Bayesian Analysis of Rayleigh Distribution

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Abstract: The Rayleigh distribution is often used in physics related fields to model processes such as sound and light radiation, wave heights, and wind speed, as well as in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year at wind turbine sites and daily average wind speed. In the present paper, we consider the estimation of the parameter of Rayleigh distribution. Bayes estimator is obtained by using Jeffrey's and extension of Jeffrey's prior under squared error loss function and Al-Bayyati's loss function. Maximum likelihood estimation is also discussed. These methods are compared by using mean square error through simulation study with varying sample sizes.

Index Terms: Rayleigh distribution, Jeffrey's prior and extension of Jeffrey's prior, loss functions.

I. Introduction

The Rayleigh distribution (RD) is considered to be a very useful life distribution. Rayleigh distribution is an important distribution in statistics and operations research. It is applied in several areas such as health, agriculture, biology, and other sciences. One major application of this model is used in analyzing wind speed data. This distribution is a special case of the two parameter Weibull distribution with the shape parameter equal to 2.This model was first introduced by Rayleigh (1980), Siddiqui (1962) discussed the origin and properties of the Rayleigh distribution. Inference for model Rayleigh model has been considered by Sinha and Howlader (1993), Mishra et al. (1996) and Abd Elfattah *et al*. (2006).

The probability density function of Rayleigh distribution is given as:

$$f(y;\theta)=\frac{y}{\theta^2}\exp\left(-\frac{y^2}{2\theta^2}\right) \qquad for\;\; y\geq 0, \theta>0 \tag{1.1}$$

Recently Bayesian estimation approach has received great attention by most researchers. Bayesian analysis is an important approach to statistics, which formally seeks use of prior information and Bayes Theorem provides the formal basis for using this information. In this approach, parameters are treated as random variables and data is treated fixed. Ghafoor et al. (2001)] and Rahul *et al. (2009)* have discussed the application of Bayesian methods. An important pre-requisite in Bayesian estimation is the appropriate choice of prior(s) for the parameters. However, Bayesian analysts have pointed out that there is no clear cut way from which one can conclude that one prior is better than the other. Very often, priors are chosen according to ones subjective knowledge and beliefs. However, if one has adequate information about the parameter(s) one should use informative prior(s); otherwise it is preferable to use non informative prior(s). In this paper we consider the extended Jeffrey's prior proposed by Al-Kutubi (2002) as:

$$g(\theta)\propto\left[I(\theta)\right]^{c_1} \qquad ,c_1 \varepsilon R^+$$

Where $I(\theta)=-nE\left[\frac{\partial^2 \log f(y;\theta)}{\partial\theta^2}\right]$ is the Fisher's information matrix.

For the model (1.1), the prior is given by

$$g(\theta)=k\left[\frac{n}{\theta^2}\right]^{c_1}$$

Where k is a constant. With the above prior, we use two different loss functions for the model (1.1), first is the squared error loss function which is symmetric, second is the Al-Bayyati's new loss function.

It is well known that choice of loss function is an integral part of Bayesian inference. As there is no specific analytical procedure that allows us to identify the appropriate loss function to be used, most of the works on point estimation and point prediction assume the underlying loss function to be squared error which is symmetric in nature. However, in-discriminate use of SELF is not appropriate particularly in these cases, where the losses are not symmetric. Thus in order to make the statistical inferences more practical and applicable, we often needs to choose an asymmetric loss function. A number of asymmetric loss functions have been shown to be functional, see Varian (1975), Zellner (1986), Chandra (2001) etc. In the present paper, we consider the above loss functions for better comparison of Bayesian analysis.

a) The first is the common squared error loss function given by:

$$l_1\left(\hat{\theta},\theta\right)=c\left(\hat{\theta}-\theta\right)^2$$

Which is symmetric, θ and $\hat{\theta}$ represent the true and estimated values of the parameter. This loss function is frequently used because of its analytical tractability in Bayesian analysis.

b) The second is the Al-Bayyati's new loss function of the form:

$$l_A\left(\hat{\theta},\theta\right)=\theta^{c_2}\left(\hat{\theta}-\theta\right)^2; \qquad c_2 \varepsilon R.$$

Which is an asymmetric loss function, for details, see Norstrom (1996). This loss function is interesting in the sense that a slight modification of squared error loss introduces asymmetry.

MATERIALS and METHODS

Maximum likelihood estimation: Maximum likelihood estimation of the parameters of Rayleigh distribution is well discussed in literature (see Cohen, (1965) and Mann et al. (1975)).

