Hamiltonian Fuzzy Cycles on K 2n+1 Fuzzy Graph

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Abstract- The concept of connectivity plays an important role in both theory and applications of Fuzzy Graphs. This paper discusses the Hamiltonian fuzzy cycle on K2n+1 fuzzy graph. Next we proved results on complement of fuzzy cycles . Finally we discusses cycles in cubic fuzzy graph.

I. Introduction

Fuzzy graph theory is now finding numerous applications modern science and technology especially in the fields of information theory, neural network, expert systems and cluster analysis, medical diagnosis, etc. Bhatta Charya [5] has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges. Rosenfeld [12] has obtained the fuzzy analogues of several basic graph- theoretic concepts like bridges, paths, cycles, trees, and connectedness and established some of the properties. The concept of decomposition of graphs into Hamiltonian cycles, Hamiltonian paths decomposition of regular graphs was introduced by Klas Markstrom [11]. we introduce the concept of decomposition of Fuzzy graphs. Next we discuss the concept of Hamiltonian path & Hamiltonian cycle of fuzzy graphs. Some important results in cubic fuzzy graph, complement of fuzzy cycles are discussed.

II. Research Elaborations

Definition 2.1: A path P in a graph G^*: (V, E) is said to be a Hamiltonian path if it covers all the vertices of G exactly once.

Definition 2.2: A cycle in a graph G^*: (V, E) is said to be a Hamiltonian cycle if it covers all the vertices of G exactly once except the end vertices.

Definition 2.3: A fuzzy graph with S as the underlying set is a pair G: (σ, μ) where σ: S → [0, 1] is a fuzzy subset, μ: S x S →[0, 1] is a fuzzy relation on the fuzzy subset σ, such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y ∈ S where ∧ stands for minium. The underlying crisp graph of the fuzzy graph G: (σ, μ) is denoted as G^*: (V, E). Where E ⊆ = V × V.

Definition 2.4: A path P of length 'n' is a sequence of distinct nodes $u_0, u_1, \ldots\ldots u_n$ such that $\mu(u_{i-1}, u_i) > 0$ i=1, 2……n is called a fuzzy path and the degree of a membership of a weakest arc is defined as it's strength.

Definition 2.5: If $u_0 = u_n$ and $n \geq 3$, then P is called a cycle and cycle P is called a fuzzy cycle (f- cycle) if it contains more than one weakest arc.

Definition 2.6: In a fuzzy graph G a fuzzy path P covers all the vertices of G exactly once then the path is called Hamiltonian fuzzy path.

Definition 2.7: In a fuzzy graph G a fuzzy cycle C covers all the vertices of G exactly once except the end vertices then the cycle is called Hamiltonian fuzzy cycle.

Definition 2.8: A fuzzy graph G: (σ, μ) is said to be Complete if $\mu(x,y)=\sigma(x)\wedge\sigma(y)$ for all x & y.

Definition 2.9: The complement of a fuzzy graph is denoted by $G^c : (\sigma^c,\mu^c)$, where $\sigma^c = \sigma$ and $\mu^c(x,y) = \wedge[(x),\sigma(y)] - \mu(x,y)$.

III. Applications and Results

Definition 3.1: K_n is a complete fuzzy graph with 'n' vertices.

Example 3.2: Consider K5 a complete fuzzy graph on five vertices in figure.1, which has two Hamiltonian fuzzy cycles. Let it be 123451 and 135241. Here the membership values of each edge are greater than zero and it contains more than one weakest arc.

