

# Effect of Kappa Distribution with General Loss Cone Distribution Function on EMIC Waves in Magneto-Plasma

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**Abstract-** Electromagnetic ion-cyclotron (EMIC) waves have been studied by kinetic approach. The effect of kappa loss-cone distribution with bi-Maxwell general loss-cone distribution function on EMIC instability in magneto-plasma is evaluated. The dispersion relation, growth rate the electromagnetic ion-cyclotron waves in low  $\beta$  case (ratio of plasma pressure to magnetic pressure), homogeneous plasma have been obtained. The wave is assumed to propagate parallel to the static magnetic field. The effect kappa loss-cone distribution with bi-Maxwell general loss-cone distribution function on EMIC waves in magnetosphere plasma is to enhance the damping/growth rate of EMIC waves in low  $\beta$  case. The results are interpreted for the space plasma parameters in magneto-plasma.

**Index Terms-** Electromagnetic ion-cyclotron waves, Plasmapause region, Solar Plasma, Kappa distribution function, Plasma density.

## I. INTRODUCTION

Particle velocity distribution functions in space plasmas often show non Maxwellian super thermal tails decreasing as a power law of the velocity. Such distributions are well fitted by the so-called kappa distribution. The presence of such distributions in different space plasma suggests a universal mechanism for the creation of such super thermal tails. The super thermal particles have impotent consequences concerning the acceleration and the temperature that are well evidenced by the kinetic approach where no closures require the distributions to be nearly maxwellians.

Plasma instabilities associated with ion and electron populations in the magnetosphere give rise to many types of hydromagnetic and ion cyclotron waves. The ultra-low frequency (ULF) magnetic field fluctuations associated with these waves are observed as geomagnetic pulsations. Observational data from spacecraft [CRRES, DE1 & 2, ISEE1 & 2, DMSP, IMP8) and numerous ground stations in Australia, New Zealand and Antarctica are used to study ULF waves in the magnetosphere and the ionosphere.

The well known Maxwellian and kappa distributions differ substantially in the high energy tail region. The drop towards zero is much more abrupt for a Maxwellian distribution when compared to that of kappa distribution with a low spectral index  $\kappa$ . Such conditions occur in space and other magnetospheric plasma and kappa distribution have been used to analyse and interpret spacecraft data in the earth's magnetospheric plasma

sheet [1] & [2] the solar wind [3] [4] Jupiter [5] and Saturn [6]. Studies of Pc1-5 waves have been undertaken near the high latitude Davis Antarctic station using a small network of magnetometers located in a square arrangement of side 120-150km. Direction of arrival determinations on daytime quasi-structured and unstructured Pc1-2 emissions show they are associated with sources on closed field lines in the outer magnetosphere. This boundary region just inside the magnetopause may also be the origin of a pronounced Pc1-2 band, possibly a consequence of electromagnetic ion cyclotron wave (EMIC) generation resulting from ion injection into the magnetosphere from the dayside cleft. It has also been shown that these waves may propagate over short distances (< 500 km) but are not propagated in the ionospheric F2 region waveguide.

The advantages of present approach and study is to its suitability for dealing and determining the effect of general loss cone distribution with kappa distribution on EMIC in magnetosphere and determines the properties of the medium (plasma) in magnetosphere. The linear theory of plasma instabilities has been thoroughly studied in the past by several authors [7-10]. Electromagnetic ion-cyclotron waves generated in the equatorial region of earth's magnetosphere as left-handed circularly (LHC) polarized waves propagate field line guided towards the ionosphere [11]. Usually the frequency range of EMIC waves are 0.1 to 5 Hz. The theoretical studies of ion heating and acceleration perpendicular to the magnetic field are common features in the auroral region.

The main aim of this study is to investigate the generation of EMIC waves in the magnetosphere and see the effect of general loss cone distribution with kappa distribution in magnetospheric plasma. The detailed description and formulae for the dispersion relation and growth rate is determined in the next section.

