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Viscous Dissipation and Mass Transfer Effects on Unsteady MHD Free Convective Flow along a moving Vertical Porous Plate in the presence of Internal Heat Generation and Variable Suction.

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Abstract- The objective of the paper is to analyze the unsteady free convective flow and mass transfer through a viscous, incompressible, electrically conducting fluid along a porous vertical isothermal non-conducting uniformly moving plate in the presence of exponentially decaying heat generation and transverse magnetic field with variable suction and viscous dissipation. The governing equations of motion, energy and concentration are transformed into ordinary differential equations using similarity parameter method. The ordinary differential equations are then solved numerically using Runge – Kutta method along with shooting technique. Numerical results for velocity, temperature and concentration are obtained for various values of physical parameter and presented graphically. Also the numerical values of skin-friction coefficient, Nusselt number and Sherwood number are obtained for various values of physical parameters discussed and presented through tables.

I. INTRODUCTION

Free convection flow is often encountered in cooling of nuclear reactors or in the study of structure of stars and planets. Along with the free convection flow, the phenomenon of mass transfer is also very common in the theories of steller structure. The study of convective flow with mass transfer along a vertical porous plate is receiving considerable attention of many researchers because of its varied applications in the field of cosmical and geophysical sciences. Gebhart and Mollendorf (1969) analyzed viscous dissipation in external natural convection flows. Soundalgekar (1972) studied viscous dissipative effects on unsteady free convective flow past a vertical porous plate with constant suction. Vajravelu (1979) investigated the natural convection at a heated semi infinite vertical plate with internal heat generation. Raptis and Tzivanidis (1981) studied mass transfer effect with variable heat transfer on accelerated vertical plate. Raptis et al. (1987) analyzed the unsteady free convective flow through a porous medium adjacent to a semi Infinite vertical plate using finite difference scheme. Sattar (1994) discussed the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Crepeau and Clarksean (1997) obtained similarity solution of natural convention with internal heat generation which decays exponentially. Unsteady free convection flow past a vertical porous plate was investigated by Helmy (1998). Acharya et al. (2000) studied free convection and mass transfer flow through a porous medium bounded by vertical infinite surface with constant suction and heat flux. Unsteady free convection flow with suction on an accelerating porous plate was discussed by Makinde et al. (2003). Ahmed (2007) observed effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Sharma and Sharma (2009) analyzed effect of mass transfer on three- dimensional unsteady mixed convective flow past an infinite vertical moving porous plate with periodic suction . Atia and Ewis (2010) studied unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient. Sharma et al. (2012) discussed mass transfer with chemical reaction in MHD mixed convection flow along a vertical stretching sheet. Venkateswarlu et al. (2013) analyzed unsteady MHD flow of a viscous fluid past a vertical porous plate under oscillatory suction velocity. Radiation and viscous dissipation effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with Hall current in the presence of chemical reaction was studied by Reddy (2014).

Aim of the paper is to investigate unsteady, laminar, two-dimensional, free convective boundary layer flow of an incompressible, viscous and electrically conducting fluid along a uniformly moving vertical non-conducting porous plate in presence of mass transfer and viscous dissipation effect with variable suction and exponentially decaying heat generation subjected to a uniform transverse magnetic field.

II. FORMULATION OF THE PORBLEM

The x-axis is taken along the vertical plate and y-axis is normal to the plate. Uniform temperature T_w at the plate and uniform species concentration C_w near the plate are maintained. Plate is moving with uniform velocity U. Magnetic field of intensity B_0 is applied in y-direction. It is assumed that the external electric field is zero, also electrical field due to polarization of charges and Hall effects are negligible. Under these assumptions along with the Boussinesq approximation, the governing equations of unsteady flow in the presence of viscous dissipation are given by

$$\frac{\partial v}{\partial y}=0 \;\Rightarrow\; v \text{ is independent of } y \Rightarrow v=v(t) \quad \ldots(1)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)$$

$$+g\beta_c(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u \quad \ldots(2)$$

$$\rho C_p\left[\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right]=\kappa\frac{\partial^2 T}{\partial y^2}+Q+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad \ldots(3)$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad \ldots(4)$$

where u and v are velocity components in x- and y-directions respectively, t is time , υ the Kinematic viscosity, ρ the fluid density, g acceleration due to gravity of the earth, β and β_c are thermal and concentration expansion coefficients respectively, σ the electrical conductivity of the fluid, C_p specific heat at constant pressure, T the temperature of fluid in boundary layer, C the species concentration in boundary layer, T_∞ fluid temperature far away from the plate, C_∞ species concentration in the fluid far away from the plate, κ the thermal conductivity, Q the rate of heat generation/absorption, μ the coefficient of viscosity, and D the mass diffusion coefficient .

The boundary conditions are

$$y=0 \;:\; u=U,\quad v=v(t),\quad T=T_w,\quad C=C_w\,;$$

$$Y\to\infty \;:\; u=0,\quad T=T_\infty,\quad C=C_\infty. \quad \ldots(5)$$

III. METHOD OF SOLUTION

In order to obtain the similarity solutions, a similarity (Schlitchting and Gersten 1999) parameter h is introduced which is time dependent length scale as given below

$$h=h(\mathrm{t})=2\sqrt{\upsilon t}, \quad \ldots(6)$$

specially used for unsteady boundary layer problems. In terms of $h(\mathrm{t})$, a solution of (1) is given by

$$v=v(t)=-V_0\frac{\upsilon}{h(t)} \quad \ldots(7)$$

where $V_0(>0)$ is suction parameter.

