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Static Deformation Due To Long Tensile Fault Embedded in an Isotropic Half-Space in Welded Contact with an Orthotropic Half-Space

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Abstract- The Airy stress function for a long tensile fault of arbitrary dip and finite width buried in a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space are obtained. This Airy stress function is used to derive closed-form analytical expressions for the stresses at an arbitrary point of the half-space caused by along vertical tensile fault of finite width. The variation of the displacement and stress fields with distance from the fault is studied numerically.

I. Introduction

Steketee (1958) introduced the theoretical formulations describing the deformation of an isotropic, homogeneous, semi-infinite medium (okada, 1992). Maruyama (1964) calculated the Green's functions for two-dimensional elastic dislocations in a semi - infinite medium and obtained surface displacements due to vertical and horizontal rectangular tensile faults in a semi-infinite Poisson solid. Freund and Barnett (1976) developed a 2-D model of dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable and obtained the relationship between fault slip and surface deformation. Davis (1983) modeled the crustal deformation associated with hydro fracture by a dipping rectangular tensile fault beneath the surface of an elastic half-space. A double-couple source mechanism modeling a shear stress cannot represent all earthquake sources. Sipkin (1986) developed the theory that the non-double couple mechanism might be due to tensile failure under high fluid pressure. Yang and Davis (1986) obtained closed analytical expressions for the displacements, strains and stresses due to a rectangular inclined tensile fault in an elastic half-space. Maruyama (1964) calculated the Green's functions for two-dimensional elastic dislocations in a semi - infinite medium and obtained surface displacements due to vertical and horizontal rectangular tensile faults in a semi-infinite Poisson solid.

Singh and Garg (1986) obtained the integral expressions for the Airy stress function in an unbounded medium due to various two- dimensional seismic sources. Beginning with these expressions, Rani et al. (1991) obtained the integral expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to various two-dimensional sources by applying the traction-free boundary conditions at the surface of the half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the Airy stress function, the displacements and the stresses at any point of the half-space caused by two-dimensional buried sources. Wu and Chou (1982) applied the generalized method of images to obtain the elastic field of an in-plane line force acting in a two –phase orthotropic medium. Singh (1986), Garg and Singh (1987), and Pan (1989a) studied the static deformation of a transversely isotropic multilayered half-space by surface loads. The problem of the static deformation of a transversely isotropic multilayered half-space by buried sources has been discussed by Pan (1989b). Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads has been investigated by Garg et al. (1991).

Singh et al. (1991) followed a similar approach to obtain closed-form analytic expressions for the displacements and the stresses at any point of either of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources. Kumar et al. (2005) obtained closed-form analytical expressions for the Airy Stress function for a tensile line source in two-welded half-spaces which are integrated analytically to derive the Airy stress function In this paper, we study the static deformation caused by various two-dimensional seismic sources located in a homogeneous, isotropic, perfectly elastic half-spaces lying over a homogeneous, anisotropic, perfectly elastic half-space with which it is in welded contact. Most anisotropic media of interest in seismology have, at least approximately, a horizontal plane of elastic symmetry. The most general system with one plane of elastic symmetry is the monoclinic system. A material having three perpendicular planes of elastic symmetry at a point is said to possess orthotropic or orthorhombic symmetry. This symmetry is exhibited by olivine and orthopyroxenes, the principal rock-forming minerals of the deep crust and upper mantle. Therefore, we assume that the lower half-space is orthotropic.

In an orthotropic material, there are nine elastic constants. The results for a tetragonal material with six constants. The results for a tetragonal material with six elastic constants, for a transversely isotropic material with five elastic constants and for a cubic material with three elastic constants can be derived as particular cases. We have verified that the results of Singh et al.

(1991) for two isotropic half-spaces in welded contact follow from the results of the present paper when the lower orthotropic half-space is replaced by an isotropic half-space

II. THEORY

Let the Cartesian co-ordinates be denoted by $(x,y,z)\equiv(x_1,x_2,x_3)$ with z-axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are welded along the plane $z=0$. The upper half-space $(z\rangle 0)$, is called medium I and the lower half-space $(z\langle 0)$ is called medium II. Medium I is assumed to be isotropic with stress-strain relation

$$p_{ij}=\lambda\delta_{ij}e_{kk}+2\mu e_{ij} \tag{1}$$

Medium II is assumed to be orthotropic with stress-strain relation

$$\begin{bmatrix} p_{11}\\ p_{22}\\ p_{33}\\ p_{23}\\ p_{31}\\ p_{12}\end{bmatrix}=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{21} & c_{23} & 0 & 0 & 0\\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{bmatrix}\begin{bmatrix} e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{31}\\ 2e_{12}\end{bmatrix}. \tag{2}$$

We consider a two-dimensional approximation in which the displacement components (u_1,u_2,u_3) are independent of x so that $\frac{\partial}{\partial x}\equiv 0$. Under this assumption the plane-strain problem $(u_1=0)$ and the antiplane-strain problem $(u_2=u_3=0)$ are decoupled and, therefore, can be solved separately. The plane-strain problem for an isotropic medium can be solved in terms of the Airy stress function U such that

$$p_{22}=\frac{\partial^2 U}{\partial z^2},\quad p_{23}=-\frac{\partial^2 U}{\partial y\partial z},\quad p_{33}=\frac{\partial^2 U}{\partial y^2}, \tag{3}$$

$$\nabla^2\nabla^2 U=0. \tag{4}$$

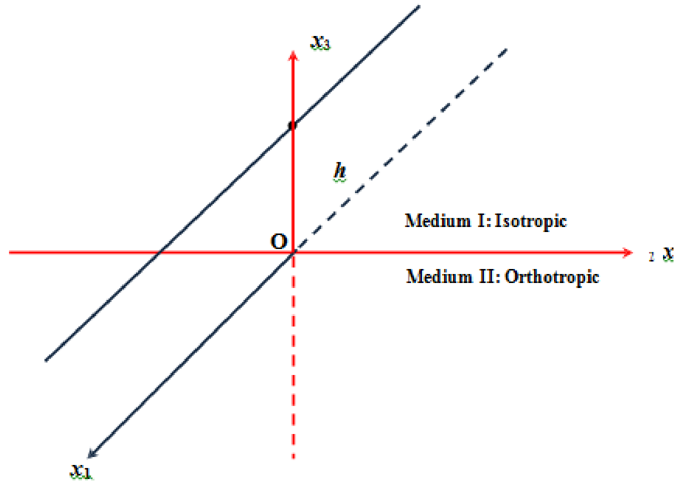

Figure 1: A two-phase medium consisting of an isotropic half space lying over an orthotropic half-space with a line source in the isotropic half-space at (0, 0, *h*)

The plane-strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* such that (Garg et al., 1991)

$$p_{22}=\frac{\partial^2 U^*}{\partial z^2},\quad p_{23}=-\frac{\partial^2 U^*}{\partial y\partial z},\quad p_{33}=\frac{\partial^2 U^*}{\partial y^2}, \tag{5}$$

$$\left(a^2\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(b^2\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)U^*=0, \tag{6}$$

where

$$\left.\begin{aligned} a^2+b^2 &=\left(c_{22}c_{33}-c_{23}^2-2c_{23}c_{44}\right)/\left(c_{33}c_{44}\right),\\ a^2b^2 &=c_{22}/c_{33}.\end{aligned}\right\} \tag{7}$$

For an isotropic medium

$$\left.\begin{aligned} c_{11}&=c_{22}=c_{33}=\lambda+2\mu\ ,\\ c_{12}&=c_{13}=c_{23}=\lambda\ ,\\ c_{44}&=c_{55}=c_{66}=\mu.\end{aligned}\right\} \tag{8}$$

This yields $a^2=b^2=1$ and eq. (6) reduces to eq. (4).

