International Journal of Scientific and Research Publications, Volume 3, Issue 11, November 2013 1
ISSN 2250-3153

Shortest Route Algorithm Using Fuzzy Graph

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Abstract- The current widespread use of location-based services and **Global Positioning System** technologies has revived interest in very fast and scalable fuzzy shortest path queries. We introduce a new shortest path query type in which dynamic constraints may be placed on the allowable set of edges that can appear on a valid fuzzy shortest path (e.g., dynamically restricting the type of roads or modes of travel which may be considered in a multimodal transportation network). Computing the shortest path between two given locations in a road network is an important problem that finds applications in various map services and commercial navigation products. Our experimental results reveal the characteristics of different techniques ,based on which we provide guidelines on selecting appropriate methods for various scenarios. Although the raw data about geography and roads may be more readily available today, computing fuzzy shortest paths is still not trivial. kruskal's algorithm allows us to compute point-to-point fuzzy shortest path queries on any road network in essentially linear time. . In a preprocessing stage, these heuristics compute some auxiliary data, such as additional edges (shortcuts) and labels or values associated with vertices or edges.

Index Terms- Fuzzy Graph, Fuzzy Shortest Path, Fuzzy Policy.

I. INTRODUCTION

We consider a fuzzy Network N = (V, E) consisting n nodes (cities) and m edges (roads) connecting the cities of a country. If we measure the crowdness that is traffic of the roads of the network for particular time duration, it is quite impossible to measure the crowdness in duration as it is not fixed, but varies from time to time. So, appropriate technique to gradation of crowdness is an interval and not a point. In this case, the network N is an fuzzy network in which the weight of the each arc (i, i+1) depends upon the crowdness.

Suppose that we want to select the fuzzy shortest route (path) between two cities. The following route fuzzy network provides the possible routes between the starting city at node A and the destination city at node U. The routes pass through intermediate cities designated by different stages. Let G be a road network (*i.e.*, a degree-bounded connected fuzzy graph) with an edge set E and a vertex set V that contains n vertices. Let each edge $e \in E$ be associated with a fuzzy weight w(e), which we assume (without loss of generality) to be the length of e. For ease of exposition, we consider undirected fuzzy graphs in this work.

II. DEFINITION

Definition 2.1: A ***fuzzy graph*** with V as the underlying set is a pair G: (A, Γ) where

A: V --→ [0,1] is a fuzzy subset, Γ : VxV ------> [0,1] is a fuzzy relation on the fuzzy subset A, such that $\Gamma(u,v) \leq A(u) \cap A(v)$ for all u,v ϵ V.

Definition 2.2: A directed ***fuzzy walk*** in a fuzzy graph is an alternating sequence of vertices and edges , $x_0, e_1, x_1, \ldots, e_n, x_n$ in which each edge e_i is $x_{i-1}x_i$.

Definition 2.3: A ***fuzzy path*** is a fuzzy walk in which all vertices are distinct.

Definition 2. 4: A fuzzy path from u to v, the v is said to be ***reachable*** from u, and the distance , D(u,v), from u to v is the length of any shortest such fuzzy path.

Definition 2.5: A fuzzy path between the point s (source) to t (sink) of fuzzy graph G is called fuzzy policy or fuzzy tree.

III. SHORTEST ROUTE ALGORITHM

To find the path of minimum distance between the point s(source) to t (sink) of fuzzy graph G can be obtained using the following steps:

Step 1: Identify the decision variables and specify objective function to be optimized for fuzzy networks.

Step 2: Start by assigning the notation x_i (i=1,2,…n) to the decision variables connected with each of the cities .

Step 3: Decompose the fuzzy network into a number of smaller sub intervals. Identify the stage variable at each stage and write down the fuzzy transformation function as a function of the state variable and decision variable at the next stage.

Step 4: Represent each x_i as stage (i+1), where S(i+1) denotes the distance between x_i and x_{i+1}.

Step 5: Assign the initial value of the fuzzy network as zero. (i.e)the value of stage (i+1) is zero.

Step 6: Write down a general recursive relationship for completing the fuzzy optimal policy of Fuzzy Network by using the fuzzy dynamic programming recursion as follows.

$D(x_i, x_{i+1})$ = min { stage (i) to stage (i+1) by multiple reaching point}.

Step 7: Construct appropriate stage to show the required values of the return function at each Stage in Fuzzy Networks.

Step 5: Determine the overall fuzzy optimal decision or policy and its value at each stage of an Fuzzy Networks.

Step 6: We get the shortest path of Fuzzy Networks .

IV. PROBLEM DEFINITION

We shall illustrate the technique with a simple example and provide the mathematical verification.

