Selection of Optimum Single Sampling Plans under Prior Binomial distribution by minimizing the Average acceptance Cost

[1] M.Nirmala, [2]R.Subramaniam and [3]G.Uma

[1]PhD Research scholar, PSG College of Arts and Science, Coimbatore -641 014.
[2]Associate Professor and Head, PSG College of Arts and Science, Coimbatore -641 014.
[3]Assistant Professor, PSG College of Arts and Science, Coimbatore -641 014.

Abstract- This paper deals with the determination of Optimum Single Sampling Plans (n,c) based on the Bayesian Prior Binomial Distribution for various values of the Lot sizes 'N' by minimizing the average acceptance cost K(N, n, c, p) subject to the condition, the cost associated with a defective item which is accepted (A_2) is very small, Producer's Risk and Consumer's Risk are minimized (or) power 1-β is maximized. The ideal Operating Characterizing Curve for a Bayesian Single Sampling Plan is also constructed by considering AQL and LQL for the optimum acceptance number 'c'.

***Index Terms*-** Bayesian Sampling Plan, Acceptance Quality Level (AQL), Limiting Quality Level (LQL), Producer's Risks (α), Consumer's Risks (β), Prior Distribution.

I. Introduction

Acceptance sampling is an inspection procedure used to determine whether to accept (or) reject a specific quantity of material. Inspection of a raw material and final product is necessary to ensure the good quality. It is well known that the acceptance sampling plans are used to reduce the cost of inspection. Acceptance sampling procedures are necessarily defensive the measures, instituted as productive devices against the threat of deterioration in quality. As such, they should be set up with the aim of discontinuance in favour of process control procedures as soon as possible. Sampling inspection plans provide decision-making procedures for controlling and improving the average quality of incoming items by specifying acceptable quality and determine when to accept (or) reject a given lot. An acceptance sampling is most useful to contact when product testing is destructive, very expensive, very time consuming (or) when product liability risks are significant [Starbird, S.A. 1994].

Because of sampling, it could happen that the lot which has a satisfactory pre – determined level of quality will be rejected and that the lot which does not have a satisfactory pre – determined level of quality will be accepted. The first situation is known as Producer's Risk (or) α and the second situation is known as Consumer's Risk (or) β. [Wadsworth et al., 2002].

The aim of the model is to find the Bayesian Optimum Single Sampling Plan (n,c) the sample size and the acceptance number by minimizing the average acceptance cost function K(N,n,c,p), which comprises cost of inspection and cost of repairing (A_2) (or) replacement of defective units subject to the condition of minimizing consumer's risk and producer's risk.

The quantity most difficult to determine is normally the cost of accepting a defective items. If for example, the items considered are to be further processed, then the cost of accepting a defective may consist of the cost of handling and identifying the defective item, cost of assembling and disassembling, damage done to other items, cost of rework and cost of renewed testing and inspection.

If however, the items are finished goods, the cost of passing a defective may involve service and replacement cost, loss of goodwill which may be difficult to measure. For various cost models are available depending on a lot size, sample sizes. In rectifying inspection plans, each item produced by a process is inspected according to some plan and then according to the number of defectives units in the sample, the lot is either accepted (or) rejected.

Guenther (1971) explained how to obtained the parameter (n,c) for single sampling plans for ordinary sampling and for Bayesian sampling, using the model of expected total quality control cost depends on cost of inspection and sampling and cost due to wrong decision, which is accepting bad lot and rejecting good one.

To improve the quality for any product and services, its customary to modernize the quality practiced and simultaneously reduced the cost for inspection and quality improvement. As a result of increasing customer quality requirement and development for new product technology may existing quality assurance practices and techniques need to be modified.

The need for such statistical and analytical techniques in a quality assurance is rapidly increasing owing to stilt competition in industry towards product quality improvement.

This paper introduces the designed model to minimize, Average acceptance cost subject to minimize the producer's and consumer's risk. Hald (1960) has derived optimal solutions for the cost function K(N,n,c,p) in the cases where the prior distribution is Rectangular, Polya and Binomial. Bayesian acceptance sampling approach is associated with the utilization of prior process history for the selection of distribution [Viz., Gamma Poisson, Beta Binomial] to describe the random fluctuation involved in acceptance sampling.

1.1 **Prior Distribution:**

The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operates. The distribution is called prior, because it is formulated prior to the taking of samples. The combination of prior knowledge represented with the prior distribution and the empirical knowledge based on the sample leads to the decision on the lot.

A complete statistical model for basic sampling inspection contains three components.

a. The prior distribution. That is the expected distribution of submitted lots according to quality.
b. The cost of sampling inspection, acceptance and rejection.
c. A class of sampling plan that usually defined by means of the restriction designed to give a production against accepting lot of poor quality.

In this paper the average acceptance cost is introduced when the proportional defective is random variable follows prior binomial distribution.

The ultimate aim of a sampling plan is to obtained the optimum single sampling plan (n,c) based on prior binomial distribution by minimizing the producer's and consumer's risk. To design the single sampling plan, two point OC curve approach is followed. The plan parameters are determined and satisfying the condition that the producer's risk and consumer's risk are minimized at optimum level of $A_{2,}$ the cost associated with a defective item which is accepted.

It is important to note that there may exist multiples of solutions, in this case the cost associated with a defective items which is accepted (A_2) and acceptance number to be optimized and then determined the minimum average acceptance cost k(N,n,c,p) by minimizing consumer's risk and producer's risk.

II. METHODOLOGY OF RESEARCH

Since the aim of research insist on finding the optimum single sampling plans (n,c) based on prior binomial distribution by minimizing the average acceptance cost K (N,n,c,p) subject to the condition A_2,the cost associated with a defective item is minimum

$$P_a(p_1) \geq 1-\alpha \qquad (1)$$

$$P_a(p_2) \leq \beta \quad (or)$$

$$P_a(p_2) \geq 1-\beta \ (Power) \qquad (2)$$

Where,

$$K(N,n,c,p) = n(s_1+s_2p) + (N-n)\{[(A_1-R_1) + (A_2-R_2)p]P(p) + (R_1+R_2p)\}$$

$$K_S(p) = S_1 + S_2\, p$$

$$K_a(p) = A_1 + A_2\, p$$

$$K_r(p) = R_1 + R_2\, p$$

p_1 = Quality level corresponding to the producer's risk which is called Acceptable Quality Level (AQL).

p_2 = Quality level corresponding to the consumer's risk which is called Limiting Quality Level(LQL).