Let there be a line source parallel to the x-axis passing through the point $(0,0,h)$ of the upper half-space $(z\rangle 0)$. As shown by Singh and Garg (1986), the Airy stress function U_0 for a line source parallel to x-axis passing through the point $(0,0,h)$ in an unbounded, isotropic medium, with Lame constants λ_1,μ_1, can be expressed in the form

$$U_0=\int_0^\infty[\left(L_0+M_0k|z-h|\right)\sin ky+\left(P_0+Q_0k|z-h|\right)\cos ky]k^{-1}e^{-k|z-h|}dk\ , \tag{9}$$

where the source coefficients L_0, M_0, P_0, Q_0 are independent of k. Singh and Garg (1986) have obtained source coefficients for various seismic sources.

For a line source parallel to the x-axis acting at the point $(0,0,h)$ of medium I $(z\rangle 0)$ which is in welded contact with medium II $(z\langle 0)$, the Airy stress function in medium I is a solution of Eq. (4) and may be taken to be of the form

$$U^{(1)} = U_0 + \int_0^\infty \left[\left(L_1 + M_1 kz \right) \sin ky + \left(P_1 + Q_1 kz \right) \cos ky \right] k^{-1} e^{-kz} \mathrm{d}k. \tag{10}$$

The Airy stress function in medium II is a solution of Eq. (6) and is of the form (assuming $a \neq b$)

$$U^{(2)} = \int_0^\infty \left[\left(L_2 e^{akz} + M_2 e^{bkz} \right) \sin ky + \left(P_2 e^{akz} + Q_2 e^{bkz} \right) \cos ky \right] k^{-1} \mathrm{d}k. \tag{11}$$

The superscript (1) denotes quantities related to medium I and the superscript (2) denotes the quantities related to medium II. The constants L_1, M_1, L_2, M_2 etc. are to be determined from the boundary conditions.

Since the half-spaces are assumed to be in welded contact along the plane $z = 0$, the boundary conditions are

$$\left. \begin{aligned} p_{23}^{(1)} = p_{23}^{(2)}, \qquad p_{33}^{(1)} = p_{33}^{(2)}, \\ u_2^{(1)} = u_2^{(2)}, \qquad u_3^{(1)} = u_3^{(2)}, \end{aligned} \right\} \tag{12}$$

at $z = 0$ .The stresses and the displacements for the isotropic medium I in terms of the Airy stress function $U^{(1)}$ are given by (Rani et al., 1991)

$$p_{22}^{(1)} = 2\frac{\partial^2 U^{(1)}}{\partial z^2}, \qquad p_{23}^{(1)} = -\frac{\partial^2 U^{(1)}}{\partial y \partial z}, \qquad p_{23}^{(1)} = \frac{\partial^2 U^{(1)}}{\partial y^2}, \tag{13}$$

$$\left. \begin{aligned} 2\mu_1 u_2^{(1)} &= -\frac{\partial U^{(1)}}{\partial y} + \frac{1}{2\alpha_1} \int (p_{22}^{(1)} + p_{33}^{(1)}) \mathrm{d}y, \\ 2\mu_1 u_3^{(1)} &= -\frac{\partial U^{(1)}}{\partial z} + \frac{1}{2\alpha_1} \int (p_{22}^{(1)} + p_{33}^{(1)}) \mathrm{d}z, \end{aligned} \right\} \tag{14}$$

where

$$\alpha_1 = (\lambda_1 + \mu_1) / (\lambda_1 + 2\mu_1). \tag{15}$$

The stresses and the displacements for the orthotropic medium II are given by (Garg et al., 1991)

$$p_{22}^{(2)} = \frac{\partial^2 U^{(2)}}{\partial z^2}, \qquad p_{23} = -\frac{\partial^2 U^{(2)}}{\partial y \partial z}, \qquad p_{33} = \frac{\partial^2 U^{(2)}}{\partial y^2}, \tag{16}$$

$$\left. \begin{aligned} u_2^{(2)} = \Delta^{-1} \int (c_{33} p_{22}^{(2)} - c_{23} p_{22}^{(2)}) \mathrm{d}y, \\ u_3^{(2)} = \Delta^{-1} \int (c_{22} p_{33}^{(2)} - c_{23} p_{22}^{(2)}) \mathrm{d}z, \end{aligned} \right\} \tag{17}$$

where

$$\Delta = c_{22} c_{33} - c_{23}^2. \tag{18}$$

Using expressions of stresses given by (Singh and Rani 1991) and Using source coefficients from the appendix for a vertical tensile fault, we obtain the following expressions of Airy stress function and stresses in both medium the expressions of stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space having long tensile line source located in the isotropic half-space are given below:

$$U^{(1)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[-\log R - X_1 \log S + \frac{2X_2 z(z+h)}{S^2} - \left\{\frac{(z-h)^2}{R^2} - 2(D-C)\log S + \frac{[(X_1 h + X_3 z)(z+h) - 2X_2 hz]}{S^2}\right.\right.$$
$$\left.\left. + \frac{4X_2 hz(z+h)^2}{S^4}\right\}\right], \tag{19}$$

$$\tau_{22}^{(1)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[\frac{-3}{R^2} + \frac{12(z-h)^2}{R^4} + \frac{(4X_2 - X_1)}{S^2} - \frac{2(X_3 - D + C)}{S^2} - \frac{2(10X_2 - X_1)(z+h)^2}{S^4}\right.$$
$$+ \frac{6h(X_1 - X_3 - 2X_2)(z+h)}{S^4} + \frac{8h(4X_2 - X_1)h(z+h)^3}{S^6} + \frac{8z(2X_2 - X_3)(z+h)^3}{S^6}$$
$$\left. + \frac{2(z+h)^2(5X_3 - 2D + 2C)}{S^6} + \frac{96X_2 hz(z+h)^2}{S^6} - \frac{96X_2 hz(z+h)^4}{S^8}\right\}\Bigg], \tag{20}$$

