ISSN 2250-3153

Abstract reasoning and Spatial Visualization in Formal Operational Stage

Sumona Datta[*], Debdulal Dutta Roy[**]

[*]Research Fellow, Psychology Research Unit, Indian Statistical Institute, Kolkata.
[**]Assistant Professor, Psychology Research Unit, Indian Statistical Institute, Kolkata.

Abstract- Formal operational stage differs from the previous stages due to the newly acquired ability to think in abstract terms and form hypotheses based on propositions (Inhelder & Piaget, 1958). As the adolescent progresses from early to late stage of development, the knowledge structure becomes more and more integrated in structure. Present study hypothesized that the ability to reason in abstract terms is associated with spatial visualization ability and this association becomes more integrated among late formal reasoners as compared to early formal reasoners. Reasoning of abstraction involves classifying objects or events into problem categories based on their properties. This is followed by making inferences in order to solve the problem which in turn requires visualizing the spatial relations among objects/events. Abstract reasoning thus, requires *conceptual knowledge* and *spatial visualization.* Spatial Visualization is the ability to visualize complex spatial relations among the constituent parts of an object and manipulate them in order to predict a possible outcome. To test this hypothesis, data were collected from 71 high school students (Boys=23; Girls=48; Mean age=14.52 years) by administering paper-pencil tests of abstract reasoning and spatial visualization. Results revealed significant association between abstract reasoning and visualization factor, with the association being stronger among late adolescents as compared to early adolescents. Results were discussed in the light of Piagetian theory of formal operational reasoning among adolescents.

Index Terms- Abstract reasoning, Spatial Visualization, Formal operations, Piaget's theory, Adolescents.

I. INTRODUCTION

One most distinguishing feature of formal operational stage is the organization of mental operations which are then performed on 'propositions' themselves and not on classes and relations that characterize the information (Inhelder & Piaget, 1958). Formal operational adolescents are able to construct hypotheses and systematically put it to test. While forming such hypotheses, unlike concrete operational child, formal operational adolescents do not have to limit their consideration to a single aspect at a time. Rather he/she is able to consider a number of variables at a time which might determine the event. The acquired information is the fed into a "combinatorial or structured whole" which further assimilates the information into "propositions" (Inhelder & Piaget, 1958). Such propositions are then arranged in all possible combinations, each offering a potential possibility. The adolescent then takes into account all the possibilities in order to decide which among them explains the real situation most appropriately. Thus the formal operational thinker has the ability to consider only the logical relations among the events while ignoring the concrete contents.

This is made possible by the development of the ability to think and reason in abstract terms, independent of concrete existence. As stated by Breuning (2003) in abstract reasoning "the problem solver examines the problem to determine whether it has certain structural properties and hence belongs to a certain class of problems" (p.232). Thus the reasoning of abstraction involves classifying objects or events into problem categories based on their properties. This is followed by making inferences in order to solve the problem. Such inference making depends on two things: first, *concepts* based on which problems are categorized and second, understanding the spatial relation among abstract representations so that they can be easily categorized and inferences can be drawn.