4.1 -TRANSPORT OPTIONS FROM CHENNAI TO KANYAKUMARI

For shortest route between the points Chennai and Kanyakumari by bus with fuzzy graph, consider the Tamilnadu map and point some cities and consider them as vertices. Point A (Chennai), located at initial stage, leads towards 7 stages with the end point U (Kanyakumari).

Fig.4.1

Here we consider each city as vertex and the distance between any two cities is represented as edge or arc. In order to develop the above problem in terms of fuzzy graph , the distance between any two cities are consider in the following manner:

Table 4.1

Distance in km	Membership grades
50-75	0.02
75-100	0.04
100-125	0.06
125-150	0.08
150-175	0.1
175-200	0.12
200-225	0.14
225-250	0.16
250-275	0.18

In order to identify the city involved in this problem, let us adopt alphabet A-CHENNAI , B-VELLORE, C-CHENGALPATTU, D-VILLUPURAM, E-THIRUVANAMALAI, F-DHARMAPURI, G-PERAMBALUR, H-NAMAKAL, I- ERODE, J-OOTY, K-THANJAVUR, L-TRICHY, M-COIMBATORE , N-PUDUKOTTAI , O-DINDIUL, P-KODAIKANAL, Q-RAMANATHAPURAM, R-MADURAI, S-TUTICORIN, T- THIRUNELVELI and U-KANYAKUMARI.

The figure 4.2 gives the distance of each cities

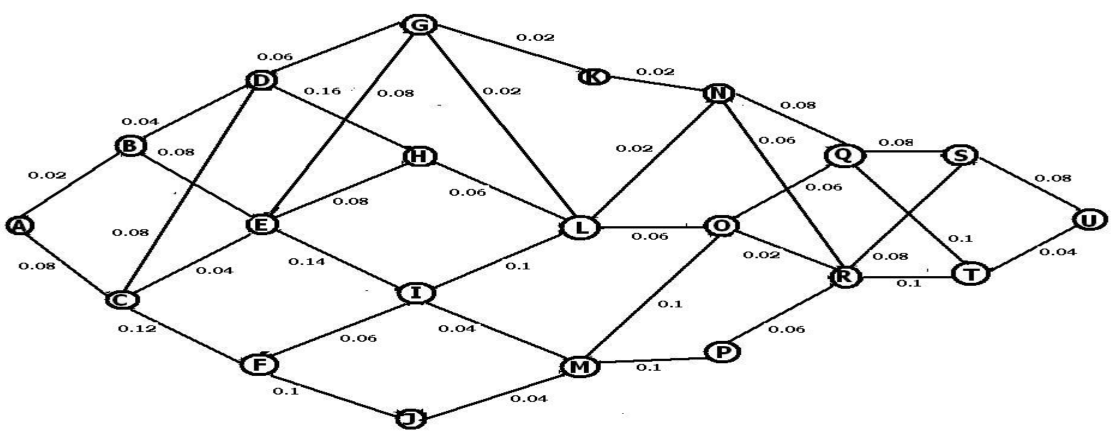

Figure 4.2- Fuzzy graph from Chennai to Kanyakumari with different routes

The decision variables do not initially have a number assigned but would be defined at each stage by an appropriate vertex on the same vertical line as the decision variable. For example, X_3 can be represented by G,H, I, or J. hence the decision variables can be the collection of vertices as follows:

X_0	:	A
X_1	:	B,C
X_2	:	D, E, F
X_3	:	G, H, I, J
X_4	:	K, L, M
X_5	:	N, O, P
X_6	:	Q, R,
X_7	:	S, T
X_8	:	U

Any route going from A to U is called fuzzy policy or fuzzy tree. For example , ACFJMPRTU is a fuzzy policy or fuzzy tree .A fuzzy subpolicy or fuzzy subtree is any shortest path. For example , CEHLOQSU, BDGKNQTU, etc. are fuzzy subpolicies or fuzzy subtrees. In this fuzzy network, it is easy to enumerate all possible fuzzy policies or fuzzy trees. Among the possible fuzzy policies we have to find the fuzzy shortest distance between the point A and U.

We start by assigning the notation

Stage i – Si – distance between X_{i-1} and X_i (i = 1,2…. 8) to the decision variables connected with each of the stages as shown in figure (4.3) .