## II. DISTRIBUTION FUNCTION

To determine the dispersion relation and the growth-rate, we consider a bi-Maxwellian plasma as

$$c^2 k^2 = \omega^2 + \frac{\pi \omega}{k} \omega_p^2 \int_0^\infty dv_\perp \int_{-\infty}^\infty \frac{f_0 dv_\parallel}{v_\parallel - v_e} \quad (1)$$

$$f_0(y, \vec{V}) = N_0 f_\perp(V_\perp) f_\parallel(V_\parallel) \quad (2)$$

We consider a general loss-cone distribution function for  $f_{\perp}(V_{\perp})$  as [11].

$$f_{\perp}(V_{\perp}) = \left[ \frac{V_{\perp}^{2j}}{\pi V_{T\perp}^{2(j+1)} j!} \right] \exp\left(-\frac{V_{\perp}^2}{V_{T\perp}^2}\right) \quad (3)$$

and  $f_{\parallel}(V_{\parallel})$  which is defined by the drifting Maxwellian [11].

$$f_{\parallel}(V_{\parallel}) = \left( \frac{1}{\sqrt{\pi} V_{T\parallel}} \right) \exp\left\{-m (V_{\parallel})^2 / V_{T\parallel}^2\right\} \quad (4)$$

Where using the value of  $V_{T\perp}^2 = (J+1)^{-1} \frac{2T_{\perp}}{m}$  for plasma and the bi Lorentzian which reduces to the anisotropic Maxwellian distribution when the spectral index  $\kappa$  tends to infinity is given by.

$$F = \frac{1}{\pi^{3/2}} \frac{\Gamma(\kappa+j+1)}{\kappa^{3/2} \Gamma\kappa - 1/2 V_{T\perp}^2 V_{T\parallel}^2} \left[ 1 + \frac{v_{\perp}^2}{\kappa v_{T\perp}^2} + \frac{v_{\parallel}^2}{\kappa v_{T\parallel}^2} \right]^{-(\kappa+j+1)} \quad (5)$$

In eq. (5)  $V_{T\perp}$  and  $V_{T\parallel}$  are related to the mass  $m$  and the temperatures  $T_{\perp}$  and  $T_{\parallel}$  respectively parallel and perpendicular to the magnetic field by

$$V_{T\perp}^2 = (J+1)^{-1} \left[ \frac{\kappa - 3/2}{\kappa} \frac{2K_B T_{\perp}}{m} \right] \quad (6)$$

And

$$V_{T\parallel}^2 = \left[ \frac{\kappa - 3/2}{\kappa} \frac{2K_B T_{\parallel}}{m} \right] \quad (7)$$

### III. DISPERSION RELATION

We consider the cold plasma dispersion relation for the EMIC wave as:

$$\frac{C^2 K_{\parallel}^2}{\omega_{pi}^2} = \frac{\omega^2}{1 - \frac{1}{\beta_{\parallel i}} \frac{\omega}{(\Omega_i - \omega)^3 \Omega_i^2}} \quad (8)$$

$$\omega_{pi}^2 = \frac{4\pi N_0 e^2}{m_i}$$

Where  $m_i$  is the plasma frequency for the ions.  $N_0$  is the plasma density of particles.

### IV. GROWTH RATE

The formula for the growth rate is given by

$$\gamma = \frac{\text{Im}[D_{zz}(K_{\parallel}, \omega)]}{\frac{\partial}{\partial \omega} \{\text{Re}[D_{zz}(K_{\parallel}, \omega)]\}} \quad (9)$$

$$\gamma = \frac{-\Omega_i \left[ \frac{T_{\perp}}{T_{\parallel}} \left( 1 - \frac{\omega}{\Omega_i} \right) + \frac{\omega}{\Omega_i} \right] \times \frac{\kappa \sqrt{\pi}}{\kappa^{j/2} \Gamma\kappa - 1/2 \left[ 1 + \left( \frac{\omega - \Omega_i}{K_{\parallel} V_{T\parallel}} \right)^2 \right]^{(\kappa+j+1)}}}{K_{\parallel} \left[ \beta_{\parallel} \left[ 2 \frac{T_{\perp}}{T_{\parallel}} \left( 1 - \frac{\omega}{\Omega_i} \right) + \left( 1 + \frac{2\omega}{\Omega_i} \right) + \frac{\omega}{\Omega_i} \left( \frac{2 - \frac{\omega}{\Omega_i}}{1 - \frac{\omega}{\Omega_i}} \right)^2 \right] \right] \left( 1 - \frac{\omega}{\Omega_i} \right)^4} \quad (10)$$

Here it is noticed that  $j$  has affected the growth rate through the temperature anisotropy as discussed for the electromagnetic wave propagating parallel to the magnetic field with kappa loss cone distribution function.

