Midpoints between the Planets and the Gravitational Force at these Midpoints

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Abstract- The midpoints determined in this paper were calculated by the use of the distance equation. The gravitation at these midpoints were determined by a modified gravitation equation. The values from both NASA and "Equations for Planetary Ellipses" were used for each midpoints. The NASA's distances and values were considered as the accepted values and the values determined in "Equations for Planetary Ellipses" are the measured values.

Index Terms- Distance, Distance Equation, Ellipses, Gravitation, Midpoint, Orbits, Planets

I. INTRODUCTION

This paper was written as an extension of my last two papers, "Equations for Planetary Ellipses" and "Models for Intersection and Collison with Earth". The use of these midpoints are unknown to me as of right now. However, the extensions of this paper could be anywhere between distances and gravitation with the planets. The units for each value are noted in the respective data tables.

II. IDENTIFY, RESEARCH AND COLLECT IDEA

The idea for this research came from the results of my last two papers, "Equations for Planetary Ellipses" and "Models for Intersection and Collison with Earth". The idea was supported by my high school teacher as a basis for my understanding of the Lagrangian Points.

III. WRITE DOWN YOUR STUDIES AND FINDINGS

The midpoint between two planets was calculated by the use of the midpoint formula. The midpoints between each pair of planets was found by the use of this equation. The values for perihelion and aphelion calculated in the paper "Equations for Planetary Ellipses" and from NASA's website, were used to find the coordinates of the midpoints. See Equation 1 for the midpoint general formula. Since the y component was always zero, the midpoint equation was simplified to just include the x values with the y values set at zero. See Equation 2.

For both perihelion and aphelion, two sets of midpoints were calculated. The first set was determined by using the data from the paper "Equations for Planetary Ellipses". These values were noted as the "measured" values in the data tables below. The data from NASA was used for the second set, noted as the "accepted" values in the below data tables.

These midpoints are located at equal distances from both planets on either side. So, the distance between one planet and the midpoint is equal to the distance between the second planet and the same midpoint. See Equation 3.

The planets are held in their orbit paths because of the gravity of the sun and the other planets pulling in opposite directions. This gravitational force has its resultant towards the sun for all the planets in our solar system. However, this resultant is small enough that the planets do not go flying into the sun. The gravitational pull of the sun on each planet was also calculated. Both the accepted and measured values for perihelion and aphelion were used. (The term measured refers to the data from the paper "Equations for Planetary Ellipses" and the term accepted refers to the data from NASA.)

The values for the gravitational force are only between the Sun, the first planet and the second planet. The forces generated by the other planets on the midpoint were not taken into account.

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad (1)$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, 0\right) \quad (2)$$

$$r^2_{Planet\ 1 \rightarrow Mdpt} = r^2_{Planet\ 2 \rightarrow Mdpt} \quad (3)$$

These midpoints are named based off of the planet that is the closest to the sun in the system. The systems that midpoints have been calculated for are Mercury-Venus, Venus-Earth, Earth-Mars, Mars-Jupiter, Jupiter-Saturn, Saturn-Uranus, and Uranus-Neptune.

The midpoints for each system were determined by the midpoint equation. The gravitation at this midpoints were determined by equation 4.

$$Gravitational\ Force\ At\ Midpoint = \left(\frac{GM_{Sun}}{r^2_{Sun\rightarrow Mdpt}} + \frac{GM_{Planet\ 1}}{r^2_{Planet\ 1\rightarrow Mdpt}}\right) - \frac{GM_{Planet\ 2}}{r^2_{Planet\ 2\rightarrow Mdpt}} \qquad (4)$$

In this equation, Planet 1 refers to the planet closest to the sun and Planet 2 is the other planet in the system. While this equation does not take into account of the forces on the midpoints by the other planets, it still represents the pull of the two planets in the system on this point. This equation also does not account for the mass of the midpoint, since it would be zero. Therefore, the units of this equation are the newton-meter.

Tables 5.1 through 6.1 are measured in Gigameters only.