To satisfy, the aim of the following notations are necessary to build a model these are

N: Lot size of product.

n: Sample Size.

p: $\left(\frac{x}{n}\right)$ percentage of defective in sample n.

x: no of defectives in sample.

S_1: Cost per item of sampling and testing.

S_2: Repair cost for a defective item found in sampling.

A_1: Cost per item associated with the handling the (N-n) items not inspected in an acceptance lot. (frequently it is Zero).

A_2: Cost associated with a defective item which is accepted (may be quite large).

R_1: Cost per item of inspecting the remaining (N-n) items in a rejected lot.

$R_{2:}$ Repair cost associated with a defective item in the (N-n) items in a lot.

α : Producer's Risk.

β : Consumer's Risk.

P(p):Probability of accepting the production with quality p and it is P(p)=P$(X \leq C)$.

Ka(p): Unit cost of acceptance
Kr(p): Unit cost of rejection
Ks(p): Unit cost of sampling
K(N,n,c,p): Average acceptance cost under Bayesian sampling

By giving different values of proportion defective 'p' for various lot sizes 'N', the probabilities of acceptance Pa(p) will be calculated with optimum acceptance number c=7 and then identified Pa(p) = 0.95, the corresponding 'p' value will be take it as Average Quality Level (AQL). Likewise, for various lot sizes 'N', the AQL values are obtained and are presented from Table 1 to 10. Also identified the point Pa(p) = 0.10, the corresponding 'p' value will be take it as Limiting quality level (LQL) and then various lot sizes 'N', the LQL values are obtained and are presented from Table 11 to 20.

The determination of Optimum Single Sampling Plan(n,c) based on Bayesian Prior Binomial Distribution for various values of 'N' were carried out through excel and are presented from table 1to 20.

Table 1: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
1000	70	0.0007	0.049	34	0.952164792	2.0014	0.1014	0.0238	165.6841713	0
1000	90	0.004	0.36	35	0.949163019	2.008	0.108	0.14	306.6396271	1
1000	90	0.009	0.81	35	0.951915796	2.018	0.118	0.315	459.6499448	2
1000	110	0.012	1.32	36	0.955921603	2.024	0.124	0.432	595.0372299	3
1000	110	0.018	1.98	36	0.950754976	2.036	0.136	0.648	778.2400273	4
1000	110	0.024	2.64	36	0.950148922	2.048	0.148	0.864	962.4728992	5
1000	130	0.025	3.25	37	0.954475231	2.05	0.15	0.925	1040.554924	6
1000	**130**	**0.031**	**4.03**	**37**	**0.949894073**	**2.062**	**0.162**	**1.147**	**1223.011726**	**7**
1000	130	0.036	4.68	37	0.954010076	2.072	0.172	1.332	1381.786969	8

Table 2: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
2000	90	0.0006	0.054	35	0.947416752	2.0012	0.1012	0.021	228.2728071	0
2000	110	0.003	0.33	36	0.956407953	2.006	0.106	0.108	424.6152221	1
2000	130	0.006	0.78	37	0.9559225	2.012	0.112	0.222	667.6332582	2
2000	150	0.009	1.35	38	0.952545396	2.018	0.118	0.342	915.734812	3
2000	150	0.013	1.95	38	0.952894463	2.026	0.126	0.494	1185.73055	4
2000	170	0.015	2.55	39	0.955851504	2.03	0.13	0.585	1378.889755	5
2000	170	0.019	3.23	39	0.955185548	2.038	0.138	0.741	1653.0377	6
2000	**190**	**0.021**	**3.99**	**40**	**0.951340586**	**2.042**	**0.142**	**0.84**	**1846.90467**	**7**
2000	190	0.025	4.75	40	0.949289145	2.05	0.15	1	2121.481349	8

Table 3: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
3000	110	0.0004	0.044	36	0.956945534	2.0008	0.1008	0.0144	272.4545279	0
3000	130	0.003	0.39	37	0.941351639	2.006	0.106	0.111	578.508396	1
3000	150	0.005	0.75	38	0.959911232	2.01	0.11	0.19	833.8597608	2
3000	170	0.008	1.36	39	0.951391629	2.016	0.116	0.312	1198.717909	3
3000	190	0.01	1.9	40	0.956775024	2.02	0.12	0.4	1473.790589	4
3000	210	0.012	2.52	41	0.957647788	2.024	0.124	0.492	1754.236137	5
3000	210	0.015	3.15	41	0.959602319	2.03	0.13	0.615	2087.485877	6
3000	**230**	**0.0175**	**4.14**	**42**	**0.948952832**	**2.035**	**0.135**	**0.735**	**2419.159606**	**7**
3000	230	0.02	4.6	42	0.956636337	2.04	0.14	0.84	2711.917857	8

Table 4: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p	K(N,n,c,p)	c
4000	130	0.0004	0.052	37	0.949318991	2.0008	0.1008	0.0148	334.2476533	0
4000	170	0.002	0.34	39	0.953932381	2.004	0.104	0.078	644.0074135	1
4000	190	0.004	0.76	40	0.958490915	2.008	0.108	0.16	982.89622	2
4000	210	0.0061	1.281	41	0.95928883	2.0122	0.1122	0.2501	1349.163674	3
4000	230	0.0086	1.978	42	0.950080472	2.0172	0.1172	0.3612	1779.760024	4
4000	230	0.011	2.53	42	0.956883319	2.022	0.122	0.462	2151.533038	5
4000	250	0.013	3.25	43	0.953414271	2.026	0.126	0.559	2527.106422	6
4000	**270**	**0.015**	**4.05**	**44**	**0.947207922**	**2.03**	**0.13**	**0.66**	**2905.53534**	**7**
4000	270	0.017	4.59	44	0.956870303	2.034	0.134	0.748	3240.443505	8