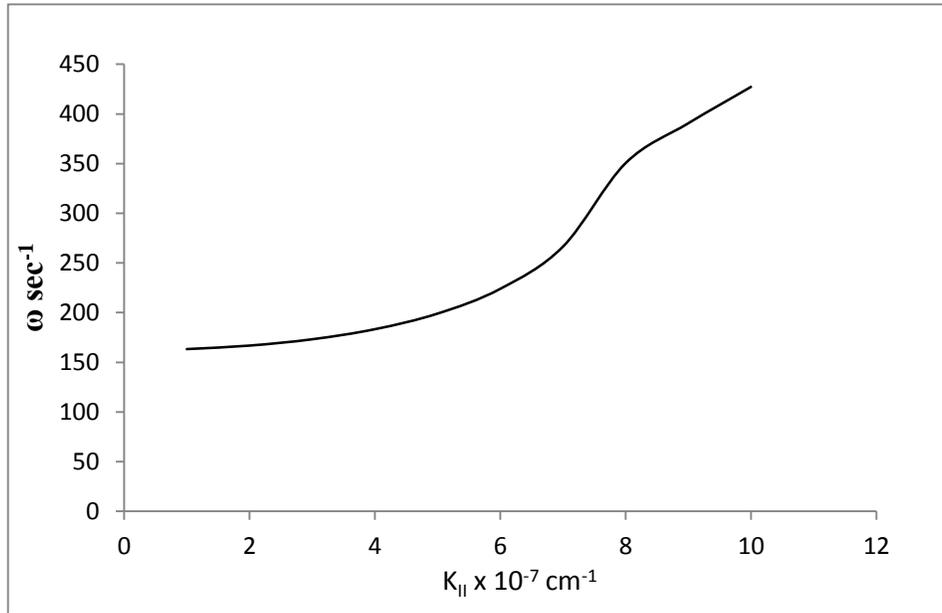
### V. RESULTS AND DISCUSSION

The role of the EMIC wave particle interaction in the auroral acceleration region is examined in the present analysis. The characteristics of the EMIC waves were derived by using auroral acceleration parameters [12, 13].

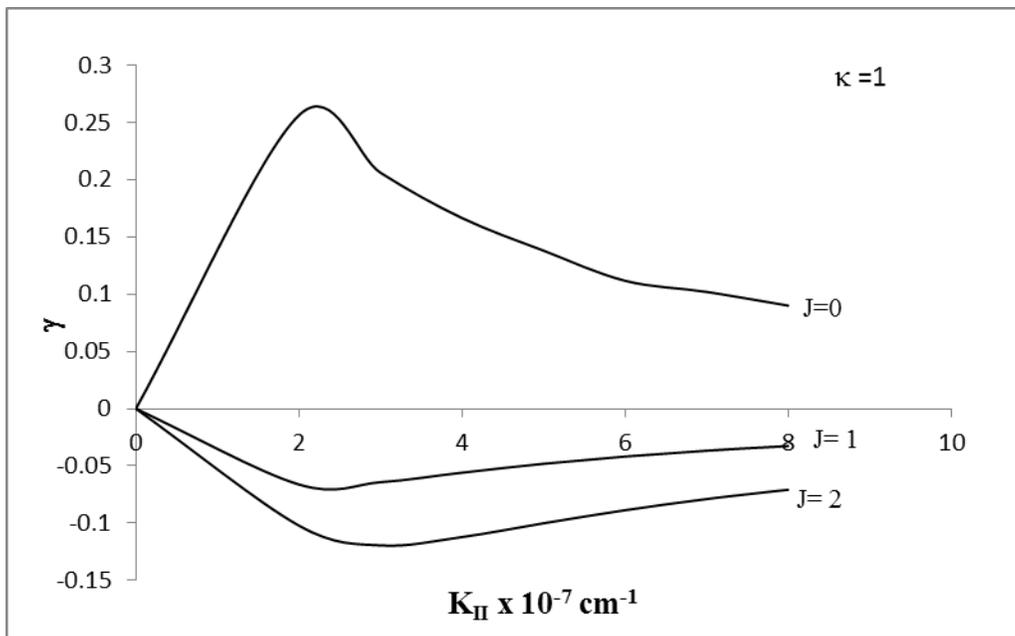
$$B_0 = 4300 \text{ nT}, \Omega_i = 412 \text{ sec}^{-2}$$

$$V_{T\parallel i} = 2 \times 10^9 \text{ cm/s}, \omega_{pi}^2 = 1.732 \times 10^6 \text{ s}^{-2}$$

