^{-2Sc\xi\eta}.$$

Also note that $F_1''(0) = 1, F_2''(0) = \frac{1}{Pr} + \frac{N}{Sc}, G_1'(0) = -2Pr, G_2'(0) = -\frac{1}{1+Pr}, H_1'(0) = -2Sc$

and $H_2'(0) = \frac{\lambda}{2} - \frac{1}{1+Sc}.$

By carrying out differentiation and making suitable substitutions, it can be shown that:

$$C_f = \frac{2}{\sqrt{2}} (\sqrt{Re_x})^{-1} f''(\xi, 0), Nu_x = -\frac{2}{\sqrt{2}} (\sqrt{Re_x}) \theta'(\xi, 0), Sh_x = -\frac{2}{\sqrt{2}} (\sqrt{Re_x}) \phi'(\xi, 0), \quad (47)$$

$$f''(\xi, 0) = 2\xi + \frac{1}{Pr} + \frac{N}{Sc}, \theta'(\xi, 0) = -2\xi Pr - \frac{1}{\xi(1+Pr)}, \phi'(\xi, 0) = -2\xi Sc - \frac{1}{\xi(1+Sc)} - \frac{\lambda}{2\xi}. \quad (48)$$

Plotting the graph of $f'(\eta)$ against η for different values of ξ gives us the velocity profile while plotting the graph of θ and ϕ against η yields the temperature and concentration profiles respectively. As equations (47) and (48) suggest, plotting the graphs of $f''(\xi), -\theta'$ and $-\phi'$ against ξ produce, respectively, the variation of the skin friction, rate of heat transfer and rate of mass transfer as the buoyancy parameter is increased.

IV. RESULTS AND DISCUSSION

In this study, a Prandtl number $Pr = 0.71$, which corresponds to air at 200 C and a Schmidt number $Sc = 0.6$ corresponding to water vapour diffusing in air were used for the reason that water vapour and air are the most commonly used fluids in industrial engineering and applications. A fixed buoyancy ratio $N = 1$ and a Grashof number $Gr = 1$ were also used. A Grashof number of unity was used in order to allow for both inertia and buoyancy forces to contribute to the flow.