Table 1- Aphelion Midpoints (Measured in Gigameters)

	Measured	Accepted	
Midpoint Name	**Mdpt Coordinates**	**Mdpt Coordinates**	**Sun Coordinates**
M2	(89.26275,0)	(89.35,0)	(11.8695,0)
E2	(130.653,0)	(130.5,0)	(0.756,0)
A2	(200.991,0)	(200.65,0)	(2.55,0)
J2	(533.3015,0)	(532.9,0)	(21.432,0)
S2	(1164.341,0)	(1165.55,0)	(38.171,0)
U2	(2256.765,0)	(2259.05,0)	(81.51,0)
N2	(3775.76,0)	(3774.65,0)	(132.02,0)

Table 2- Aphelion Midpoints (Measured in Meters)

	Measured	Accepted	(Not in Meters)
Midpoint Name	**Mdpt Coordinates**	**Mdpt Coordinates**	**Sun Coordinates**
M2	(8.9263E+10,0)	(8.9350E+10,0)	(11.8695,0)
E2	(1.3065E+11,0)	(1.3050E+11,0)	(0.756,0)
A2	(2.0099E+11,0)	(2.0065E+11,0)	(2.55,0)
J2	(5.3330E+11,0)	(5.3290E+11,0)	(21.432,0)
S2	(1.1643E+12,0)	(1.1656E+12,0)	(38.171,0)
U2	(2.2568E+12,0)	(2.2591E+12,0)	(81.51,0)
N2	(3.7758E+12,0)	(3.7747E+12,0)	(132.02,0)

Table 3- Perihelion Midpoints (Measured in Gigameters)

	Measured	Accepted	
Midpoint Name	**Mdpt Coordinates**	**Mdpt Coordinates**	**Sun Coordinates**
M2	(76.63725,0)	(76.75,0)	(11.8695,0)
E2	(127.347,0)	(127.3,0)	(0.756,0)
A2	(177.009,0)	(176.85,0)	(2.55,0)
J2	(473.6985,0)	(473.55,0)	(21.432,0)
S2	(1044.66,0)	(1046.55,0)	(38.171,0)
U2	(2043.235,0)	(2046.95,0)	(81.51,0)
N2	(3594.24,0)	(3592.9,0)	(132.02,0)

Table 4- Perihelion Midpoints (Measured in Meters)

	Measured	Accepted	(Not in Meters)
Midpoint Name	**Mdpt Coordinates**	**Mdpt Coordinates**	**Sun Coordinates**
M2	(7.6637E+10,0)	(7.6750E+10,0)	(11.8695,0)
E2	(1.2735E+11,0)	(1.2730E+11,0)	(0.756,0)
A2	(1.7701E+11,0)	(1.7685E+11,0)	(2.55,0)
J2	(4.7370E+11,0)	(4.7355E+11,0)	(21.432,0)
S2	(1.0447E+12,0)	(1.0466E+12,0)	(38.171,0)
U2	(2.0432E+12,0)	(2.0470E+12,0)	(81.51,0)
N2	(3.5942E+12,0)	(3.5929E+12,0)	(132.02,0)

Table 5.1- Distance between Midpoint and Sun at Aphelion (Measured Values Used for Midpoint Coordinates)			
Midpoint Name	**Midpoint Coordinates**	**Sun Coordinates**	**Distance between Sun and Midpoint**
M2	89.2628	11.8695	77.3933
E2	130.653	0.756	129.897
A2	200.991	2.55	198.441
J2	533.302	21.432	511.87
S2	1164.34	38.171	1126.169
U2	2256.77	81.51	2175.26
N2	3775.76	132.02	3643.74

Table 5.2- Distance between Midpoint and Sun at Aphelion (Accepted Values Used for Midpoint Coordinates)			
Midpoint Name	**Midpoint Coordinates**	**Sun Coordinates**	**Distance between Sun and Midpoint**
M2	89.35	11.8695	77.4805
E2	130.5	0.756	129.744
A2	200.65	2.55	198.1
J2	532.9	21.432	511.468
S2	1165.55	38.171	1127.379
U2	2259.05	81.51	2177.54
N2	3774.65	132.02	3642.63