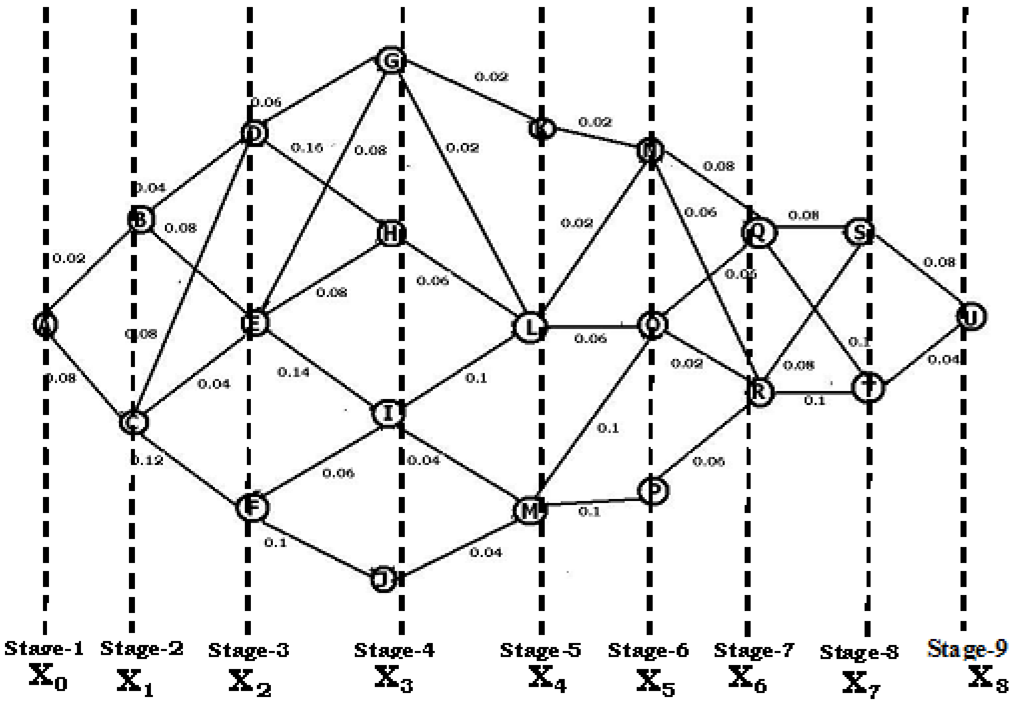

Figure 4.3- Stage wise representation as per Algorithm

Let $S1(X_0 , X_1)$ be the distance between the point X_0 and X_1 .It is apparent that the value assigned to decision variable X_1 is dependent upon the value of X_0 , which in this case is A for an initial value of zero. There is only one starting point in this network , namely , A , but in more complex networks, there could be multiple starting points resulting in a variety of values of X_0 . Similarly for $S2 (X_1 , X_2)$ be the distance between the point X_1 and X_2 . Hence $Sj(X_i , X_j)$ represents the distance between the point X_i and X_j respectively (i=0,1…….. 8 and j= 0,1……8).

The total fuzzy shortest distance for this stage is

$D (X_0 , X_1 ,X_2 , X_3 ,X_4 , X_5 ,X_6 , X_7 , X_8) = S1(X_0 , X_1)+ S2(X_1 , X_2)+ S3(X_2,X_3)+ S4(X_3,X_4)+ S5(X_4 , X_5)+ S6(X_5 , X_6)+ S7(X_6 , X_7)+ S8(X_7 , X_8)$.

The fuzzy shortest distance for the stage 1 which terminates B or C . The choice is a simple one because the distance at X_0 or A is zero.

Therefore S1(A, B) =0.02
S1(A, C) =0.08

$D (X_0 , X_1) = \min(0.02,0.08)= 0.02$ with X_1=B

These values appear at the vertices in figure(4.3) to indicate the values at each vertex of X_1 . let us examine the fuzzy shortest distance at stage 2 namely $S2(X_1 , X_2)$

$D_{S1, S2}(D)= \min(0.02+0.04, 0.08+0.08)=0.06$ with X_1 =B
$D_{S1, S2}(E)= \min(0.02+0.08, 0.08+0.04)=0.1$ with X_1 =B
$D_{S1, S2}(F)= \min(0.02+\infty, 0.08+0.12)=0.2$ with X_1 =C.

The fuzzy shortest distance at X_2 can be shown in the vertices of figure(4.3). Thus for $S2(X_1, X_2)$ the least distance is :
ABD if one stops at D = 0.06
ABE if one stops at E= 0.1
ACF if one stops at F = 0.2.

Next, moving to X_3 ,we have to determine the fuzzy shortest distance at the vertices G,H,I and J.

$D_{S1, S2, S3}(G)$= min(0.06+0.06, 0.1+0.08, 0.2+∞)=0.12 with X_2 =D

$D_{S1, S2, S3}(H)$= min(0.06+0.16, 0.1+0.14, 0.2+∞)=0.22 with X_2 =D

$D_{S1, S2, S3}(I)$ = min(0.06+∞, 0.1+0.14, 0.2+0.06)=0.24 with X_2 =E

$D_{S1, S2, S3}(J)$ = min(0.06+∞, 0.1+∞, 0.2+0.1)=0.3 with X_2 =F.

