ISSN 2250-3153

Steady state and transient Brillouin gain in narrow band-gap magnetized semiconductors

V. Pal Singh[*], M. Singh[**]

[*]Department of Physics, Singhania University, Pacheri Bari, Jhunjhunu - 333515, India
[**]Department of Physics, Amity University, Sector-125, Noida-201301 (U.P.), India

Abstract: Using the hydrodynamic model of semiconductor plasmas, a detailed analytical investigation is made to study both the steady-state and the transient Brillouin gain in narrow band gap magnetized one-component centrosymmetric semiconductor viz. n-InSb under off-resonant laser irradiation. Using the fact that origin of stimulated Brillouin scattering (SBS) lies in the third-order (Brillouin) susceptibility ($\chi_B^{(3)}$) of the medium, we obtained an expression of the threshold pump electric field (E_{th}), the resultant gain coefficients (steady-state as well as transient $g_{B,TB}$) and optimum pulse duration (τ_p) for the onset of SBS. The application of a strong magnetic field not only lowers E_{th} but also enhances $g_{B,TB}$. The carrier heating by the intense pump modifies the electron collision frequency and hence the nonlinearity of the medium which in turn enhances $g_{B,TB}$ significantly. The Brillouin gain is found to be maximizing when the generated acoustic wave suffers no dispersion in the medium. The enhanced g_{TB} can be greatly used in the compression of scattered pulses.

Index Terms: Stimulated Brillouin scattering, Semiconductor plasmas, Carrier heating, Narrow band gap semiconductor

I. INTRODUCTION

Following the advent of lasers, nonlinear optics (NLO) has emerged as a multidisciplinary subject of great breadth and richness, attracting the interest of researchers in basic as well as applied fields. The subject area covered by NLO can be divided into two broad categories: (i) steady-state NLO effects, and (ii) coherent transient optical effects. A large number of NLO effects (parametric interactions, stimulated scatterings and their applications to parametric amplifiers and oscillators in the phenomena like optical phase conjugation etc.) occur as a consequence of either cw laser operation or with lasers having pulse durations much longer than the dephasing or recombination times of the medium. Such NLO phenomena can be broadly defined as steady-state NLO effects. Moreover, sufficient large interaction times allow the control of light with light and therefore, all optical signal processing phenomena fall into the class of steady-state NLO effects. The origin of such mechanisms in the crystals lies largely in the presence of free-carrier states and the photo-generation of carriers. The study of transient coherent optical effects viz., optical nutation, and free induction transparency has become a very important area of research in the recently developed subfield of coherent optical spectroscopy. These effects arise when the material response to an incident intense laser light field is slower than the rate of variation of light intensity. In other words, if the pump pulse duration is much smaller than the dephasing time of the resonant excited state, the medium can keep in memory the light induced coherence for some time after the switching off of the excited pulse.

Out of the several NLO effects, the nonlinear scattering of laser radiation in gaseous and solid state plasmas aroused a great deal of interest in the past few years [1-5] on account of the fact that the absorption of laser radiation in plasma greatly depends on these processes. When an intense light beam interacts with an active medium, strong optical amplification of the scattered wave occurs at Stoke's shifted frequency. Such phase coherent processes are called stimulated scattering processes. Amongst these, the study of stimulated Brillouin scattering (SBS) in solids has been the subject of intensive investigations (both theoretical and experimental) due to its manifold technological applications in a wide range of optical communication and optoelectronics applications. Some of these are: distributed fiber optic Brillouin sensors, laser induced fusion, pulse squeezing and optical phase conjugation (OPC) [6]. In laser-induced fusion experiments, SBS is of great concern because it can significantly redirect the pump energy away from the target and thus adversely affects the energy absorption [7]. It is, therefore, desirable to minimize SBS process in these experiments. For OPC, SBS is preferred over other stimulated scattering processes because it initiates at low threshold pump intensity, suffers negligible frequency shifts and offers high conversion efficiency [8]. In this method, the incident wave serves as both the pump for inciting the nonlinear process and distorted wave to be conjugated. Il'ichey [9] has reported that the OPC-SBS conversion efficiency can be expressed in terms of parameters of the scattering medium and excitation intensity. Using a frequency doubled Nd: YAG laser as a pump and Rhodamine 6G as a Brillouin medium, it was experimentally demonstrated that OPC-SBS reflectivity is maximum only at a specific intensity of an incident pump beam [10]. The combination of SBS and four-wave mixing (FWM), termed Brillouin-enhanced four-wave mixing (BEFWM), has recently received great attention as it can yield phase conjugate signals with extremely high reflectivity [11].

Various aspects of SBS and its consequent instabilities have been investigated in gaseous plasmas [1, 2, 12, 13]. But the practical utilization of semiconductors in drew the attention of many solid-state physicists to examine the role of semiconductors in the areas such as spectroscopy, lasers, device fabrication etc. Moreover, in the search for optical memories and switching elements, one found that the optical properties of these materials change strongly when electrons are excited optically. The electrical properties of semiconductors lie in between those of metals with nearly free electrons and insulators with tightly bound electrons. This intermediate situation makes semiconductors attractive as nonlinear devices in electronics as well optics because their properties can be influenced easily by fields, compositions and micro-structuring. Hence, the supremacy of semiconductors as active media in laser communication, modern optoelectronic devices, optical computing [14, 15] and all optical signal processing [16] is unquestionable and hence the understanding of the mechanisms of transient effects in these crystals appears to be of fundamental significance.

SBS is caused by coherent interaction of an intense pump, scattered and acoustic waves in a medium. The internally generated acoustic and scattered waves propagate along specific directions and amplify when the intensity of pump beam exceeds a threshold value [17]. The origin of SBS lies in the third-order optical susceptibility of the medium, also known as Brillouin susceptibility ($\chi_B^{(3)}$). The steady-state and transient Brillouin gain coefficients ($g_{B,TB}$) being directly dependent upon $\chi_B^{(3)}$, an enhancement in Brillouin gains is possible if one can achieve larger $\chi_B^{(3)}$ in the nonlinear medium. Recently, one of the present authors [18] have shown keen interest in the application of an external magnetic field to enhance remarkably $\chi^{(3)}$ in III-V semiconductors. In the recent past, a significant amount of research work on SBS and its consequent instabilities in magnetized doped semiconductors have been reported by several groups [19-23]; the theoretical predictions and experimental measurements are far apart [24]. Several experiments, with short laser pulses of low intensity, suggest that SBS starts below the theoretically estimated threshold value, whereas some experiments with high-intensity radiation reveal that SBS signal levels saturate at much lower values than their theoretically predicted values.

In crystalline solids, there always exists a thermally excited acoustic wave (AW) which can scatter the incident light of any arbitrary intensity and gives rise to spontaneous Brillouin scattering. However, when the pump intensity reaches a certain threshold value, it induces electrostrictive AW that gives rise to SBS. The interaction of this intense pump with carriers in a semiconductor results in an appreciable increase in the carrier temperature due to their high mobility, low effective mass, long free path and slow rate of energy transfer to the lattice.