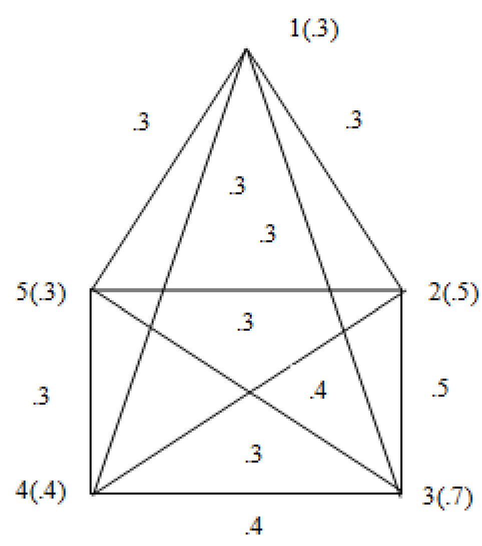

Fig. 1.K_5 Fuzzy graph

Example3.3: In figure .2 K_7 is decomposed into $3c_7$ .That is three hamiltonian fuzzy cycles. Lable the vertices clockwise around the circle and always go to the next vertex in the first copy of the Hamiltonian fuzzy cycle (c_7) .

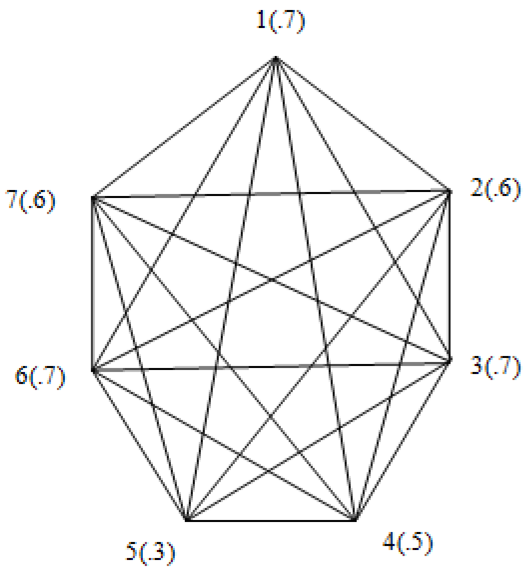

Fig.2. K_7 Fuzzy Graph

We get 12345671, 13572461, 14736251.

We can't extend this to K_9. As it turns out, K_9 is decomposable into C_9 any way. We just need a more clever trick called the turning trick.

Theorem 3.4: For any $n \geq 1$, K_{2n+1} fuzzy graph is decomposable into n Hamiltonian fuzzy cycles C_{2n+1}.

Proof: Let $G = K_{2n+1}$ be a fuzzy graph. Label the vertices ∞,0,1,2,3,….,2n-1. In this fuzzy graph form the Hamiltonian fuzzy cycles as follows.

(C_1) is a fuzzy cycle that is
(C_1) ∞ ,0,2n-1,1,2n-2,2,2n-3,…n-1,n,∞
(C_2) ∞ ,1,0,2,2n-1,3,2n-2, …,n,n+1,∞
(C_3) ∞ ,2,1,3,0,4,2n-1,…..n+1,n+2,∞
.
.
.
(C_n) ∞,n-1,n-2,n,n-3,n+1,n-4,…..2n-2,2n-1,∞ .
(so each time we add 1 to every label and find the remainder modulo 2n).

For example, label vertices of K_9 with ∞ ,0,1,2,3,4,5,6,7 (n=4) and decompose it into

∞	0	7	1	6	2	5	3	4	∞.
∞	1	0	2	7	3	6	4	5	∞.
∞	2	1	3	0	4	7	5	6	∞.
∞	3	2	4	1	5	0	6	7	∞.

(you can visualize this by putting vertices 0 through 2n-1 clockwise along a circle with ∞ outside with circle.)

Corollary 3.5: For any $n \geq 1$,K_{2n} is decomposable into n Hamiltonian fuzzy paths P_{2n-1} .

Proof: Take the composition of K_{2n+1} of theorem 1 and delete the ∞ vertex.K_{2n+1} will become $K_{2n,}$ While each Hamiltonian fuzzy path P_{2n-1} of K_{2n}

IV. Cycles in Regular Fuzzy Graph

Definition 4.1. A fuzzy graph G is called r – regular if every vertex of G has degree r

Definition 4.2: A 3- regular fuzzy graph is called cubic fuzzy graph

Example 4.3

In the following example figure 3 is a cubic fuzzy graph.

