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Analytical Proof of Bounded Rationality Theory on Unitas SACCO Members in Kenya: Bayesian Approach

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Abstract- The first of the four fundamental assumptions in economics and standard finance is that humans are rational actors, which does not hold good all the time. This anomaly begot behavioural finance that recognizes instances of irrational decision making in human beings. Rationality bounds in financial decision making as espoused in bounded rationality theory need to be determined to reflect how humans actually behave rather than how they should behave; to pave way for modification of classical economics and standard finance theories. This analytical proof of bounded rationality utilizes LOT-R parameterized cumulative prospect theory decision weights function to transform subjective to objective probabilities. Human intrinsic perceptions and self proclaimed prospects measured on a 9 point Likert scale are converted into subjective probabilities. I construct a rationality measuring instrument on a 0 – 1 scale using the probabilities of making a rational decision, of observing economic well being increase after an irrational decision and that of observing economic well being increase after a rational decision as inputs. Thereafter, I show that the multiperiod model forms an absolutely converging sequence in the open interval (0, 1), hence bounded. The instrument is a Bayesian learning model whereby wealth movement is the observable dimension variable and the rationality to be determined is the unobservable dimension variable. Stochastic discrete time case in a binomial setting is explored. The concept of entropy from the second law of thermodynamics in physical chemistry – information theory version is expected to feature prominently and to guide recommendations likely to yield global lessons.

Index Terms- rational, irrational, entropy, information theory, perception, human intrinsic

I. Introduction

Consistent departure from rational decision making by humans led to the birth of behavioural economics and behavioural finance as pointed out by various behavioural economists like Robert Shiller (1994). This notion formed the basis of the work of Simon (1996) who propounded the theory of bounded rationality characterization in decision making processes. If rationality is limited as opposed to complete as assumed by standard finance, it means there is inherent irrationality in financial decision making. Quantification of rationality after the economic agents indefinitely update their Bayesian learning in a binomial setting is the subject matter of this study. By reason of most forecasts for phenomena being stochastic rather than deterministic, the study considered a stochastic environment as more realistic for use.

Behavioural finance is the study of the influence of psychology on the behaviour of financial practitioners and the subsequent effect on markets Sewell (2001). Most other scholars like Shefrin (2000), Thaler (2000), describe behavioural finance as the interaction of psychology with financial actions and performance of “practitioners” (all types of investors). Behavioural finance was born of inadequacy of efficient market hypothesis in financial markets. As such, the discipline’s definition was initially contextualized in financial markets where the actors are investors (Ross, 2014). However, it is clear that human biases and errors are not only exercised by investors or restricted to financial markets. This observation led to a panel of behavioural proponents expanding the definition in 2009. They defined behavioural finance as the study of how psychology impacts financial decisions in households, markets and organizations (Werner De Bondt, Gulnur Muradoglu, Hersh Shefrin, and Sotiris K. Staikouras, 2009). This is the definition that this study predominantly adopted, since it covers households (individuals) and does not confine its application to financial markets. Behavioral finance combines the disciplines of psychology and economics to explain why and how people make seemingly irrational or illogical decisions when they spend, invest, save and borrow money (Belsky and Gilovich, 1999). This additional part of expenditure and savings was particularly important to this study.

Unitas SACCO, initially Murang’a Tea SACCO was at the beginning operating on an agricultural common bond. Two decades later, the SACCO has expanded its common bond to offer financial services to formally employed members of the public; essentially gaining the financial services common bond. Agricultural and Financial services common bonds account for about 82% of registered deposit-taking SACCOs in Kenya (Gweyi, Ndwiga and Karagu, 2005). Three questions are posed to the respondent. First, they were required to state their level of agreement with the statement of being complete rational actors. Secondly, they were required to state their level of agreement with the claim that they are usually lucky to post economic increase after making irrational financial decisions and lastly, state their level of agreement with the claim that they do not post any economic increase after making rational financial decisions. After observing economic increase, economic agents update to reinforce good decisions or update by avoiding bad decisions after observing an economic decrease.

The rest of this paper is divided into three sections: first, statement of the problem, which forms part of introduction and contains the objective of the paper. Secondly, review of literature on bounded rationality theory, Bayesian decision theory and prospect theory. Finally, methodology section that presents Sample data, then develops the Bayesian learning process culminating in Bayesian rationality model and the findings that confirm limitation of human rationality.

1.1 Statement of the problem

Consensus has been struck by scholars in the last three decades that the assumption of humans as complete rational actors, onto which standard finance is anchored, is no longer a valid argument. Moreover, it has been established that deviation from complete rationality is consistent (Shiller, 1994). This means that standard finance models are inaccurate both as descriptive and predictive instruments. However, it does not mean that all decisions are irrational (Binmore, 2015). The problem is determination of whether complete rationality level in financial decision making is achievable through learning over time to eradicate all instances of irrationality in Unitas SACCO. The research aimed to establish how individual cognitive and affective dispositions, reactions and inclinations to economic environment determined the level of rationality in Kenyan SACCOs. Behavioural finance proponents increasingly feel that a vital component of human nature has not been factored in both macro and microeconomic models (Simon, 1996); which would otherwise yield better financial models. It is known that education is directly proportional to rationality (Katsikopoulos, 2014), but the exact mathematical function has not been determined. Had the function been determinable, human resource practitioners would project financial decision making productivity more precisely, including the rate of training required to reach a certain level of rationality. Derivation of actual average rationality level as a function of age was pertinent, which can alleviate grave micro-level financial planning fallacy effects (Kahneman and Tversky, 1974). The objective of this paper is to show quantitatively that rationality of humans as financial decision makers is bounded.