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n having the probability density function as

$$f(y;\theta)=\frac{y}{\theta^2}\exp\left(-\frac{y^2}{2\theta^2}\right) \qquad for \;\; y\geq 0, \theta>0$$

The likelihood function is given by

$$L(y\mid\theta)=\frac{\prod_{i=1}^{n} y_i}{\theta^{2n}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}}$$

$$\therefore \qquad Log\, L(y\mid\theta)=\sum_{i=1}^{n}\log y_i - 2n\log\theta - \frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}$$

The ML estimator of θis obtained by solving the

$$\frac{\partial}{\partial\theta}\log L\left(y\mid\theta\right)=0$$

$$\Rightarrow \qquad -\frac{2n}{\theta}+\frac{\sum_{i=1}^{n} y_i^2}{\theta^3}=0$$

$$\Rightarrow \qquad \hat{\theta} = \sqrt{\frac{\sum_{i=1}^{n} y_i^2}{2n}} \qquad (2.1)$$

Bayesian estimation of Rayleigh distribution under Jeffrey's prior by using Squared Error Loss Function

Consider there are n recorded values, $\underline{y} = (y_1, \ldots y_n)$ from (1.1). We consider the extended Jeffrey's prior as:

$$g(\theta) \propto \sqrt{[I(\theta)]}$$

Where $[I(\theta)] = -nE\left[\frac{\partial^2 \log f(y;\theta)}{\partial \theta^2}\right]$ is the Fisher's information matrix. For the model (1.1), the prior distribution is given by

$$g(\theta) \propto \frac{1}{\theta}$$

The likelihood function is given by

$$L(y;\theta) = \frac{\prod_{i=1}^{n} y_i}{\theta^{2n}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}}$$

The posterior density function is given by:

$$P(\theta \mid \underline{y}) \quad \propto \quad L(y;\theta) g(\theta)$$

$$\therefore \qquad P(\theta \mid \underline{y}) \quad \propto \quad \frac{\prod_{i=1}^{n} y_i}{\theta^{2n}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}} \frac{1}{\theta}$$

$$\Rightarrow \qquad P(\theta \mid \underline{y}) \quad = k \frac{1}{\theta^{2n+1}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}}$$

Where $k^{-1} = \int_0^\infty \frac{1}{\theta^{2n+1}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}} d\theta$

$$\Rightarrow \qquad k^{-1} = \frac{\Gamma n}{2^{1-n}\left(\sum_{i=1}^{n} y_i^2\right)^n}$$

$$\Rightarrow \qquad k = \frac{2^{1-n}\left(\sum_{i=1}^{n} y_i^2\right)^n}{\Gamma n}$$

From (3.2), the posterior density of θ is given by

$$p(\theta \mid \underline{y}) = \frac{2^{1-n}\left(\sum_{i=1}^{n} y_i^2\right)^n}{\theta^{2n+1}\Gamma n} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}}$$

$$\Rightarrow \qquad p(\theta \mid \underline{y}) = \frac{2\left(\frac{\sum_{i=1}^{n} y_i^2}{2}\right)^n}{\theta^{2n+1}\Gamma n} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}} \tag{2.2}$$

Estimation under squared error loss function:

By using a squared error loss function $L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$ for some constant c, the risk function is:

$$R(\hat{\theta}) = \int_0^\infty c(\hat{\theta}-\theta)^2 \, p(\theta \mid \underline{y}) \, d\theta$$

$$\Rightarrow \qquad R(\hat{\theta}) = c\hat{\theta}^2 + \frac{c}{n-1}\left(\frac{\sum_{i=1}^{n} y_i^2}{2}\right) - \frac{2c\hat{\theta}\,\Gamma\left(\frac{2n-1}{2}\right)}{\Gamma(n)}\sqrt{\frac{\sum_{i=1}^{k} y_i^2}{2}}$$

Now $\frac{\partial R(\hat{\theta})}{\partial \theta} = 0$, Then the Bayes estimator is

$$\hat{\theta}_1 = \frac{\Gamma\left(\frac{2n-1}{2}\right)}{\Gamma n}\sqrt{\frac{\sum_{i=1}^{k} y_i^2}{2}} \tag{2.3}$$

b) Bayesian estimation of Rayleigh distribution under Extension of Jeffrey's prior by using Squared Error Loss Function