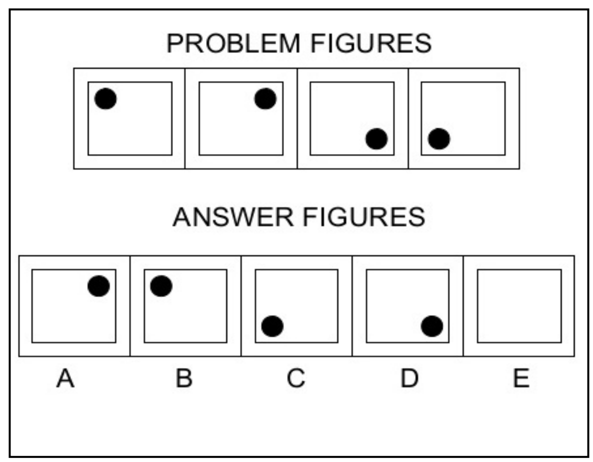

Figure 1. Problem of Abstract Reasoning

This categorization is done based on some rules which are peculiar to a particular category. For example: What comes after 3,5,7,9? The answer is 11. Here one has to have the knowledge of numbers which are again abstract representations. In addition to solve the problem, one needs to know the concept of 'odd numbers'. Now while solving the problem, one need to visualize the number line and has to comprehend the spatial relation among the numbers so as to arrive at the immediate next odd number. Categorization is the process by which concepts (abstract mental representation), based on some rules, determine if a new entity is a representative of a category or not (Rips, Smith & Medin, 2012). Once an individual develops the concept of an event and is able to categorize a new member based on the rules of the category, he/she can predict if a new entity/event is a member of a particular category. This facilitates not only categorizing but also explains which new course a particular event is going to take. For example in Fig. 1, the question concerns after the first four problem figures. The answer is B. Simply the dot is rotating clockwise and the dot will continue to rotate in a circle and will not rotate in the opposite direction. Thus to solve the problem one needs the knowledge of clockwise rotation. Again, it also requires understanding the spatial relation among the dot and the square box. In the problem figures the dot is always placed at the corners and not in other positions. Thus the next figure would be one where the dot will be placed similarly at the corner. This requires the ability to visualize the box and the dot in the correct order. Thus abstraction requires spatial visualization ability.

Spatial visualization is the ability to deal with complex, multistep manipulations of spatially presented information (Linn & Peterson, 1985). Others however argue that it is the "ability in manipulating visual patterns, as indicated by level of difficulty and complexity in visual stimulus material that can be handled successful, without regard to the speed of task solution" (Lohman, 1979). Spatial visualization is thereby considered as the most extensively studied factor from a cognitive psychological perspective (Carpenter & Just, 1986). As described by Carroll (1993), this factor requires processes for apprehension, encoding and mental manipulation of spatial forms. This factor loads highest on tests like Spatial Relations Test (Bennet, Seashore and Wesman, 1972) items of which require subjects to hold on a visual array of information, comprehend the relations among the information units, apply mental operations to perform transformations of the spatial relations in order to predict an inference and evaluation of predicted outcome.

Development of Conception of Space

As studied by Piaget & Inhelder (1956), the conception of space develops through different stages and comes to a full bloom at the end of formal operational stage. Like all other thought processes, the conception of space is also characterized by different mental operations which enable the adolescents to make sense of spatial relations. For example, the concept that synthesis of the whole is the reverse product of unlimited subdivisions develops through various stages of development. While in the pre-operational stage (upto 6-7 years), the child fails to consider unlimited subdivisions of a whole due to lack of imagination, the concrete operational child (beginning at 7-8 years) can consider a large number of subdivisions but still do not consider it to be infinite. Moreover, they can not generalize the subdivisions beyond vision or tangible size. It is only during the formal operational stage (11-12 years onwards) that the adolescent's thinking processes are finally free of "quasi-perceptual notions" of previous stages. The adolescent can now easily consider the fact that synthesis of the whole is the reverse product of unlimited subdivisions and this conception becomes more and more consolidated as the adolescent progresses towards the end of formal operational stage (16-17 years). As documented by Sorby & Baartmans (2000) development of spatial visualization takes place through three different stages (Bishop, 1978). The first stage involves learning *topological visualization* which enables children to comprehend topological relationships among object. This is followed by the second stage where they acquire *projective representation ability*. This stage facilitates them to visualize an object from a different perspective. Finally in the third stage, they learn to combine their projective representation ability with their concepts of measurement. The projective representation ability appears during adolescence and continues to mature till adulthood is reached. This point to the fact that formal operations appearing during early adolescence also plays an important role in the development of the second and third stage of spatial visualization development.