II. Review of Literature

2.1 Bounded Rationality Theory

As late as early 2015, Ken Binmore in the *Handbook of Game Theory* recounts 22 essays on 22 different notions of rationality cited in the *Oxford Handbook of Rationality* and notes that all these are divergent from the neoclassical rationality notion which he strongly supports. This neoclassical orthodoxy is the rationality depicted in the state preference theory of Arrow and Debreu (1959) which forms the reference point of rationality view in this study. He further agrees with Simon (1976) that the neoclassical rationality orthodoxy is deficient in that it is substantive; that is it is concerned with what decisions are made rather than how they are made (Binmore, 2015). According to classical and neoclassical economic theories including the rational choice theory (1961), the main goal of decision making is to be rational by first collecting all the relevant information regarding the issue under investigation, evaluate all alternatives and finally choose the optimal one (Kalantari, 2010); this is grossly incongruent with efficient market hypothesis in that even at the eve of decision taking, there will still be information not factored in the decision to be taken.

The combined assumptions of rationality made by classical economists do not hold all the time; perhaps only to a given extent, leading to bounded rationality (Simon, 1996b). Numerous contributions in bounded rationality have since been made with the notion taking various dimensions. Bounded rationality has been described as incapable of speaking with one voice; by reason of having been researched in various fields such as finance, economics, psychology, engineering, and management. There are multiple views of bounded rationality as many authors including Rubinstein (1998) have pointed out (Katsikopoulos, 2014). Katsikopoulos has distinguished two cultures in discussing bounded rationality: the idealistic and the heuristic (pragmatic) cultures. In idealistic, utility theory has been modified by including elements of decision weights function, while in pragmatic culture, people are assumed to ignore information and use simple rules of thumb.

This study acknowledges the two cultures and aims to explore the limit of the human person's financial decision making rationality limits when all learning has taken place. It posits that humans as financial decision making agents cannot sustain wealth creation if they are not sufficiently rational; a person suffering from mental disorder cannot run a wealth creating entity. A certain minimum level of rationality is imperative. Rationality can be enhanced through nudging or education (Katsikopoulos, 2014). Whichever the choice, this study intends establish whether it is possible to achieve complete rationality in life.

In the world over, lots of investments are usually made to educate nationals of various countries and to train employees of organizations to enhance productivity. This seeks to equip them with theoretical reasoning in a structured manner. Not forgetting that all life is about learning, persons also acquire theoretical reasoning from general interaction with the environment; mainly fellow human beings. The theoretical reasoning so acquired is aimed at equipping the individual with rational beliefs about the world using rational inferences (Koehler and Harvey, 2004). But even after the acquisition, the actors may decide to utilize the information (rational beliefs) in their subsequent action (which Koehler and Harvey call judgement), or not. Persons who will update and those who will not update the information subsequently are regarded rational. This is known as instrumental rationality in experimental psychology (Koehler and Harvey, 2004). Instrumental rationality avoids condemnation of individuals for not updating information so acquired from the environment. It argues that the difference is mainly caused by different individual goals. The notion of bounded rationality has been interpreted by this study in the light of the rate of updating new learning (financial information) in individuals and organizations. It is this rate whose optimization leads to achievement of individual and organizational goals.

From the field of cognitivism in psychology, a number of theorists claim that most of our mental life is devoted to the task of creating and updating mental situation models that allows us to navigate through life. Since these mental situation models are the causal mediators of stimulus – response relationships, we must study these mental models to predict and explain behaviour (Hastie and Pennington, 1995). Hastie and Pennington further concede that there is considerable agreement that cognitive analysis occurs at one level of a system of theoretical levels that comprises levels above the cognitive level (e.g., a level at which theories concerned with optimally rational solutions to behavioral-environmental problems are framed) and levels below the cognitive level (e.g., a level at which cognitive: processes are implemented in the neural medium of the brain; J. R. Anderson, 1987; Marr, 1982; Newell, 1990; Pylyshyn, 1984). This admission portends that rates of updating may not be equal to rate at which new information is availed to the brain neither is it regular.

In this study, the researcher's contention is that the difference in magnitude between the level of cognitive analysis and implementation has not been determined. Different organizations may invest equal amounts of money in training equally qualified and experienced employees only to post differing results. The updating rate parameter has been grossly ignored and for a long time. Sub-Saharan countries are characterized by huge educational and exposure disparities. Without cognizance of the updating rate, governments and organizations are likely to waste resources due to poor planning. For instance, if the citizenry of a certain region update financial lessons after 8 months on average, designing a public policy awareness programme that lasts less than 8 months means a waste of funds; learning cycles should be scheduled in multiples of 8 months to allow the target group to update information so received into their financial lifestyles effectively. The key assumption in this paper is that economic agents update regularly.

2.2 Bayesian Decision Theory

This theory is invariably the most important tool in this study which will factor in the human decision making learning processes through assumed regular updating. An example is a survivor who swims to an island after their ship sinks. He initially sees little chances of human habitation in the island given that no house-like structures exist. However, the sight of human foot prints substantially raises hope human existence to the survivor leading to subsequent revision of his subjective belief of human existence. This is the process of updating. Bayesian decision theory stresses accumulation of knowledge about parameters in a synthesis of prior knowledge with the data at hand. Bayesian methods are used in econometrics, including applications in linear regression, serial correlation in time series and simultaneous equations have been developed since 1960s; with the seminal work of Box and Tiao (1973) and Zellner (1971) (Congdon, 2003).