Literature survey reveals that no schematic attempt has so far been made to explore the influence of carrier heating on SBS process in narrow band-gap magnetized semiconductors. In the present paper, by using a hydrodynamic model of semiconductor plasma, we intend to study the influence of the pump-induced carrier heating on the steady-state and transient gain coefficients of the Brillouin mode. The stimulus for the present study stems from the fact that the carrier heating by the pump can remarkably modify the nonlinearity of the medium and hence the related phenomena. In the wake of high-power lasers, such an investigation becomes even more important because it may lead to a better understanding of the scattering mechanisms in solids and gaseous plasmas and thus may prove to be a step forward towards filling the gap between theory and experimental observations. The ambient temperature of the crystal is assumed to be maintained at 77 K. The semiconductor is assumed to be immersed in a strong magnetic field, which may appreciably lower the SBS threshold and hence may enhance the Brillouin gain coefficient.

II THEORETICAL FORMULATIONS

This section deals with the theoretical formulation of complex effective third-order (Brillouin) susceptibility $\chi_B^{(3)}$, and there from the steady-state and transient Brillouin gain coefficients $g_{B,TB}$ for the Stokes component of the scattered electromagnetic wave in a Brillouin active medium. We consider the propagation of a hybrid pump wave

$$\vec{E}_0 = (E_{0x}\hat{i} + E_{0y}\hat{j})\exp[i(k_0 x - \omega_0 t)] \qquad (1)$$

in a homogeneous electrostrictive n-type III–V semiconductor viz. n-InSb embed in a uniform static magnetic field $\vec{B}_s = B_{0x}\hat{i} + B_{0z}\hat{k}$ in a direction making an angle θ with the *x*-axis, as shown in Fig. 1.

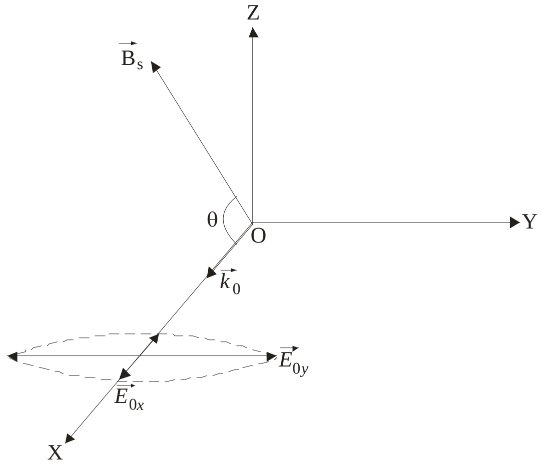

Figure 1: Geometry of SBS in magnetic field.

The authors have chosen this particular field geometry because most of the reported cases correspond to the propagation of a pump wave exactly parallel to the applied magnetic field. Such an exact parallel propagation may not be experimentally feasible. Moreover, the electric field of the pump considered is either perpendicular or parallel to the propagation directions. This, again, is not the case in realistic situations [25]. For a finite solid state plasma, $\vec{E}_0$ must have components that are both parallel and perpendicular to the propagation direction. Thus the most realistic case (that the authors have considered here) is to consider a hybrid mode propagating obliquely to the external magnetic field.

According to the classical description of SBS, the intense pump $(\omega_0, \vec{k}_0)$ induces electrostrictive force and drives an AW $(\omega_a, \vec{k}_a)$ in the medium. This AW in turn plays the role of an induced density-modulated grating for the pump and gives rise to Brillouin scattered light $(\omega_1, \vec{k}_1)$. Thus the pump light, acoustic and the scattered light waves can couple with one another in an electrostrictive medium. In favourable conditions, a significant growth of both the AW and scattered light waves is possible at the expense of the pump intensity, provided the pump intensity exceeds the threshold value.

The equation of the generated AW in an electrostrictive medium is given by

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{C}{\rho}\frac{\partial^2 u(x,t)}{\partial x^2} + 2\Gamma_a \frac{\partial u(x,t)}{\partial t} = \frac{\gamma}{2\rho}\frac{\partial}{\partial x}[(E_e)_x (E_1^*)_x],$$

(2)

where $\vec{E}_e = \vec{E}_0 + (\vec{v}_0 \times \vec{B}_s)$ represents the effective electric field which includes the Lorentz force $(\vec{v}_0 \times \vec{B}_s)$ in the presence of external static magnetic field $\vec{B}_s$, in which $\vec{v}_0$ being the oscillatory fluid velocity of an electron of effective mass m and charge e at pump frequency ω_0; $u(x,t) = u\exp[i(k_a x - \omega_a t)]$ is the lattice displacement. C, ρ, Γ_a and γ are the elastic constant, material density, phenomenological damping parameter and electrostrictive coefficient of the medium, respectively. E_1 is the space charge electric field. The term on the right-hand side of Eq. (2) is the driving force per unit material density and originates from the electrostrictive mechanism. In hydrodynamic approximation ($kl \ll 1$; k the wave number, and l the carrier mean free path), we consider electrons are mobile carriers while holes are at rest due to their heavy mass. The condition implies that the sound wavelength is much greater than the average distance the electron travels between collisions so that the motion of the carriers under the influence of the external field is averaged out.

The other basic equations of the analysis are:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 + \left(\vec{v}_0 \cdot \frac{\partial}{\partial x}\right)\vec{v}_0 = \frac{e}{m}\vec{E}_e \tag{3}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0 \cdot \frac{\partial}{\partial x}\right)\vec{v}_1 + \left(\vec{v}_1 \cdot \frac{\partial}{\partial x}\right)\vec{v}_0 = \frac{e}{m}[\vec{E}_1 + \vec{v}_1 \times \vec{B}_s] \tag{4}$$

$$\vec{P}_{es} = -\gamma \vec{E}_e \frac{\partial u^*}{\partial x} \tag{5}$$

$$\frac{\partial n_1}{\partial t} + n_e \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \tag{6}$$

$$\frac{\partial E_1}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{\gamma E_e}{\varepsilon}\frac{\partial^2 u^*}{\partial x^2} \tag{7}$$

Eqs. (3) and (4) represent the carrier motion under the influence of the pump ($\vec{E}_0$), scattered ($\vec{E}_1$) and static magnetic ($\vec{B}_s$) fields; $\vec{v}_0$ and $\vec{v}_1$ are the zeroth and first-order electron fluid velocities. Here ν the momentum-transfer collision frequency of electrons. In Eq. (5), $\vec{P}_{es}$ is the electrostrictive polarization produced due to modulation of the dielectric constant of the medium under the electrostrictive action of the pump. Eq. (6) is the continuity equation of the electrons, in which n_e and n_1 are the equilibrium and perturbed electron densities, respectively. The strong space-charge $\vec{E}_1$ developed due to migration of charge carriers under the influence of the pump and magnetic fields is determined from Poisson equation [Eq. (7)]. Here ε is the dielectric permittivity of the medium expressed as $\varepsilon = \varepsilon_0 \varepsilon_L$; ε_0 and ε_L are the free-space permittivity and lattice dielectric constant of the medium, respectively.

