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FIND OUT nth ORDER ROOT OF A NUMBER BY DIVISION METHOD (USING THIS THEOREM CALCULATION 3rd, 4th, 5th ROOT OF ANY REAL NUMBER & DEDUCE $(1+f(x))^{1/n}$ expansion where f(x) is an algebraic function)

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Abstract- Find out nth order root of any real number by division method
Using nth order root method calculation of cubic, 4th, 5th order root using value of nCr.
& also deduce expansion of (1+f(x))^1/n where f(x)is an algebraic function .There are three main step in this method named PRE STEP,RAM STEP ,SHYAM STEP. For finding calculation of nth order root PRE STEP used only time and remain two step are calculated as per requirement in cycle e.g.
PRE STEP –RAM STEP –SHYAM STEP –RAM STEP –SHYAM STEP……………..as per requirement

I. INTRODUCTION

Here I am going to present a much simpler and understandable technique for Finding nth order root (cubic root, 4th, 5th order root) of a number which can easily be used to make the students understand the concept at primary level also.

The concept does not use any advanced mathematics concept like calculus, etc. (for cubic, 4th ,5th order root) which is most popular used method for this purpose.

Finding cubic root of a number has always been a fascination for me as I was told by teacher that there is no perfect way to find cubic root .There are methods like prime factor method (only acceptable for whole cube number) calculus(needs higher &advanced mathematics)etc.

The proposed "K. K. METHOD" for finding nth order root (especially for cube root 4th order root, 5th order root) not only overcomes these demerits but also is very easy to understand & solve. And because of the limitations of the number available for this purpose, I have always been trying to develop one.

II. METHODOLOGY

Here I am describing method for calculating nth order root of any number
GENERAL STEP OF CALCULATION OF nth ORDER ROOT:-STEP.1) Make pair of n digit if you want to calculate nth order root …CASE 1)..If number is left to decimal then make right to left
Case.2)..If number is right to decimal then left to right …..
As we make pair of 2 digit in calculation of square root e.g.
For square root 2453831455.0015275754
For cubic root 42527893412.012345600
For 4th 12435786352123.12345680
For 5th 124253678963123.456375860 and so on

STEP .2) There are two main step named here 1st one is "RAM "& 2ND one is "SHYAM" eg. p.5) 1154

```
   PRE STEP   1        1536800264        1= 1^2
              2         1                  2=2×1
              3         0536
1.  RAM          1        331              331=331×1
STEP             3        205800          3=3×1×1
331         189875          189875=37975×5
2.  SHYAM        2        15925264        2=2×1
```

STEP 3 15925264 3=3×1, 15925264=3981316×4
363 0
25 25=5^2
1 165 165=3×11×5
37975
50 50=25×2
2 165 165=165×1
39675
16 16=4^2
1380 1380=3×115×4
3981316

STEP.3) you have to fill up n-1 place when you are in "STEP RAM" which is very understandable
By an Intermediate educated student. As I describe we have to fill up n-1 place row wise
That are right of "PRE STEP (actually it is combination of RAM&SHYAM STEP) needs some Information factorial if we are interest more than 5 th order root …
Here I am describing the rule-
1. 1^{ST} row is (next quotient) $^{n-1}$; from right to n-1 place right to left (same for next).
2. 2^{nd} row is $^{n}C_{n-1}\times1^{st}$ quotient (initial to final if more than two step are already done) $\times(2^{nd}$ quotient $)^{n-2}$; from right to n-2 place
3. 3^{rd} row $^{n}C_{n-2}\times1^{st}$ quotient$^{2}\times$ (initial to final if more than two step are already done) $\times2^{nd}$ Quotient $^{n-3}$ …hence so on up to n-1 row wise place ………last one is $^{n}C_{2}\times1^{st}$ quotient^{n-2}(initial to final)$\times2^{nd}$ quotient(next)

After it add all of them with column wise .see above example.
STEP.4) Now in "SHYAM STEP" 1.it start with (n-1) × value of RAM STEP & write down it from unity
Of RAM STEP.
2. (n-2) ×value of 2^{nd} row from 10^{th} place
3. (n-3) ×value of rd row from 100 th place. Hence so on up to n-1 term
After it add them and guess what the next quotient is & repeat "RAM STEP "again
And so on repeat these two step …
Here many example is given of cubic root, 4th root 5^{th} root &also method for cubic root, 4th order, and 5th order root.