SACCO cooperators as individuals make financial decisions, towards increasing their wealth in line with the super-ordinate goal of a firm; that is current wealth maximization. These decisions are made at convenient intervals of time (discrete). The most important argument here is that for wealth to increase, the decisions made must be sufficiently and consistently rational. Besides, updating of the information learned should take place to improve subsequent decision quality. Further, some of the decisions made may be irrational by reason of insufficient information hereinafter referred to as informational irrationality. Bayesian statistics deals with two dimensions: one is the observable variable dimension (OVD) and the other is unobservable variable dimension (UVD) (Bolstad, 2007). Increase/decrease in wealth is observable while rationality is unobservable. Yet, the decision maker cannot for a specific instance, tell whether a rational decision caused the increase or an irrational one or the increase just happened (say out of previous decisions or was just a windfall). Quantities for these dimensions are to be solved using Bayes theorem.

2.3 Prospect theory

Prospect Theory is a psychological account that describes how people as individuals make decisions under conditions of uncertainty. These may involve decisions about anything where the outcome of the decision is risky and uncertain. The decisions range from deciding whether or not to: enroll for a Doctorate Program, buy a lottery ticket, undergo chemotherapy treatment, to marry a particular prospective partner, or to invest in life insurance among others. Prospect Theory predicts that people go through two distinct stages when deciding between risky options like these. In the first stage, decision makers are predicted to edit a complicated decision into a simpler decision, usually specified in terms of gains versus losses.

In the second stage, financial decision making agents choose between the edited options available to them. This choice is based on two dimensions: the apparent value of each option, and the weight subjectively assigned to those values or options. These two results into the overall value and its weight are then combined by the decision maker, and the option with the highest combined value is chosen by the decision maker. The most interesting feature of prospect theory for most psychologists is that it predicts when and why people will make decisions that differ from perfectly rational or normative decisions, and has therefore featured prominently in explanations of why people make a variety of evidently outright bad decisions in daily life.

Prospect Theory was a notable departure from existing theories before the 1970s dominated by normative theories that prescribe how people "ought" to make decisions in a perfectly rational way, by offering a descriptive theory of how people actually make decisions, rather than how they ought to do so. The simplest way to choose between risky options is to choose the option with the highest expected value, (the likelihood that an option will occur, multiplied by the value of that option).

Imagine, for instance, that you are deciding whether to pay $1 for a lottery ticket that offers a 10% chance of winning $10. The expected value of this lottery ticket is $1 (0.1 x $10), the same as the cost of the ticket. Rationally speaking, you should therefore be perfectly indifferent about buying this ticket or not. The problem, noted by both economists and psychologists, is that rational theories did not always describe people's actual behavior accurately. It was noted that few people would actually purchase the lottery ticket. The certain loss of a dollar simply does not compensate for the 10% change of winning $10 and a 90% change of winning nothing.

In general, research found that people were more averse to taking risks that the expected value of outcomes would predict. Propounded by two Israelis, Daniel Kahneman and Amos Tversky (1979), prospect theory captured observed human behavioural preferences and was able to answer question why people go for insurance and engage in gambling at the same time. A modification of the original prospect theory was done by the authors in 1992 and named cumulative prospect theory to avoid the possibility of choosing dominated gambles limitation that

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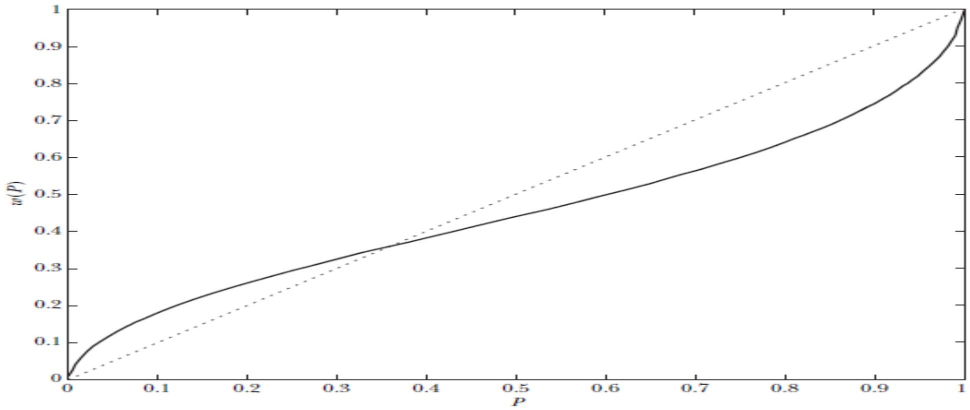
characterized the original prospect theory. Following are the value and decision weights functions.



Notes: The graph plots the probability weighting function proposed by Tversky and Kahneman (1992) as part of cumulative prospect theory, namely $w(P) = P^\delta/(P^\delta + (1-P)^\delta)^{1/\delta}$, where P is an objective probability, for two values of δ. The solid line corresponds to $\delta = 0.65$, the value estimated by the authors from experimental data. The dotted line corresponds to $\delta = 1$, in other words, to linear probability weighting.