2.1. Total induced current density

The interaction of the pump with the electrostrictively generated AW produces an electron density perturbation, which in turn drives an electron-plasma wave and induces nonlinear current density in the medium. In a doped semiconductor, the equation of this electron-plasma wave is obtained by using the standard approach [26]. Differentiating Eq. (6) with respect to time and using Eqs. (3) and (7), one gets on simplification,

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 - \frac{n_e \gamma k_a^2 u^*}{\varepsilon}\frac{\bar{\omega}_p^2}{\omega_p^2}\bar{E} = -ik_1 n_1 \bar{E} \tag{8}$$

where $\bar{E} = \frac{e}{m}(E_e)_x$, $\bar{\omega}_p^2 = \omega_p^2\left(1 - \frac{\nu^2 + \omega_{cx}^2}{\nu^2 + \omega_{cz}^2}\right)$, $\omega_p = \left[n_e e^2 / m\varepsilon\right]^{\frac{1}{2}}$ (electron-plasma frequency) and $\omega_{cx,z}\left(= eB_{sx,z}/m\right)$ are the components of the electron cyclotron frequency $\omega_c\left(=\sqrt{\omega_{cx}^2 + \omega_{cz}^2}\right)$. We neglect the Doppler shift under the assumption that $\omega_0 \gg \nu \gg \vec{k}_0 . \vec{v}_0$.

The process of SBS may also be described as the annihilation of a pump photon and simultaneous creation of one scattered photon and one induced phonon. From this viewpoint, the conservation of energy and momentum requires that $\hbar\omega_0 = \hbar\omega_1 + \hbar\omega_a$ and $\hbar\vec{k}_0 = \hbar\vec{k}_1 + \hbar\vec{k}_a$. These relations are commonly termed as phase-matching conditions and determine the frequency shift and direction of propagation of the scattered light. By assuming a long interaction path for the interacting waves we may consider the resonant Stokes component ($\omega_1 = \omega_0 - \omega_a$, $\vec{k}_1 = \vec{k}_0 - \vec{k}_a$) only, and neglect the off-resonant higher-order component [27]. Moreover, for a spatially uniform pump $\left|\vec{k}_0\right| = \left|\vec{k}_a\right|$ we may assume $\left|\vec{k}_a\right| = \left|\vec{k}_1\right| = k$ (say). The perturbed electron density produced in the medium will have two components, which may be recognized as fast and slow components. The fast component (n_f) corresponds to the Stokes component of the scattered light and varies as $\exp[i(k_1 x - \omega_1 t)]$; whereas the slow component (n_s) is associated with the AW and varies as $\exp[i(k_a x - \omega_a t)]$ such that $n_1 = n_f + n_s$. Using the phase matching conditions and rotating wave approximation, the coupled wave equations for n_f and n_s are obtained from Eq. (8) as:

$$\frac{\partial^2 n_f}{\partial t^2} + \nu\frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f - \frac{n_e \gamma k^2}{\varepsilon}\frac{\bar{\omega}_p^2}{\omega_p^2}u^* \bar{E} = -ik\bar{E}n_s^* \tag{9a}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu\frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s = -ik\bar{E}n_f^* . \tag{9b}$$

Eqs. (9a) and (9b) indicate that the pump couples the generated AW and Stokes component of the scattered light with each other in an electrostrictive medium. Thus it is obvious that the presence of the pump field is the fundamental necessity for SBS to occur. The coupled wave equations are solved and are simplified for n_s, which is given by

$$n_s = \frac{n_e \gamma^2 k^2 \bar{\omega}_p^2 (E_e)(E_1^*)_x}{2\rho\varepsilon\omega_p^2(\delta_a^2 + 2i\Gamma_a\omega_a)}\left[1 - \frac{(\delta_1^2 + i\nu\omega_1)(\delta_2^2 - i\nu\omega_a)}{k^2\bar{E}^2}\right]^{-1},$$

(10)

where $\delta_a^2 = \omega_a^2 - k_a^2 v_a^2$, in which $v_a = \sqrt{C/\rho}$ is the acoustic velocity in the medium; $\delta_1^2 = \bar{\omega}_p^2 - \omega_1^2$ and $\delta_2^2 = \bar{\omega}_p^2 - \omega_a^2$.

From the above expression, it is clear that n_s depends upon the input pump field. The density perturbations thus produced affects the propagation characteristics of the generated waves.

The resonant Stokes component of the current density may be obtained from the relation:

$$\vec{J}_{cd}(\omega_1) = n_e e\vec{v}_{1x} + n_s^* e\vec{v}_{0x}\,, \tag{11}$$

which yields

$$\vec{J}_{cd}(\omega_1) = \frac{\varepsilon\bar{\omega}_p^2(\nu+i\omega_1)}{\omega_1^2}\vec{E}_{1x} + \frac{\gamma^2 k^2\bar{\omega}_p^2(\nu+i\omega_0)\left|(\vec{E}_e)_x\right|^2(\vec{E}_1)_x}{2\rho\omega_0^2(\delta_a^2-2i\Gamma_a\omega_a)}\left[1-\frac{(\delta_1^2-i\nu\omega_1)(\delta_2^2+i\nu\omega_a)}{k^2\bar{E}^2}\right]^{-1} \tag{12}$$

The first term of Eq. (12) represents the linear component of the induced current density while the second-term represents the nonlinear coupling amongst the three interacting waves via the nonlinear current density.

2.2. Threshold pump amplitude and effective Brillouin susceptibility

To begin with, let us treat the induced polarization $\vec{P}_{cd}(\omega_1)$ as the time integral of the current density $\vec{J}_{cd}(\omega_1)$, then one gets

$$\vec{P}_{cd}(\omega_1) = \int \vec{J}_{cd}(\omega_1)dt = \frac{-\vec{J}_{cd}(\omega_1)}{i\omega_1}\,. \tag{13}$$

Using Singh et.al. [28], we obtain the nonlinear induced polarization using perturbed current density

$$\vec{P}_{cd}(\omega_1) = \frac{b\gamma^2k^2\bar{\omega}_p^2(\omega_0\delta_a^2+2\nu\Gamma_a\omega_a+2i\Gamma_a\omega_a\omega_0)\cos^2\phi}{2\rho\omega_1\omega_0^2(\delta_a^4+4\Gamma_a^2\omega_a^2)} \times\left[1-\frac{(\delta_1^2-i\nu\omega_1)(\delta_2^2+i\nu\omega_a)}{k^2\bar{E}^2}\right]^{-1}\left|\vec{E}_0\right|^2(\vec{E}_1)_x \tag{14}$$

where $b = \frac{(\omega_0^2-\omega_{cx}^2)^2+4\nu^2\omega_0^2}{(\omega_0^2-\omega_c^2)^2+4\nu^2\omega_0^2}$, and ϕ is the inclination of pump electric field with x-axis.