Value of nCr for square root -1^{st} row = 1
For cubic root 1st row =1;2^{nd} row=3
For 4^{th} root 1^{st} row=1 ;2^{nd} row=4;3^{rd} =6
For 5^{th} root 1^{st} row =1;2^{nd} row=5;3^{rd} =10;4^{th} row=10

cofficient of
RAM STEP

METHODOLOGY:- Method for calculating cubic root.
STEP.1) Make pair of 3 digit ---CASE 1. Right to left if number is left of decimal
Case 2.Left to right if it is right to decimal
12432578635412.012453680000002

STEP.2) Whatever is quotient we have to put square of quotient at the place of divisor.eg if quotient is 8 then put 8^2=64

STEP 3) after it, you have to put below the dividend eg 8×64=512& subtract it and put net three digit.

p.1) 8 p.2) 12
64 512 8^2=64 1 1728 1^2=1
512 64×8 2 1 1×2=2
0 3 0728
4 728 2^2=4 ,364×2
6 000 3×2×1=6
364

STEP 4) Now you have to add doble of divisor in the divisor eg. 1+2=3

STEP 5) Since it is a method of cubic root you have to fill up two place after divisor+2×divisor .see eg .2 1^{st} place is square of of next quotient eg .4=2^2& 2^{nd} one is 3×1^{st} quotient (initial to last one is called 1^{st} one)$\times2^{nd}$ one (that is presently quotient by which we want to divide)

1st one is started 2 place right to divisor+divisor×2
2nd one is started 1 place right in next line, after it add all of them column wise.

STEP.6) Now after addition multiplied it with 2nd quotient and put below in dividend.(see eg.2,3
And subtract it and put next three digit.

STEP.7) Now again put 2× square of 2nd quotient from place of unity and next line below the present divisor 3×1st quotient (initial to last one is called 1st one)×2nd after one place of unity
i.e 10th place.

STEP8) Now again start division with quotient .put square of quotient 2place right of sum after step seven ;3×1st quotient (initial to final –now there is two digit see eg p.5)×2nd quotient one place right to sum of divisor after step 7 and sum up all of them column wise.
After it multiplied by quotient and put to dividend & subtract it, put next three digit to divison.
At the place of divisor if all digit in dividend is over then calculation of cubic root is finished
If not then repeat step 7&8 …… see more example …

p.1)

	8	
64 3	512	8^2=64
	512	64×8
	0	

p.2)

	12	
1 3	1728	1^2=1
2	1	1×2=2
3	0728	
4	728	2^2=4 ,364×2
6	000	3×2×1=6
364		

p.3)

	67	
36 3	300763	36=6^2
72	216	216=36×6,72=36×2
108	084763	
49	84763	84763=12109×7
126	00000	3×6×7=126
12109		

p.4)

	569	
25 3	184220009	25=5^2
50	125	50=25×2,125=25×5
75	059220	
36	50616	36=6^2,50616=8436×6
90	08604009	
8436	8604009	8604009=956001×9
72	0	72=36×2
90		90=90×1
9408		
81		81=9^2
1512		1512=3×56×9
956001		

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p.5)

	1154	
1	3 | 1536800264	1= 1^2
2	1	2=2×1
3	0536	
1	331	331=331×1
3	205800	3=3×1×1
331	189875	189875=37975×5
2	15925264	2=2×1
3	15925264	3=3×1,15925264=3981316×4
363	0	
25		25=5^2
165		165=3×11×5
37975		
50		50=25×2
165		165=165×1
39675		
16		16=4^2
1380		1380=3×115×4
3981316		

P.6)

	101	
1	3 | 1030301	1=1^2
2	1	2=2×1
3	0030301	
0	30301	0^2=0; 30301=30301×1
0	00000	
300		
0		0×2=0
0		0×1=0
300		
1		1^2=1
30		3×10×1=30
30301		

p.7)