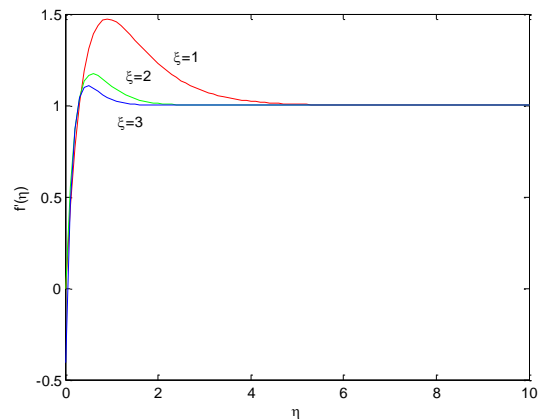


Figure 2: Velocity profile

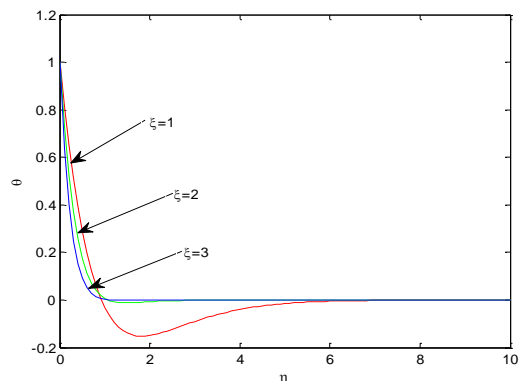


Figure 3: Temperature Profile

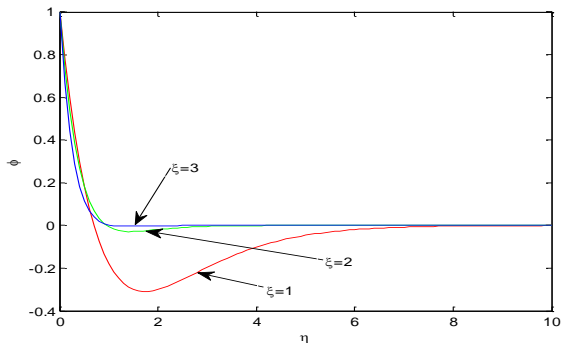


Figure 4: Concentration Profile

Figures 2 to 4 show profiles obtained for the velocity, temperature and concentration respectively. In the vicinity of the vertical wall, Figure 2 illustrates that increase in buoyancy is associated with a significant increase in velocity, as expected. However further away from the plate, there is a reduction in the velocity of the fluid. Figures 3 and 4 show that an increase in buoyancy is accompanied by a decrease in fluid temperature and concentration. Figures 2 to 4 also illustrate that the boundary layer is significantly reduced by increasing the buoyancy parameter ξ .

The decrease in the concentration of the chemical species can be attributed to diffusion of chemical species into the fluid and the increased velocity of the fluid as the buoyancy is increased. As the velocity of the fluid particles increases due to an increase in buoyancy, the particles of the diffusing fluid are immediately carried away, and that explains the reduced concentration close to the wall.

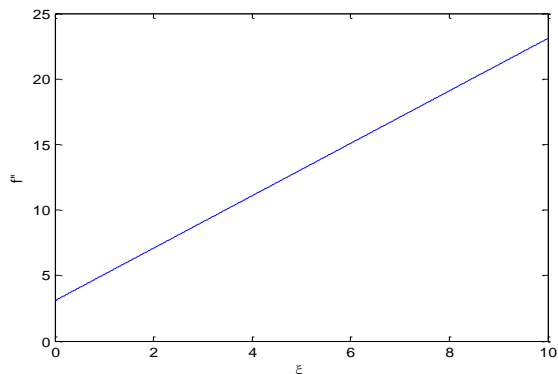


Figure 5: Variation of skin friction with buoyancy

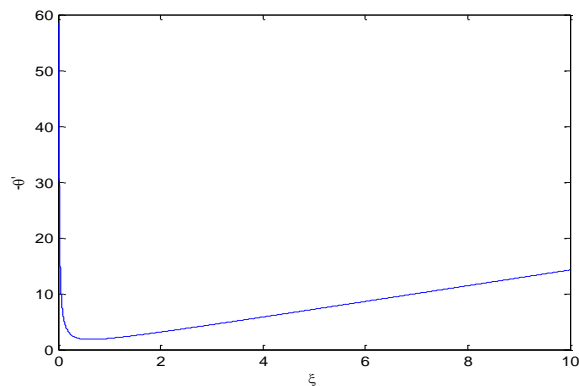


Figure 6: Variation of the rate of heat transfer with buoyancy

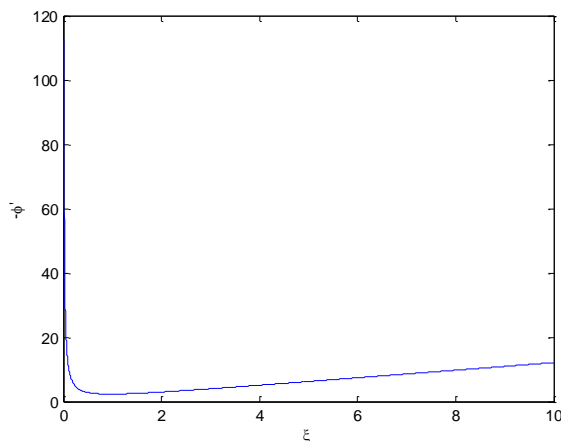


Figure 7: Variation of the rate of mass transfer with buoyancy

Figures 5 to 7 show the effects of varying the buoyancy on the skin friction and rates of heat and mass transfer. It is evident from the figures that an increase in buoyancy is accompanied by a linear increase in skin friction, while the rates of heat and mass transfer fall rapidly from very large values close to the wall down to a minimum value and then start to increase as the buoyancy parameter becomes larger. It is also noticed that the increase of the rate of heat transfer is more pronounced than the heat of mass transfer as the buoyancy parameter increases.