$$\tau_{23}^{(1)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[\frac{6y(z-h)}{R^4} - \frac{8y(z-h)^3}{R^6} + \frac{2y[(X_3 - 2X_2)(2z+h) + 2(C-D)(z+h) + X_1 z]}{S^4}\right.$$
$$\left. + \frac{8y[(2X_2 - X_3)z + (2X_2 - X_3)h](z+h)^2}{S^6} + \frac{48X_2 hyz(z+h)}{S^6} - \frac{96X_2 hyz(z+h)^3}{S^8}\right], \tag{21}$$

$$\tau_{33}^{(1)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[\frac{1}{R^2} + \frac{[2(D-C) - X_1]}{S^2} + \frac{2y^2[X_1 - 2(D-C)]}{S^4} + \frac{2hz(4X_2 - X_3)}{S^4}\right.$$
$$\left. + \frac{2[X_1 h + (X_3 - 2X_2)z](z+h)}{S^6} + \frac{8y^2(z+h)[(2X_2 - X_3)z - X_1 h]}{S^6} - \frac{96X_2 hy^2 z(z+h)^2}{S^8}\right\}\Bigg], \tag{22}$$

$$U^{(2)} = -\frac{\mu_1 bds}{2\pi(1-\sigma)}\left[(A+C)\ln T^2 + (B-D)\ln H^2 + 2h\left(\frac{A(h-az)}{T^2} + \frac{B(h-bz)}{H^2}\right)\right], \tag{23}$$

$$\tau_{22}^{(2)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[\frac{a^2(A+C)}{T^2} + \frac{b^2(B-D)}{H^2} - 2a^2 y^2 \frac{(A+C)}{T^4} + 2b^2 y^2 \frac{(D-B)}{H^4}\right.$$
$$\left. - 2h\left(\frac{a^2 A(h-az)}{T^4} + \frac{b^2 B(h-bz)}{H^4}\right) + 8hy^2\left(\frac{a^2 A(h-az)}{T^6} + \frac{b^2 B(h-bz)}{H^6}\right)\right], \tag{24}$$

$$\tau_{23}^{(2)} = \frac{\mu_1 bds}{2\pi(1-\sigma)}\left[-2ay\frac{(A+C)(h-az)}{T^4} + 2by\frac{(D-B)(h-bz)}{H^4} - 2hy\left(\frac{aA}{T^4} - \frac{bB}{H^4}\right)\right.$$
$$\left. + 8hy\left(\frac{aA(h-az)^2}{T^6} + \frac{bB(h-bz)^2}{H^6}\right)\right\}\Bigg], \tag{25}$$

$$\tau_{33}^{(2)} = -\frac{\mu_1 bds}{2\pi(1-\sigma)}\left[\frac{(A+C)}{T^2} + \frac{(B-D)}{H^2} - 2y^2\left(\frac{(A+C)}{T^4} + \frac{(B-D)}{H^4}\right)\right.$$
$$\left. - 2h\left(\frac{A(h-az)}{T^4} + \frac{B(h-bz)}{H^4}\right) + 8hy^2\left(\frac{A(h-az)}{T^6} + \frac{B(h-bz)}{H^6}\right)\right], \tag{26}$$

where

$$\left.\begin{aligned} R^2 &= y^2 + (z-h)^2, & T^2 &= y^2 + (h-az)^2, \\ S^2 &= y^2 + (z+h)^2, & H^2 &= y^2 + (h-bz)^2, \end{aligned}\right\} \tag{27}$$

and

$$\left.\begin{aligned}
X_1 &= 2(A+B)-1,\\
X_2 &= A(1+a)+B(1+b)-1,\\
X_3 &= 2D(1+b)-2c(1+a)+1,\\
A &= \alpha_1(1-b+2\mu_1 s_2-2\mu_1 r_2)/W,\\
B &= \alpha_1(a-1+2\mu_1 r_1-2\mu_1 s_1)/W,\\
C &= (1+b-b\alpha_1+2\mu_1\alpha_1 s_2)/W,\\
D &= (1+a-a\alpha_1+2\mu_1\alpha_1 s_1)/W,\\
W &= (1+a-\alpha_1+2\mu_1\alpha_1 r_1)(1+b-b\alpha_1+2\mu_1\alpha_1 s_2)\\
&\quad -(1+a-a\alpha_1+2\mu_1\alpha_1 s_1)(1+b-\alpha_1+2\mu_1\alpha_1 r_2).
\end{aligned}\right\} \tag{28}$$

We define the following dimensionless quantities $Y=\frac{y}{h},\quad Z=\frac{z}{h}$, where L is the finite width of the tensile fault. Let the dimensionless stresses be denoted by $P_{ij}^{(1)}, P_{ij}^{(2)}$. Then,

$$P_{ij}^{(1)}=\frac{\pi h^2}{\mu_1 bds}\tau_{ij}^{(1)},\qquad P_{ij}^{(2)}=\frac{\pi h^2}{\mu_1 bds}\tau_{ij}^{(2)}, \tag{29}$$

From equations (20) - (22) and (24) - (26), we get the following expressions for the dimensionless stresses for the two mediums for a vertical tensile line source:

$$\begin{aligned}
P_{22}^{(1)}=\frac{1}{2(1-\sigma)}\Bigg[&\frac{-3}{E^2}+\frac{12(Z-1)^2}{E^4}+\frac{(4X_2-X_1)}{F^2}-\frac{2(X_3-D+C)}{F^2}-\frac{2(10X_2-X_1)(Z+1)^2}{F^4}\\
&+\frac{6(X_1-X_3-2X_2)(Z+1)}{F^4}+\frac{8(4X_2-X_1)(Z+1)^3}{F^6}+\frac{8Z(2X_2-X_3)(Z+1)^3}{F^6}\\
&+\frac{2(Z+1)^2(5X_3-2D+2C)}{F^6}+\frac{96X_2Z(Z+1)^2}{F^6}-\frac{96X_2Z(Z+1)^4}{F^8}+\frac{(3-4X_2+X_1)}{(Y^2+Z^2)}\\
&+\frac{2(X_3-D+C)}{(Y^2+Z^2)}+\frac{2Z^2(10X_2-X_1-6)}{(Y^2+Z^2)^2}-\frac{2Z^2\left\{4Z^2(2X_2-X_3)+(5X_3-2D+2C)\right\}}{(Y^2+Z^2)^3}\Bigg],
\end{aligned} \tag{30}$$

$$\begin{aligned}
P_{23}^{(1)}=\frac{1}{2(1-\sigma)}\Bigg[&\frac{6Y(Z-1)}{E^4}-\frac{8Y(Z-1)^3}{E^6}+\frac{2Y[(X_3-2X_2)(2Z+1)+2(C-D)(Z+1)+X_1Z]}{F^4}\\
&+\frac{8Y[(2X_2(Z+1)-(X_3Z+X_1)](Z+1)^2}{F^6}+\frac{48X_2YZ(Z+1)}{F^6}-\frac{96X_2YZ(Z+1)^3}{F^8}\\
&-\frac{2YZ[3+2(X_3-2X_2)+2(C-D)+X_1)]}{(Y^2+Z^2)^2}+\frac{8YZ^3(1-2X_2+X_3)}{(Y^2+Z^2)^3}\Bigg],
\end{aligned} \tag{31}$$