	1001	
1	3 | 1003003001	
2	1	2=2×1
30000	0003003001	
1	3003001	3003001=3003001×1
300	0000000	300=3×100×1
3003001		

p.8

		9867	
81	3	960628317363	
162		729	162=2×81,729=81×9
243		231628	
64		212192	64=8^2,212192=26524×8
216		19436317	216=3×9×8
26524		17393256	17393256=2898876×6
128		2043061363	128=2×64
216		2043061363	216=216×1, 2043061363=291865909×7
28812		0	
36			36=6^2
1764			1764=3×98×6
2898876			
72			72=2×36
1764			1764=1764×1
2916588			
49			49=7^2
20706			20706=3×986×7
291865909			

p.9)

		1.816...........	
1	3	6. 000000000	
2		1	2=2×1
3		5000	
64		4832	64=8^2, 4832=8×604
24		168000	24=3×1×8
604		97714	97741=97741×1 continued

p.10)

		12856	
1	3	2124797718016	
2		1	2=2×1
3		1124	
4		728	4=2^2,728=364×2
6		396797	6=3×1×2
364		369152	369152=46144×8
8		27645718	8=2×4
6		24672125	6=6×1,24672125=5×4934425
432		2973593016	
64		2973593016	8^2=64, 2973593016=6×495598836
288		0000000000	288=3×8×12
46144			
128			128=64×2
288			288=288×1
49152			
25			25=5^2
1920			1920=3×128×5
4934425			
50			50=25×2
1920			
4953675			
36			
23130			
495598836			

p.11)

		4.11711......	
16		69.787690000000	
32		64	32=16×2,64=8^2
48		5787	
1		4921	1=1^1
12		866690	12=3×4×1 continued

p.11)

		4.11711......	
16	3	69.787690000000	
32		64	32=16×2,64=8^2
48		5787	
1		4921	1=1^1,4921=4921×1
12		866690	12=3×4×1
4921		505531	505331=505331×1
2		361159000	2=2×1
12		355338613	12= 12×1,3506763×7
5043		5820387000	
1		5085030211	49=7^2,508503011=1×508503011
123		735356789000	123=41×3×1
505531		508516606431	508516606431=1×508516606431
2		226840182569	2=2×1
123			123=123×1
506963			
49			49=7^2
8631			8631=3×7×411
50762659			
98			98=2×49
8631			8631=8631×1
50849067			
1			1=1^2
12351			12351=3×4117×1
5085030211			
2			2=1×2
12351			12351=1×12351
5085153713			
1			1=1^2
123513			123513=3×41171×1
508516606431			

Example of 4th order root :-

p.1)

		11	
1	4	14641	
3		1	3=3×1; 1=1^3
4		04641	
1		4641	1=1^3;4641=1*4641
4		0000	4=4*1*1^2
6			6=6*1^2*1
4641			

```
p.2)                 12
        1      4√  20736
        3          1            3=3*1;1=1^3
        4          10736
          8        10736        10736=2*5368;8=2^3
         16        00000         16=4*2^2*1
        12                      12=6*1^2*2
        5368
```

```
p.3)                 98
        729      4√ 92236816     729=9^3
        2187        6561          2187=3*729;6561=729*9
        2916        26626816
            512     26626816     512=8^3;26626816=3328352*8
          2304      00000000      2304=4*9*8^2
        3888                      3888=6*9^2*8
        3328352
```

```
p.4)                101
        1        ∜104060401
        3         1
        4000      04060401
              1     4060401
            40       000000
          600
        4060401
```

```
p.5)                 111
        1        ∜151807041

        3         1
        4         05180
          1        4641
         4           5397041
        6            5397041
        4641         0000000
          3
         8
        6
        5324
             1
           44
         726
        5397041
```

```
p.6)           119
        1    ∜20053392
        3     1
        4     10053
          1    4641
         4      54123921
        6       54123921
        4641        0
         3
        8
        6
        5324
          729
         3564
        6534
        6013769
```

```
p.7)               179
        1       ∜1026625681
        3        1
        4        92662
          343    73521
         196     191415681
         42      191415681
        10503        0
         1029
         392
         42
        19652
             729
            5508
         15606
        21268409
```

p.8)