From the above relation we can determine the nature of the threshold for the onset of SBS process by setting $P_{cd}(\omega_1)=0$. This condition yields,

$$E_{th} = \frac{m}{ek}\frac{\delta_1\delta_2(\omega_0^2-\omega_c^2)}{[(\omega_0^2-\omega_{cx}^2)\cos\phi+\nu\omega_{cz}\sin\phi]}\,. \tag{15}$$

This equation reveals that E_{th} is strongly influenced by the material parameters (n_e,ν), the magnetic field (ω_c) and the geometry (θ, ϕ) of the magnetic and pump fields.

In addition to the polarization $\vec{P}_{cd}$, the system should also possess electrostrictive polarization $\vec{P}_{es}$, arising due to the interaction of the pump wave with the AW generated in the medium. This is due to the fact that the scattering of light from the AW affords a convenient means of controlling the frequency, intensity and direction of an optical beam. This type of control makes possible a large number of applications involving the transmission, display and processing of information. The electrostrictive polarization is obtained from Eqs. (2) and (5) as:

$$\vec{P}_{es}(\omega_1) = \frac{b\gamma^2k^2(\delta_a^2+2i\Gamma_a\omega_a)\cos^2\phi}{2\rho(\delta_a^4+4\Gamma_a^2\omega_a^2)}\left|\vec{E}_0\right|^2(\vec{E}_1)_x \tag{16}$$

Thus, the total polarization induced at the Stokes component for a pump amplitude well above the threshold value is given by:

$$\vec{P}(\omega_1) = \vec{P}_{cd}(\omega_1)+\vec{P}_{es}(\omega_1)$$

$$= \frac{b\gamma^2k^2\cos^2\phi}{2\rho(\delta_a^4+4\Gamma_a^2\omega_a^2)} \times\left[(\delta_a^2+2i\Gamma_a\omega_a)+\frac{\bar{\omega}_p^2(\omega_0\delta_a^2+2\nu\Gamma_a\omega_a+2i\Gamma_a\omega_a\omega_0)}{\omega_1\omega_0^2}\right]\left|\vec{E}_0\right|^2(\vec{E}_1)_x \tag{17}$$

Now it is well known that, the origin of SBS process lies in that component of $\vec{P}(\omega_1)$ which depends on $\left|\vec{E}_0\right|^2(\vec{E}_1)_x$. The corresponding effective third-order susceptibility (also known as Brillouin susceptibility $\chi_B^{(3)}$) is given by:

$$\chi_B^{(3)} = \chi_{es}^{(3)}+\chi_{cd}^{(3)} = \frac{b\gamma^2k^2\cos^2\phi}{2\rho(\delta_a^4+4\Gamma_a^2\omega_a^2)} \times\left[(\delta_a^2+2i\Gamma_a\omega_a)+\frac{\bar{\omega}_p^2(\omega_0\delta_a^2+2\nu\Gamma_a\omega_a+2i\Gamma_a\omega_a\omega_0)}{\omega_1\omega_0^2}\right] \tag{18}$$

Here, the higher-order contributions like $\chi^{(5)},\chi^{(7)},\ldots$ are neglected because the susceptibility rapidly converges with respect to the pump amplitude $\vec{E}_0$.

2.3. Carrier heating and modified nonlinearity

To incite SBS, the fundamental requirement is to apply a pump field above the threshold value. When this high-intensity pump traverses a high mobility semiconductor, the carriers acquire momentum and energy from the pump and as a result they (here electrons) acquire a temperature (T_e) somewhat higher than that of the lattice (T_0). This field-dependent rise in the electron temperature, in turn, modifies the electron collision frequency (ECF) through the relation [29]:

$$\nu = \nu_0\left(\frac{T_e}{T_0}\right)^{1/2}, \tag{19}$$

where ν_0 is the ECF in the absence of the pump, i.e. at $T_e = T_0$. The temperature ratio (T_e/T_0) can be readily obtained from energy balance equation in the following manner.

The power absorbed per electron from the pump may be obtained as [30]:

$$\frac{e}{2}\mathrm{Re}\left[\vec{v}_{0x}.\vec{E}_e^*\right] = \frac{be^2\nu_0\cos^2\phi}{2m\omega_0^2}\left|\vec{E}_0\right|^2 \tag{20}$$

where the asterisk denotes the complex conjugate while Re stands for the real part of the quantity concerned. The x-component of $\vec{v}_{0x}$ used in the above relation may be evaluated from Eq. (3).

Following Conwell [31], the power dissipation per electron in collisions with the POP may be expressed as:

$$\left(\frac{\partial\in}{\partial t}\right)_{diss} = \left(\frac{2k_B\theta_D}{m\pi}\right)^{1/2} eE_{PO}(x_e)^{1/2}K_0\left(\frac{x_e}{2}\right) \times\exp\left(\frac{x_e}{2}\right).\frac{\exp(x_0-x_e)-1}{\exp(x_0)-1} \tag{21}$$

where $x_{0,e} = \frac{\hbar\omega_l}{k_BT_{0,e}}$; $\hbar\omega_l$ is the energy of the POP and is given by $\hbar\omega_l = k_B\theta_D$ and θ_D is the Debye temperature of the crystal. $\vec{E}_{PO} = \frac{me\hbar\omega_l}{\hbar^2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon}\right)$ is the field of POP scattering potential; ε and ε_∞ are the static and high frequency dielectric permittivities of the medium, respectively. K_0 $(x_e/2)$ is the zeroth-order Bessel function of the first kind.

In steady-state, the power absorption per electron from the pump is just equal to the power lost in collisions with POP. For moderate heating of the carriers, equations (19) and (20) yield

$$\frac{T_e}{T_0} = 1 + \alpha\left|\vec{E}_0\right|^2, \tag{22}$$

where

$$\alpha = \frac{be^2\nu_0\tau\cos^2\phi}{2m\omega_0^2}, \tag{23a}$$

in which

$$\tau^{-1} = \left(\frac{2k_B\theta_D}{m\pi}\right)^{1/2} eE_{PO}K_0\left(\frac{x_0}{2}\right)\frac{x_0^{1/2}\exp(x_0/2)}{\exp x_0 - 1}. \tag{23b}$$

Thus the modified ECF may be obtained as:

$$\nu = \nu_0\left(1 + \alpha\left|\vec{E}_0\right|^2\right)^{1/2} \approx \nu_0\left(1 + \frac{1}{2}\alpha\left|\vec{E}_0\right|^2\right). \tag{24}$$

By incorporating this modified ECF in Eq. (17), one can obtain the third-order (Brillouin) susceptibility modified by the carrier heating as:

$$\chi_B^{(3)} = \left[\chi_B^{(3)}\right]_r + i\left[\chi_B^{(3)}\right]_i$$
$$= \frac{b\gamma^2k^2\cos^2\phi}{2\rho\varepsilon_0(\delta_a^4 + 4\Gamma_a^2\omega_a^2)}$$
$$\times\left[\left(\delta_a^2 + \frac{\bar{\omega}_p^2(\omega_0\delta_a^2 + 2\nu_0\Gamma_a\omega_a)}{\omega_0^2\omega_1}\right) + 2i\Gamma_a\omega_a\left(1 + \frac{\bar{\omega}_p^2}{\omega_0\omega_1}\right)\right] + \Phi \tag{25}$$

where subscripts r and i to the quantities represent the real and imaginary parts, respectively. $\Phi = \frac{\alpha\varepsilon_L\bar{\omega}_p^2\nu_0}{2\omega_1^3}$ may be termed as ''carrier heating parameter''. It is also evident from the above equation that the carrier heating considerably influences $\chi_B^{(3)}$ of the medium.