Development of Abstract reasoning

Formal reasoning is characterized by logical reasoning advancing from concrete to abstract forms (Lister, 2011). How and why such ability to reason is acquired has been so far studied from two different approaches. From the Piagetian point of view, such acquisition is an outcome of brain maturation. Needless to mention, formal operational reasoning is the most abstract form of reasoning in Piaget's view (1952, 1954). Also, since this ability to think in abstract terms is a result of brain maturation, a child exhibiting a certain level of abstract reasoning in a particular domain will also exhibit equally potential abstract reasoning ability in many other domains (Marini & Case, 1994). The neo-piagetian approach however differs from the piagetian viewpoint in that irrespective of age, reasoning in terms of abstraction progresses as one gains expertise in problem-specific domain (Lister, 2011). Thus a person may be novice in a particular type of abstraction-based reasoning task, while might be an expert in solving another type of abstraction based task. According to neo-piagetians (Case, 1985), adolescents can reason out at abstract levels as because the formal stage of development allows them to integrate the results of two different types of lower-order reasoning processes which are typically concrete operational by nature. As observed by Susac, Bubic, Vrlanc & Plaininic (2014), while solving numerical abstract reasoning problems, early adolescents were less accurate and slower in solving equations with symbols than those with numbers while late adolescents could solve both types of problems equally. The researchers concluded that late adolescents could solve the problems as they had already reached a stage of abstract reasoning. Thus a transition occurs at the age of 15-16 years when adolescents learn to use abstract strategies instead of concrete strategies while solving problems.

Present Study

Thus the only thing that differentiates formal operational abstract reasoning is the ability to think in abstract terms. This essentially requires the ability to visualize relations among objects or events, form hypothetical mental models, manipulate the relations by applying mental operations and deduce inferences. Also as the formal operational stage progresses abstract reasoning becomes more and more systematic. Needless to say the ability to visualize spatial relations also develops as this facilitates the ability to think in abstract terms.

Although there are some evidences suggesting the possible connection between spatial visualization and abstract reasoning, there hardly exists any empirical study testifying this hypothesis. The present endeavor is based on the assumption that there exists a substantial association between spatial visualization and abstract reasoning as because theory suggests that spatial visualization underlies abstract reasoning ability. Also there are reasons to believe that this association intensifies as the formal operational child enters into late adolescence from early phase. Thus objective of the present study was to examine the association between abstract reasoning ability and spatial visualization ability in formal operational adolescents and how this association varies among early and late formal thinking.

Objectives of the study

There are two objectives of the present study:

1. To examine the association between abstract reasoning and spatial visualization ability of formal operational adolescents.
2. To examine whether this association differs among early formal operational reasoners and late formal operational reasoners.

II. Method

Participants

Participants were 71 high school students (Boys=23; Girls=48; Mean age=14.52 years). Among them, 52% were of age range 13-14 years while 48% were of age range 15-17 years.

Instruments

Abstract Reasoning: The Abstract Reasoning Test of Differential Aptitude Test (DAT) battery was administered in group testing situation. The test has 50 multiple-choice items, each having five options, out of which only one option is correct. A score of 1 is given on choosing the correct option.

Spatial Visualization: The Space Relation Test of Differential Aptitude Test (DAT) battery was used to measure this ability. This test has 40 multiple-choice items, each having five options out of which there can be more than one correct answer. A score of 1 can be obtained on choosing any one of the correct option.

In both the tests, total score is computed by summating the number of correct responses. The standard formula for calculating the total score as given in the DAT manual has not been followed. This is because here the emphasis is on assessing the ability and not on determining the position of the participant in a given population as can be obtained by comparing his/her score with the given norm.

Procedure

For data collection, permission was sought from local high schools. After obtaining permission, rapport was established with the students. Data were collected in groups. Total time taken for each session was almost 60 minutes. After data collection, participants were provided with small incentives in the form of pens and chocolates. Data were then analyzed and results were discussed.