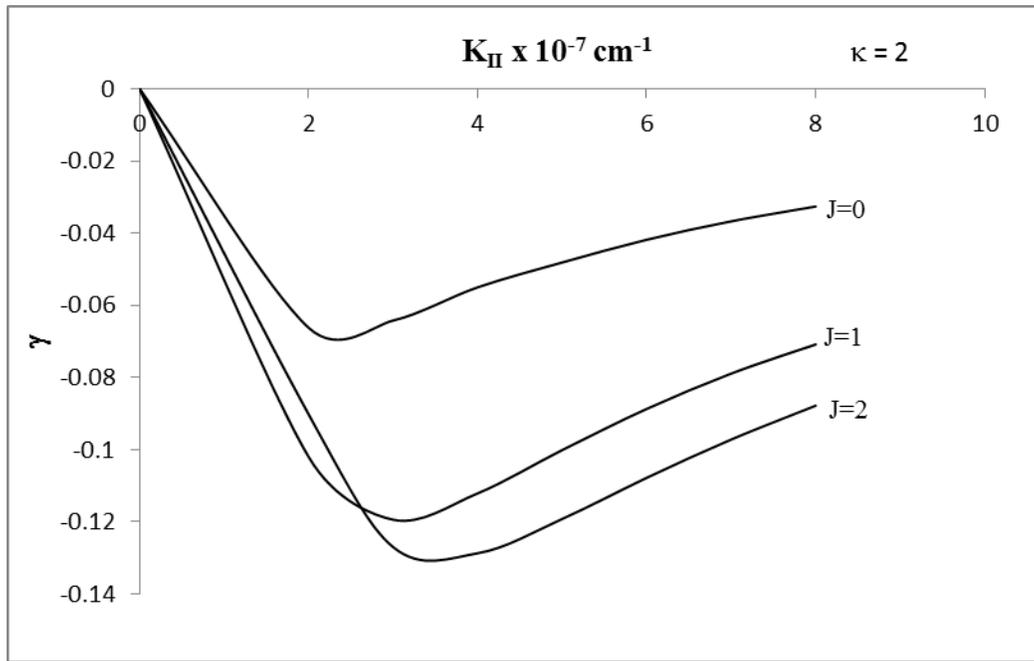
Fig. 1 Shows the variation of frequency of plasma ( $\omega$ ) in  $\text{sec}^{-1}$  versus wave vector ( $k_{\parallel}$ ) in  $\text{cm}^{-1}$  for magnetospheric plasma. It is found that the frequency ( $\omega$ ) is linearly increases with the increasing of the parallel wave vector ( $k_{\parallel}$ )  $\text{cm}^{-1}$  and the variation shows by the straight line.



**Fig. 1** Variation of wave frequency ( $\omega$ ) versus wave vector ( $K_{||}$ )  $\text{cm}^{-1}$ .



**Fig. 2** Variation of growth rate ( $\gamma$ ) versus wave vector ( $K_{||}$ )  $\text{cm}^{-1}$  for different values of general loss-cone distribution function  $J$  at  $\kappa = 1$ .



**Fig. 3 Variation of growth rate ( $\gamma$ ) versus wave vector ( $K_{II}$ )  $\text{cm}^{-1}$  for different values of general loss-cone distribution function  $J$  at  $\kappa=2$ .**

Fig. 2, 3 predict the variation of the growth rate ( $\gamma$ ) with the wave vector  $K_{II}$  ( $\text{cm}^{-1}$ ) for different values of general loss-cone distribution index  $J = 0, 1, 2$  and kappa distribution  $\kappa=1$  &  $2$  respectively. The steepness of loss-cone distribution i.e. for the Maxwellian distribution the growth rate slightly increases with the particular value of wave number ( $K_{II}$ ). The steepness of loss-cone distribution also introduced a peak in the growth rate which may be due to wave-particle resonance interaction at  $K_{II}$ . Further increase in the steepness of the loss-cone distributions shifts the resonance condition towards the lower side of the wave number ( $K_{II}$ ) and the peak value of the growth rate also decreases. Under this condition for the increasing magnetic activity, magnetospheric convection is enhanced and the location of the flow separator moves rapidly earthward. In such conditions, the outer plasma sphere becomes strongly structured, and plasmaspheric material will appear beyond the plasma pause. The large-amplitude electric fields represent a common feature on auroral field lines at altitudes above a few thousand kilometers, as shown by a number of spacecraft such as Dynamics Explorer 1, Viking, Polar and FAST and in the return current region at altitudes as low as 800 km, as was first shown by Freja.

## VI. CONCLUSIONS

In the present work, we have conducted a comprehensive mathematical analysis. The effects of a general loss-cone distribution with kappa distribution function are also incorporated in the auroral acceleration region to discuss EMIC wave's emission.

The concluding remarks of this study are as follows:

1. It is found that the effect of increasing the values of general loss-cone distribution with kappa distribution is to enhance the growth rate of EMIC waves, may be due a shifting of the resonance condition.
2. The behavior studied for the EMIC waves may be of importance in the electromagnetic emission in the auroral acceleration region. The result of the study is also applicable to the plasma devices that have the steep loss-cone distribution.

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