Figure 2.1: cumulative Prospect Theory Decision Weights function
Source: Wakker (2010), *Prospect theory for Risk and Ambiguity*

Under cumulative prospect theory, by contrast, the gamble is evaluated as:

$$\upsilon(*) = \sum_{i=m}^{n} \pi_i \, \mathrm{U}(x_i)$$ ………………………………………….. (2.1)

, where $v(\cdot)$, the value function, and π_i are decision weights. Decision weights are determined by the following single parameter equation:

$$\omega P = \frac{P^{\delta}}{\left\{P^{\delta} + (1-P)^{\delta}\right\}^{1/\delta}}$$ …………………………….. (2.2)

, where ωP is subjective decision weight while P is the objective probability. δ is a measure of individual optimism or pessimism. This formulation illustrates the four elements of prospect theory:

1. Reference dependence,
2. Loss aversion,
3. Diminishing sensitivity, and
4. Probability weighting.

First, in prospect theory, people derive utility from gains and losses, measured relative to some reference point, rather than from absolute levels of wealth. We are more attuned to changes in attributes such as brightness, loudness, and temperature than we are to their absolute magnitudes. This explains why this study is structured around changes in net worth of an entity the premise on which subjective prospects will be solicited.

In cumulative prospect theory, the weighting function is applied to cumulative probabilities. Notably, probability weighting leads the individual to overweight the tails of any distribution. Under cumulative prospect theory, the unlikely state of the world in which the individual gains or losses $5,000 is over weighted in his mind, thereby explaining these choices. Kahneman and Tversky emphasize that the transformed probabilities π_i do not represent erroneous beliefs; rather, they are decision weights. Subsequent to Tversky and Kahneman's (1992) paper on cumulative prospect theory, several studies have used more sophisticated techniques, in conjunction with new experimental data, to estimate the value function $v(\cdot)$ and the weighting function $w(\cdot)$ more accurately (Gonzalez and Wu 1999; Abdellaoui 2000; Bruhin, Fehr-Duda, and Epper 2010). They provide especially strong support for subjective probability weighting. On the strength of this evidence, this study will use the decision weights function in transforming the rationality determinants which are subjective prior probabilities in respect of wealth increase or decrease and rationality level prospects of the units of observation into objective probabilities instead of the calibration procedure used by Ramsey (1926). Its enormous advantage is that all subjective probabilities can be transformed by the continuous decision weights function for use in Bayesian learning model described in the following section under Bayesian decision theory.

III. Methodology

The greatest drawback of using Bayesian probabilities on humans is the subjective nature of their responses which introduces a huge element of bias (Wang, 2003; Binmore, 2015). By transforming observed responses through cumulative prospect theory decision weights function (Kahneman & Tversky, 1992), into objective probabilities, objective conclusions were made. This is a single parameter function, where the parameter is a measure of the respondents level of optimism/pessimism on a Life Orientation Test – Revised (Scheire, 1994). Derivation of the parameter necessitated inclusion of a ten-question test and appropriate scoring as shown in appendix. The 9 point Likert is transformed into a 0-8 continuous scale by deducting 1 from every observation, without which the 0-1 interval would be lost.

LOT-R mean value is interpolated within 0.61 and 0.69 to obtain the value of the decision weights function.

3.1 Data presentation

Table 3.1: Summary of data collected

S/N	Ran No.	Mean Age	Gender	Employment status	Education Level	LOT-R score/24	Current Rationality/8	Pr(Inc|Rat=0) April 2016	Pr(Dec|Rat=1) April 2016
1	135	42	M	SE	Diploma	21	8	2	1
2	174	47	M	E		16	7	3	6
3	155	32	M	SE		19	9	7	7
4	188	37	F	SE	H. School	17			
5	191	47	F	SE	Diploma	21	7	0	7
6	178	37	M	SE	H. School	24	8	8	8
7	181	32	F	SE	H. School	20	8	8	8
8	176	42	F	SE	H. School	19	7	7	0
9	175	37	F	SE	Diploma	12	8	7	8
10	182	37	F	SE	H. School	16	8	2	
11	180	32	F	SE	Diploma	15	5	3	1
12	190	32	F	SE	H. School	20			
13	177	37	F	SE	H. School	17	7		
14	179	47	F	SE	H. School	15	8	8	1
15	156		F	SE	H. School		7		
16	157	47	F	SE	H. School	17	7	7	1
17	170	27	F	SE		11	7		
18	158	37	M	SE	H. School	14	7	3	5
19	172	37	F	SE	H. School	13			
20	171	37	F	SE	H. School	13			
21	173	37	F	SE	H. School	14			
22	139	57	M	E	H. School	20	7	1	6
23	167	42	M	SE		23	7	7	6
24	142	32	M	SE	H. School	12	7	0	7
25	143	32	M	SE	H. School	23	7	7	0
26	85	27	M	SE	H. School	13	3	1	6
27	138	42	F	SE	H. School	19	7	0	6
28	152	37	M	SE	Diploma	17	7	0	0
29	145		F	SE	Diploma	15	4		
30	149	27	M	SE		17	7		
31	148	37	F	SE	H. School	15	7		
32	140	37	F	SE		11	6	7	7
33	153	32	M	SE		20	7	7	8
34	141	27	M	SE	Diploma	15	5	2	4
35	166	32	F	SE	Diploma	16	7	1	2
36	147	32	F		Diploma	16			
37	168	37	F	SE	H. School	19	7	7	
38	169	47	M	SE		18	8	6	