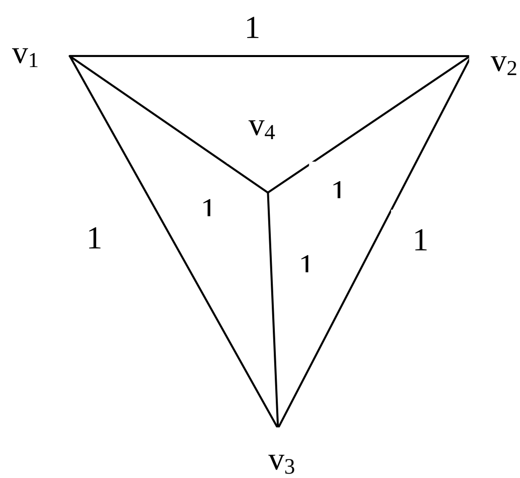

Fig.3. Cubic Fuzzy Graph

Definition 4.4 . A fuzzy graph g is called r – regular if every vertex of G has degree r

Definition 4.5: 1-factor of a fuzzy graph is a spanning 1-regular fuzzy sub graph of G.

Example 4.6

In the following example figure 4 is a 1-regular fuzzy graph.

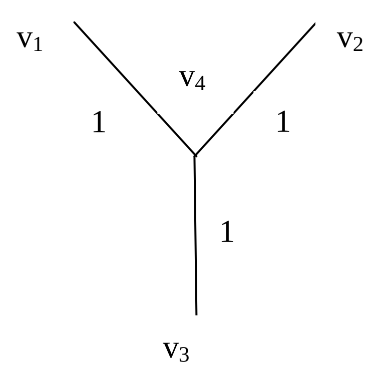

Fig.4. 1-Regular Fuzzy Graph

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Theorem 4.7: For any $n \geq 1$, K_{2n} is decomposable into 2n-1 1-factors.

Proof: label vertices ∞, 0,1,2,… ,2n-2. start with a 1-factor

∞,0; 1,2n-1; 2,2n-2; . . . n-1,n

use the turning trick to obtain the following decomposition:

∞,0; 1,2n-1; 2,2n-2; . . . n-1,n

∞,1; 2,0; 3,2n-2; . . . n, n+1

.

.

∞ ,2n-2; 0,2n-3; 1,2n-4; . . . n-2,n-1.

Theorem 4.8: Let G (σ,μ) be a fuzzy graph where G^*: (V,E) is an odd cycle. Then G is regular iff μ is a constant function.

Proof: If μ is a constant function , say $\mu(uv)=c$, for all $uv \in E$, then $d(v)=2c$, for every $v \in V$.

So G is regular.

Conversely, suppose that G is a k-regular fuzzy graph . Let $e_1,e_2,\ldots,e_{2n+1}$ be the edges of G^* in that order.

Let $\mu(e_1) = k_1$. Since G is k-regular,

$\mu(e_2) = k-k_1$

$\mu(e_3) = k- (k-k_1)=k_1$

$\mu(e_4) = k- k_1$

and so on.

Therefore $\mu(e_i) = k_1$, if i is odd

$= k-k_1$ if i is even

Hence $\mu(e_1) = \mu(e_{2n+1})=k_1$.

so if e_1 and e_{2n+1} incident at a vertex u , then $d(u)=k$. so $d(e_1) + d(e_{2n+1}) = k$

ie $k_1 + k_1 = k$

$2k_1 = k$

$k_1 = k/2$.

Hence $k-k_1 = k/2$. so $\mu(e_i) = k/2$, for all i. Hence μ is a constant function.

Example 4.9

In the following example figure 5 G (σ,μ) be a fuzzy graph where G^*: (V,E) is an odd cycle. Also G is regular if μ is a constant function.