2.4. Steady state and transient Brillouin gain coefficients of the Stokes component

The steady state-Brillouin gain coefficient of the Stokes component in the presence of a pump well above the threshold value is obtained as:

$$g_B(\omega_1) = -\frac{k}{2\varepsilon_L}\left[\chi_B^{(3)}\right]_i\left|\vec{E}_0\right|^2$$
$$= -\frac{k}{2\varepsilon_L}\left[\frac{b\gamma^2k^2\Gamma_a\omega_a\cos^2\phi}{\rho\varepsilon_0(\delta_a^4 + 4\Gamma_a^2\omega_a^2)}\left(1 + \frac{\bar{\omega}_p^2}{\omega_1\omega_0}\right) + \Phi\right]\left|\vec{E}_0\right|^2 \tag{26}$$

This relation can be used to study the dependence of $g_B(\omega_1)$ on the material parameters, the field strength and their geometry, dispersion characteristics of the generated AW, etc.

Due to the threshold nature of the stimulated scattering processes, in general, they start at high excitation intensity and therefore the pulsed lasers are often used in stimulated scattering experiments. One should therefore consider the time dependence of the output (i.e. transient solutions of the coherent scattering processes) in crystals irradiated by pulsed lasers. From Eq. (26), one may infer that a high-power laser source is the only pump which yields large $g_B(\omega_1)$. Hence, the laser pump source for SBS should be either in the form of a pulse with a time duration of the order of ~ 10^{-9} s for the Q-switched lasers or in the form of a pulse train with an individual pulse duration ~ 10^{-12} s for mode-locked lasers. Since these time durations are comparable to or smaller than the acoustic phonon life time $(\tau_a \approx \Gamma_a^{-1})$, under such circumstances the product $g_B\Gamma_a$ is more natural gain parameter instead of steady-state gain coefficient g_B alone. This product gives an idea about compression of the Stokes pulse [32]. In addition, it is found that transient coherent scatterings are of significance in many experimental situations as well as in the study of transient gain not only in predicting the threshold pump intensity correctly for onset of coherent scattering but also in predicting the optimum pulse durations for which these instabilities can be observed. From the above discussion, it is clear that the stimulated scatterings should be dealt with under transient regime. Therefore, we will extend the above formulations to study the transient behavior of Brillouin medium by using the product of steady-state gain and phonon lifetime. To do so, we consider the pump pulse duration $(\tau_p) \leq$ acoustic phonon lifetime (τ_a) and following Carman etal.. [33], the transient gain of coherent scattering medium can be expressed as:

$$g_{TB} = (2g_Bx\Gamma_a\tau_p)^{1/2} - \Gamma_a\tau_p\,;\,\Gamma_a\tau_p < g_BL, \tag{27}$$

where x is the interaction length.

Here, it is worth pointing out that the interaction length (L) becomes very small for backward scattering because the Stokes pulse and the laser pulse travel in opposite directions and hence, their overlap region cannot exceed the length of the laser pulse; viz., for a typical pico-second pulse laser, the interaction length is of the order of a millimeter. Thus following Wang [34], for very short durations ($\tau_p \leq 10^{-10}$s) the interaction length should be replaced by $c_1\tau_p/2$ (where c_1 is the velocity of light in the crystal medium).

Consequently by making g_{TB} = 0 in Eq. (27), we can obtain the threshold-pump intensity for the onset of transient SBS as:

$$I_{th} = \frac{\Gamma_a}{2G_Bc_1}, \tag{28}$$

where $G_B = g_B/I_p$ is the steady state Brillouin gain coefficient per unit pump intensity and $I_p = (1/2)\eta\varepsilon_0c\left|\vec{E}_0\right|^2$.

Using $\Gamma_a = 2\times10^{10}\,\mathrm{s}^{-1}$ and $g_B = 5\times10^4\,\mathrm{m}^{-1}$ at $I_p = 3.55\times10^8\,\mathrm{Wm}^{-2}$ for a centrosymmetric semiconductor-plasma and Eq. (28), we obtain the threshold value of pump intensity for the onset of Raman instability as 10^6 Wm^{-2}.

However, for comparatively long pulse duration ($\tau_p \geq 10^{-9}$s), the cell length can be taken equal to x, and under such circumstances, we find

$$g_{TB} = (\Gamma_a\tau_p)^{1/2}[-(\Gamma_a\tau_p)^{1/2} + (g_Bx)^{1/2}]. \tag{29}$$

Using the above equation, we may obtain the optimum value of pulse duration $\tau_{p,opt}$, above which no transient gain could be achieved. This can be obtained by making g_{TB} = 0, which yields:

$$\tau_{p,opt} \approx \frac{g_B}{\Gamma_a}. \tag{30}$$

The values of $\tau_{p,opt}$ suggest that optimum pulse duration can be increased by increasing the pump intensity. A calculation for centrosymmetric semiconductor-plasma using the values given earlier and $x = 10^{-4}$ m, gives $\tau_{p,opt} = (7.5\times10^{-15} I_p)$s .

III RESULTS AND DISCUSSION

In order to establish the validity of the present model and to study SBS process, we have chosen a narrow band gap centrosymmetric crystal (n-InSb) at 77 K as the medium which is assumed to be irradiated by 10.6 µm CO_2 laser of frequency $\omega_0 = 1.78\times10^{14}$ s .The physical parameters chosen are [28]:

$n_e = 10^{22} - 10^{24}\,\mathrm{m}^{-3}$, $m = 0.0145 m_e$ (m_e the free mass of electron), $\varepsilon_L = 17.8$, $\varepsilon_1 = 15.8$, $\rho = 5.8\times10^3\,\mathrm{kg\,m}^{-3}$, $\omega_a = 10^{11}\,\mathrm{s}^{-1}$, $k = 10^6 - 10^9\,\mathrm{m}^{-1}$, $v_s = 4\times10^3\,\mathrm{ms}^{-1}$, $\Gamma_a = 2\times10^{10}\,\mathrm{s}^{-1}$, $\gamma = 5\times10^{-10}\,\mathrm{s}^{-1}$, $\theta_D = 278$K, $\omega_c = 0.1 - 0.9\omega_0$.

Using the material parameters given above, the nature of dependence of the threshold pump amplitude E_{th} on magnetic field strength (in terms of ω_c/ω_0) with AW number k as a parameter [Eq. (15)] is investigated in the n-InSb crystal and is plotted in Fig. 2. In all the cases, E_{th} starts with a relatively high value (5.75×10^7, 4.60×10^7 and $2.75\times10^7\,\mathrm{Vm}^{-1}$ for $k = 2\times10^7$, 2.5×10^7 and $4\times10^7\,\mathrm{m}^{-1}$ respectively) at $\omega_c = 0.1\omega_0$ and decreases continuously with increasing ω_c. For strong magnetic field, when the medium becomes a magneto-plasma ($\omega_c \approx \omega_0$), E_{th} becomes independent of k and the curves nearly coincide.