We consider the extended Jeffrey's prior are given as:

$$g(\theta) \propto [I(\theta)]^{c_1} \qquad , c_1 \varepsilon R^+$$

Where $[I(\theta)] = -nE\left[\frac{\partial^2 \log f(y;\theta)}{\partial \theta^2}\right]$ is the Fisher's information matrix. For the model (1.1), $g(\theta) \propto \frac{1}{\theta^{2c_1}}$

The likelihood function is given by

$$L(y;\theta) = \frac{\prod_{i=1}^{n} y_i}{\theta^{2n}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}}$$

The posterior density function is given by:

$$P(\theta \mid \underline{y}) \quad \propto \quad L(y;\theta) g(\theta)$$

$$\therefore \qquad P(\theta \mid \underline{y}) \quad \propto \quad \frac{\prod_{i=1}^{n} y_i}{\theta^{2n}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}} \frac{1}{\theta^{2c_1}}$$

$$\Rightarrow \qquad P(\theta \mid \underline{y}) \quad = k \frac{1}{\theta^{2n+2c_1}} e^{-\frac{\sum_{i=1}^{n} y_i^2}{2\theta^2}} \tag{2.4}$$

Where $k^{-1} = \int_0^\infty \frac{1}{\theta^{2n+2c_1}} e^{-\frac{\sum_{i=1}^n y_i^2}{2\theta^2}} d\theta$

$$\Rightarrow \qquad k^{-1} = \frac{\Gamma\left(n + \frac{c_1}{2} - \frac{1}{2}\right)}{2^{1-n+\frac{c_1}{2}-\frac{1}{2}} \left(\sum_{i=1}^n y_i^2\right)^{n+\frac{c_1}{2}-\frac{1}{2}}}$$

$$\Rightarrow \qquad k = \frac{2^{1-n+\frac{c_1}{2}-\frac{1}{2}} \left(\sum_{i=1}^n y_i^2\right)^{n+\frac{c_1}{2}-\frac{1}{2}}}{\Gamma\left(n + \frac{c_1}{2} - \frac{1}{2}\right)}$$

From (3.6), the posterior density of θ is given by

$$p(\theta \mid \underline{y}) = \frac{2^{1-n+\frac{c_1}{2}-\frac{1}{2}} \left(\sum_{i=1}^n y_i^2\right)^{n+\frac{c_1}{2}-\frac{1}{2}}}{\Gamma\left(n + \frac{c_1}{2} - \frac{1}{2}\right)} \frac{1}{\theta^{2n+2c_1}} e^{-\frac{\sum_{i=1}^n y_i^2}{2\theta^2}}$$

$$\Rightarrow \qquad p(\theta \mid \underline{y}) = \frac{2\left(\frac{\sum_{i=1}^n y_i^2}{2}\right)^{n+\frac{c_1}{2}-\frac{1}{2}}}{\theta^{2n+2c_1}\Gamma\left(n + \frac{c_1}{2} - \frac{1}{2}\right)} e^{-\frac{\sum_{i=1}^n y_i^2}{2\theta^2}} \qquad (2.5)$$

Estimation under squared error loss function:

By using a squared error loss function $L(\hat{\theta}, \theta) = c(\hat{\theta} - \theta)^2$ for some constant c, the risk function is:

$$R(\hat{\theta}) = \int_0^\infty c(\hat{\theta} - \theta)^2 p(\theta \mid \underline{y})\, d\theta$$

$$\Rightarrow \qquad R(\hat{\theta}) = c\hat{\theta}^2 + \frac{c\Gamma\left(n + c_1 - \frac{3}{2}\right)}{\Gamma\left(n + c_1 - \frac{1}{2}\right)}\left(\frac{\sum_{i=1}^n y_i^2}{2}\right) - \frac{2c\hat{\theta}\,\Gamma(n + c_1 - 1)}{\Gamma\left(n + c_1 - \frac{1}{2}\right)}\sqrt{\frac{\sum_{i=1}^k y_i^2}{2}}$$

$$\Rightarrow \qquad R(\hat{\theta}) = c\hat{\theta}^2 + \frac{c}{n-1}\left(\frac{\sum_{i=1}^n y_i^2}{2}\right) - \frac{2c\hat{\theta}\Gamma\left(\frac{2n-1}{2}\right)}{\Gamma(n)}\sqrt{\frac{\sum_{i=1}^k y_i^2}{2}}$$