39	73	37	M	SE	H. School	16	7	6	1
40	74	37	F	SE	H. School	15	4	6	1
41	72	37	F	SE	H. School	19	8	8	1
42	76	27	F	SE	H. School	20	8	7	0
43	75	32	F	SE	H. School	23	8	0	0
44	55	37	F	SE		23	8	6	0
45	48	37	F	SE	H. School	19	8	1	1
46	185	37	M	SE	H. School	20	3	2	0
47	52	32	M	SE	Diploma	15	6	8	1
48	50	32	F	E	Diploma	24	2	2	0
49	49	42	M	SE	Diploma	14	6	5	3
50	154			SE	H. School	24	8	6	0
51	54	42	F	SE	H. School	19	8	0	0
52	46	37	M		Bachelors	20	8	3	2
53	51	37	M	SE	H. School	13	7	4	7
54	187	42	M	SE	Diploma	21	8	7	6
55	45	32	M	SE	H. School	14	8	1	1
56	183	32	M	SE	H. School	21	7	1	1
57	53	32	M	SE	Diploma	17	6	7	1
58	86	42	M	SE	H. School	13	8	7	8
59	44	45	F	SE	H. School	15	7	0	3
60	47	27	M	SE	H. School	11	8	0	3
61	11	37	F	SE	H. School	16	7	5	1
62	18	37	F	SE	H. School	22	7	1	1
63	16	42	F	SE	H. School	16	9	5	0
64	19	47	F	SE	H. School	21	6	7	5
65	41	52	M	SE	H. School	14	7	0	0
66	10	37	F	SE	Diploma	19	8	7	7
67	7	47	F	SE	Diploma	19	7	5	7
68	8	37	F	SE	Diploma	19	6	7	1
69	6		M	SE	H. School	17	1	0	0
70	39		M		H. School	23	8	8	0
71	136	52	M	SE	H. School	18	7	1	2
72	82	47	M	SE	H. School	14	7	1	2
73	83	52	M	SE	H. School	15	8	0	0
74	81						8	0	1
75	137	57	M	SE	H. School	15	8	2	4
76	37	37	M	SE	h. School	16	8	1	1
77	42	52	M	SE	H. School	12	7	0	3
78	43	37	M	SE	H. School	11	8	0	6
79	25	37	M	E	Diploma	18	8	8	1
80	20	42	F	SE	H. School	21	8	7	1
81	26	37	F	SE	H. School	22	8	8	0
82	77	37	M	SE	H. School	22	8	5	0
83	27	47	F	SE	H. School	22	8	8	0

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84	5	37	F	SE	H. School	21	8	1	1
85	21	32	M	SE	Bachelors	20	7	4	3
86	33	42	F	SE	H. School	17	7	1	0
87	69	37	F	SE	H. School	19			
88	31	52	F	SE	Diploma	18	8	1	7
89	32	52	F	SE	H. School	16	8	0	0
90	2	22	M	SE	Diploma	20	8	8	0
91	13	42	M	SE	H. School	17	8	1	0
92	14	37	F	SE	H. School	17			
93	96	42	M	SE	H. School	19	7	7	1
94	4	32	F	SE	Bachelors	13	7	7	5
95	24	47	F	SE	H. School	13	3	0	4
96	1	42	M	SE	H. School	19	8	2	0
97	12	47	F	SE	H. School	12	4	1	7
98	71	32	F	SE	H. School	20	7	3	2
99	150	27	M	SE	Diploma	8	0	8	0
100	67	32	F	SE	H. School	18	0	6	4
101	97	42	F	SE	Diploma	17	8	7	8
102	95	42	M	SE	H. School	14	7	5	6
103	91	37	M	SE	H. School	20	7	0	0
104	92	27	M	SE	H. School	19	6	7	7
105	93	37	M	SE	H. School	13	6	8	
106	89	42	M	SE	H. School	17	7	1	6
107	94	32	M	SE	H. School	13	5	5	1
108	90	47	F	SE	H. School	20	7	7	6
109	88	42	M	SE	H. School	20	7	3	1
110	87	27	M	SE		17	7	7	7
111	23	37	M	SE	Diploma	17	2	5	7
						***17.29**	**6.7777**	**4.0103**	**2.914**
						δ=0.63	**r=.847**	**q=.501**	**p=.6357**

Notes to the data:

1. Ran refers to the assigned serial number on the questionnaire
2. SE means self employed while E means employed
3. H. School refers to high school; M, F stand for male and female genders
4. Gaps in the table represent questions not responded to.
5. δ=0.6324 is obtained thus: 0.69- (17.29/24)*(0.69-0.61)
6. r = 6.777/8, q = 4.0103/8 and p = 1- 2.914/8 and represent subjective probabilities.

3.2 Data Processing Algorithm for Bounded Rationality (Sample data: n =111)
Step 1: Averaging of subjective probability

$$\Pr = \frac{1}{111}\sum_{1}^{111} x_i$$

P_s(inc/Rat=0) = 0.501, P_s(inc/Rat=1) = 0.636, P_s(Rat=1) =0.847 **.**

Step 2: Averaging LOT-R scores out of 24 the group scores

$$Lot - R = \frac{1}{24} \times \frac{1}{111}\sum_{1}^{111} x_i = \mathbf{17.29}$$

Step 3: Linear interpolated delta parameter between 0.61 (optimism) and 0.69 (pessimism)

$$\delta = 0.69 - \frac{17.29}{24}(0.69 - 0.61) = 0.6324$$

, **approximately 0.63**, into equation:

$$P_S = \frac{P_O{}^{\delta}}{\left\{P_O{}^{\delta} + (1-P_O)^{\delta}\right\}^{1/\delta}} \quad \text{……………………………………………………………. (3.1)}$$

, where P_s = subjective probabilities; P_o = objective probabilities.