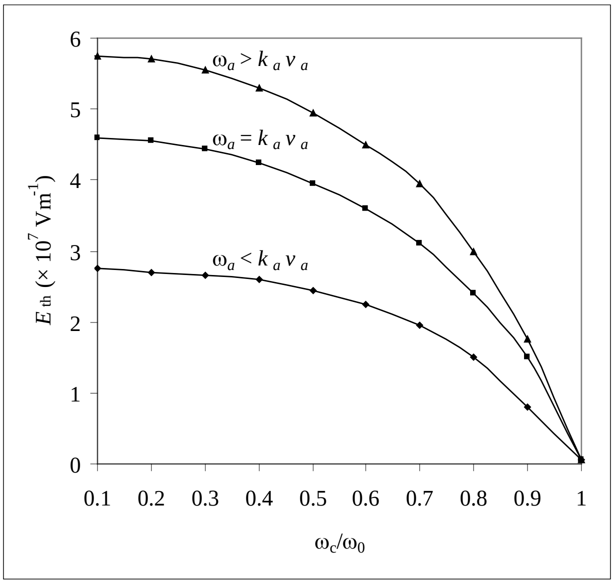

Figure 2: Dependence of threshold pump amplitude E_{th} on magnetic field strength (in terms of ω_c/ω_0) for $k = 2\times10^7$, 2.5×10^7 and $4\times10^7\,\mathrm{m}^{-1}$. Here $n_e = 10^{24}\,\mathrm{m}^{-3}$, $\omega_a = 10^{11}\,\mathrm{s}^{-1}$, $v_a = 4\times10^3\,\mathrm{ms}^{-1}$, $\phi = \pi/6$, $\theta = \pi/4$.

Fig. 3 illustrates the dependence of E_{th} on the magnetic field inclination θ for various carrier concentrations. In the presence of a longitudinal field ($\theta = 0$), a heavily doped ($n_e = 10^{24}\,\mathrm{m}^{-3}$) medium requires an order larger threshold field ($E_{th} = 5.7\times10^7\,\mathrm{Vm}^{-1}$) in comparison with that of moderately doped ($n_e = 10^{22}\,\mathrm{m}^{-3}$) medium ($E_{th} = 7.5\times10^6\,\mathrm{Vm}^{-1}$). However, as θ increases E_{th} decreases and the gap between the E_{th} curves narrows down. For $\theta \to 90^\circ$; the two curves approach each other which indicates that the presence of a transverse magnetic field makes E_{th} almost independent of carrier concentration.

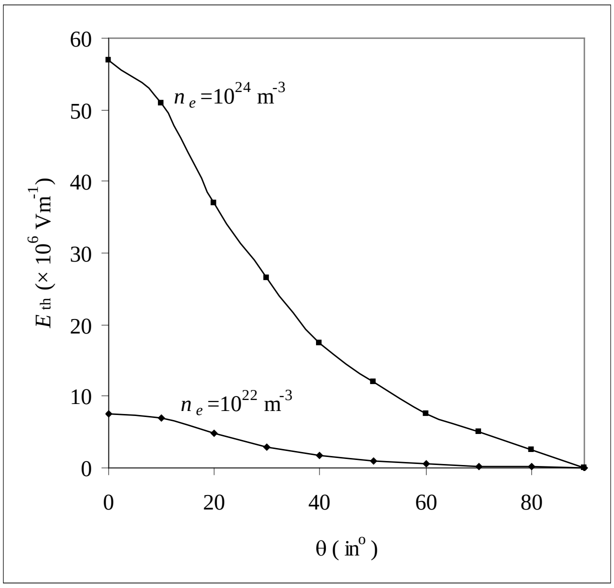

Figure 3: Dependence of threshold pump amplitude E_{th} on magnetic field inclination θ for $n_e = 10^{22}$ and $10^{24}\,\mathrm{m}^{-3}$. Here $\omega_c = 0.9\omega_0$, $k = 2.5\times10^7\,\mathrm{m}^{-1}$, $\phi = \pi/6$.

In Fig. 4, we present the influence of the AW number k on the steady-state Brillouin gain of the Stokes mode. Here g_B [Eq. (26)], and g_{BO} [Eq. (26) with $\Phi = 0$] denote the steady-state gain coefficients with and without incorporating carrier heating by the pump, respectively. The gain coefficients are found to be very sensitive to the dispersion characteristics of the AW. In anomalous ($\omega_a > k_a v_a$) and normal ($\omega_a < k_a v_a$) dispersion regimes of the AW, the gain coefficient g_{BO} is negative (which signifies absorption of the Stokes mode) and decreases with increase in k, whereas gain coefficient g_B is small but positive and nearly remains constant. However, as we approach the dispersion-less regime ($\omega_a \approx k_a v_a$); both g_B and g_{BO} increase very sharply, acquiring maximum values $g_B = 1.1\times10^4\,\mathrm{m}^{-1}$, $g_{BO} = 8\times10^3\,\mathrm{m}^{-1}$ in the presence of a non-dispersive AW ($\omega_a = k_a v_a$). The smallness of g_{BO} over g_B may be attributed to the increase in ECF [Eq. (24)] due to the carrier heating. This increase in ECF apparently results in an increase in energy transfer of the pump and Stokes mode and subsequently enhances g_B through the parameter Φ [Eq. (26)].

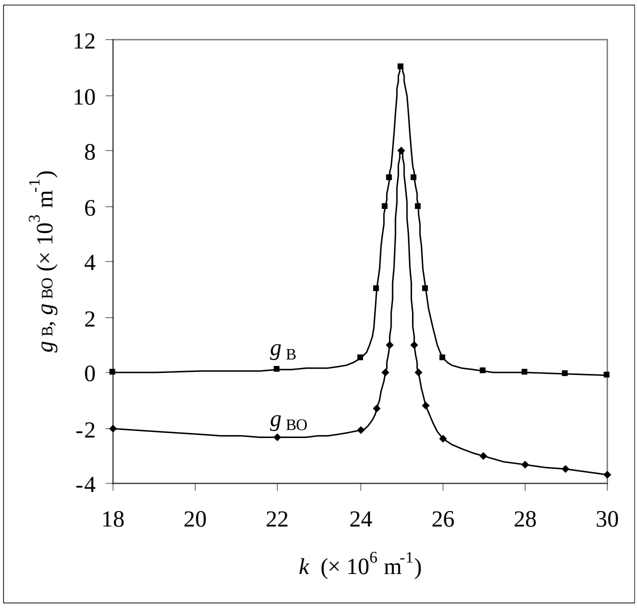

Figure 4: Variation of Brillouin gain coefficients g_B and g_{BO} as a function of acoustic wave number k. Here $E_0 = 2.5\times10^7\,\mathrm{Vm}^{-1}$, $n_e = 10^{24}\,\mathrm{m}^{-3}$, $\omega_c = 0.9\omega_0$, $\phi = \pi/6$, $\theta = \pi/4$.

