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Global Accurate Domination in Graphs

V.R.Kulli [*], M.B.Kattimani [**]

[*] Department of Mathematics, Gulbarga University, Gulbarga 585 106, India
[**] Department of Mathematics, The Oxford College of Engineering, Hosur Road, Bangalore 560 068, India

Abstract- A dominating set D of a graph $G = (V, E)$ is an *accurate dominating set*, if $V–D$ has no dominating set of cardinality $|D|$. An accurate dominating set D of a graph G is a *global accurate dominating set*, if D is also an accurate dominating set of $\overline{G}$. The *global accurate domination number* $\gamma_{ga}(G)$ is the minimum cardinality of a global accurate dominating set. In this paper, some bounds for $\gamma_{ga}(G)$ are obtained and exact values of $\gamma_{ga}(G)$ for some standard graphs are found. Also a Nordhaus-Gaddum type result is established.

Index Terms- accurate dominating set, global accurate dominating set, global accurate domination number.

Mathematics Subject Classification: 05C.

I. INTRODUCTION

All graphs considered here are finite, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [1].

A set D of vertices in a graph $G = (V, E)$ is a *dominating set* of G if every vertex in $V – D$ is adjacent to some vertex in D. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set.

A dominating set D of a graph G is an *accurate dominating set*, if $V – D$ has no dominating set of cardinality $|D|$. The *accurate domination number* $\gamma_a(G)$ of G is the minimum cardinality of an accurate dominating set. This concept was introduced by Kulli and Kattimani in [2].

A dominating set D of a graph G is a *global dominating set* if D is also a dominating set of $\overline{G}$. The *global domination number* $\gamma_g(G)$ of G is the minimum cardinality of a global dominating set, [5].

In [4], Kulli and Kattimani introduced the concept of global accurate domination as follows:

An accurate dominating set D of a graph G is a *global accurate dominating set*, if D is also an accurate dominating set of $\overline{G}$. The *global accurate domination number* $\gamma_{ga}(G)$ of G is the minimum cardinality of a global accurate dominating set.

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. A γ_a-set is a minimum accurate dominating set.

II. RESULTS

We characterize accurate dominating sets which are global accurate dominating sets.

Theorem1. An accurate dominating set D of a graph G is a global accurate dominating set if and only if the following conditions hold:

(i) for each vertex $v \in V – D$, there exists a vertex $u \in D$ such that u is not adjacent to v,

(ii) there exists a vertex $w \in D$ such that w is adjacent to all vertices in $V – D$.

Theorem 2. Let G be a graph such that neither G nor $\overline{G}$ have an isolated vertex. Then

i) $$\gamma_{ga}(G) = \gamma_{ga}(\overline{G}); \quad (1)$$

ii) $$\frac{\left(\gamma_a(G) + \gamma_a(\overline{G})\right)}{2} \le \gamma_a(G) \le \gamma_a(G) + \gamma_a(\overline{G}). \quad (2)$$

Theorem 3. Let G be a graph such that neither G nor $\overline{G}$ have an isolated vertex. Then

$$\gamma_a(G) \le \gamma_{ga}(G) \quad (3)$$

Proof: Every global accurate dominating set is an accurate dominating set. Thus (3) holds.

Theorem4. Let G be a graph such that neither G nor $\overline{G}$ have an isolated vertex. Then

$$\gamma_g(G) \le \gamma_{ga}(G). \quad (4)$$

Proof: Every global accurate dominating set is a global dominating set. Thus (4) holds.

Exact values of $\gamma_{ga}(G)$ for some standard graphs are given in Theorem 5.

Theorem 5.

i) $$\gamma_{ga}(K_p) = p. \quad (5)$$

ii) $$\gamma_{ga}(C_p) = \left\lfloor \frac{p}{2} \right\rfloor + 1, \quad \text{if } p \ge 3. \quad (6)$$

iii) $\gamma_{ga}(P_p) = \left\lfloor \frac{p}{2} \right\rfloor + 1$, if $p \geq 2$. (7)

iv) $\gamma_{ga}(K_{m,n}) = m+1$, if $m \leq n$. (8)

v) $\gamma_{ga}(W_p) = \left\lfloor \frac{p}{2} \right\rfloor + 1$, if $p \geq 5$. (9)

vi) For any regular graph G,

$\gamma_{ga}(G) = \left\lfloor \frac{p}{2} \right\rfloor + 1$, if $p \geq 2$. (10)

Now we obtain an upper bound for $\gamma_{ga}(G)$.