Table 5: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
5000	150	0.0003	0.045	38	0.955991028	2.0006	0.1006	0.0114	374.4191616	0
5000	190	0.0018	0.342	40	0.953431505	2.0036	0.1036	0.072	734.0822249	1
5000	210	0.0039	0.819	41	0.950148264	2.0078	0.1078	0.1599	1175.118051	2
5000	230	0.0059	1.357	42	0.951526118	2.0118	0.1118	0.2478	1613.274023	3
5000	250	0.0079	1.975	43	0.9502807	2.0158	0.1158	0.3397	2064.647282	4
5000	270	0.0097	2.619	44	0.950420975	2.0194	0.1194	0.4268	2491.913998	5
5000	270	0.012	3.24	44	0.953953012	2.024	0.124	0.528	2955.92789	6
5000	**290**	**0.014**	**4.06**	**45**	**0.946500798**	**2.028**	**0.128**	**0.63**	**3428.925417**	**7**
5000	310	0.015	4.65	46	0.953690795	2.03	0.13	0.69	3743.773505	8

Table 6: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
6000	150	0.0003	0.045	38	0.955991028	2.0006	0.1006	0.0114	389.744762	0
6000	190	0.0018	0.342	40	0.953431505	2.0036	0.1036	0.072	807.5537893	1
6000	230	0.0035	0.805	42	0.952159552	2.007	0.107	0.147	1298.758425	2
6000	250	0.0054	1.35	43	0.952227062	2.0108	0.1108	0.2322	1804.502101	3
6000	270	0.007	1.89	44	0.957321179	2.014	0.114	0.308	2261.177369	4
6000	290	0.009	2.61	45	0.951025244	2.018	0.118	0.405	2817.511639	5
6000	310	0.011	3.41	46	0.942460724	2.022	0.122	0.506	3380.238984	6
6000	**330**	**0.012**	**3.96**	**47**	**0.952270029**	**2.024**	**0.124**	**0.564**	**3746.723268**	**7**
6000	330	0.014	4.62	47	0.955125717	2.028	0.128	0.658	4265.248292	8

Table 7: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
7000	170	0.0003	0.051	39	0.950271399	2.0006	0.1006	0.0117	450.2075598	0
7000	210	0.0017	0.357	41	0.949728129	2.0034	0.1034	0.0697	905.4803604	1
7000	250	0.003	0.75	43	0.959744201	2.006	0.106	0.129	1366.000287	2
7000	270	0.005	1.35	44	0.952191772	2.01	0.11	0.22	1987.907569	3
7000	290	0.007	2.03	45	0.945242765	2.014	0.114	0.315	2623.858369	4
7000	310	0.008	2.48	46	0.959963277	2.016	0.116	0.368	3019.38289	5
7000	330	0.01	3.3	47	0.949930854	2.02	0.12	0.47	3684.613578	6
7000	**350**	**0.0115**	**4.025**	**48**	**0.948405829**	**2.023**	**0.123**	**0.552**	**4231.65957**	**7**
7000	370	0.013	4.81	49	0.944812267	2.026	0.126	0.637	4785.957824	8

Table 8: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
8000	170	0.0003	0.051	39	0.950271399	2.0006	0.1006	0.0117	466.3284324	0
8000	230	0.0015	0.345	42	0.952676646	2.003	0.103	0.063	964.9080984	1
8000	270	0.003	0.81	44	0.951389895	2.006	0.106	0.132	1552.210341	2
8000	290	0.0047	1.363	45	0.950775656	2.0094	0.1094	0.2115	2174.642039	3
8000	310	0.006	1.86	46	0.959595148	2.012	0.112	0.276	2695.203016	4
8000	330	0.008	2.64	47	0.948689268	2.016	0.116	0.376	3446.876138	5
8000	350	0.009	3.15	48	0.959101741	2.018	0.118	0.432	3912.858293	6
8000	**370**	**0.011**	**4.07**	**49**	**0.945603114**	**2.022**	**0.122**	**0.539**	**4687.634885**	**7**
8000	390	0.012	4.68	50	0.951869633	2.024	0.124	0.6	5181.014482	8

Table 9: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
9000	190	0.0003	0.057	40	0.944585992	2.0006	0.012	0.1006	529.088291	0
9000	250	0.0014	0.35	43	0.951441353	2.0028	0.0602	0.1028	1045.550236	1
9000	270	0.003	0.81	44	0.951389895	2.006	0.132	0.106	1682.946478	2
9000	310	0.0044	1.364	46	0.950641171	2.0088	0.2024	0.1088	2341.436319	3
9000	330	0.006	1.98	47	0.949670794	2.012	0.282	0.112	3034.719784	4
9000	350	0.0074	2.59	48	0.952346474	2.0148	0.3552	0.1148	3678.566398	5
9000	370	0.009	3.33	49	0.947842557	2.018	0.441	0.118	4407.101649	6
9000	**390**	**0.0102**	**3.978**	**50**	**0.951071103**	**2.0204**	**0.51**	**0.1204**	**5014.926168**	**7**
9000	410	0.0114	4.674	51	0.952131093	2.0228	0.5814	0.1228	5635.000472	8

Table 10: Determination of Optimum Sample Size 'n', , acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Acceptable Quality Level (AQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
10000	210	0.0002	0.042	41	0.958865753	2.0004	0.1004	0.0082	537.4913346	0
10000	250	0.0014	0.35	43	0.951441353	2.0028	0.1028	0.0602	1107.818834	1
10000	330	0.0024	0.792	45	0.953927076	2.0048	0.1048	0.108	1704.518319	2
10000	330	0.0041	1.353	47	0.951793598	2.0082	0.1082	0.1927	2486.724826	3
10000	350	0.0056	1.96	48	0.951379813	2.0112	0.1112	0.2688	3223.896474	4
10000	370	0.007	2.59	49	0.952311153	2.014	0.114	0.343	3943.103215	5
10000	390	0.0084	3.276	50	0.951350896	2.0168	0.1168	0.42	4681.000575	6
10000	**410**	**0.0097**	**3.977**	**51**	**0.95108473**	**2.0194**	**0.1194**	**0.4947**	**5396.074731**	**7**
10000	430	0.011	4.73	52	0.949105685	2.022	0.122	0.572	6124.323632	8