III. Results

Age wise difference in abstract reasoning and spatial visualization ability

Before looking for association among abstract reasoning and spatial visualization as also the age wise variance in this association, it is important to check if the early adolescents and late adolescents actually differ among themselves with respect to their abstract reasoning ability and spatial visualization ability.

Results suggest that late adolescents (Mean=24.65; *SD*=13.53) performed better than the early adolescents (Mean=14.46; *SD*=7.76) in abstract reasoning ability; this mean difference being statistically significant F (1, 69) = 15.45, p<0.000 with effect size being large η^2=0.183. Again, late adolescents (Mean=22.79; *SD*=6.57) outperformed early adolescents (Mean=18.57; *SD*=5.84) in their ability of spatial visualization. This mean difference is also found to be statistically significant F (1, 69) = 8.23, p<0.01 with effect size being moderate, η^2=0.106. Thus age explains almost 18% variance in the abstract reasoning ability and 11% variance in the spatial visualization ability of the adolescents (Table 1).

Table 1.
Descriptive statistics & Analysis of Variance showing age wise difference in Abstract Reasoning ability and Spatial Visualization ability of the adolescents (*N*=71).

Ability	Age groups	Mean	SD	F value	df	p-value	Eta squared value (η^2)
Abstract Reasoning	Early adolescents (*n*=37)	14.46	7.76	15.45**	1, 69	0.000	0.183
	Late adolescents (*n*=34)	24.65	13.53				
Spatial Visualization	Early adolescents (*n*=37)	18.57	5.84	8.23**	1, 69	0.005	0.106
	Late adolescents (*n*=34)	22.79	6.57				

**p value<0.01

Association among abstract reasoning and spatial visualization

With age significantly affecting the abstract reasoning and spatial visualization abilities of the adolescents, the two age groups can now be separately considered for further analysis of association between the two abilities.

Overall, a moderate association between spatial visualization ability and abstract reasoning ability is obtained *r* (69) = 0.66, p<0.000. Further this association was checked by taking two age groups into consideration (Table 2). By selecting cases separately for early adolescent group (age 13-14 years) and late adolescent group (age 15-17 years) Pearson correlation was done. Results revealed higher association among late adolescents *r* (32) = 0.76, p<0.000 as compared to that among early adolescents *r* (35) = 0.39, p<0.02. Further the two coefficients are found to be significantly differing among each other, as evident from the z value, *z* = 2.35, p value = 0.02 (two tailed).

Table 2.
Correlation coefficients and z-value showing association among Abstract Reasoning and Spatial Visualization ability (*N*=71).

	Association among Spatial Visualization & Abstract Reasoning			
	r	**df**	**p value**	**z value**
Overall association	0.66**	69	0.000	
• **Early adolescents**	0.39*	35	0.02	2.35*
• **Late adolescents**	0.76**	32	0.000	

*p value<0.05 **p value<0.01

IV. Discussion

As mentioned in the introduction the first question of interest was whether spatial visualization in object-space is associated with abstract reasoning ability among formal operational adolescents. As revealed in results there exists a moderately significant association between spatial visualization and abstract reasoning. Thus spatial visualization is a correlate of abstract reasoning. As a matter of fact often highly developed spatial skills are attributed for better reasoning ability. As proposed by Byrne & Johnson-Laird (1989) “human reasoning relies on the construction and manipulation of mental models”. This follows a three step mechanism: constructing a model based on the relations described in the premise, devising a probable conclusion compatible with the model and finally trying to falsify the conclusion by constructing further alternative mental models based on the premises (Johnson-Laird, 1983). Mohring, Newcombe & Frick (2015) investigated the association between spatial skills and propositional reasoning ability among 4- and 5-year olds. Their results indicated that errors made in spatial reasoning task were closely associated with the children’s propositional reasoning ability after controlling for age and verbal intelligence. Geary, Saults, Liu & Hoard (2000) found that male advantage in arithmetic reasoning is mediated by male advantage in spatial skills. Thus there is no denying the fact that spatial reasoning underlies ability to think in abstract terms. Present results add to this vast literature by studying the association among abstract reasoning and spatial visualization ability.

