FIND OUT nth ORDER ROOT OF A NUMBER BY DIVISION METHOD (USING THIS THEOREM CALCULATION 3rd, 4th, 5th ROOT OF ANY REAL NUMBER & DEDUCE (1+f(x))^{1/n} expansion where f(x) is an algebraic function)

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Abstract- Find out nth order root of any real number by division method

Using nth order root method calculation of cubic, 4th, 5th order root using value of nCr.

& also deduce expansion of (1+f(x))^{1/n} where f(x) is an algebraic function. There are three main step in this method named PRE STEP, RAM STEP, SHYAM STEP. For finding calculation of nth order root PRE STEP used only time and remain two step are calculated as per requirement in cycle e.g.

PRE STEP – RAM STEP – SHYAM STEP – RAM STEP – SHYAM STEP……………..as per requirement

I. INTRODUCTION

Here I am going to present a much simpler and understandable technique for finding nth order root (cubic root, 4th, 5th order root) of a number which can easily be used to make the students understand the concept at primary level also.

The concept does not use any advanced mathematics concept like calculus, etc. (for cubic, 4th, 5th order root) which is most popular used method for this purpose.

Finding cubic root of a number has always been a fascination for me as I was told by teacher that there is no perfect way to find cubic root. There are methods like prime factor method (only acceptable for whole cube number) calculus(needs higher & advanced mathematics) etc.

The proposed “K. K. METHOD” for finding nth order root (especially for cube root 4th order root, 5th order root) not only overcomes these demerits but also is very easy to understand & solve. And because of the limitations of the number available for this purpose, I have always been trying to develop one.

II. METHODOLOGY

Here I am describing method for calculating nth order root of any number

GENERAL STEP OF CALCULATION OF nth ORDER ROOT: -STEP.1) Make pair of n digit if you want to calculate nth order root …CASE 1)…If number is left to decimal then make right to left

Case.2)…If number is right to decimal then left to right …..

As we make pair of 2 digit in calculation of square root e.g.

For square root 2453831455.0015275754
For cubic root 42527893412.012345600
For 4th 12435786352123.12345680
For 5th 124253678963123.4563758600 and so on

STEP .2) There are two main step named here 1st one is “RAM “& 2nd one is “SHYAM” eg. p.5)

| PRE STEP | 1 | 1536800264 | 1= 1^2 |
| 2 | 1 | 2=2×1 |
| 3 | 0536 |
| 1. RAM | 1 | 331 | 331=331×1 |
| 331 | 189875 | 189875=37975×5 |
| 2. SHYAM | 2 | 15925264 | 2=2×1 |
STEP 3) you have to fill up n-1 place when you are in “STEP RAM” which is very understandable

By an Intermediate educated student. As I describe we have to fill up n-1 place row wise
That are right of “PRE STEP (actually it is combination of RAM & SHYAM STEP) needs some Information factorial if we are interest more than 5 th order root …

Here I am describing the rule-
1. 1st row is (next quotient) \( n-1 \); from right to n-1 place right to left (same for next).
2. 2nd row is \( \binom{n}{n-1} \times \) quotient (initial to final if more than two step are already done) \( \times \) (2nd quotient) \( n-2 \); from right to n-2 place
3. 3rd row \( \binom{n}{n-2} \times \) quotient \( \times \) (initial to final if more than two step are already done) \( \times \) 2nd quotient

Quotient \( n-3 \) … hence so on up to n-1 row wise place ………last one is \( \binom{n}{2} \times \) quotient \( n-2 \) (initial to final) \( \times \) 2nd quotient(next)

After it add all of them with column wise .see above example.

STEP 4) Now in “SHYAM STEP” 1.it start with (n-1) \( \times \) value of RAM STEP & write down it from unity

Of RAM STEP.
2. (n-2) \( \times \) value of 2nd row from 10th place
3. (n-3) \( \times \) value of rd row from 100th place. Hence so on up to n-1 term

After it add them and guess what the next quotient is & repeat “RAM STEP “again
And so on repeat these two step …

Here many example is given of cubic root, 4th root 5th root & also method for cubic root, 4th order, and 5th order root.

Value of \( \binom{n}{r} \) for square root
-1st row = 1
For cubic root 1st row = 1; 2nd row = 3
For 4th root 1st row = 1; 2nd row = 4; 3rd = 6
For 5th root 1st row = 1; 2nd row = 5; 3rd = 10; 4th row = 10

METHODOLOGY:- Method for calculating cubic root.