Table 11: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
1000	70	0.033	2.31	34	0.09546698	2.066	0.166	1.122	383.8777829	0
1000	90	0.043	3.87	35	0.096569451	2.086	0.186	1.505	472.9113459	1
1000	90	0.059	5.31	35	0.093987327	2.118	0.218	2.065	546.9710801	2
1000	110	0.06	6.6	36	0.098030381	2.12	0.22	2.16	598.2592554	3
1000	110	0.072	7.92	36	0.095776071	2.144	0.244	2.592	653.145171	4
1000	110	0.084	9.24	36	0.092006482	2.168	0.268	3.024	702.6771792	5
1000	130	0.08	10.4	37	0.097284951	2.16	0.26	2.96	735.5223507	6
1000	**130**	**0.09**	**11.7**	**37**	**0.092798464**	**2.18**	**0.28**	**3.33**	**773.2407254**	**7**
1000	130	0.099	12.87	37	0.093998187	2.198	0.298	3.663	820.1843928	8

Table 12: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
2000	90	0.033	2.97	35	0.093391235	2.052	0.152	0.91	610.2099624	0
2000	110	0.043	4.73	36	0.090513788	2.072	0.172	1.296	745.2838703	1
2000	130	0.059	7.67	37	0.094742848	2.082	0.182	1.517	847.5207834	2
2000	150	0.06	9	38	0.090664506	2.09	0.19	1.71	919.9485899	3
2000	150	0.072	10.8	38	0.096323244	2.106	0.206	2.014	1019.181987	4
2000	170	0.084	14.28	39	0.098795435	2.108	0.208	2.106	1082.150138	5
2000	170	0.08	13.6	39	0.0924205	2.124	0.224	2.418	1142.070156	6
2000	**190**	**0.09**	**17.1**	**40**	**0.092387104**	**2.124**	**0.224**	**2.48**	**1186.249806**	**7**
2000	190	0.099	18.81	40	0.095463079	2.136	0.236	2.72	1262.20582	8

Table 13: Determination of Optimum Sample Size 'n' , acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
3000	110	0.021	2.31	36	0.096849214	2.042	0.142	0.777	812.7328342	0
3000	130	0.03	3.9	37	0.095737632	2.06	0.16	1.17	1004.514674	1
3000	150	0.036	5.4	38	0.090707072	2.072	0.172	1.44	1128.797218	2
3000	170	0.039	6.63	39	0.09863957	2.078	0.178	1.599	1253.672125	3
3000	190	0.042	7.98	40	0.095911423	2.084	0.184	1.764	1338.827537	4
3000	210	0.044	9.24	41	0.096746284	2.088	0.188	1.848	1411.070739	5
3000	210	0.05	10.5	41	0.095806094	2.1	0.2	2.15	1520.233053	6
3000	**230**	**0.051**	**11.73**	**42**	**0.096029193**	**2.102**	**0.202**	**2.244**	**1586.173764**	**7**
3000	230	0.056	12.88	42	0.098746024	2.112	0.212	2.352	1658.346681	8

Table 14: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	ka(p	K(N,n,c,p)	c
4000	130	0.018	2.34	37	0.094296031	2.036	0.136	0.666	984.4105887	0
4000	170	0.023	3.91	39	0.095768703	2.046	0.146	0.897	1182.462395	1
4000	190	0.028	5.32	40	0.096927909	2.056	0.156	1.12	1341.000701	2
4000	210	0.032	6.72	41	0.093929086	2.064	0.164	1.312	1463.677938	3
4000	230	0.035	8.05	42	0.092823574	2.07	0.17	1.47	1571.928337	4
4000	230	0.04	9.2	42	0.099373149	2.08	0.18	1.68	1718.95516	5
4000	250	0.042	10.5	43	0.09674642	2.084	0.184	1.806	1799.460101	6
4000	**270**	**0.043**	**11.61**	**44**	**0.102852628**	**2.086**	**0.186**	**1.892**	**1911.490356**	**7**
4000	270	0.048	12.96	44	0.096014439	2.096	0.196	2.112	1983.184467	8

Table 15: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
5000	150	0.015	2.25	38	0.10361785	2.03	0.13	0.57	1156.120491	0
5000	190	0.02	3.8	40	0.104991998	2.04	0.14	0.8	1394.307598	1
5000	210	0.025	5.25	41	0.102163584	2.05	0.15	1.025	1577.193121	2
5000	230	0.029	6.67	42	0.097318964	2.058	0.158	1.218	1719.064147	3
5000	250	0.032	8	43	0.095946377	2.064	0.164	1.376	1847.363291	4
5000	270	0.034	9.18	44	0.101180358	2.068	0.168	1.496	1988.558348	5
5000	270	0.039	10.53	44	0.095599636	2.078	0.178	1.716	2098.462495	6
5000	**290**	**0.04**	**11.6**	**45**	**0.103717731**	**2.08**	**0.18**	**1.8**	**2242.387028**	**7**
5000	310	0.042	13.02	46	0.094020266	2.084	0.184	1.932	2279.789422	8

Table 16: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
6000	150	0.015	2.25	38	0.10361785	2.03	0.13	0.57	1331.712345	0
6000	190	0.02	3.8	40	0.104991998	2.04	0.14	0.8	1603.602317	1
6000	230	0.023	5.29	42	0.099577941	2.046	0.146	0.966	1784.143069	2
6000	250	0.027	6.75	43	0.09271351	2.054	0.154	1.161	1935.834402	3
6000	270	0.03	8.1	44	0.090683424	2.06	0.16	1.32	2075.754584	4
6000	290	0.032	9.28	45	0.09602845	2.064	0.164	1.44	2234.659445	5
6000	310	0.034	10.54	46	0.095684782	2.068	0.168	1.564	2357.047191	6
6000	**330**	**0.036**	**11.88**	**47**	**0.090818575**	**2.072**	**0.172**	**1.692**	**2441.71081**	**7**
6000	330	0.039	12.87	47	0.101247454	2.078	0.178	1.833	2645.090925	8

Table 17: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
7000	170	0.014	2.38	39	0.091007155	2.028	0.546	0.128	1478.819968	0
7000	210	0.019	3.99	41	0.090212414	2.038	0.779	0.138	1757.63961	1
7000	250	0.021	5.25	43	0.102637432	2.042	0.903	0.142	1996.222831	2
7000	270	0.025	6.75	44	0.092940463	2.05	1.1	0.15	2157.214847	3
7000	290	0.027	7.83	45	0.10654885	2.054	1.215	0.154	2387.554291	4
7000	310	0.03	9.3	46	0.095211359	2.06	1.38	0.16	2486.096071	5
7000	330	0.032	10.56	47	0.094929272	2.064	1.504	0.164	2623.45885	6
7000	**350**	**0.034**	**11.9**	**48**	**0.090147515**	**2.068**	**1.632**	**0.168**	**2718.64015**	**7**
7000	370	0.035	12.95	49	0.097996823	2.07	1.715	0.17	2896.815756	8