Now $\frac{\partial R(\hat{\theta})}{\partial \theta} = 0$, Then the Bayes estimator is

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$$\hat{\theta}_2 = \frac{\Gamma(n+c_1-1)}{\Gamma\left(n+c_1-\frac{1}{2}\right)}\sqrt{\frac{\sum_{i=1}^{k} y_i^{\,2}}{2}} \tag{2.6}$$

Remark 1: Replacing c_1= 1/2 in (2.6), the same Bayes estimator is obtained as in (2.3) corresponding to the Jeffrey's prior. By Replacing c_1= 3/2 in (2.6), the Bayes estimator becomes the estimator under Hartigan's prior (Hartigan (1964)). By Replacing c_1= 0 in(2.6), thus we get uniform prior.

c) Bayesian estimation of Rayleigh distribution under Jeffrey's prior by using Al-Bayyati's new loss function.

This section uses a new loss function introduced by Al-Bayyati (2002). Employing this loss function, we obtain Bayes estimators using Jeffrey's and extension of Jeffrey's prior information.

Al-Bayyati introduced a new loss function of the form:

$$l_A(\hat{\theta},\theta) = \theta^{c_2}(\hat{\theta}-\theta)^2 ; c_2 \varepsilon R.$$

Here, this loss function is used to obtain the estimator of the parameter of the Rayleigh distribution.

By using the Al-Bayyati's loss function, we obtained the following risk function:

$$R(\hat{\theta}) = \int_0^\infty \theta^{c_2}(\hat{\theta}-\theta)^2 p(\theta \mid \underline{y})\, d\theta$$

$$\Rightarrow \quad R(\hat{\theta}) = \hat{\theta}^2 \frac{\Gamma\left(\frac{2n-c_2}{2}\right)}{\Gamma(n)}\left[\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right]^{\frac{c_2}{2}} + \frac{\Gamma\left(\frac{2n-c_2-2}{2}\right)}{\Gamma(n)}\left(\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right)^{\frac{c_2+2}{2}} - \frac{2\hat{\theta}\Gamma\left(\frac{2n-c_2-1}{2}\right)}{\Gamma(n)}\left[\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right]^{\frac{c_2+1}{2}}$$

Now $\frac{\partial R(\hat{\theta})}{\partial \theta} = 0$, Then the Bayes estimator is

$$\hat{\theta}_3 = \frac{\Gamma\left(\frac{2n-c_2-1}{2}\right)}{\Gamma\left(\frac{2n-c_2}{2}\right)}\sqrt{\left(\frac{\sum_{i=1}^{k} y_i^{\,2}}{2}\right)} \tag{2.7}$$

Remark 2: Replacing c_2 = -2 in (2.7), we get the Bayes estimator under quadratic loss function with Jeffrey's prior. By Replacing c_2 =0 in (2.7), the Bayes estimator becomes the estimator under squared error loss function with Jeffrey's prior that reduced to (2.3) By Replacing c_2 =1 in (2.7), thus we get uniform prior.

Bayesian estimation of Rayleigh distribution under the extension of Jeffrey's prior by using Al-Bayyati's new loss function.

By using the Al-Bayyati's loss function, we obtained the following risk function:

$$R(\hat{\theta}) = \int_0^\infty \theta^{c_2}(\hat{\theta}-\theta)^2 \pi_2(\theta/\underline{y})\, d\theta$$

$$\Rightarrow \quad R(\theta) = \hat{\theta}^2 \frac{\Gamma\left(\frac{2n+2c_1-c_2-1}{2}\right)}{\Gamma\left(\frac{2n+2c_1-1}{2}\right)}\left[\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right]^{\frac{c_2}{2}} + \frac{\Gamma\left(\frac{2n+2c_1-c_2-3}{2}\right)}{\Gamma\left(\frac{2n+2c_1-1}{2}\right)}\left(\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right)^{\frac{c_2+2}{2}} - \frac{2\hat{\theta}\Gamma\left(\frac{2n+2c_1-c_2-2}{2}\right)}{\Gamma\left(\frac{2n+2c_1-1}{2}\right)}\left[\frac{\sum_{i=1}^{n} y_i^{\,2}}{2}\right]^{\frac{c_2+1}{2}}$$

Now $\frac{\partial R(\hat{\theta})}{\partial \theta} = 0$, Then the Bayes estimator is

$$\hat{\theta}_4 = \frac{\Gamma\left(\frac{2n+2c_1-c_2-2}{2}\right)}{\Gamma\left(\frac{2n+2c_1-c_2-1}{2}\right)} \sqrt{\left(\frac{\sum_{i=1}^{k} y_i^2}{2}\right)} \tag{2.8}$$

Remark 3: Replacing c_1= 1/2 and c_2 = 0 in (2.8), we get the Bayes estimator under squared error loss function with Jeffrey's prior which is same as (2.3). By Replacing c_1= 1/2 and c_2 =-2 in (2.8), we get the Bayes estimator under Quadratic loss function with Jaffrey prior.. By Replacing c_1= 0 and c_2 =0 in (2.8), thus we get uniform prior.