As Fig. 4 indicates, the steady-state Brillouin gain is very sensitive to the dispersion characteristics of the generated AW. In the presence of a dispersive AW, i.e. when $\omega_a \neq k_a v_a$, g_B is very small because of the phase mismatch of the interaction waves. However, the presence of a non-dispersive AW significantly enhances the energy transfer from the pump to the Stokes mode and thus maximizes g_B which may be obtained from Eq. (26) as:

$$[g_B]_{\max} = \frac{k}{2\varepsilon_L}\left[\frac{b\gamma^2 k^2\cos^2\phi}{4\rho\varepsilon_0\Gamma_a\omega_a}\left(1+\frac{\overline{\omega}_p^2}{\omega_1\omega_0}\right)+\Phi\right]\left|\vec{E}\right|^2 \tag{31}$$

Fig. 5 depicts the effect of magnetic field on the $[g_B]_{\max}$ and $[g_{BO}]_{\max}$. For weak magnetic fields ($\omega_c = \omega_0$), both the gain coefficients are nearly constant but suddenly start increasing very sharply as ω_c approaches ω_0. This is due to the modulation of the parameter $\overline{\omega}_p$ in Eq. (31) that results in large energy transfer from the pump to the generated waves. The carrier heating by the pump is found to modify the SSBG significantly particularly in the weak magnetic field regimes. However, a strong magnetic field $\omega_c > 0.5\omega_0$ significantly influences the carriers and causes a considerable reduction in the parameter Φ through $\overline{\omega}_p$. As a result, the curves corresponding to $[g_B]_{\max}$ and $[g_{BO}]_{\max}$ continue to come closer.

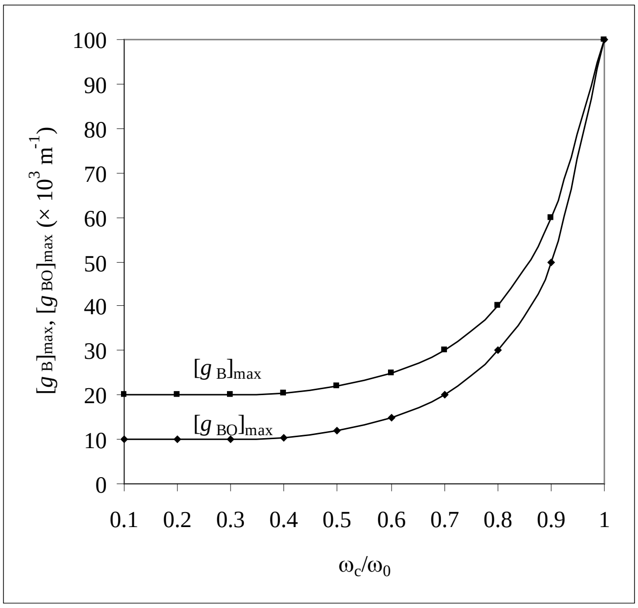

Fig. 5: Variation of steady state Brillouin gain coefficients g_B and g_{BO} as a function of magnetic field strength (in terms of ω_c/ω_0). Here $E_0 = 2.5\times10^7\,\mathrm{Vm}^{-1}$, $k = 2.5\times10^7\,\mathrm{m}^{-1}$, $n_e = 10^{24}\,\mathrm{m}^{-3}$, $\omega_c = 0.9\omega_0$, $\phi = \pi/6$, $\theta = \pi/4$.

Fig. 6 depicts the influence of pump pulse duration τ_p on the transient Brillouin gain of Stoke's mode. Here g_{TB} [Eq. (29)], and g_{TBO} [Eq. (29) with $\Phi = 0$] denote the transient Brillouin gain coefficients with and without incorporating carrier heating by the pump, respectively. To draw this behaviour, we have considered pulse durations in the range $10^{-12} \le \tau_p \le 10^{-8}\,\mathrm{s}$ and pump field intensity $I_p = 2\times10^9\,\mathrm{W\,cm}^{-2}$. The interaction length is the cell length x or $c_1\tau_p/2$, whichever is shorter. The gain coefficients are found to be vanishingly small at shorter pulse durations with $\tau_p \le 0.1\,\mathrm{ns}$. For further increase in τ_p, transient gain coefficients increases very rapidly ($g_{TBO} = 4.4\,\mathrm{m}^{-1}$, $g_{TB} = 10\,\mathrm{m}^{-1}$). The enhancement in g_{TB} may be attributed to increase in ECF which results in an increase in energy transfer of the pump and Stokes mode and subsequently enhances g_{TB} through the parameter Φ [Eq. (29)].

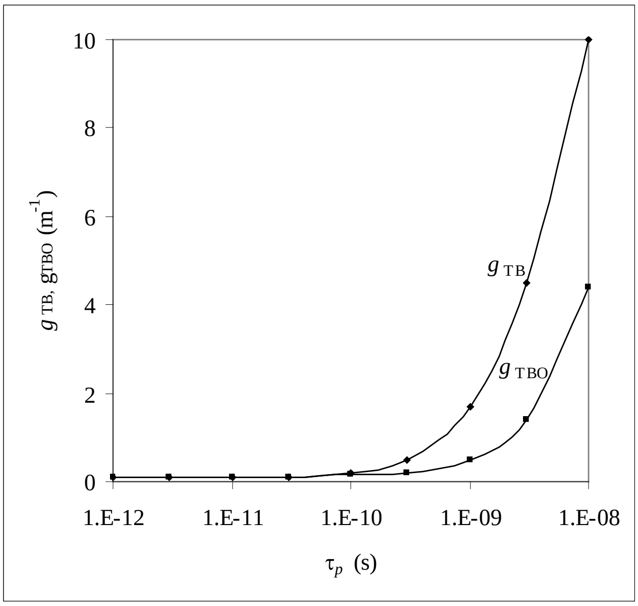

Fig. 6: Variation of transient Brillouin gain coefficients g_{TB} and g_{TBO} as a function of pump pulse duration τ_p. Here $I_p = 2\times10^9\,\mathrm{W\,cm^{-2}}$, $k = 2.5\times10^7\,\mathrm{m^{-1}}$, $n_e = 10^{24}\,\mathrm{m^{-3}}$, $\omega_c = 0.9\omega_0$, $\phi = \pi/6$, $\theta = \pi$.

Thus incorporation of carrier heating in the analysis not only makes our model realistic and the analysis more reliable but may also considerably minimize discrepancies between the experimental observations and theoretical predictions.