Table 18: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
8000	170	0.014	2.38	39	0.091007155	2.028	0.128	0.546	1644.860959	0
8000	230	0.017	3.91	42	0.096456345	2.034	0.134	0.714	1943.690162	1
8000	270	0.02	5.4	44	0.092512494	2.04	0.14	0.88	2162.189967	2
8000	290	0.023	6.67	45	0.098016015	2.046	0.146	1.035	2390.82039	3
8000	310	0.026	8.06	46	0.093302891	2.052	0.152	1.196	2554.0692	4
8000	330	0.028	9.24	47	0.098621507	2.056	0.156	1.316	2752.455274	5
8000	350	0.03	10.5	48	0.098151144	2.06	0.16	1.44	2906.096005	6
8000	**370**	**0.032**	**11.84**	**49**	**0.093093537**	**2.064**	**0.164**	**1.568**	**3012.266373**	**7**
8000	390	0.033	12.87	50	0.101961487	2.066	0.166	1.65	3220.475542	8

Table 19: Determination of Optimum Sample Size 'n', acceptance number (c) and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	p	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
9000	190	0.012	2.28	40	0.100883345	2.024	0.124	0.48	1793.406486	0
9000	250	0.0159	3.975	43	0.091658255	2.0318	0.1318	0.6837	2103.829172	1
9000	270	0.02	5.4	44	0.092512494	2.04	0.14	0.88	2370.649212	2
9000	310	0.0219	6.789	46	0.091004896	2.0438	0.1438	1.0074	2566.162989	3
9000	330	0.024	7.92	47	0.101489036	2.048	0.148	1.128	2821.31174	4
9000	350	0.026	9.1	48	0.10663794	2.052	0.152	1.248	3043.970328	5
9000	370	0.028	10.36	49	0.105684217	2.056	0.156	1.372	3216.058627	6
9000	**390**	**0.03**	**11.7**	**50**	**0.099947612**	**2.06**	**0.16**	**1.5**	**3334.135582**	**7**
9000	410	0.032	13.12	51	0.090797126	2.064	0.164	1.632	3399.962657	8

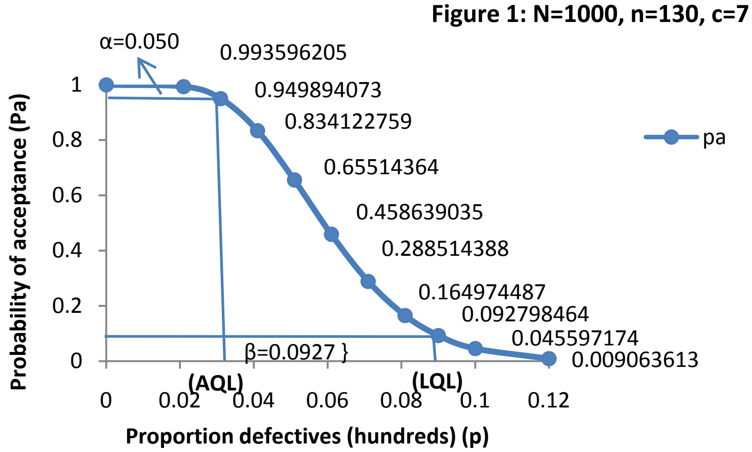
Table 20: Determination of Optimum Sample Size 'n', acceptance number (c) , and minimum average acceptance cost K(N,n,c,p) based on Limiting Quality Level (LQL). (A_1=0, R_1=0.1, R_2=2, S_1=S_2=2)

N	n	P	np	A_2	P(p)	Ks(p)	Kr(p)	Ka(p)	K(N,n,c,p)	c
10000	210	0.011	2.31	41	0.097998908	2.022	0.122	0.451	1934.645662	0
10000	250	0.015	3.75	43	0.10988575	2.03	0.13	0.645	2326.763823	1
10000	330	0.016	5.28	45	0.101097205	2.032	0.132	0.72	2521.834663	2
10000	330	0.02	6.6	47	0.102795384	2.04	0.14	0.94	2822.225087	3
10000	350	0.023	8.05	48	0.094194247	2.046	0.146	1.104	2995.797559	4
10000	370	0.025	9.25	49	0.098434766	2.05	0.15	1.225	3222.021306	5
10000	390	0.027	10.53	50	0.09699343	2.054	0.154	1.35	3395.799804	6
10000	**410**	**0.029**	**11.89**	**51**	**0.091161241**	**2.058**	**0.158**	**1.479**	**3513.866155**	**7**
10000	430	0.03	12.9	52	0.100903211	2.06	0.16	1.56	3768.901222	8

For various lot sizes 'N', the Optimum Single Sampling Plan (n,c) by using Prior Binomial Distribution, the Operating Characteristics Curve are presented from figure 1 –10

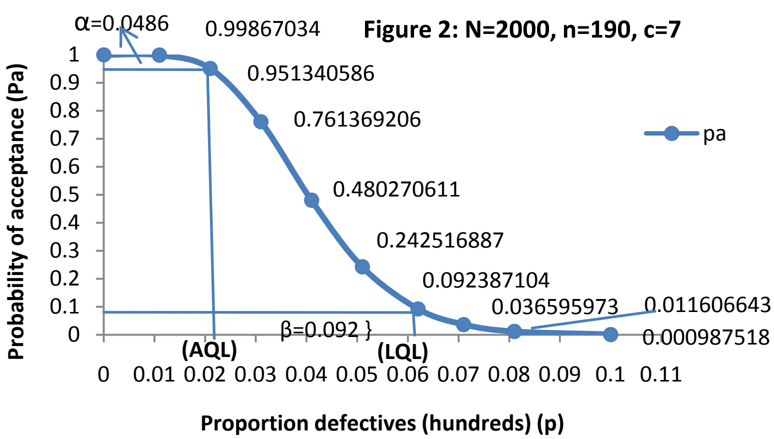
Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.021	0.993596205
0.031(AQL)	**0.949894073**
0.041	0.834122759
0.051	0.65514364
0.061	0.458639035
0.071	0.288514388
0.081	0.164974487
0.09(LQL)	**0.092798464**
0.1	0.045597174
0.12	0.009063613