STEP 1) Make pair of 3 digit ---CASE 1. Right to left if number is left of decimal
Case 2.Left to right if it is right to decimal
12432578635412.012453680000002

STEP 2) Whatever is quotient we have to put square of quotient at the place of divisor. eg if quotient is 8 then put \( 8^2=64 \)

STEP 3) after it, you have to put below the dividend \( 8 \times 64 = 512 \& \) subtract it and put net three digit.

\[
\begin{array}{c|c} 
8 & 512 \\
512 & 64 \times 8 \\
0 & 3 \\
4 & 728 \\
6 & 000 \\
364 & 3 \times 2 \times 1 = 6
\end{array}
\]

STEP 4) Now you have to add doble of divisor in the divisor eg. 1+2=3

STEP 5) Since it is a method of cubic root you have to fill up two place after divisor+2×divisor .see eg .2. 1st place is square of of next quotient eg \( 4=2^2 \& \) 2nd one is \( 3 \times 1^4 \) quotient (initial to last one is called 1st one) \( \times \) 2nd one (that is presently quotient by which we want to divide)
1st one is started 2 place right to divisor + divisor × 2
2nd one is started 1 place right in next line, after it add all of them column wise.

STEP.6) Now after addition multiplied it with 2nd quotient and put below in dividend. (see eg. 2, 3
And subtract it and put next three digit.

STEP.7) Now again put 2 × square of 2nd quotient from place of unity and next line below the present divisor 3 × 1st quotient (initial to last one is called 1st one) × 2nd after one place of unity i.e 10th place.

STEP.8) Now again start division with quotient. put square of quotient 2 place right of sum after step seven; 3 × 1st quotient (initial to final – now there is two digit see eg p.5) × 2nd quotient one place right to sum of divisor after step 7 and sum up all of them column wise.
After it multiplied by quotient and put to dividend & subtract it, put next three digit to division.
At the place of divisor if all digit in dividend is over then calculation of cubic root is finished
If not then repeat step 7 & 8 …… see more example …

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<tr>
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<th>p.2) 12</th>
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<tr>
<td>64</td>
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</tr>
<tr>
<td>512</td>
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<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>512</td>
<td>1 × 2 = 2</td>
</tr>
<tr>
<td>0</td>
<td>0728</td>
</tr>
<tr>
<td>0</td>
<td>728</td>
</tr>
<tr>
<td></td>
<td>2 × 2 = 4, 364 × 2</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>000</td>
</tr>
<tr>
<td></td>
<td>3 × 2 × 1 = 6</td>
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p.5)  
\[
\begin{array}{c|c}
1 & 1154 \\
2 & \underline{1536800264} \\
3 & 0536 \\
1 & \underline{331} \\
3 & \underline{205800} \\
331 & \underline{189875} \\
2 & \underline{15925264} \\
3 & \underline{15925264} \\
363 & 0 \\
25 & 25=5^2 \\
165 & 165=3\times11\times5 \\
37975 & 50=25\times2 \\
165 & 165=165\times1 \\
39675 & 16=4^2 \\
1380 & 1380=3\times115\times4 \\
3981316 & \\
\end{array}
\]

p.6)  
\[
\begin{array}{c|c}
1 & 101 \\
2 & \underline{1030301} \\
3 & 0030301 \\
0 & \underline{30301} \\
0 & 00000 \\
300 & 0 \\
0 & 0 \\
300 & 0 \\
1 & 1^2=1 \\
30 & 3\times10\times1=30 \\
30301 & \\
\end{array}
\]

p.7)  
\[
\begin{array}{c|c}
1 & 1001 \\
2 & \underline{1003003001} \\
30000 & 0003003001 \\
0 & \underline{3003001} \\
0 & 0000000 \\
300 & 0 \\
3003001 & \\
\end{array}
\]

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p.8

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p.9

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p.10

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<td>4</td>
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<td>8=2×4</td>
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<td>6</td>
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<td>6=6×1,2482125=5×4934425</td>
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<td>432</td>
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<tr>
<td>64 = 8×2, 4832=8×604</td>
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<tr>
<td>288 = 3×8×12</td>
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<td>46144</td>
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<td>128=64×2</td>
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<td>49152</td>
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<tr>
<td>25 = 5×2</td>
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<tr>
<td>1920 = 3×128×5</td>
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<td>4934425</td>
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<td>50 = 25×2</td>
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p.11

| 0.117111 |
| 16 | 69.78769000000 |
| 32 | 64 |
| 48 | 5787 |
| 1 | 4921 |
| 12 = 3×4×1 |
| continued |
Example of 4^{th} order root :-

p.11) 

\[ \begin{array}{c|c}
16 & 3 \\
32 & 64 \\
48 & 5787 \\
1 & 4921 \\
12 & 866690 \\
4921 & 505531 \\
2 & 361159000 \\
12 & 355338613 \\
5043 & 5820387000 \\
1 & 5085030211 \\
123 & 735356789000 \\
505531 & 508516606431 \\
2 & 226840182569 \\
123 & 123=123\times1 \\
506963 & 49 \\
8631 & 69742 \\
50762659 & 98 \\
8631 & 8631=3\times7\times411 \\
50849067 & 98=2\times49 \\
1 & 8631=8631\times1 \\
12351 & 1=1^\times2 \\
5085030211 & 12351=3\times4117\times1 \\
2 & 2=1\times2 \\
12351 & 12351=1\times12351 \\
5085153713 & 1=1^\times2 \\
1 & 123513=3\times41171\times1 \\
508516606431 & 3=3\times1; 1=1^\times3 \\
\end{array} \]
p.2)