The study further purported to examine how the association between abstract reasoning and spatial visualization ability vary across early and late adolescent groups. Results showed that the association is stronger in case of late adolescents as compared to the early adolescents. In solving abstract reasoning problems, formal operational adolescents form propositions based on the conceptual knowledge and the spatial knowledge they acquire and thereby predict the outcome. With maturity these knowledge structures become more integrated and therefore prediction is more accurate. The high association between abstract reasoning and visuospatial reasoning among late adolescents indicates the existence of a more integrated knowledge structure as compared to the early adolescents. This finding is in line with the Piagetian theory of development of spatial skills wherein late adolescents learn to combine their previously developed projective representations with their newly acquired concepts of measurement in order to solve reasoning based problems (Sorby & Baartman, 2000). As a matter of fact during late formal operational stage, adolescents master to combine previously matured lower-order reasoning processes in order to form complex knowledge structures which enable them to solve reasoning problems more efficiently (Case, 1985; Lister, 2011).

According to Inhelder & Piaget (1958), this integrated structure is an outcome of the various mental operational possibilities implied in the successive stages. This ultimately leads to a state of equilibrium in formal thinking, which exists as a set of possibilities of operations and operational schemata. As the child progresses from one stage to another, this equilibrium widens in its range, with more and more integrated form of understanding. This is not only because the child's abilities grow, but also because as the abilities grow, they also become more and more associated among each other, in order to form a more integrated complex structure. This is evident from the present study findings where the early reasoners and late reasoners not only differed in each of their abilities of abstract reasoning and spatial visualization, but also when the association among the two abilities was considered.

V. Implications of the Study

Previous research have consistently investigated individual differences in spatial visualization factor as also abstract reasoning in relation to their applications in other areas (Breuning, 2003; Miyake et al., 2001; Sorby, 1999; Marini & Case, 1994). However, hardly any attempt has been made before to investigate the role played by spatial visualization in abstract reasoning or as to how this association varies across formal operational stage of development. Present results not only point to the fact that spatial visualization is associated with abstract reasoning but also indicate how this association develops during the formal operational stage. The finding that the association between visualization factor and abstract reasoning is more integrated during late adolescence as compared to early adolescence directly supports piagetian and neo-piagetian theories claiming better developed formal operations appearing later in adolescence. This finding can be further investigated to explore how this integration materializes during late adolescence.

VI. Limitations & Future Research

Before generalizing the results readers must keep in mind the limitations of the present study. First, for assessing abstract reasoning and spatial visualization paper-pencil tests were used. Results would have been much more concrete had reasoning tasks been used instead of paper-pencil tests. That would have allowed studying even the differences in strategies used to solve the problems and resulted in better integration of present results. Inhelder & Piaget had studied spatial skills and abstract reasoning by providing cognitive tasks to solve (1958). Second, number of participants was very small in the present study, representing the early and late adolescents, thus complicating the process of generalization. Further studies should investigate the same problem in a larger population so as to explore the association in more details.

In conclusion it can be said that it would be too much to claim that the present study has a lot to contribute in the field of research in reasoning processes, but this study is definitely a small step in providing empirical evidences for the association between spatial visualization and abstract reasoning as well as how this association develops to form an integrated structure during late adolescence.

References

[1] Bennett, G. K., Seashore, H. G., & Wesman, A. G., Form T, Differential Aptitude Tests, Space Relations, 1972. New York: Psychological Corporation.

[2] Bishop, J. E., Developing Students' Spatial Ability. Science Teacher, 45(8), 1978, 20-23.