V. CONCLUSION

The study considered the double diffusive heat and mass transfer processes over a permeable vertical plate in the presence of wall suction and chemical reaction. The equations of flow were derived from the basic principles of mass conservation, energy conservation, heat and mass diffusion. The equations were non-dimensionalised by use of appropriate approximations and the resulting non-linear differential equations then solved by means of asymptotic expansions in the limit of large buoyancy. The variation of buoyancy with velocity, temperature, concentration, skin friction, rates of heat and mass transfer were presented graphically and analyzed.

Results obtained showed that in the vicinity of the plate wall, an increase in buoyancy causes an increase in the velocity of the fluid. Furthermore, the results indicate that the skin friction, heat and mass transfer rates are enhanced by an increase of buoyancy.

Profiles obtained also indicated that an increase buoyancy is accompanied by a decrease in fluid temperature as well as fluid concentration. It was also noticed that the concentration and thermal boundary layers are reduced as a consequence of increasing the buoyancy.

Results showed that even though an increase in buoyancy leads to a linear increase in skin friction, it was noted that an increase in buoyancy causes a sharp decrease in the rates of heat and mass transfer for very small values of the buoyancy parameter. However as the buoyancy parameter becomes larger, the rates of heat and mass transfer increase proportionally with the buoyancy.

REFERENCES

- [1] Alam MS, Rahman MM & Samad MA(2006). Numerical study of the combined free-forced convection and mass transfer flow past a vertical plate in a porous medium with heat generation and thermal diffusion. *Nonlinear Analysis: Modeling and Control* 11(4):331-343.
- [2] Aman F & Ishak A (2012). Mixed Convection Boundary Layer Flow towards a Vertical Plate with a Convective Surface Boundary Condition. *Mathematical Problems in Engineering* Volume Article ID 453457
- [3] Bachok N, Ishak A & Pop I (2010). Mixed convection boundary layer flow over a permeable vertical flat plate embedded in an anisotropic porous medium. *Mechanical Problems in Engineering*
- [4] Bachok N & Ishak A (2009). Mixed convection boundary layer flow over a permeable vertical cylinder with prescribed surface heat flux. *European Journal of Scientific Research* 34(1):46-54.
- [5] Beg OA, Beg TA, Bakier AY & Prasad VR (2009). Chemically reacting mixed convective heat and mass transfer along inclined and vertical plates with Soret and Daffour. *Int. Journal of Applied Math and Mech* 5(2):39-57.
- [6] Chamkha AJ (2010). Heat and mass transfer of a Non-Newtonian fluid flow over a permeable wedge in porous media with variable wall temperature and concentration and heat source or sink. *WEAS Transactions on Heat and Mass Transfer 1: Volume 5*.
- [7] Chaudhary RC & Kumar A (2008). Heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with hall effect. *Latin American Applied Research* 38:17-26.
- [8] Ferdows & Al-Mdallal, QM (2012). Effects of Order of chemical reaction on a boundary layer flow with heat and mass transfer over a linearly stretching sheet. *American Journal of Fluid Dynamics* 2(6): 89-94.
- [9] Gorla RSR, Chamkha AJ & Rashad AM (2011). Mixed convective boundary layer flow in a porous medium saturated with a nanofluid: Natural convection dominated regime. *Springer Open Journal, Nanoscale Research Letters* 6: 207-714.
- [10] Guria M & Jana RN (2005). Hydrodynamic effect on the three-dimensional flow past a vertical porous plate. *International Journal of Mathematics and Mathematical Sciences* 20:3359-3372.
- [11] Hossain MA & Rees DAS(1999). Combined heat and mass transfer in natural convection flow from a wavy surface. *Acta Mathematica* 136:133-141.
- [12] Lan WM, Lin TF & Yang CJ (1988). Combined heat and mass transfer in natural convection between vertical parallel plates. *Warme und Stoffubtragung* : Springer-Verlag 23:69-76.