IV CONCLUSIONS

The present work deals with the analytical investigations of steady-state and transient Brillouin gain in narrow band-gap magnetized semiconductors duly shined by a pulsed 10.6 µm CO_2 laser. The role of carrier heating by the pump has been examined at length. The detailed analysis enables one to draw the following conclusions:

1. The carrier heating by the pump appreciably enhances the ECF and hence transfers energy from the pump to the scattered light. As a result, the steady state as well as transient Brillouin gain coefficients are considerably enhanced. Thus, incorporation of carrier heating by the pump may help in reducing discrepancies between theory and experimental measurements. This may be an important step towards the appropriate interpretation of experimental measurements in solid and gaseous plasmas.
2. A significant reduction in the SBS threshold and enhancement of steady state and transient gain coefficients can be achieved by applying a strong magnetic field in the transverse direction. This is because it maximally influences the carriers and enhances the parameter Φ appreciably by modifying the carrier parameter $\bar{\omega}_p$. As a result, the carrier current density induced polarization $\vec{P}_{cd}$ is considerably increased.
3. A heavily doped narrow band-gap III–V semiconductor immersed in a transverse strong magnetic field yields maximum Brillouin gain when irradiated by a longitudinal pump, provided the generated AW has dispersion-less propagation in the medium.
4. Semiconductor plasmas duplicate gaseous plasmas as far as phenomena of waves and instabilities are concerned. From this viewpoint, semiconductors may be used as a compact and convenient substitute for gaseous plasmas on account of their considerable ease of operation, liberty of manipulating the material parameters over a wide range, and lack of confinement problems. Thus, the present study may also be used to develop a clearer understanding of stimulated scattering mechanisms and their threshold values encountered in laser induced plasmas.

ACNKOWLEDGEMENTS

One of the authors (M.S.) acknowledges Prof. P. Aghamkar, Department of Physics, Chaudhary Devi Lal University, Sirsa for useful suggestions to carry out this work.

REFERENCES

1. R. Menzel, *Photonics: Linear and Non Linear Interactions of Laser Light and Matter*. Berlin: Springer-Verlag, 2001.
2. K. Rottwitt, Peter T. Lichtenberg, *Nonlinear Optics: Principles and Applications*. CRC Press: Institute of Physics, 2014.
3. K. P. Maheshwari and G. Tarey, Phys. Status Sol. (b) 141 (1987) 333.
4. S.Y. Chen, A. Maksimchuk and D. Umstadter, Nature 396 (1998) 653.
5. S. Banerjee, A. R. Valenzuela, R. C. Shah, A. Maksimchuk, and D. Umstadter, J. Opt. Soc. Am. B 20 (2003) 182.
6. M J Damzen, V Vlad, Anca Mocofanescu, V Babin, *Stimulated Brillouin Scattering: Fundamentals and Applications*. CRC Press, Institute of Physics, 2003.
7. J.A. Stamper, *Laser Interaction and Related Phenomena*. New York: Plenum, 1974 Vol 3B.
8. B. Ya. Zeldovich, N.F. Pilipetsky and V.V. Shkunov, *Principles of Phase Conjugation*. Berlin: Springer, 1985.
9. N.N. Il'ichey, Quantum Electron. 29 (1999) 811.
10. S.K. Hyum, D. Kyong, L. Gwon, L.C. Buyong and K. Jongmin, Opt. Lett. 25 (2000) 399.
11. A.M. Scott and K.D. Ridley, IEEE J. Quantum Electron, QE-25 (1989) 438.
12. Z J Liu, L Hao, J Xiang, S P Zhu, C Y Zheng, L H Cao and X T He, Plasma Phys. Control. Fusion 54 (2012) 095004.
13. L.B. Yehud, D. Belker, G. Ravnitzki and A.A. Ishaaya, Opt. Lett. 39 (2014) 1026.
14. C.H. Lee, *Picosecond Optoelectronic Devices*, C.H. Lee, Ed. New York: Academic, 1984, pp. 1-9, 119-188, 219-284.
15. H.N. Gibbs, *Optical Bistability: Controlling Light with Light*. Orlando: Academic, 1985.
16. B.S. Wherrett, *Optical Computing*, B.S. Wherrett and F.P.A. Tooley, Eds. Edinburg: SUSSP, 1989 pp. 1-21.
17. R.L. Sutherland, *Handbook of Nonlinear Optics*, 2nd Ed. London: Marcel Dekker, 2003, Ch. 13.
18. M. Singh, P. Aghamkar and S. Duhan, Chin. Phys. Lett. 25 (2008) 3276.
19. S. Guha and N. Apte, Phys. Status Sol. (b) 96 (1979) 569.
20. M. Salimullah, T. Ferdousi and F. Majid, Phys. Rev. B 50 (1994) 14104.
21. K.L. Jat, S. Mukherjee and S. Ghosh, Physica Scripta 50 (1994) 507.

22. P. Aghamkar, M. Singh, N. Kishore, S. Duhan and P.K. Sen, Semicond. Sci. Technol. 22 (2007) 749.
23. G. Brodin and L. Stenflo, J. Plasma Phys. 79 (2013) 983.
24. M. Ostermeyer, H.J. Kong, V.I. Kovalev, R.G. Harrisson, A.A. Fotiadi, P. Me´gret, M. Kalal, O. Slezak, J.W. Yoon, J.S. Shin, D.H. Beak, S.K. Lee, Z. Lu, S. Wang, D. Lin, J.C. Knight, N.E. Kotova, A. Straber, A. Scheikh-Obeid, T. Riesbeck, S. Meister, H.J. Eichler, Y. Wang, W. He, H. Yoshida, H. Fujita, M. Nakatsuka, T. Hatae, H. Park, C. Lim, T. Omatsu, K. Nawata, N. Shiba, O.L. Antipov, M.S. Kuznetsov, and N.G. Zakaharov, Laser Part. Beams, 26 (2008) 297.
25. M.C. Steele and B. Vural, *Wave interactions in Solid State Plasmas.* New York: McGraw Hill, 1969, p. 105.
26. P.K. Sen, Phys. Rev. B 21 (1980) 3604 (1980).
27. A. Yariv, *Optical Electronics in Modern Communications.* Oxford: Oxford University Press, 1997, pp. 479.
28. M. Singh and P. Aghamkar, J. Mod. Opt. 55 (2008) 1251.
29. A. C. Beer, *Galvanometric Effects in Semiconductors: Solid State Physics*, Suppl. 9. New York: Academic Press, 1963.
30. M.S. Sodha, A.K. Ghatak and V.K. Tripathi, *Self-Focusing of Laser Beams in Dielectrics, Plasmas and Semiconductors.* New Delhi: Tata McGraw, 1974, pp. 55-62.
31. E.M. Conwell, *High Field Transport in Semiconductors.* New York: Academic Press, 1967, pp. 159.
32. D. Pohl, W. Kaiser, Phys. Rev. B 1 (1970) 31.
33. R.L. Carman, F. Shimizu, C.S. Wang, and N. Bloembergen, Phys. Rev. A 2 (1970) 60.
34. C.H. Wang, in: H. Rabin, C.L. Tang (Eds.), *Quantum Electronics*, Vol. 1, Part A, New York: Academic Press, 1975, pp. 455-472.

First Author – V. Pal Singh, Research Student, Department of Physics, Singhania University, Pacheri Bari, Jhunjhunu - 333515, India
Second Author – M. Singh, (Ph.D.), Ex. Assistant Professor, Department of Physics, Amity University, Sector-125, Noida-201301 (U.P.), India.