$$\begin{aligned}
P_{33}^{(1)}=\frac{1}{2(1-\sigma)}\Bigg[&\frac{1}{E^2}+\frac{[2(D-C)-X_1]}{E^2}+\frac{2X_1\left\{Y^2+(Z+1)\right\}}{F^4}-\frac{4Y^2(C+D)}{F^4}+\frac{2Z(Z+1)(X_3-2X_2)}{F^4}\\
&+\frac{2Z(4X_2-X_3)}{F^4}+\frac{8Y^2(Z+1)[(2X_2-X_3)Z-X_1]}{F^6}-\frac{96X_2Y^2Z(Z+1)^2}{F^8}-\frac{[1+2(D-C)-X_1]}{(Y^2+Z^2)}\\
&-\frac{2Y^2[X_1-2(D-C)]}{(Y^2+Z^2)^3}-\frac{2Z^2(X_3-2X_2)}{(Y^2+Z^2)^2}-\frac{8Y^2Z^2(2X_2-X_3)}{(Y^2+Z^2)^3}\Bigg],
\end{aligned} \tag{32}$$

$$P_{22}^{(2)} = \frac{1}{2(1-\sigma)}\left[\frac{a^2(A+C)}{G^2} + \frac{b^2(B-D)}{J^2} - 2a^2Y^2\frac{(A+C)}{G^4} + 2b^2Y^2\frac{(D-B)}{J^4} - 2\left(\frac{a^2A(1-aZ)}{G^4} + \frac{b^2B(1-bZ)}{J^4}\right) + 8Y^2\left(\frac{a^2A(1-aZ)}{G^6} + \frac{b^2B(1-bZ)}{J^6}\right) - \frac{a^2(A+C)}{(Y^2+a^2Z^2)} - \frac{b^2(B-D)}{(Y^2+b^2Z^2)} + \frac{2a^2Y^2(A+C)}{(Y^2+a^2Z^2)^2} + \frac{2b^2Y^2(B-D)}{(Y^2+b^2Z^2)^2}\right], \tag{33}$$

$$P_{23}^{(2)} = \frac{1}{2(1-\sigma)}\left[-\frac{2aY(1-aZ)(A+C)}{G^4} + \frac{2Yb(1-bZ)(D-B)}{J^4} - \frac{2aAY}{G^4} - \frac{2bBY}{J^4} + \frac{8aAY(1-aZ)^2}{G^6} + \frac{8bBY(1-bZ)^2}{J^6} - \frac{2a^2YZ(A+C)}{(Y^2+a^2Z^2)^2} + \frac{2b^2YZ(D-B)}{(Y^2+b^2Z^2)^2}\right], \tag{34}$$

$$P_{33}^{(2)} = -\frac{1}{2(1-\sigma)}\left[\frac{(A+C)}{G^2} + \frac{(B-D)}{J^2} - 2Y^2\left(\frac{(A+C)}{G^4} + \frac{(B-D)}{J^4}\right) - 2\left(\frac{A(1-aZ)}{G^4} + \frac{B(1-bZ)}{J^4}\right) + 8Y^2\left(\frac{A(1-aZ)}{G^6} + \frac{B(1-bZ)}{J^6}\right) - \frac{(A+C)}{(Y^2+a^2Z^2)} - \frac{(B-D)}{(Y^2+b^2Z^2)} + \frac{2Y^2(A+C)}{(Y^2+a^2Z^2)^2} + \frac{2Y^2(B-D)}{(Y^2+a^2Z^2)^2}\right], \tag{35}$$

III. DISCUSSION AND CONCLUSION

We have derived the results when an isotropic half-space (medium I) lies over an orthotropic half-space (medium II). The results when medium II is tetragonal can be obtained on putting

$$c_{22} = c_{11}, \qquad c_{23} = c_{13}, \qquad c_{55} = c_{44}, \tag{36}$$

The results when medium II is transversely isotropic follow by taking

$$c_{22} = c_{11}, \qquad c_{23} = c_{13}, \qquad c_{55} = c_{44}, \qquad c_{66} = (c_{11} - c_{12})/2. \tag{37}$$

Similarly, the results when medium II is cubic are obtained on taking

$$c_{22} = c_{33} = c_{11}, \qquad c_{12} = c_{13} = c_{23}, \qquad c_{44} = c_{55} = c_{66}, \tag{38}$$

When medium II is isotropic,

$$\left.\begin{aligned} c_{11} &= c_{22} = c_{33} = \lambda_2 + 2\mu_2, \\ c_{12} &= c_{13} = c_{23} = \lambda_2, \\ c_{44} &= c_{55} = c_{66} = \mu_2. \end{aligned}\right\} \tag{39}$$

This is a degenerate case for which $a = b = 1$ (see Eq. (7)). However, we have verified that, when medium II is replaced by an isotropic medium, the results of the present paper, in the limit, coincide with the results of Singh et. al.(1991) for two isotropic half-spaces in welded contact.

For numerical calculations we assume that medium II is transversely isotropic and use the values of the elastic constants given by Anderson (1961). For beryl,

$$c_{11}/c_{44} = 4.13, \quad c_{33}/c_{44} = 3.62, \quad c_{12}/c_{44} = 1.47, \quad c_{13}/c_{44} = 1.01. \tag{40}$$

This yield $a = 1.7018,\ b = 0.6276.$

For ice,

$$c_{11}/c_{44} = 4.70, \quad c_{33}/c_{44} = 4.96, \quad c_{12}/c_{44} = 2.27, \quad c_{13}/c_{44} = 1.60. \tag{41}$$

and $a = 1.8019,\ b = 0.5402$. For the isotropic medium I, we assume that $\lambda_1 = \mu_1$. We further assume that $c_{44} / \mu_1 = 2$. When medium II is also isotropic, we take $\lambda_2 = \mu_2$ for numerical work.