- [13] Makakula ZG, Sibanda P, Motsa SS & Shateyi S (2011). On new numerical techniques for the MHD flow past a shrinking sheet with heat and mass transfer in the presence of a chemical reaction. *Hindawi Publishing Corporation: Mathematical problems in Engineering*
- [14] Makinde OD & Sibanda P (2011). Effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation. *International Journal of Numerical Methods for Heat & Fluid Flow* 21(6): 779 - 792.
- [15] Mohamed RA, Osman ANA & Abo-Dahab SM (2012). Unsteady MHD double-diffusive convection boundary-layer flow past a radiate hot vertical surface in porous media in the presence of chemical reaction and heat sink. *Meccanica: Springer Science+Business*
- [16] Moorthy MBK & Senthilvadivu K (2011). Effect of variable viscosity on a convective heat and mass transfer by natural convection from a horizontal surface in a porous medium. *WEAS Transactions on Mathematics* 6 (10): 210-218.
- [17] Moorthy MBK & Senthilvadivu K (2012). Effect of variable viscosity on a convective heat and mass transfer by natural convection from a vertical surface in a porous medium. *WEAS Transactions on Mathematics* 9 (11): 751-759.
- [18] Motsa SS & Sibanda P (2012). On the solution of MHD flow over a non-linear stretching sheet by an efficient semi-analytical technique. *Int. J. Numer. Meth. Fluids* 68:1524-1537.
- [19] Motsa SS, Sibanda P & Marewo GT (2012). On a new analytical method for flow between two inclined walls. *Numer Algor* 68:1524-1537.
- [20] Muthucumaraswamy R (2002). Effects of suction on heat and mass transfer along a moving vertical surface in the presence of a chemical reaction. *Forsch Ingenieurwes* 67:129-132.
- [21] Muthucumaraswamy R, Chandrakala P & Raj A (2006). Radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. *Int Journal of Applied Mechanics and Engineering* 11(3): 639-646.
- [22] Nazar R & Amin N(2003). Free convection boundary layer flow over a non-isothermal vertical flat plate. *Journal Teknologi* 39:61-74.
- [23] Olasusi E & Sibanda P (2006). On variable laminar convective flow properties due to a porous rotating disk in a magnetic field. *Rom. Journ. Phys.* 51(9-10): 937-950.
- [24] Ouachria Z, Rouichi F & Haddad D (2012). Double diffusion effects on vertical plate embedded in porous media. *Frontiers in Heat and Mass Transfer* 3: 023004.
- [25] Patil PM, Roy S, & Chamka AJ (2009). Double diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and a chemical reaction. *Turkish Journal of Eng. and Env. Science* 33: 193-205.
- [26] Shateyi S (2008). Thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. *Hindawi Publishing: Journal of Applied Mathematics* Article ID 414830.
- [27] Shateyi S & Motsa S (2011). Unsteady Magneto hydrodynamic Convective Heat and Mass Transfer Past an Infinite Vertical Plate in a Porous Medium with Thermal Radiation, Heat Generation/Absorption and Chemical Reaction. *Advanced Topics in Mass Transfer - Prof. Mohamed El-Amen* : 141-162.
- [28] Shateyi S, Sibanda P & Motsa SS (2009). Convection from a stretching surface with suction and power-law variation in species concentration. *Into Journal of Heat and Mass Transfer* 45: 1099-1106.
- [29] Sri Haru Babu V & Ramana Reddy GV (2011). Mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation. *Advances in Applied Science Research* 2(4):138-146.
- [30] Tomer NS, Singh P & Kumar M(2011). Effect of Variable viscosity on Convective Heat Transfer along an Inclined Plate Embedded in Porous Medium with an Applied Magnetic Field. *World Academy of Science, Engineering and Technology* 51: 992-996.
- [31] Uddin MJ, Khan WA, Ahmed I & Ismail AI (2012). MHD Free Convective Boundary Layer Flow of a Nanofluid past a Flat Vertical Plate with Newtonian Heating Boundary Condition. *PLoS ONE* 7 (11): e49499. doi:10.1371/journal.pone.0049499

AUTHORS

First Author – Mangwiro Magodora, Department of Mathematics & Physics, Bindura University, P. Bag 1020, Bindura, Zimbabwe

Second Author – Kisswell Basira, Department of Mathematics & Physics, Bindura University, P. Bag 1020, Bindura, Zimbabwe

Third Author – Precious Sibanda, School of Mathematical Sciences, University of KwaZulu-Natal, P. Bag X01, Pietermaritzburg, Scottsville 3209, South Africa