Figure 1: N=1000, n=130, c=7



Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.011	0.99867034
0.021(AQL)	**0.951340586**
0.031	0.761369206
0.041	0.480270611
0.051	0.242516889
0.062(LQL)	**0.092387104**
0.071	0.036595973
0.081	0.011606643
0.1	0.000987518

Figure 2: N=2000, n=190, c=7



Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.009	0.99872966
0.0175(AQL)	**0.948952832**
0.029	0.648488722
0.038	0.351031612
0.046	0.166167589
0.051(LQL)	**0.096029193**
0.061	0.027472996
0.071	0.006625358
0.081	0.00139006
0	1

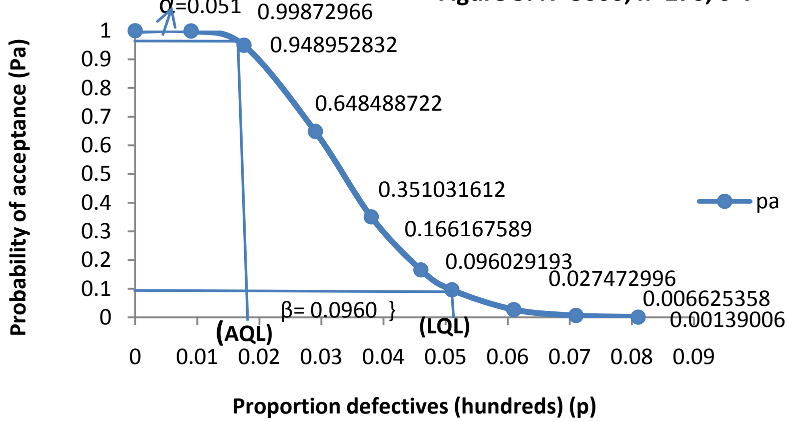


Figure 3: N=3000, n=270, c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.01	0.9936765
0.015(AQL)	**0.947207922**
0.02	0.823609333
0.025	0.636388777
0.03	0.436778401
0.035	0.269294091
0.039	0.170894648
0.043(LQL)	**0.102852628**
0.05	0.037888275

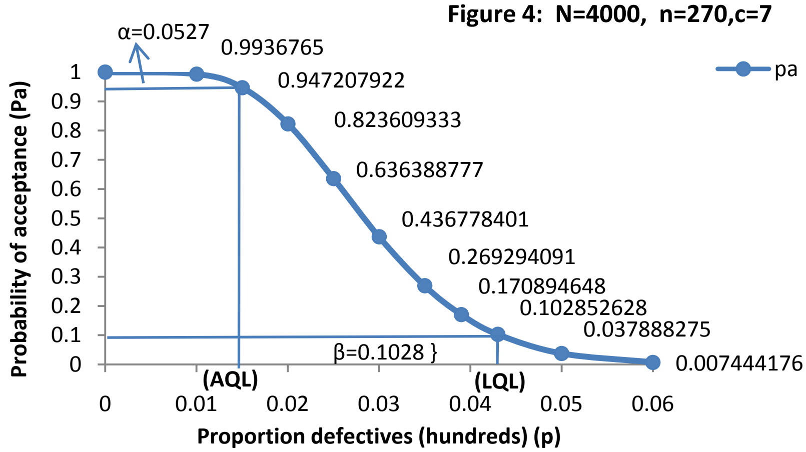


Figure 4: N=4000, n=270,c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.005	0.999873473
0.01	0.990499652
0.014(AQL)	**0.946500798**
0.02	0.772645794
0.025	0.561052756
0.031	0.321385182
0.04(LQL)	**0.103717731**
0.045	0.048896249
0.052	0.015146153

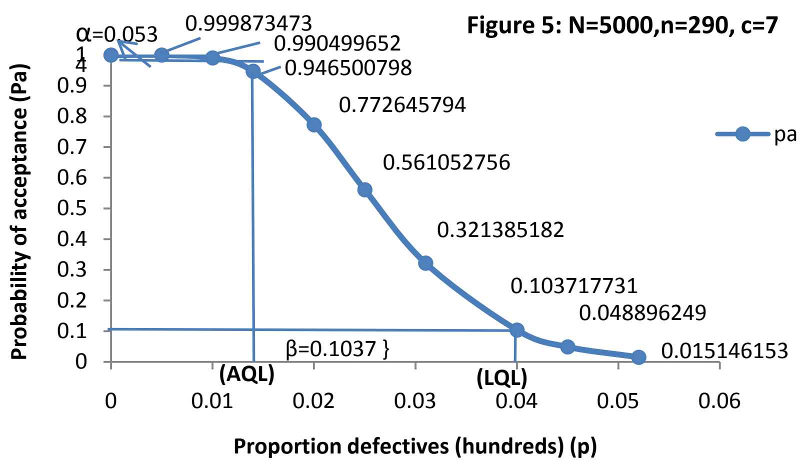


Figure 5: N=5000,n=290, c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.004	0.999932759
0.008	0.994395785
0.012(AQL)	**0.952270029**
0.015	0.873387954
0.02	0.658685262
0.025	0.41650417
0.03	0.225269429
0.036(LQL)	**0.090818575**
0.04	0.045479092
0.045	0.017669518

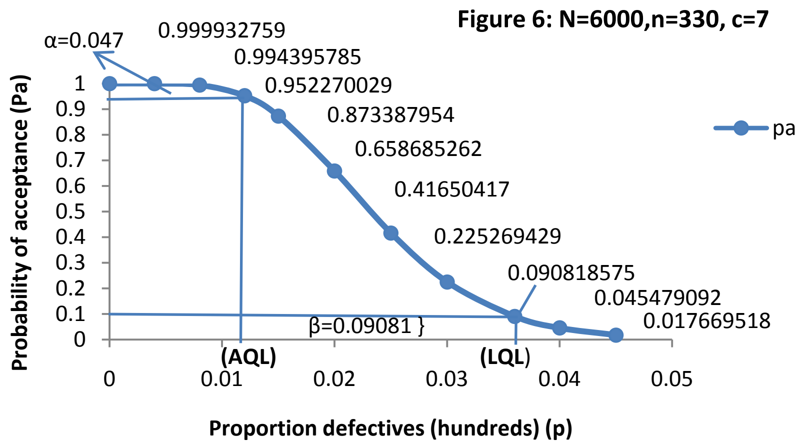


Figure 6: N=6000,n=330, c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.01	0.97393602
0.0109(AQL)	**0.960021145**
0.0115	0.948405829
0.015	0.840766342
0.02	0.598718904
0.025	0.351236742
0.034(LQL)	**0.090147515**
0.04	0.029203153
0.05	0.003346043