	979
729	∜918609150481
2187	6561
2916	26250915
343	22919281
1764	33316340481
3402	33316340481
3274183	0
1029	
3528	
3402	
3650692	
729	
31428	
508086	
3701815609	

p.9)

	1111
1	∜1523548331041
3	1
4	05235
1	4641
4	5944833
6	5397041
4641	5477921041
3	5477921041
8	0
6	
5324	
1	
44	
726	
5397041	
3	
88	
726	
5470524	
1	
444	
73926	
5477921041	

Example of 5th order root:-

	11	
1	161051	
4	1	
5	×	61051
1		61051
5		0
10		
10		
61051		

	121
1	25937424601
4	1
5	159374
16	148832
40	105424601
40	105424601
20	0
74416	
64	
120	
80	
20	
103680	
1	
60	
1440	
17280	
1054224601	

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DEDUCTION OF $(1+F(X))^{1/n}$ USING THIS THEOREM:

- Expansion of $(1+x)^{1/2}$

$1+x/2-x^2/8+x^3/16-5x^4/128$……………………………………

1 $2 \,|\, 1+x$

1 1

$2+x/2$ $0 \quad x$

$X/2$ $x+x^2/4$

$2+x-x^2/8$ $-x^2/4$

$-x^2/8$ $-x^2/4-x^3/8+x^4/64$

$2+x-x^2/4+x^3/16$ $x^3/8-x^4/64$

$X^3/16$ $x^3/8+x^4/16-x^5/64+x^6/256$

$2+x-x^2/4+x^3/8-5x^4/128$ $-5x^4/64-x^5/64+x^6/256$

$-5x^4/128$ $-5x^4/64-5x^5/128+5x^6/512-5x^7/1024+25x^8/16384$

- Expansion of $(1+x)^{1/3}$

$1+x/3-x^2/9+5x^3/81-10x^4/243$………..

1 $3 \,|\, 1+x$

2 1

$3+(x^2/9+x)$ x

$2x^2/9+x$ $x+x^2/3+x^3/27$

$3+2x+x^2/3+x^4/81-x^2/3$ $-x^2/3-x^3/27$

$2x^4/81-x^2/3$ $-x^2/3-2x^3/9-x^6/729$

$3+2x-x^2/3+x^4/27+25x^6/6561-5x^3/27$ $5x^3/27+x^6/729$

$50x^6/6561-5x^3/27$ $5x^3/27+10x^4/81-5x^5/243-25x^6/2187+5x^7/2187+125x^9$…..

$3+2x$………. $-10x^4/81$………….

- Expansion of $(1+x+x^2)^{1/2}$

$1+x/2+3x^2/8-3x^3/16+3/128x^4$……………………

1 $2 \,|\, 1+x+x^2$

1 1

$2+x/2$ $x+x^2$

$x/2$ $x+x^2/4$

$2+x+3x^2/8$ $3x^2/4$

$3x^2/8$ $3x^2/4+3x^3/8+9x^4/64$

$2+x+3x^2/4-3x^3/16$ $-3x^3/8-9x^4/64$

$-3x^3/16$ $-3x^3/8-3x^4/16-9x^5/64+9x^6/256$

$2+x+3x^2/4-3x^3/8$ $3x^4/64+9x^5/64-9x^6/256$

- Expansion of $(1+x)^{1/n}$

$1+x/n-{}^nc_2(x/n)^2/n+(2*{}^nc_2{}^2*(x/n)^3*1/n-{}^nc_3(x/n)^3)/n$…………………

1 $1+x$

$(n-1)1$ 1

$n+{}^nc_2x/n+{}^nc_3(x/n)^2........+{}^nc_n(x/n)^{n-1}$ x

${}^nc_2x/n+2{}^nc_3(x/n)^2+$……….$(n-1){}^nc_n(x/n)^{n-1}$ $x+{}^nc_2(x/n)^2$………………..

$n+2\,{}^{n}c_2 x/n$……………………….. $-{}^{n}c_2(x/n)^2$………………………….

III. CONCLUSION

Now by this method you can find nth order (especially Cubic) root of any number easily. I think that this method is added in primary level education syllabus to study.

FUTURE SCOPE OF THE WORK:-I will try to make it more easy & understandable.

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