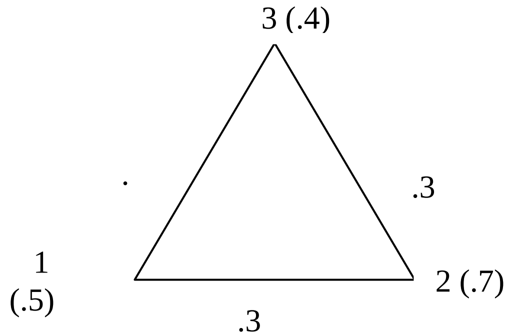

Fig.5.Fuzzy Graph

Theorem 4.10: **:** Let G (σ,μ) be a fuzzy graph where G^*: (V,E) is an even cycle. Then G is regular iff μ is a constant function or alternate edges have same membership values.

Proof: If either μ is a constant function or alternate edges have same membership values, then G is a regular fuzzy graph. Conversely, Suppose G is a k-regular fuzzy graph. Let $e_1,e_2,\ldots,e_{2n}$ be the edges of even cycle G^* in that order. Proceeding as in theorem,

$\mu(e_i) = k_1$, if i is odd

$= k-k_1$ if i is even

If $k_1= k-k_1$, then μ is a constant function.

If $k_1 \neq k-k_1$, then alternate edges have same membership values.

Example 4.11

In the following example figure 6 is G (σ,μ) be a fuzzy graph where G^*: (V,E) is an even cycle. Also G is regular if μ is a constant function or alternate edges have same membership values.

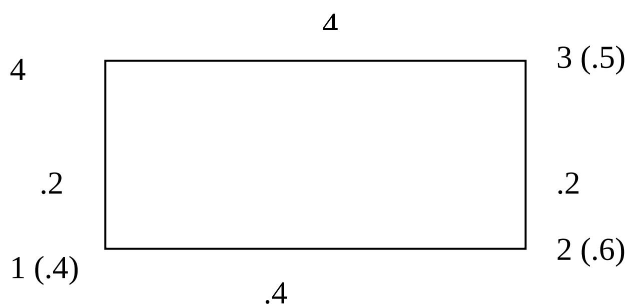

Fig.6. Fuzzy Graph

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Theorem 4.12 : Let G be a cubic fuzzy graph where G^* is a cycle. Then G is a fuzzy cycle. It cannot be a fuzzy tree.
Proof : Assume that G be a cubic fuzzy graph where G^* is a cycle. That is G be a 3- regular fuzzy graph on a cycle G^* , then by theorem theorem , either μ is a constant function or alternate edges have same membership values. So there does not exist a unique edges xy such that $\mu(xy)=\wedge\{\mu(uv)/\ \mu(uv)>0\}$.

Therefore G is a fuzzy cycle.
Hence by lemma , G cannot be a fuzzy tree.

Example 4.13

In the following example figure 7 is a cubic fuzzy graph and G^* is a cycle. Then G is a fuzzy cycle. Also it cannot be a fuzzy tree.

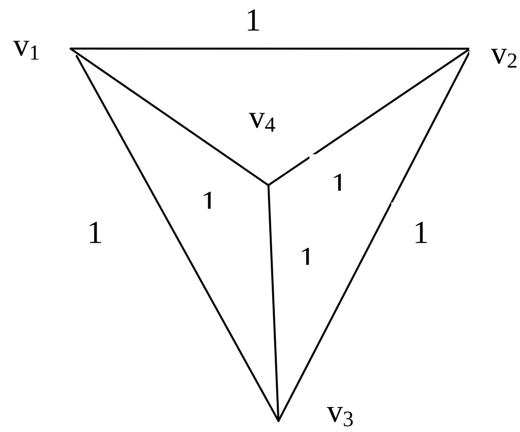


Fig.7. Cubic Fuzzy Graph

V. COMPLEMENT OF FUZZY CYCLES

Construct the complement of fuzzy cycles on 3, 4, 5 vertices as fuzzy cycles by choosing the membership values of vertices and edges suitably