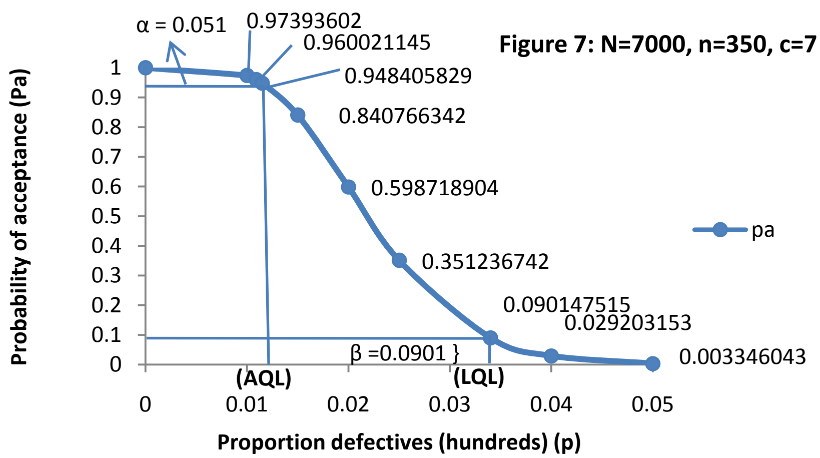


Figure 7: N=7000, n=350, c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.006	0.997991709
0.011(AQL)	**0.945603114**
0.016	0.756108669
0.021	0.484328511
0.026	0.252948016
0.032(LQL)	**0.093093537**
0.039	0.022893
0.05	0.001727565
0.06	0.00012257

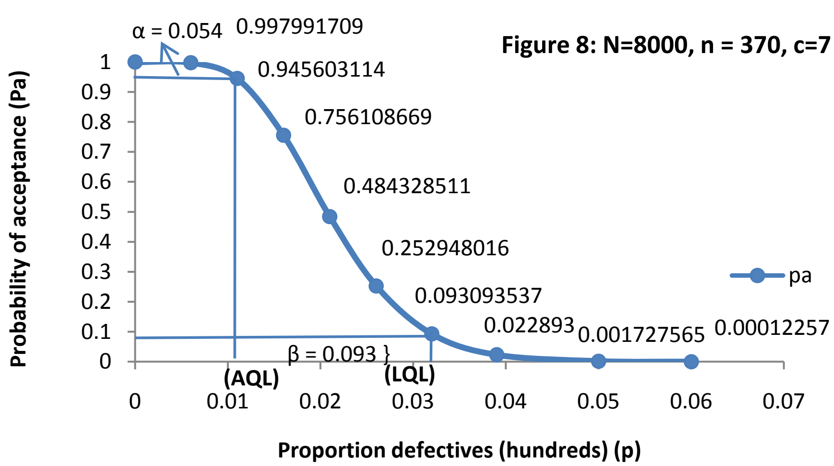
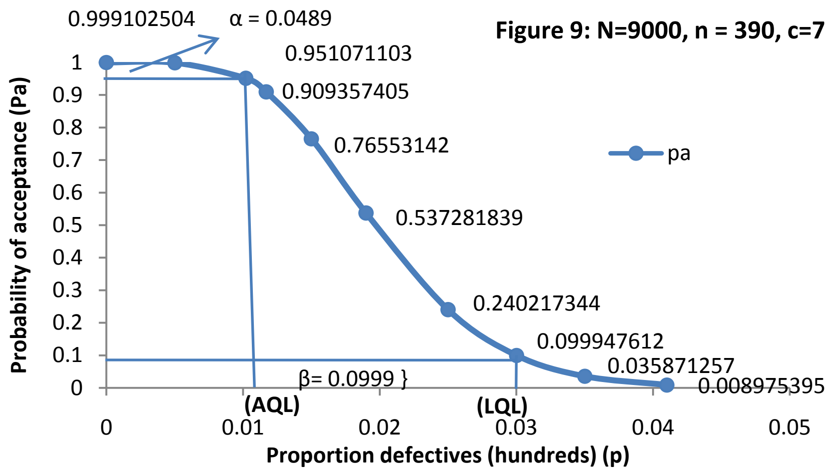


Figure 8: N=8000, n = 370, c=7

Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.005	0.999102504
0.0102(AQL)	**0.951071103**
0.0117	0.909357405
0.015	0.76553142
0.019	0.537281839
0.025	0.240217344
0.03(LQL)	**0.099947612**
0.035	0.035871257
0.041	0.008975395



Proportion Defective (p)	Probability of 7 (or) less defective(Pa)
0	1
0.004	0.999706538
0.0097(AQL)	**0.95108473**
0.011	0.913728808
0.015	0.723994061
0.02	0.423760085
0.029(LQL)	**0.091161241**
0.035	0.024182074
0.051	0.000325916
0.05	0.000437455

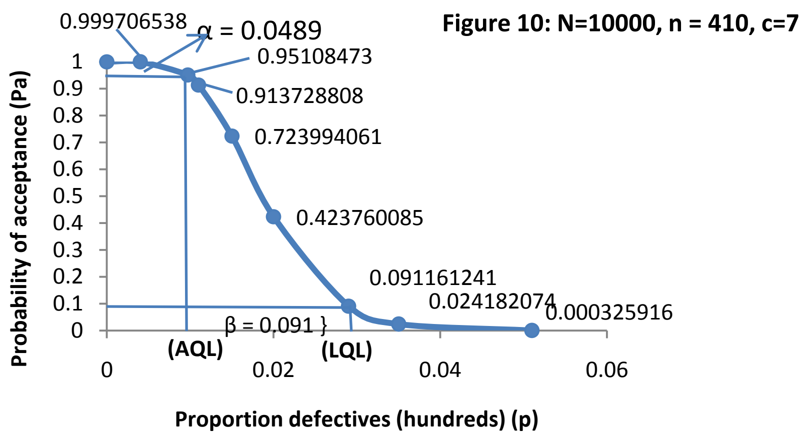


Table 21: The following table represents the Optimum Single sampling plan (n,c) based on Prior Binomial Distribution.