Simulation Study

In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The scale parameter is estimated for Rayleigh distribution with Maximum Likelihood and Bayesian using Jeffrey's & extension of Jeffrey's prior methods. For the scale parameter we have considered θ= 1.0 and 1.5. The values of Jeffrey's extension were c_1 = 0.5, 1.0, 1.5 and 2.0. The value for the loss parameter a = ±1.0 and ±2.0. This was iterated 5000 times and the scale parameter for each method was calculated. A simulation study was conducted R-software to examine and compare the performance of the estimates for different sample sizes with different values for the Extension of Jeffreys' prior and the loss functions.The results are presented in tables for different selections of the parameters and c extension of Jeffrey's prior.

Table 1: Mean Squared Error for $(\hat{\theta})$ under Jeffrey's prior.

n	θ	θ_{ML}	θ_{SL}	θ_{NL}				Remarks
				C2=1.0	C2=-1.0	C2=2.0	C2=-2.0	
25	1.0	0.0248	0.0212	0.0191	0.0234	0.0171	0.0257	C2=2
	1.5	0.0332	0.0373	0.0415	0.0338	0.0463	0.0310	C2=-2
50	1.0	0.0110	0.0122	0.0132	0.0114	0.0142	0.0106	C2=-2
	1.5	0.0129	0.0136	0.0144	0.0129	0.0153	0.0124	C2=-2
100	1.0	0.0026	0.0024	0.0024	0.0025	0.0023	0.0026	C2=2
	1.5	0.0107	0.0098	0.0093	0.0103	0.0089	0.0109	C2=2

Table 2: Mean Squared Error for $(\hat{\theta})$ under extension of Jeffrey's prior.

n	θ	C_1	θ_{ML}	θ_{SL}	θ_{NL}				Remarks
					C2=1.0	C2=-1.0	C2=2.0	C2=-2.0	
	1.0	0.5	0.0248	0.0212	0.0191	0.0234	0.0171	0.0257	C2=2.0
		1.0	0.0248	0.0232	0.0210	0.0255	0.0189	0.0278	C2=2.0
		1.5	0.0248	0.0252	0.0229	0.0276	0.0207	0.0300	C2=2.0
		2.0	0.0248	0.0274	0.0250	0.0298	0.0227	0.0322	C2=2.0
25	1.5	0.5	0.0334	0.0374	0.0415	0.0339	0.0464	0.0310	C2=-2.0
		1.0	0.0334	0.0330	0.0364	0.0301	0.0406	0.0278	C2=-2.0
		1.5	0.0334	0.0292	0.0321	0.0269	0.0356	0.0227	C2=-2.0
		2.0	0.0334	0.0261	0.0284	0.0243	0.0313	0.0231	C2=-2.0

50	1.0	0.5	0.0111	0.0122	0.0131	0.0113	0.0141	0.0105	C2=-2.0
		1.0	0.0111	0.0112	0.0120	0.0104	0.0130	0.0097	C2=-2.0
		1.5	0.0111	0.0104	0.0111	0.0096	0.0120	0.0090	C2=-2.0
		2.0	0.0111	0.0095	0.0103	0.0088	0.0110	0.0083	C2=-2.0
	1.5	0.5	0.0130	0.0136	0.0144	0.0129	0.0153	0.0124	C2=-2.0
		1.0	0.0130	0.0128	0.0134	0.0122	0.0142	0.0118	C2=-2.0
		1.5	0.0130	0.0120	0.0125	0.0116	0.0132	0.0113	C2=-2.0
		2.0	0.0130	0.0115	0.0118	0.0111	0.0124	0.0109	C2=-2.0
100	1.0	0.5	0.0027	0.0026	0.0025	0.0026	0.0024	0.0027	C2=2.0
		1.0	0.0027	0.0026	0.0026	0.0027	0.0025	0.0028	C2=2.0
		1.5	0.0027	0.0026	0.0025	0.0027	0.0024	0.0027	C2=2.0
		2.0	0.0027	0.0027	0.0026	0.0028	0.0025	0.0029	C2=2.0
	1.5	0.5	0.0108	0.0099	0.0094	0.0104	0.0090	0.0110	C2=2.0
		1.0	0.0108	0.0104	0.0098	0.0096	0.0093	0.0114	C2=2.0
		1.5	0.0108	0.0108	0.0102	0.0113	0.0097	0.0119	C2=2.0
		2.0	0.0108	0.0113	0.0108	0.0119	0.0102	0.0125	C2=2.0