Figure 1 shows that in the model considered here, medium I is isotropic and medium II is orthotropic. Numerical calculations of stress components *P22*, *P23* and *P33* is done and their figures are drawn by considering the medium II as beryl, ice or isotropic. Figures 2(a) - 2(c), 3(a) - 3(c) and 4(a) - 4(c) show the variation of stress components *P22*, *P23*, and *P33* with distance from the fault. Figures 2(a), 2(b) and 2(c) show the variation of the dimensionless stress component *P22* with distance from the fault when source is at depth *x3* = − *h*/2 , *x3* = 2*h* and *x3* = 5*h* respectively. These figures depict the comparison of stress component when medium II is beryl or ice and comparison is also made in case medium II is isotropic. The figures show that when *x3* is negative, then near the fault, *P22* is negative(Compressive stress) in case of beryl and positive (tensile stress) in case of ice, and when *x3* is positive, then the value of *P22* is more in case of beryl. As we move away from the fault i.e. when *x2* tends to infinity, *P22* tends to zero in all the cases. Figures 3(a), 3(b) and 3(c) depict the behavior of stress component *P23* depending on the distance from the fault when source is located at the depth *x3* = −h/2, *x3* = 2*h* and *x3* = 5*h* respectively. All these figures show that the value of *P23 is* negative near the fault but as we move away, it becomes positive. Again when *x3* is negative, then the value of *P23* is less for beryl than ice, but when *x3* is positive, the trend get reversed. Similar behavior is observed for the stress component *P22*. Figures 4(a), 4(b) and 4(c) depict the behavior of stress component *P33* depending on the distance from the fault when same is located at the depth *x3* = − h/2, *x3* = 2*h* and *x3* = 5*h* respectively. Here also, the situation is similar to that for *P22* and *P23* in case medium II is beryl, ice or isotropicThe comparison is also made when medium II is beryl, ice or isotropic. These figures show that the value of stress components do not differ so much for beryl and ice but the difference in values is significant when both media are isotropic.

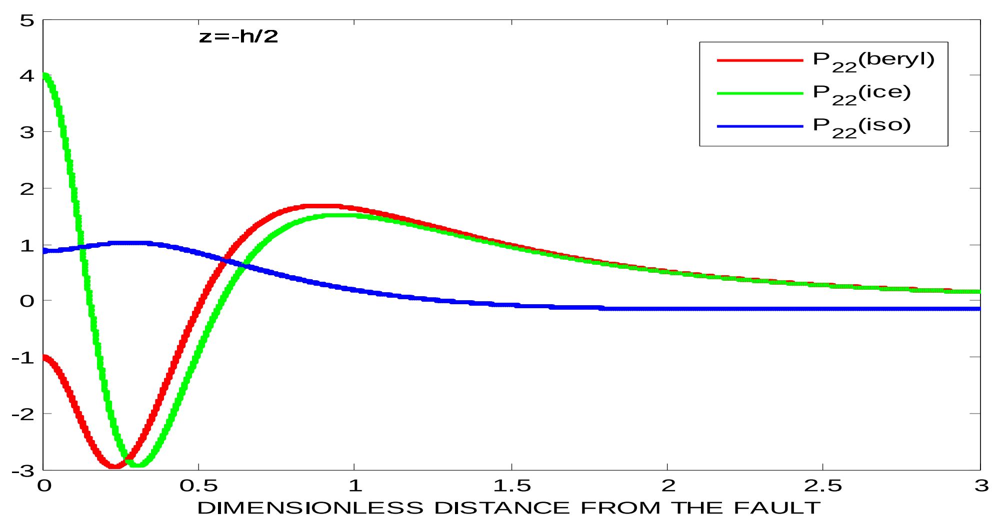

Figure 2(a): Variation of dimensionless normal stress *P22* with distance from the fault at *x3* = - *h*/2.

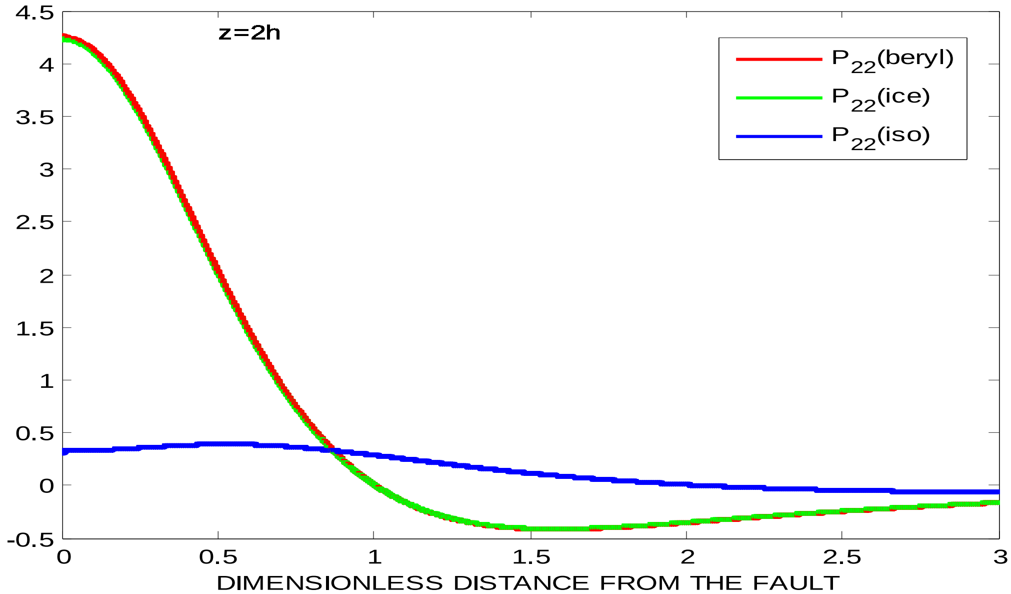

Figure 2(b): Variation of dimensionless normal stress *P22* with distance from the fault at *x3 =2h.*

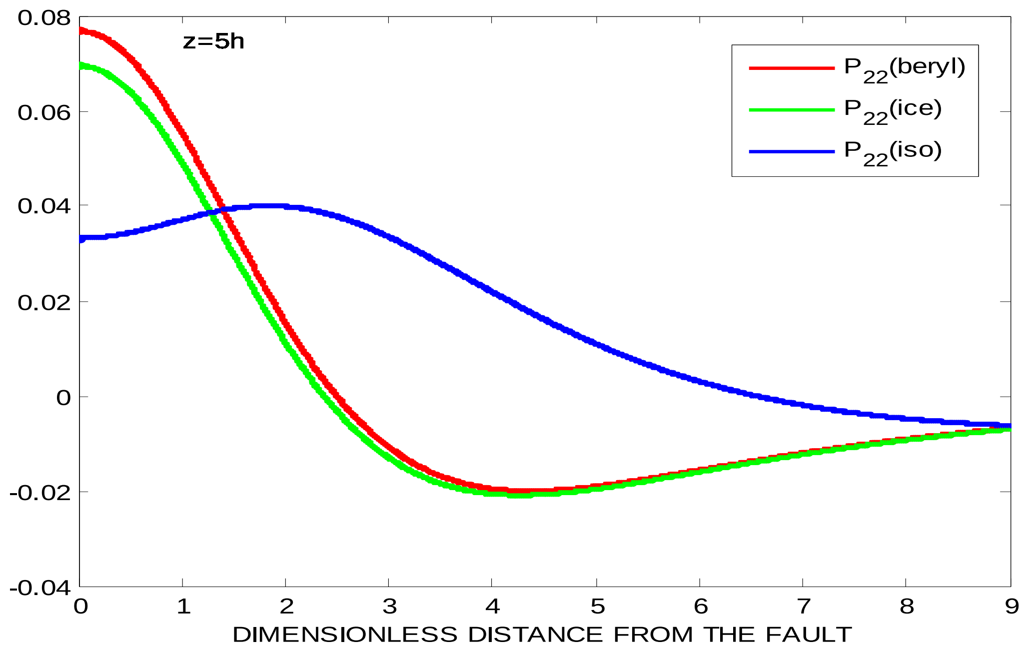

Figure 2(c): Variation of dimensionless normal stress *P22* with distance from the fault at *x3* = 5*h.*