Lot Size N	Sample Size n	Average Lot Quality p	Average Acceptance Cost K(N,n,c,p)	Acceptance Number c	A_2
1000	130	0.129	657.83	6	37
	130	0.13	621.3717	7	37
	130	0.13	642.8208	8	37
2000	170	0.09	910.9353	6	39
	190	0.1	967.6134	7	40
	190	0.1	979.0334	8	40
3000	210	0.07	1174.291	6	41
	230	0.08	1231.032	7	42
	230	0.08	1253.642	8	42
4000	250	0.06	1411.443	6	43
	270	0.07	1485.83	7	44
	270	0.07	1507.125	8	44
5000	270	0.054	1655.763	6	44
	290	0.058	1689.352	7	45
	310	0.062	1742.351	8	46
6000	310	0.05	1845.489	6	46
	330	0.06	1957.026	7	47
	330	0.06	1974.662	8	47
7000	330	0.049	2116.640	6	47
	350	0.05	2113.95	7	48
	370	0.05	2169.36	8	49
8000	350	0.04	2276.208	6	48
	370	0.05	2332.658	7	49
	390	0.05	2382.454	8	50
9000	370	0.039	2524.08	6	49
	390	0.04	2540.015	7	50
	410	0.05	2603.997	8	51
10000	390	0.039	2695.53	6	50
	410	0.04	2703.509	7	51
	430	0.04	2800.238	8	52

From the above table clearly shows that the average lot quality 'p' is higher at c=7 and the corresponding average acceptance cost K(N,n,c,p) are very less for various lot sizes 'N' at the minimum level of A_2, cost of handling defective items in assembling and disassembling.

Table 22: The following table shows the consolidated **AQL**, **LQL** and **Average Lot Quality** values based on the optimum values of the sample size **'n'** and acceptance number **'c'**.

N	n	AQL	LQL	Quality Level		Average Acceptance Cost k(N,n,c,p)			1-α	1-β	A_2	c
				Based on Producer's Risk	Based on Consumer's Risk	LQL Based	Average Lot Quality p	AQL Based				
1000	130	0.031	0.09	0.949894073	0.092798464	773.2407254	621.37	1223.011726	0.95	0.908	37	7
2000	190	0.021	0.09	0.951340586	0.092387104	1186.249806	967.61	1846.90467	0.952	0.908	40	7
3000	270	0.0175	0.051	0.948952832	0.096029193	1586.173764	1231.03	2419.159606	0.949	0.904	42	7
4000	270	0.015	0.043	0.947207922	0.102852628	1911.490356	1485.83	2905.53534	0.948	0.898	44	7
5000	290	0.014	0.04	0.946500798	0.103717731	2242.387028	1653.72	3428.925417	0.947	0.897	45	7
6000	330	0.012	0.036	0.952270029	0.090818575	2441.71081	1974.66	3746.723268	0.953	0.91	47	7
7000	350	0.0115	0.034	0.948405829	0.090147515	2718.64015	2113.95	4231.65957	0.949	0.91	48	7
8000	370	0.011	0.032	0.945603114	0.093093537	3012.266373	2332.65	4687.634885	0.946	0.907	49	7
9000	390	0.0102	0.03	0.951071103	0.099947612	3334.135582	2540.01	5014.926168	0.952	0.901	50	7
10000	410	0.0097	0.029	0.95108473	0.091161241	3513.86615	2703.50	5396.074731	0.952	0.909	51	7

The above table clearly indicates that, for different lot sizes 'N' the selection of Optimum Bayesian Single Sampling Plans (n,c) based on prior binomial distribution consist of minimum average acceptance cost K (N,n,c,p) at c=7 by minimizing A_2, cost of handling defective items in assembling and disassembling and also producer's risk and consumer's risk are at minimum level. The average acceptance cost K (N,n,c,p) associated with the average lot quality 'p' always lies between the cost associated with AQL and LQL. Therefore the determination of single sampling plan (n,c) based on Bayesian prior binomial distribution is optimum and this single sampling plans satisfies the above mentioned conditions (1) and (2).

Table 23: The following table shows the estimated **AQL, LQL**, **Pa(p)** and Average Acceptance Cost **K (N, n, c, p)** values when **α=5%** and **β= 10%.**

Lot Size (N)	Sample Size(n)	α	α value in percentage	β	β value in percentage
1000	130	0.050	5.0	0.092	9.2
2000	190	0.048	4.8	0.092	9.2
3000	230	0.051	5.1	0.096	9.6
4000	270	0.052	5.2	0.102	10.2
5000	330	0.053	5.3	0.103	10.3
6000	330	0.047	4.7	0.090	9.0
7000	350	0.051	5.1	0.090	9.0
8000	370	0.054	5.4	0.093	9.3
9000	390	0.048	4.8	0.099	9.9
10000	410	0.048	4.8	0.091	9.1

The values of AQL and LQL are converted into percentages and are presented in table 23. A better single sampling plan would have a lower producer's risk and a lower consumer's risk. When AQL and LQL values are lies within (α = 0.05, β = 0.10) then the plan is better single sampling plan. Hence in our case, for various lot sizes 'N', the percentages values of AQL and LQL are lies within α = 5%, β = 10%. This indicates that the selection of single sampling plans based on prior binomial distribution is further strengthened to be optimum.

III. Conclusion

In this study, various lot sizes 'N', Bayesian single sampling plans based on prior Binomial distribution, the optimum design parameters (n,c) are determined using the two points on the Operating Characteristics approach. In the proposed plan, the average acceptance cost K (N,n,c,p) is minimized based on the condition that the producer's risk and consumer's risk are minimum and also A_2 the cost of handling the defective item in assembling and disassembling are minimum at the acceptance number c=7.

Authors

First Author – M.Nirmala, PhD Research scholar, PSG College of Arts and Science, Coimbatore -641 014. Email: nirmalastat2005@gmail.com

Second Author – R.Subramaniam, Associate Professor and Head, PSG College of Arts and Science, Coimbatore -641 014. Email: psgstat@yahoo.com

Third Author – G.Uma, Assistant Professor, PSG College of Arts and Science, Coimbatore -641 014. Email: amug072000@gmail.com

Correspondence Author – M.Nirmala, nirmalastat2005@gmail.com, 98422 31867.

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