[3] Breuning, M., The Role of Analogies and Abstract Reasoning in Decision-Making: Evidence from the Debate over Truman's Proposal for Development Assistance. International Studies Quarterly, 47(2), 2003, 229-245.

[4] Byrne, R. M., & Johnson-Laird, P. N., Spatial reasoning. Journal of memory and language, 28(5), 1989, 564-575.'

[5] Carpenter, P. A., & Just, M. A., Spatial ability: An information processing approach to psychometrics. In R. J. Sternberg (Ed.), Advances in the psychology of human intelligence, Vol. 3, 1986, Hillsdale, NJ: Erlbaum.

[6] Carroll, J. B., Human cognitive abilities: A survey of factor-analytic studies. Cambridge University Press, 1993.

[7] Case, R., Intellectual development. Birth to adulthood, 1985.

[8] Geary, D. C., Saults, S. J., Liu, F., & Hoard, M. K., Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. Journal of Experimental child psychology, 77(4), 2000, 337-353.

[9] Johnson-Laird, P. N., Mental models: Towards a cognitive science of language, inference, and consciousness (No. 6). Harvard University Press, 1983.

[10] Linn, M. C., & Petersen, A. C., Emergence and characterization of sex differences in spatial ability: A meta-analysis. Child development, 1985, 1479-1498.

[11] Lister, R., Concrete and other neo-Piagetian forms of reasoning in the novice programmer. In Proceedings of the Thirteenth Australasian Computing Education Conference-Volume 114 (pp. 9-18), 2011, January. Australian Computer Society, Inc..

[12] Lohman, D., Spatial Ability: A review and reanalysis of the correlational literature (Tech. Rep. No. 8). Stanford, CA: Stanford University, School of Education, 1979.

[13] Marini, Z., & Case, R., The development of abstract reasoning about the physical and social world. Child Development, 1994, 65(1), 147-159.

[14] Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M., How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. Journal of Experimental Psychology: General, 2001, 130, 621−640.

[15] Möhring, W., Newcombe, N. S., & Frick, A. (2015). The relation between spatial thinking and proportional reasoning in preschoolers. Journal of experimental child psychology, 2015, 132, 213-220.

[16] Piaget, J., The origins of intelligence in children (Vol. 8, No. 5, p. 18). New York: International Universities Press, 1954.

[17] Piaget, J., The construction of reality in the child (M. Cook, trans.). New York: Basic Books, 1954. (Originally published, 1937.)

[18] Piaget, J. & Inhelder, B., "The Child's Conception of Space." Trans. FJ Langdon & JL Lunzer. London: Routledge & Kegan Paul (1956).

[19] Rips, L. J., Smith, E. E., & Medin, D. L., Concepts and categories: memory, meaning, and metaphysics. The Oxford Handbook of Thinking and Reasoning, 2012, 177-209.

[20] Sorby, S. A., Developing 3-D spatial visualization skills. Engineering Design Graphics Journal, 1999, 63(2), 21-32.

[21] Sorby, S. A., & Baartmans, B. J., The Development and Assessment of a Course for Enhancing the 3-D Spatial Visualization Skills of First Year Engineering Students. Journal of Engineering Education, 2000, 89(3), 301-307.

[22] Susac, A, Bubic, A, Vrbanc, A, Planinic, M., Development of abstract mathematical reasoning: the case of algebra. Frontiers in Human Neuroscience. 2014, 8: 679.

AUTHORS

First Author: Sumona Datta, Senior Research Fellow, Psychology Research Unit, Indian Statistical Institute, Kolkata. (sumonadatta.psych@gmail.com)
Second Author: Dr. Debdulal Dutta Roy, Assistant professor, Psychology Research Unit, Indian Statistical Institute, Kolkata. (dduttaroy@gmail.com)

Corresponding Author: Sumona Datta, Senior Research Fellow, Psychology Research Unit, Indian Statistical Institute, Kolkata. (sumonadatta.psych@gmail.com) (09432175390)