\[
\begin{array}{c|c}
1 & 12 \\
3 & 20736 \\
4 & 10736 \\
8 & 10736 \\
16 & 00000 \\
12 & 5368 \\
\end{array}
\]

\[
\begin{array}{c|c}
3 & 1 \\
8 & 10736 \\
16 & 00000 \\
12 & 6 \times 1^2 \times 2^2 \\
\end{array}
\]

p.3)

\[
\begin{array}{c|c}
729 & 4 \\
2187 & 6561 \\
2916 & 266266816 \\
512 & 266266816 \\
2304 & 00000000 \\
3888 & 3328352 \\
\end{array}
\]

\[
\begin{array}{c|c}
729 & 9^3 \\
2187 & 3 \times 729; 6561 = 729 \times 9 \\
2916 & 512 \times 8^3; 266266816 = 3328352 \times 8 \\
512 & 2304 = 4 \times 9 \times 8^2 \\
2304 & 3888 = 6 \times 9 \times 2^8 \\
3888 & 3328352 \\
\end{array}
\]

p.4)

\[
\begin{array}{c|c}
1 & \frac{1}{2} \\
3 & 1 \\
4000 & 4000 \\
1 & 4000 \\
40 & 00000 \\
600 & 4000 \\
\end{array}
\]

\[
\begin{array}{c|c}
1 & 101 \\
3 & 1 \\
4 & 05180 \\
1 & 6461 \\
4 & 5397041 \\
6 & 5397041 \\
\end{array}
\]

p.5)

\[
\begin{array}{c|c}
1 & 111 \\
3 & 1 \\
4 & 05180 \\
1 & 6461 \\
4 & 5397041 \\
6 & 5397041 \\
\end{array}
\]

\[
\begin{array}{c|c}
3 & 8 \\
6 & 5324 \\
44 & 729 \\
729 & 5397041 \\
\end{array}
\]

p.6)

\[
\begin{array}{c|c}
1 & \frac{1}{2} \\
3 & 1 \\
4 & 10053 \\
1 & 4641 \\
4 & 54123921 \\
6 & 54123921 \\
\end{array}
\]

\[
\begin{array}{c|c}
4641 & 0 \\
3 & 8 \\
6 & 5324 \\
729 & 10503 \\
3564 & 1029 \\
6534 & 392 \\
6013769 & 42 \\
\end{array}
\]

p.7)

\[
\begin{array}{c|c}
1 & \frac{1}{2} \\
3 & 1 \\
4 & 92662 \\
343 & 73521 \\
196 & 191415681 \\
42 & 191415681 \\
\end{array}
\]

\[
\begin{array}{c|c}
10503 & 0 \\
1029 & 19652 \\
392 & 729 \\
15606 & 5508 \\
21268409 & 21268409 \\
\end{array}
\]

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### Example of 5th order root:

1. 
   - 1  |  161051
   - 4  |  1
   - 5  |  61051
   - 1  |  61051
   - 5  |  0
   - 10
   - 10
   - 61051

2. 
   - 1  |  121
   - 4  |  25937424601
   - 5  |  159374
   - 16  |  148832
   - 40  |  105424601
   - 20  |  0
   - 74416
   - 64
   - 120
   - 80
   - 20
   - 103680
   - 1
   - 60
   - 1440
   - 17280
   - 1054224601
DEDUCTION OF \((1+F(X))^{1/n}\) USING THIS THEOREM:

- Expansion of \((1+x)^{1/2}\)

\[
1 + x/2 + x^2/8 + x^3/16 - 5x^4/128 \ldots
\]

- Expansion of \((1+x)^{1/3}\)

\[
1 + x/3 - x^2/9 + 5x^3/81 - 10x^4/243 \ldots
\]

- Expansion of \((1+x+x^2)^{1/2}\)

\[
1 + x/2 + 3x^2/8 - 3x^3/16 + 3/128x^4 \ldots
\]

- Expansion of \((1+x)^{1/n}\)

\[
1 + x/n - c_n(x/n)^2/n + (2n^2c_2^2(x/n)^3 + 1/n^2c_3(x/n)^3)/n \ldots
\]
n+2 \( n \), \( c_2 \), \( x/n \), \( c_2(x/n)^2 \)

### III. CONCLUSION

Now by this method you can find nth order (especially Cubic) root of any number easily. I think that this method is added in primary level education syllabus to study.

**FUTURE SCOPE OF THE WORK:** I will try to make it more easy & understandable.

### REFERENCES


### AUTHORS

**First Author** – Krishna Kumar Kharwar, 3rd year CIVIL ENGINEERING 14/CE/15, National Institute Of Technology, Durgapur