The fuzzy shortest distance at X_3 can be shown in the vertices of figure(4.3). Thus for $S3(X_2, X_3)$ the least distance is
ABDG if one stops at G = 0.12
ABEH if one stops at H= 0.22
ACFI if one stops at I = 0.24
ACFJ if one stops at J= 0.3.

Similarly proceeding , we get the idea of fuzzy shortest route using

$S4(X_3, X_4)$, $S5(X_4, X_5)$,$S6(X_5, X_6)$, $S7(X_6, X_7)$ and $S8(X_7, X_8)$ as shown in figure 4.4

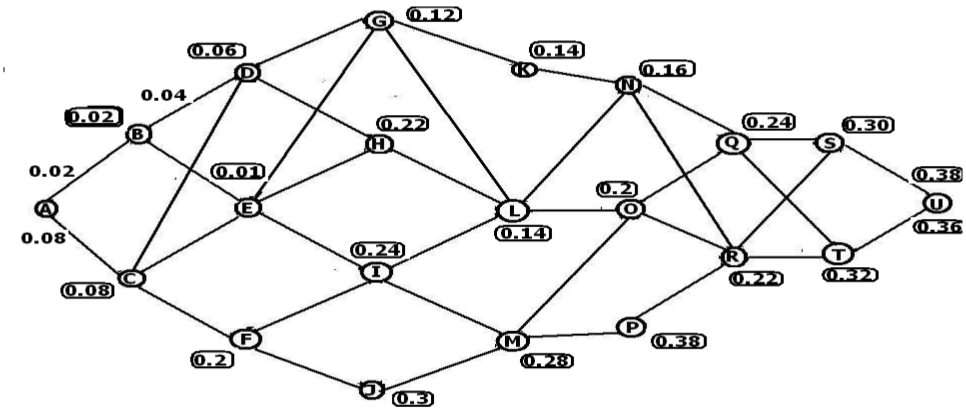

Figure 4.4
Fuzzy Shortest path from Chennai to Kanyakumari by Bus

The fuzzy shortest route from A to U are (i) A-B-D-G-K-N-R-T-U (value =0.36) and
(ii) A-B-D-G-L-N-R-T-U (value =0.36)

Hence the fuzzy shortest route among the cities are

(i) CHENNAI -CHENGALPATTU -VILLUPURAM -PERAMBALUR -THANJAVUR-PUDUKOTTAI-MADURAI-THIRUNELVELI -KANYAKUMARI.

(ii) CHENNAI -CHENGALPATTU -VILLUPURAM -PERAMBALUR -TRICHY-PUDUKOTTAI-MADURAI-THIRUNELVELI -KANYAKUMARI.

4.2 -*Chennai to Kanyakumari By Train:* Take a direct train from Chennai to Kanyakumari. Several direct express trains ply from Chennai covering the distance is approximately 14 hours. Some of the trains are Kanyakumari Express (12633) and Thirukkural Express (12642). Stations covered during the journey are Chennai Egmore->Tambaram->Chengalpattu->Melmaruvattur->Tindivanam->Villupuram Jn->Vridhachalam Jn->Tiruchirapalli->Dindigul Jn->Madurai Jn->Virudunagar Jn->Satur->Kovilpatti->Tirunelveli Jn->Valliyur->Nagercoil Jn->Kanyakumari.

Fuzzy shortest route from Chennai to Kanyakumari by train
The shortest route from A to U is A-B-D-G-L-N-R-T-U (value =0.34)

Hence the fuzzy shortest route among the cities are

CHENNAI – CHENGALPATTU – VILLUPURAM – ARIYALUR – TRICHY - DINDIGUL - MADURAI-THIRUNELVELI -KANYAKUMARI

V. CONCLUSION

The value of uncertain fuzzy shortest route among the cities from Chennai to Kanyakumari by Bus is 0.36 and by Train is 0.34. The recursive procedure described here was carried out from A towards U. This same procedure is often utilized in the reverse direction –from U to A -which leads to an equivalent solution. It is useful for solving several different types of network problems. Fundamentally, it consists of finding optimal fuzzy sub-policies or fuzzy subtrees and cumulating these through various cities and stages until optimal fuzzy policy or fuzzy tree or fuzzy shortest route is found large number of algorithms are available to solve the fuzzy shortest route problem. The algorithm by kruskal's[5] is most efficient for complete fuzzy graph for uncertainty.

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