Example 5.1: n=3, G and G^c are both fuzzy cycles

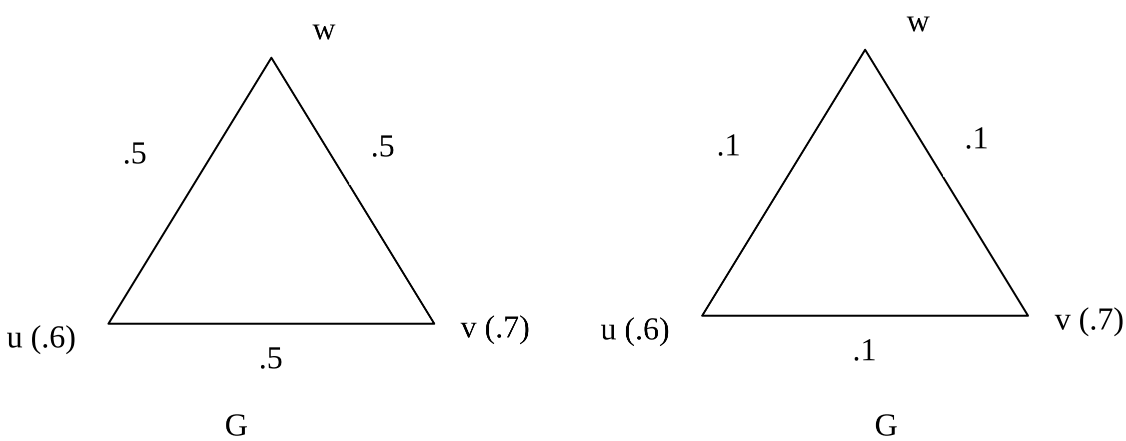
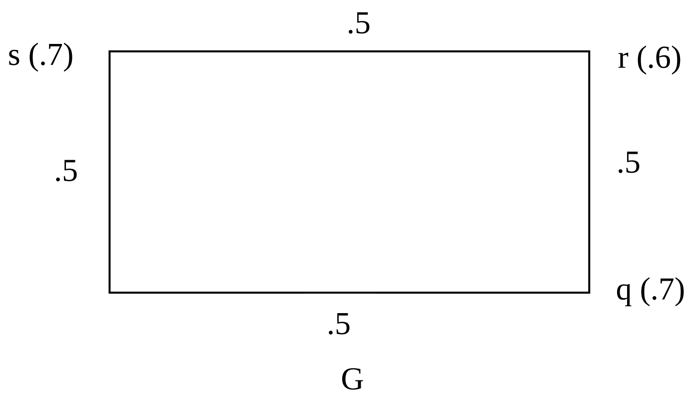


Fig.8. G and G^c

Example 5.2: n=4, G and G^c are both fuzzy cycles



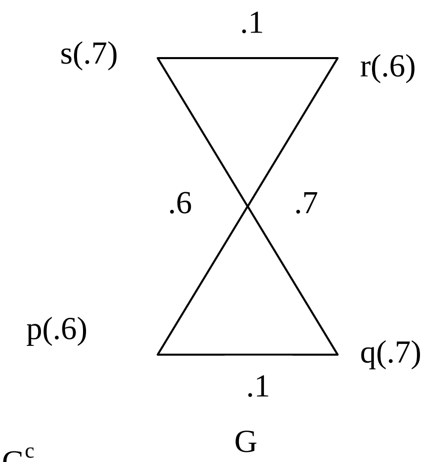
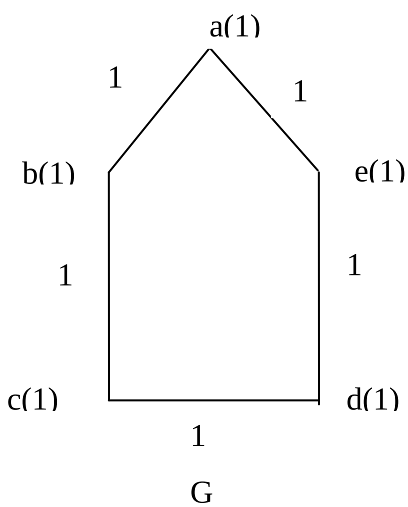