ML= Maximum Likelihood, SL=Squared Error Loss Function, NL= New Loss Function,

In table 1, Bayes estimation with New Loss function under Jeffrey's prior provides the smallest values in most cases especially when loss parameter C_2 is ± 2. Similarly, in table 2, Bayes estimation with New Loss function under extension of Jeffrey's prior provides the smallest values in most cases especially when loss parameter C_2 is ± 2 whether the extension of Jeffrey's prior is 0.5, 1.0, 1.5 or 2.0. Moreover, when the sample size increases from 25 to 100, the MSE decreases quite significantly.

CONCLUSION

In this article, we have primarily studied the Bayes estimator of the parameter of the Rayleigh distribution under the extended Jeffrey's prior assuming two different loss functions. The extended Jeffrey's prior gives the opportunity of covering wide spectrum of priors to get Bayes estimates of the parameter - particular cases of which are Jeffrey's prior and Hartigan's prior.

Wehave also addressed the problem of Bayesian estimation for the Rayleigh distribution, under asymmetric and symmetric loss functions and that of Maximum Likelihood Estimation. From the results, we observe that in most cases, Bayesian Estimator under New Loss function (Al-Bayyati's Loss function) has the smallest Mean Squared Error values for both prior's i.e, Jeffrey's and an extension of Jeffrey's prior information.

REFRENCES

ABD Elfattah, A.M; Hassan, A.S. and Ziedan, D.M.(2006). Efficiency of Maximum likelihood estimators under different censored sampling Schemes for Rayleigh distribution, *Interstat.*

Al-Bayyati. (2002). Comparing methods of estimating Weibull failure models using simulation.Ph.D. Thesis, College of Admistration and Economics, Baghdad University, Iraq.

Al-Kutubi, H. S. (2005). On comparison estimation procedures for parameter and survival function.

Chandra, M. J. (2001). Statistical Quality Control. CRC Press, Boca Raton.

Cohen, A. C., 1965. Maximum likelihood estimation in Weibull distribution based on complete and on censored samples. Technometrics, 7: 579-588.

Hartigan, J. A. (1964). Invariant Prior Distribution. Ann. Math. Statist., 34, 836-845

Lalitha,S. and Mishra, A. (1996). Modified maxcimum likelihood estimation for Rayleigh distribution, Comm.Stat.Theory Methods, 25,389-401.

Mann, N. R., Schaffer, R. E., and Singpurwala, N.D., 1975. Methods for Statistical Analysis of Reliability and Life Data. John Wiley & Sons, New York.

Norstrom, J. G. (1996). The use of Precautionary Loss Functions in Risk Analysis. IEEE Transactions on reliability, (3), 400-403.

Rahul, GP. Singh and O.P. Singh, (2009). Population project OF Kerala using Bayesian methodology .Asian J. Applied Sci., 2: 402-413

Rayleigh, J. (1980). On the resultant of a large number of vibrations of the same pitch and of arbitrary phase, Philos. Mag.;10,73-78.

Siddiqui, M.M. (1962). Some problems conectede with Rayleigh distributions,J. Res. Nat. Bur. Stand. 60D, 167-174.

Sinha, S.K. and Howlader, H . A. (1993). Credible and HPD intervals of the parameter and reliability of Rayleigh distribution, IEEE Trans. Reliab;32 . 217-220.

Varian, H. R. (1975). A Bayesian approach to real estate assessment, Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage (eds: S.E. Fienberg and A.Zellner), North-Holland, Amsterdam, 195-208.

Zellner, A. (1986). Bayesian estimation and prediction using asymmetric loss function. J.Amer. Statist. Assoc., 81, 446-451.exponential distribution using simulation. Ph.D Thesis, Baghdad University, Iraq