Step 4: Cumulative Prospect Theory Decision Weights Function Transformed Probabilities (by iteration):

$$P_S = \frac{P^{0.63}}{\left\{P^{0.63} + (1-P)^{0.63}\right\}^{1/0.63}}$$

P_o(inc/Rat=0) = 0.623 = **q**; therefore P_o(dec/Rat=0) = 0.377 = **1-q**. P_o(inc/Rat=1) = 0.811 = **p,** therefore P_o(dec/Rat=1) = 0.189 = **1-p.**
P_o(Rat=1) =0.9665 = **r**, therefore P_o(Rat=0) = 0.0335 = **(1-r).**

Step 5:

a) **Evolution of rationality through increase/decrease in economic wealth**

Bayesian learning process using objective probabilities:

i) If a wealth increase is observed, we apply:

$$P(Rat=1\,|\,inc) = \frac{P(Rat=1)P(inc\,|\,Rat=1)}{P(Rat=1)P(inc\,|\,Rat=1) + P(Rat=0)P(inc\,|\,Rat=0)} \quad \text{……….... (3.2)}$$

, where P(Rat=1) =0.357, P(inc/Rat=1) = 0.5, P(Rat=0) = 0.643, P(inc/Rat=0) = 0.357 to give **0.9741**. This becomes the new prior in the next financial decision to be made. Meanwhile, Rationality (Γ) = P(Rat=1)(1) + P(Rat=0)(0) =0.9741(1) = 0.9741

ii) If another wealth increase is observed, we apply:

$$P(Rat=1\,|\,inc,inc) = \frac{P(Rat=1)P(inc\,|\,Rat=1)^2}{P(Rat=1)P(inc\,|\,Rat=1)^2 + P(Rat=0)P(inc\,|\,Rat=0)^2} \quad \text{….. (3.3)}$$

Substituting data values gives **0.9800.** Then, Γ = P(Rat=1)(1) + P(Rat=0)(0) =0.9800(1) = 0.9800

iii) If a wealth decrease is observed, we apply:

$$P(Rat=1\,|\,\text{dec},inc,inc) = \frac{P(Rat=1)P(inc\,|\,Rat=1)^2 P(dec\,|\,Rat=1)}{P(Rat=1)P(inc\,|\,Rat=1)^2 P(dec\,|\,Rat=1) + P(Rat=0)P(inc\,|\,Rat=0)^2 P(dec\,|\,Rat=0)}$$

……………………………………………………………………………………… (3.4)

Working out with data values gives **0.9608.** Again, Γ = P(Rat=1)(1) + P(Rat=0)(0) =0.9608(1) = 0.9608.

iv) If instead a wealth decrease was observed the first time, we apply:

$$P(Rat=1\,|\,dec) = \frac{P(Rat=1)P(dec\,|\,Rat=1)}{P(Rat=1)P(dec\,|\,Rat=1) + P(Rat=0)P(dec\,|\,Rat=0)} \quad \text{…………. (3.5)}$$

, to obtain a rationality level of **0.9353,** at which point

Γ = P(Rat=1)(1) + P(Rat=0)(0) =0.9353(1) = 0.9353.
The evolution summary can be depicted thus:

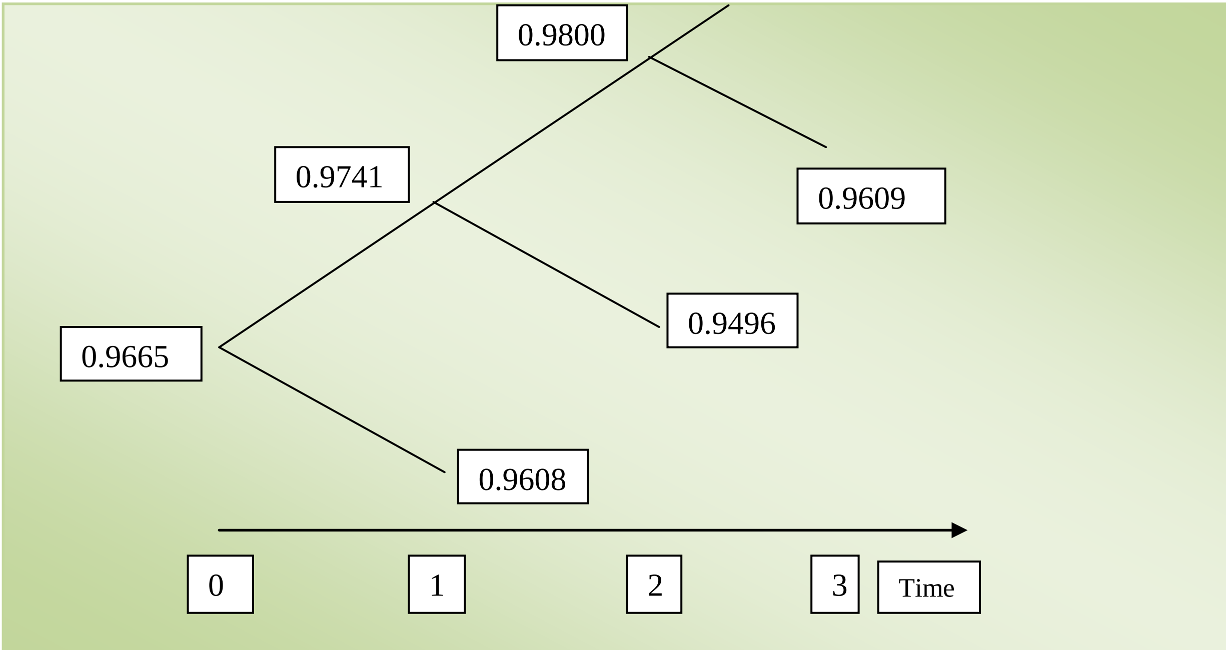

Figure 3.1: Evolution of Rationality with time

Step 5:

b) Evolution of rationality through increase/decrease in economic wealth

We define:

$$p = P(\text{inc} \mid Rat = 1), \quad 1 - p = P(dec \mid Rat = 1)$$

$$q = P(inc \mid Rat = 0), \quad 1 - q = P(dec \mid Rat = 0)$$

, where 'i' and 'd' are respective numbers of increases and decreases.

$$P(Rat = 1 \mid i, d) = \frac{P(Rat = 1)\, p^i (1-p)^d}{P(Rat = 1)\, p^i (1-p)^d + P(Rat = 0)\, q^i (1-q)^d}$$

$$P(Rat = 0 \mid i, d) = \frac{P(Rat = 0)\, q^i (1-q)^d}{P(Rat = 1)\, p^i (1-p)^d + P(Rat = 0)\, q^i (1-q)^d}$$

………………………… (3.6)

Taking the posterior odds (the ratio of the probability that Rationality = 1 to the probability that Rationality = 0), we get:

$$\frac{P(Rat = 1 \mid i, d)}{P(Rat = 0 \mid i, d)} = \frac{P(Rat = 1)\, p^i (1-p)^d}{P(Rat = 0)\, q^i (1-q)^d}$$

……….…………………………………………….. (3.7)

We then take the natural logs of both sides to arrive at:

$$\ln\left(\frac{P(Rat = 1 \mid i, d)}{P(Rat = 0 \mid i, d)}\right) = \ln\left(\frac{P(Rat = 1)}{P(Rat = 0)}\right) + \ln\left(p^i (1-p)^d\right) - \ln\left(q^i (1-q)^d\right)$$

$$= \ln\left(\frac{P(Rat = 1)}{P(Rat = 0)}\right) + i \ln p + d \ln(1-p) - i \ln q - d \ln(1-q)$$

Taking the mean of the odds ratio and the limit as a + b goes to infinity degenerates into:

$$\lim_{(i+d)\to\infty}\left\{\frac{1}{i+d}\ln\frac{P(Rat=1|i,d)}{P(Rat=0|i,d)}\right\}=\lim_{(i+d)\to\infty}\left\{\underbrace{\frac{1}{i+d}}_{\text{Goes to 0 as } i+d\to\infty\text{, so the term}\to 0}\ln\left(\frac{P(Rat=1)}{P(Rat=0)}\right)+\underbrace{\frac{i}{i+d}}_{\text{If Rat = 1, then this goes to q by definition of q, a.s.}}\ln\left(\frac{p}{q}\right)+\underbrace{\frac{d}{i+d}}_{\text{Goes to 1- q by definition a.s.}}\ln\left(\frac{(1-p)}{(1-q)}\right)\right\}$$

$$\lim_{(i+d)\to\infty}\left\{\frac{1}{i+d}\ln\frac{P(Rat=1|i,d)}{P(Rat=0|i,d)}\right\}=q\ln\left(\frac{p}{q}\right)+(1-q)\ln\left(\frac{1-p}{1-q}\right)$$ …………………..….. (3.8)

The right hand side of equation 3.11 represents an expression form of statistical entropy; a concept borrowed from the second law of thermodynamics in physical chemistry. Statistical entropy (also known as relative entropy or *Kullback Leibler's divergence*) measures the distance between two probability distributions as stated in equation 3.10.

$$I_q(p)=q\ln\left(\frac{q}{p}\right)+(1-q)\ln\left(\frac{1-q}{1-p}\right)$$ …………………………………………………... (3.9),

Entropy derives from the second law of thermodynamics as a measure of randomness or disorder of an isolated system, formulated by Ludwig Boltzman in 1896.

$S=k_B\ln\Omega$, where Ω , is number of microstates in the system and k_B is Boltzman constant.
Relative statistical entropy is stated as,

$$D(q\,\|\,p)=\sum_{x\in\chi}q(x)\ln\frac{q(x)}{p(x)}=I_{(q)}p$$ ………… (3.10)

It has the following properties:

$I_q(p)\geq 0\ \ \forall q,\ p$

$I_q(q)=0\ \ \forall q,\ p$

$I_q(p)\neq 0\ \text{ if } q\neq p$

For the left hand side to be finite and negative,

$$\lim_{(i+d)\to\infty}\frac{1}{i+d}\ln\frac{P(Rat=1|i,d)}{P(Rat=0|i,d)}=q\ln\left(\frac{p}{q}\right)+(1-q)\ln\left(\frac{1-p}{1-q}\right)=-I_q(p)>-\infty$$

$\ln\dfrac{P(Rat=1|i,d)}{P(Rat=0|i,d)}=-\infty$, hence $P(Rat=1|i,d)$ = 0.