Fig.9. G and G^c

Example 5.3: n=5, G and G^c are both fuzzy cycles



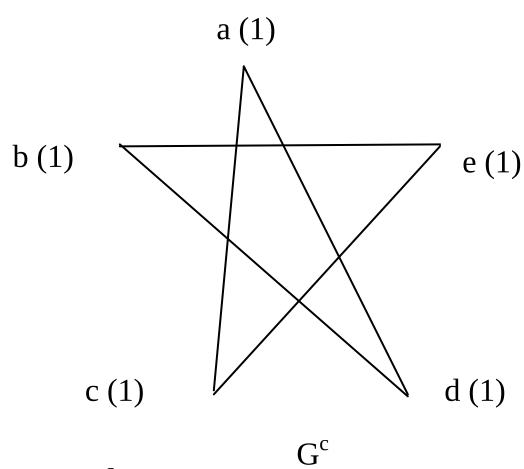


Fig.10. G and G^c

Theorem 5.4 Let G :(σ,μ) be a fuzzy graph such that G^* is a cycle with more than five vertices. Then $(G^*)^c$ cannot be a cycle.

Proof: Given G^* is a cycle having n vertices where n≥6. Then G^* will have exactly n edges. since all the vertices of G are also present in G^c. Therefore number of vertices in G^c is n. Let the vertices of G and G^c be $v_1, v_2, \ldots, v_n$. Then G^c must contain atleast the following edges.

$(v_1, v_3), (v_1, v_4), (v_1, v_5), \ldots, (v_1, v_n); (v_2, v_4), (v_2, v_5), \ldots, (v_2, v_n); (v_3, v_5), (v_3, v_6), \ldots, (v_3, v_n)$

since n≥6 the total number of edges in G^* will be greater than n. Thus G^c will not be a cycle.

Corollary: Let G be fuzzy cycle with 6 or more vertices. Then G^c will not be fuzzy cycle.

VI. CONCLUSION

In this paper we have introduced the concept of fuzzy cycle, fuzzy paths in a fuzzy graph. A complete analysis of Hamiltonian fuzzy cycle in K_{2n+1} fuzzy graph is presented. Next we are studying the properties of regular fuzzy graph and complement of fuzzy cycles.

REFERENCES

[1] J.N. Mordeson, P.S. Nair, Fuzzy graphs and fuzzy hyper graphs, Physica – veriag, 2000.

[2] G.Nirmala and M.Vijaya , Fuzzy graphs on composition, Tensor and Normal products, International journal of scientific and research publications , vol 2 ,Issue 6 , 2012.

[3] L.A. Zadeh, Fuzzy sets, information control 8 (1965) 338- 353.

[4] G.Nirmala and M.Vijaya , Application of fuzzy back propagation network, The PMU journal of humanities and sciences, vol 2 , no 2, (jul – dec 2011) pp 99-107.

[5] Bhattacharya,p Some remarks on fuzzy graphs, Pattern Recognition Lett 6(1987) 297- 302.

[6] G.Nirmala and M.Vijaya , Fuzzy Number – Imprecise aspects of real world problems, Advances in Mathematics Scientific Developments and Engineering Applications. Narosa Publishing House.

[7] Frank Harary ,Graph theory , Narosa /Addison Wesley. Indian student edition 1988.

[8] A.Nagoor Gani and V.T.Chandrasekaran, A First Look at Fuzzy Graph Theory, Allied Publishers, 2010.

[9] G.Nirmala and M.Vijaya , Strong Fuzzy Graphs on Composition ,Tensor and Normal products , Accepcted and published in, Arabhatta Journal of Mathematics and Informatics, Vol 4 , No .2 , Nov 2012

[10] A.Nagoor Gani and K. Radha, On Regular Fuzzy Graphs, Journal of physical sciences, Vol . 1233-40 (2010).

[11] Klas Markstrom ,Even cycle decompositions of 4- Regular graphs and line graphs.

[12] A.Rosenfeld, Fuzzy graphs, in: LA.Zadeh, K.S. Fu,M.Shimura(Eds).Fuzzy sets and their applicationsto cognitive and decision processes, Academic press, New York, 1975 pp, 77-95.

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