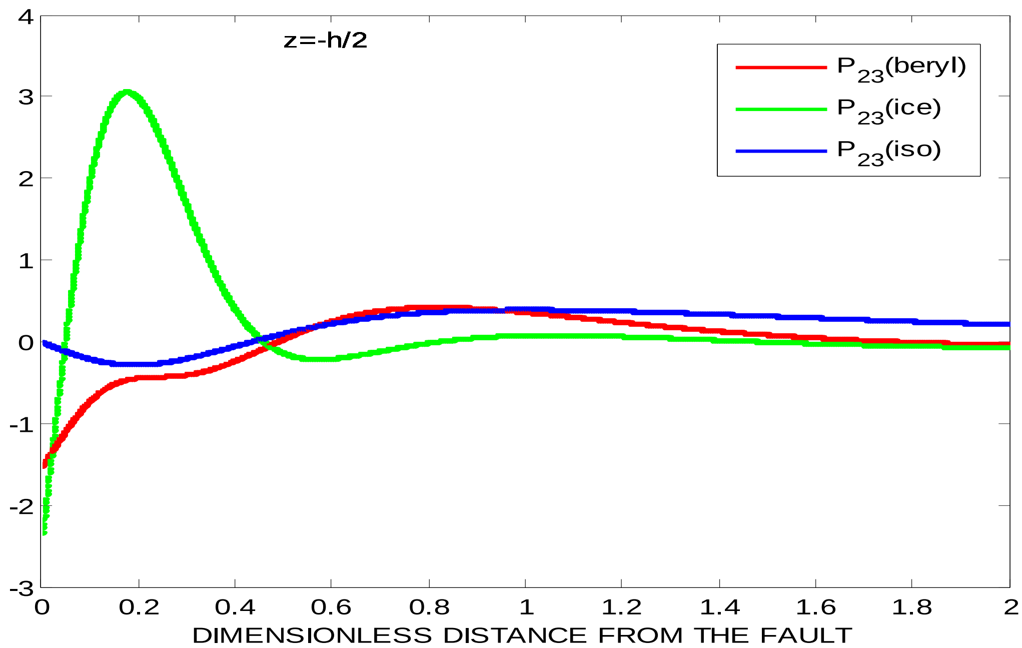

Figure 3(a): Variation of dimensionless shear stress *P*23 with distance from the fault at *x3* = -*h*/2.

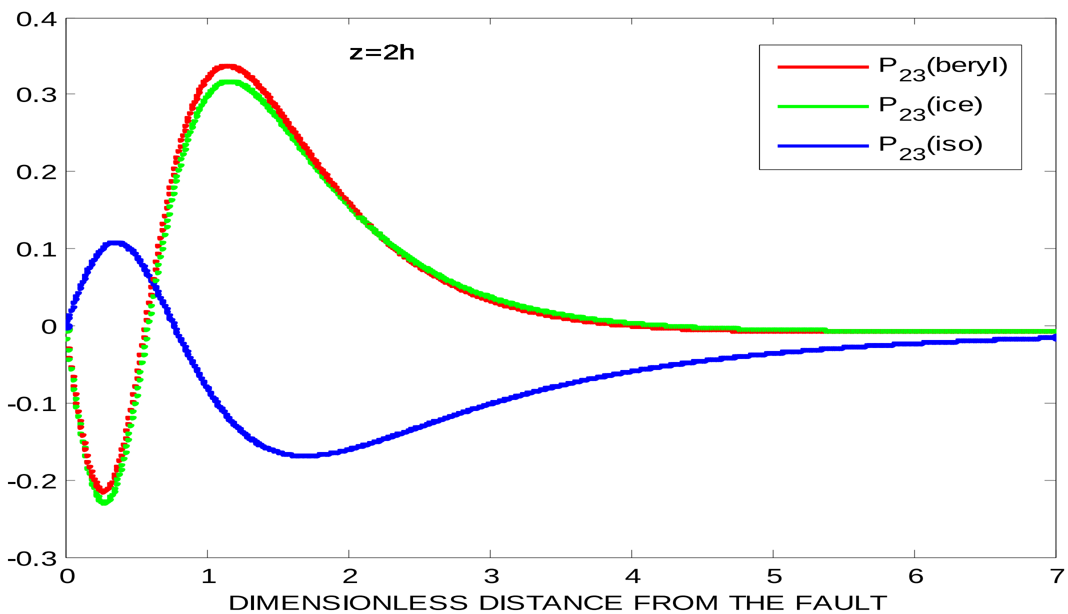

Figure 3(b): Variation of dimensionless shear stress *P*23 with distance from the fault at *x3* = 2*h*.

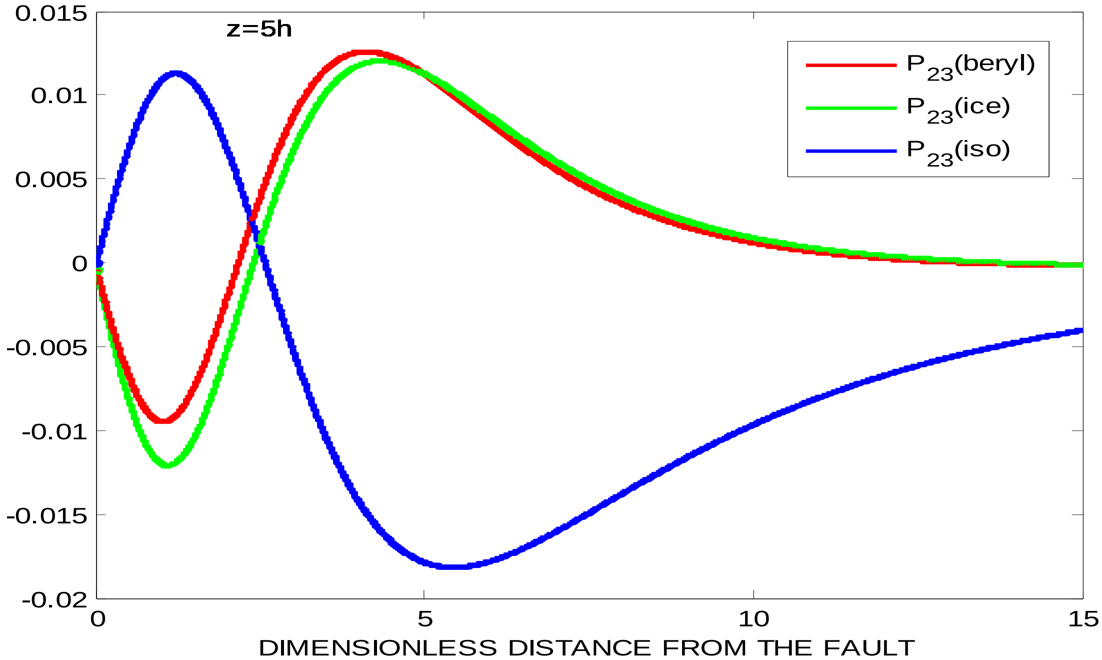

Figure 3(c): Variation of dimensionless shear stress *P*23 with distance from the fault at *x3* = 5*h*.