3.3 Conditions for bounded rationality from the data

The condition of rationality boundedness can only be fulfilled if and only if $0<p<1$ and also $0\leq q<1$. Moreover, if $q=p$, and $r=1$, it would mean that rationality quantity can take a value of 1. Therefore, additionally, we need to show that $r<1$ from the data under investigation.

But this is not a sufficient lifetime boundedness proof. Using the data, the mean values for q and p are 0.501 and 0.636 respectively. We need to show that the population mean values for q and p are less than 1 analytically. We may test the hypothesis about population mean values for q and p .but this may not yield desirable results even if we use 100% confidence level of the Z-statistic. This is because the normal distribution is asymptotic about the horizontal axis at +∞ and -∞. We therefore the following argument:

3.3.1q and p values across the population

Given a population of 126,000 members, assume that q and p take the maximum value of 1 (for all members other than the sampled 111), since they are probabilities. The population mean can then be worked out thus;

$$\bar{q} = \frac{1}{126,000}\left\{\sum_{i=1}^{111} q_i + \sum_{i=112}^{126000} q_i\right\}$$

$$\bar{q} = \frac{1}{126,000}\left\{55.611 + \sum_{i=112}^{126000} q_i\right\}$$

$$\bar{q} = \frac{1}{126,000}\{55.611 + 125889\} = 0.99956 < 1$$

$$\bar{p} = \frac{1}{126,000}\left\{\sum_{i=1}^{111} p_i + \sum_{i=112}^{126000} p_i\right\}$$

$$\bar{p} = \frac{1}{126,000}\left\{70.596 + \sum_{i=112}^{126000} pi\right\}$$

$$\bar{p} = \frac{1}{126,000}\{70.596 + 125889\} = 0.99968 < 1$$

From the data, it is clear that neither q nor p is a monotonically increasing function with age as shown in the graphs below. Actually, data values suggest a martingale. This justifies the use of arithmetic mean for q and p in statistical entropy calculations.

3.3.2 q and p values across time

Having shown that mean population values for q and p are less than 1, we need to show that even across time (financial decision making life) of the economic agent these values never get to 1. A similar argument like the one employed above holds. We argue that the economic agent takes financial decisions for a countably finite number of times during his/her life time. This claim is supported by the fact that first, the economic agent does not exist forever. He does so for a number of years. Further in each year, he makes a finite number of financial decisions. Of course the agent does not take decisions while asleep, while taking meals, sporting among other time intervals. Given that this particular instance where the data was collected, the agent recorded q and p values less than 1, any other mean value for the same parameters that will include this instance values will result in q and p values less than 1, even if in all other instances q and p values rise to 1, that is $q_i = p_i = 1$. Symbolically;

$$\bar{q} = \frac{1}{n}\left\{0.99956 + \sum_{i=1}^{n-1} q_i\right\} < 1 \text{ a.s.}$$

$$\bar{p} = \frac{1}{n}\left\{0.99968 + \sum_{i=1}^{n-1} p_i\right\} < 1 \text{ a.s.}$$

3.3.3 Less than unity condition for the prior r

The final condition that guarantees rationality boundedness in case $p = q < 1$, is that the prior probability initially in the learning process has to be less than 1. At the moment, the population under study is operating in less than 100% level of rationality, precisely at 84.7%. The potential problem is in case $p = q < 1$, and the current rationality level is optimal at 100%, then we shall have failed to prove boundedness. Luckily, rationality is already suboptimal so that even if $p = q < 1$, rationality level will remain bounded.

IV. Conclusion

As Jones (1999) noted, humans are intendedly rational but fail occasionally to act rationally. This article sought to show quantitatively that people operate below 100% level of rationality and that it is not possible to attain the 100% capacity of rationality. This fact may be used by administration managers, professional planners, and human resource managers to set and/or revise their expectations from their human subjects by especially predicting when they are likely to be irrational and taking appropriate measures ahead of time. Moreover, rationality measure should be recognized and used both as a macroeconomic and microeconomic variable to determine more precisely expectations in dealing with workers and planning the economy at the macro level, not to mention its indispensability at personal level.

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Appendix

LIFE ORIENTATION TEST - REVISED

Please answer the following questions about yourself by indicating the extent of your agreement using the following scale; indicate as appropriate in the box at the end of each question.

[0]= strongly disagree [1] = disagree [2] = neutral [3] = agree [4] = strongly agree

1. When I am not sure of things to come, I usually expect the best ☐
2. It is easy for me to relax ☐
3. If I sense something can go wrong with me, it will go wrong ☐
4. I am always expecting good things about my future ☐
5. I enjoy my friends a lot ☐
6. It is important for me to keep busy ☐
7. It is very unlikely that things go my way ☐
8. I do not get upset too easily ☐
9. It is difficult for good things to happen to me ☐
10. Overall, I expect good things to happen to me than bad ☐

Notes:

a) Questions 2, 5, 6 and 8 are fillers. They do not contribute to the overall score. They serve to increase objectivity of the respondent lest he or she may withhold information they sense they are being tested;

b) Questions 3, 7, and 9 are scored in the reverse e.g. a response of 3 score a 1, while that of 4 scores a zero;

c) Overall highest score is a four for each question totaling 24 for questions 1, 3, 4, 7,9 and 10