Theorem 6. Let G be a graph such that neither G nor $\bar{G}$ have an isolated vertex. Then

$$\gamma_{ga}(G) \leq p - 1. \quad (11)$$

Proof: Clearly G has two nonadjacent vertices u and v such that u is adjacent to some vertex in $V - \{u\}$. This implies that $V - \{u\}$ is a global accurate dominating set of G. Thus

$$\gamma_{ga}(G) \leq |V - \{u\}|$$

or $$\gamma_{ga}(G) \leq p - 1.$$

Next we characterize graphs G that have global accurate domination number equal to the order of G.

Theorem 7. For any graph G,

$$\gamma_{ga}(G) = p \quad (12)$$

if and only if $G = K_p$ or $G = \bar{K}_p$.

Proof: Suppose (12) holds. Assume $G \neq K_p$, $\bar{K}_p$. Then G has at least three vertices u, v and w such that u and v are adjacent and w is not adjacent to one of u or v. Suppose w is not adjacent to u. Then this implies that $V - \{u\}$ is a global accurate dominating set of G. This is a contradiction. This proves necessity.

Converse is obvious.

Theorem 8. Let D be an accurate dominating set of G. If there exist two vertices $u \in V - D$ and $v \in D$ such that u is adjacent only to the vertices of D and v is adjacent only to the vertices of $V - D$. Then

$$\gamma_{ga}(G) \leq \gamma_a(G) + 1. \quad (13)$$

Proof: Let D be a γ_a-set of G. If these exists a vertex u in $V - D$ such that u is adjacent only to the vertices of D, then $D \cup \{u\}$ is a global accurate dominating set of G. Thus

$$\gamma_{ga}(G) \leq |D \cup \{u\}|$$
$$\leq |D| + 1$$

or $$\gamma_{ga}(G) \leq \gamma_a(G) + 1.$$

In a graph G, a vertex and an edge incident with it are said to *cover* each other. A set of vertices that covers all the edges of G is a *vertex cover* of G. The *vertex covering number* $\alpha_0(G)$ of G is the minimum number of vertices in a vertex cover. A set S of vertices in G is *independent* if no two vertices in S are adjacent. The *independence number* $\alpha_0(G)$ of G is the maximum cardinality of an independent set of vertices. The *clique number* β_0 (G) of G is the maximum order among the complete subgraphs of G.

Theorem 9. Let G be a graph without isolated vertices. Then

$$\gamma_{ga}(G) \leq \alpha_0(G) + 1 \quad (14)$$

Proof: Let S be a maximum independent set of vertices in G. Then for any vertex $v \in S$, $\{V - S\} \cup \{v\}\}$ is a global accurate dominating set of G. Thus

$$\gamma_{ga}(G) \leq |(V - S) \cup \{v\}|$$
$$\leq |V - S| + 1$$
$$\leq p - \beta_0(G) + 1$$

or $$\gamma_{ga}(G) \leq \alpha_0(G) + 1.$$

Now we obtain a Nordhaus-Gaddum type result.

Theorem 10. Let G be a graph such that neither G nor $\bar{G}$ have an isolated vertex. Then

$$\gamma_{ga}(G) + \gamma_{ga}(\bar{G}) \leq p + \gamma_0(G) - w(G) + 2.$$

Proof: By Theorem 9, $\gamma_{ga}(G) \leq \alpha_0(G) + 1$.

Therefore $$\gamma_{ga}(\bar{G}) \leq \alpha_0(\bar{G}) + 1.$$
$$\leq p - \beta_0(\bar{G}) + 1$$
$$\leq p - \omega(G) + 1$$

Hence $$\gamma_{ga}(G) + \gamma_{ga}(\bar{G}) \leq p + \alpha_0(G) - \omega(G) + 2$$

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AUTHORS

First Author – V.R.KULLI, Department of Mathematics, Gulbarga University, Gulbarga 585 106, India, e-mail: vrkulli@gmail.com

Second Author – M.B.KATTIMANI, Department of Mathematics, The Oxford College of Engineering, Hosur Road, Bangalore 560 068, India