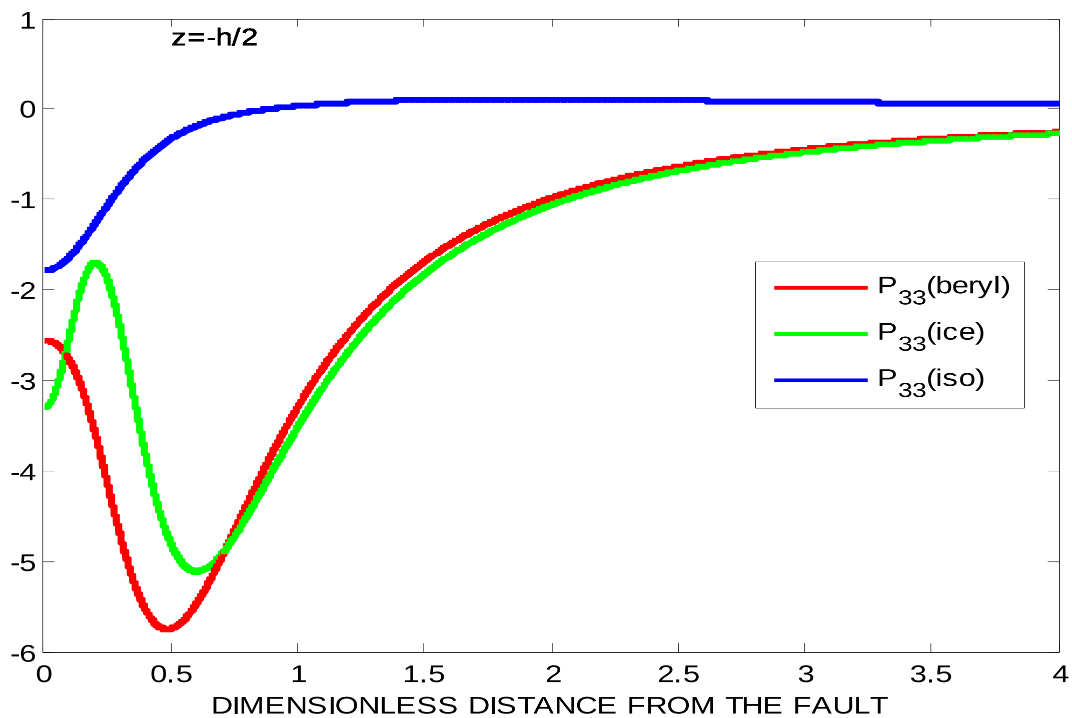

Figure 4 (a): Variation of dimensionless shear stress *P*33 with distance from the fault at *x3* = -*h*/2.

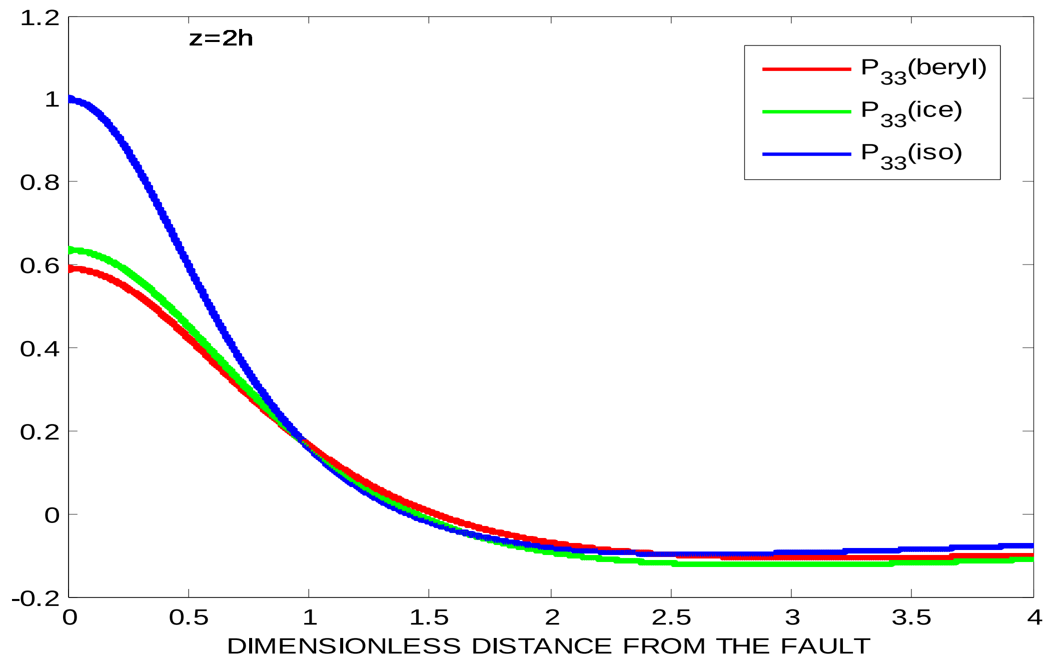

Figure 4(b): Variation of dimensionless shear stress *P*33 with distance from the fault at *x3* = 2*h*.