Table 5.3- Distance between Midpoint and Sun at Perihelion (Measured Values Used for Midpoint Coordinates)			
Midpoint Name	**Midpoint Coordinates**	**Sun Coordinates**	**Distance between Sun and Midpoint**
M2	76.63725	11.8695	64.76775
E2	127.347	0.756	126.591
A2	177.009	2.55	174.459
J2	473.6985	21.432	452.2665
S2	1044.66	38.171	1006.489
U2	2043.235	81.51	1961.725
N2	3594.24	132.02	3462.22

Table 5.4- Distance between Midpoint and Sun at Perihelion (Accepted Values Used for Midpoint Coordinates)			
Midpoint Name	**Midpoint Coordinates**	**Sun Coordinates**	**Distance between Sun and Midpoint**
M2	76.75	11.8695	64.8805
E2	127.3	0.756	126.544
A2	176.85	2.55	174.3
J2	473.55	21.432	452.118
S2	1046.55	38.171	1008.379
U2	2046.95	81.51	1965.44
N2	3592.9	132.02	3460.88

Table 6.1- Distance between Midpoint and *Both* Planets at Aphelion (Measured Coordinates are Used)			
Midpoint Name	**Midpoint Coordinates**	**Planet Coordinates**	**Distance between Planet and Midpoint**
M2	89.2628	69.7695	19.4933
E2	130.653	108.756	21.897
A2	200.991	152.55	48.441
J2	533.302	249.432	283.87
S2	1164.34	817.171	347.169
U2	2256.77	1511.51	745.26
N2	3775.76	3002.02	773.74

Table 6.2- Distance between Midpoint and *Both* Planets at Aphelion (Accepted Coordinates are Used)			
Midpoint Name	**Midpoint Coordinates**	**Planet Coordinates**	**Distance between Planet and Midpoint**
M2	89.35	69.7695	19.5805
E2	130.5	108.756	21.744
A2	200.65	152.55	48.1
J2	532.9	249.432	283.468
S2	1165.55	817.171	348.379
U2	2259.05	1511.51	747.54
N2	3774.65	3002.02	772.63

Table 6.3- Distance between Midpoint and *Both* Planets at Perihelion (Measured Coordinates are Used)			
Midpoint Name	**Midpoint Coordinates**	**Planet Coordinates**	**Distance between Planet and Midpoint**
M2	76.63725	46	30.63725
E2	127.347	107.5	19.847
A2	177.009	147.1	29.909
J2	473.6985	206.6	267.0985
S2	1044.66	740.5	304.16
U2	2043.235	1352.6	690.635
N2	3594.24	2741.3	852.94

Table 6.4- Distance between Midpoint and *Both* Planets at Perihelion (Accepted Coordinates are Used)			
Midpoint Name	**Midpoint Coordinates**	**Planet Coordinates**	**Distance between Planet and Midpoint**
M2	76.75	46	30.75
E2	127.3	107.5	19.8
A2	176.85	147.1	29.75
J2	473.55	206.6	266.95
S2	1046.55	740.5	306.05
U2	2046.95	1352.6	694.35
N2	3592.9	2741.3	851.6

IV. GET PEER REVIEWED

The math aspect of this paper was reviewed by Mr. Bruce Peachey by my asking. He told me to make sure that the equations and the units fit. He reminded me of using dimensional analysis. My math teacher, Mrs. Kimberly Waterbury, also reviewed some of the midpoint calculations before they made their way into this paper.

V. IMPROVEMENT AS PER REVIEWER COMMENTS

The changes made to this paper was a review of the math equations to make sure the values were correct. The units for the equations was reviewed by dimensional analysis. The gravitational equation units were reviewed and originally noted as just Newtons, but under review they were changed to Newton-meters.

VI. CONCLUSION

The midpoints above in the tables are based off of just a two planet system. However, the midpoint(s) would change if the number of bodies in the system was increased. This is leading up to a re-structuring of the way the Lagrangian Points are looked at.

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