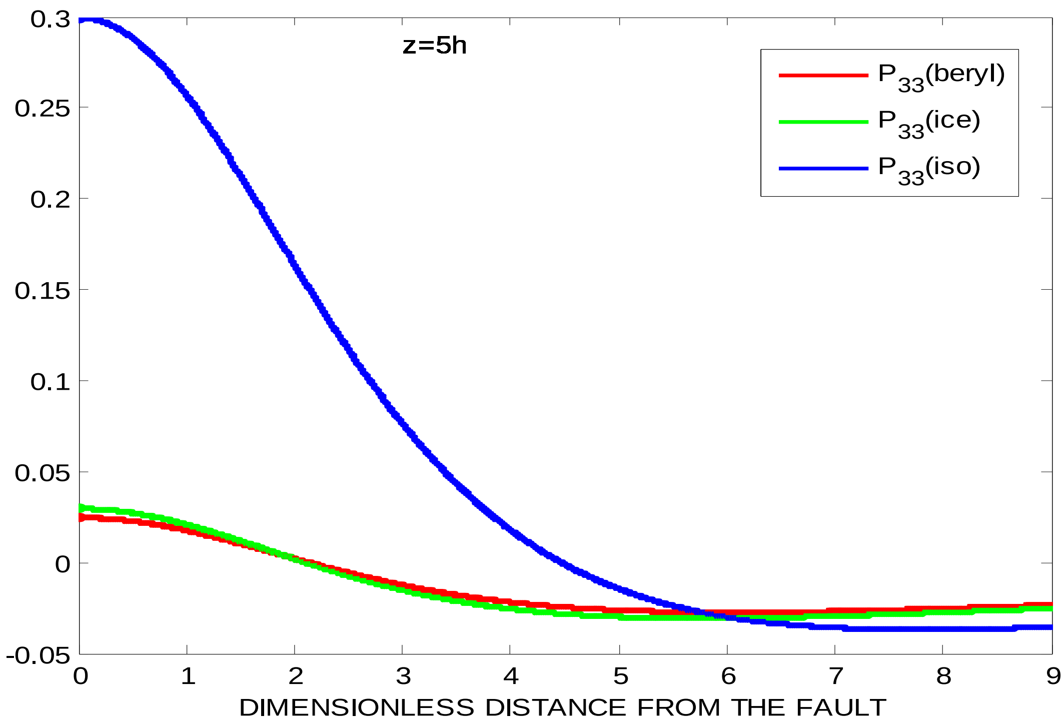

Figure 4(c): Variation of dimensionless shear stress *P*33 with distance from the fault at *x3* = 5*h*.

APPENDIX

1. Vertical dip-slip fault

$$L_0 = P_0 = Q_0 = 0, \qquad M_0 = \pm \frac{\mu b ds}{2\pi(1-\sigma)}$$

2. Vertical tensile fault

$$L_0 = M_0 = 0, \qquad P_0 = -Q_0 = \frac{\mu b \mathrm{ds}}{2\pi(1-\sigma)}$$

3. Horizontal tensile fault

$$L_0 = M_0 = 0, \qquad P_0 = Q_0 = \frac{\mu b \mathrm{ds}}{2\pi(1-\sigma)}$$

The upper sing is for $x_3 > h$,the lower sign is for $x_3 < h$, b is the magnitude of the displacement dislocation and ds is the width of the line fault.

REFERENCES

[1] Bonafede, M. & Danesi, S., 1997. Near-field modifications of stress induced by dyke injection at shallow depth, *Geophys. J. Int.,* **130,** 435-448.

[2] Bonafede, M. & Rivalta, E., 1999. The tensile dislocation problem in a layered elastic medium, *Geophys. J. Int.,* **136,** 341-356.

[3] Davis, P. M., 1983. Surface deformation associated with a dipping hydrofracture, *Geophys. J. Res.,* **88,** 5826-5836.

[4] Dundurs, J. & Hetenyi, M., 1965. Transmission of force between two semi-infinite solids, *ASME Journal of Applied Mechanics,* **32,** 671-674.

[5] Garg, N. R., Singh, S. J. & Manchanda, S., 1991. Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads, *Proc. Ind. Acad. Sci. (Earth Planet. Sci.),* **100,** 205-218.

[6] Heaton, T. H. & Heaton, R. E., 1989. Static deformation from point forces and point force coupls located in welded elastic Poissonian half-spaces: implications for seismic moment tensors, *Bull. Seism. Soc. Am,.* **79,** 813-841.

[7] Kumar, A., Singh, S. J. & Singh, J., 2005. Deformation of two welded half-spaces due to a long inclined tensile fault, *J. Earth Syst. Sci.,* **114,** 97-103.

[8] Kumari, G. Singh, S. J. & Singh, K., 1992. Static deformation of two welded half-spaces caused by a point dislocation source, *Phys. Earth. Planet. Inter,* **73,** 53-76.

[9] Maruyama, T., 1964. Statical elastic dislocations in an infinite and semi-infinite medium, *Bull. Earthquake Res. Inst.,* **42,** 289-368.

[10] Pan, E., 1989a. Static response of a transversely isotropic and layered half-space to general surface loads, *Phys. Earth Planet. Inter.,* **54,** 353-363.

[11] Pan, E., 1989b. Static response of a transversely isotropic and layered half-space to general surface loads, *Phys. Earth Planet. Inter.,* **58,** 103-117.

[12] Rani, S. & Singh, S. J., 1992. Static deformation of two welded half-spaces due to dip-slip faulting, *Proc. Indian Acad. Sci.(Earth Planet. Sci.)* **101** 269-282.

[13] Rani, S., Singh, S. J. & Garg, N. R., 1991 Displacements and stresses at any point of a uniform half-space due to two-dimensional buried sources, *Phys. Earth Planet. Inter.,* **65,** 276-286.

[14] Rongved, L., 1955. Force interior to one of the two joined semi-infinite solids, *in Proc. Of the 2nd Midwestern Conf on Solid Mech.,ed,* Bogdanoff J L *Purdue University, Indiana, Res., Ser.,***129,** 1-13.

[15] Singh, M. & Singh, S. J., 2000. Static deformation of a uniform half-space due to a very long tensile fault, *ISET J. Earthquake Techn.* **37,** 27-38.

[16] Singh, S. J. & Garg, N. R., 1985. On two-dimensional elastic dislocations in a multilayered half-space, *Phy. Earth Planet. Int.,* **40,** 135-145.

[17] Singh, S. J. & Rani, S., 1991. Static deformation due to two-dimensional seismic sources embedded in an isotropic half-space in welded contact with an orthotropic half space, *J. Phys. Earth,* **39,** 599-618.

[18] Singh, S. J., Rani, S. & Garg, N. R., 1992. Displacements and stresses in two welded half-spaces caused by two dimensional sources, *Phys. Earth Planet. Int.,* **70,** 90-101.

[19] Singh, S. J. & Garg, N. R., 1986 On the representation of two-dimensional seismic sources, *Acta Geophy. Pol.,* **34,** 1-12.

[20] Singh, S. J., 1986. Static deformation of a transversely isotropic multilayered half-space by general surface loads, *Phys. Earth Planet. Inter.,* **42,** 263-273.

[21] Sipkin, S. A., 1986. Interpretation of non-double- couple earthquake mechanisms derived from moment tensor inversions, *J. Geophys. Res.,* **91,** 531-547.

[22] Wu, R.S. & Chou, Y.T., 1982. Line force in a two-phase orthotropic medium, *ASME. Appl. Mech.,* **49,** 55-61.

[23] Yang, X. M. & Davis, P. M., 1986. Deformation due to a rectangular tensile crack in an elastic half-space, *Bull. Seism. Soc. Am.,* **76,** 865-881.

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