Introducing the following dimensionless quantities

$$\eta=\frac{y}{h},\quad u=Uf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\; Gr=\frac{g\beta h^2(T_w-T_\infty)}{U\upsilon}$$

$$Gm=\frac{g\beta_c h^2(\mathrm{C}_w-C_\infty)}{U\upsilon},\qquad M=\frac{\sigma B_0^2 h^2}{\upsilon\rho},$$

$$\Pr=\frac{\mu C_p}{\kappa},\qquad Q=\frac{S\kappa(T_w-T_\infty)}{h^2}e^{-\eta},$$

$$Ec=\frac{U^2}{C_p(T_w-T_\infty)},\qquad Sc=\frac{\upsilon}{D} \quad \ldots(8)$$

into the equations (2) to (4) , we obtain

$$f''+(2\eta+V_0)f'+Gr\theta+GmC-MF=0, \quad \ldots(9)$$

$$\theta''+\Pr(2\eta+V_0)\theta'+Se^{-\eta}+Ec\Pr f'^2=0, \quad \ldots(10)$$

$$\phi''+Sc(2\eta+V_0)\phi'=0, \quad \ldots(11)$$

where η is similarity variable, Gr is the Grashof number, Gm is modified Grashof number, M is the magnetic parameter, $\Pr$ is Prandtl number, Ec is Eckert number, Sc is Schmidt number.

Following Crepeau and Clarkseon (1997), the fluid has internal volumetric rate of heat generation Q as given below

$$Q=\frac{SK(T_w-T_\infty)}{h^2}e^{-\eta}, \quad \ldots(12)$$

where S is the rate of heat generation/absorption parameter.

The corresponding boundary conditions are reduced to

$$\eta=0 \;:\; f=1 \quad \theta=1 \quad \phi=1;$$

$$\eta\to\infty \;:\; f=0 \quad \theta=0 \quad \phi=0. \quad \ldots(13)$$

The governing equations (9) to (11) are non – linear second order coupled differential equations and solved under the boundary conditions (13) using Runge –Kutta fourth order technique (Jain et al. 1985 ; Jain 2000) along with shooting technique (Conte and Boor 1981). Substituting the first order system as given below

$$f=f_1,\qquad f'=f_2,\qquad f''=f_2'$$

$$\theta=f_3,\qquad \theta'=f_4,\qquad \theta''=f_4'$$

$\phi = f_5$, $\phi' = f_6$ $\phi'' = f_6'$ …(14)

$$f_2' = -\left(2\eta + V_0\right) f_2 - Grf_3 - Gmf_5 + Mf_1 \quad \dots(15)$$

$$f_4' = -\Pr\left(2\eta + V_0\right) f_4 - Se^{-\eta} - Ec\Pr\left(f_2\right)^2 \quad \dots(16)$$

$$f_6' = -Sc\left(2\eta + V_0\right) f_6 \quad \dots(17)$$

with the boundary conditions

$$f_1(0) = 1, \quad f_3(0) = 1, \quad f_5(0) = 1,$$
$$f_1(\infty) = 0, \quad f_3(\infty) = 0, \quad f_5(\infty) = 0, \quad \dots(18)$$

To solve equations (15) to (17) with equation (18) as an initial value problem we also need the values of $f_2(0), f_4(0)\ and\ f_6(0)$ ie $f'(0), \theta'(0)$ and $\phi'(0)$, but no such values are available. The initial guess values for $f'(0), \theta'(0)\ and\ \phi'(0)$ are chosen and using the fourth-order Runge - Katta method, the solutions are obtained. We compare the calculated values of $f(\eta), \theta(\eta), and\ \phi(\eta)$ at a finite value of $\eta \to \infty$ with the given boundary conditions $f(\infty) = 0$, $\theta(\infty) = 0$ and $\phi(\infty) = 0$, and adjust the values of $f'(0), \theta'(0), and\ \phi'(0)$ to give a better approximation for the solution. The step – size is taken as $\Delta\eta = 0.01$. The process is replaced until the results corrected up to the desired accuracy level of 10^{-4}.

IV. SKIN-FRICTION COEFFICIENT

Skin-friction coefficient at the plate is given by

$$C_f = \frac{2\upsilon}{Uh} f'(0) = 2\left(\text{Re}\right)^{-1} f'(0) \quad \dots(19)$$

where $\text{Re} = \frac{Uh}{\upsilon}$ is the Reynolds number.

NUSSELT NUMBER

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = \frac{2q\sqrt{\upsilon t}}{\kappa(T_w - T_\infty)} = -\theta'(0) \quad \dots(20)$$

where $q = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is heat flux per unit area.

SHERWOOD NUMBER

The rate of mass transfer in the terms of the Sherwood number is given by

$$Sh = \frac{2J\sqrt{\upsilon t}}{D(C_w - C_\infty)} = -\phi'(0) \quad \dots(21)$$

where $J = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$ is rate of mass flux .

V. RESULT AND DISCUSSION

Numerical values have been carried out for velocity profiles, temperature profiles, species concentration profiles, skin – friction coefficient, rate of heat transfer in terms of Nusselt number and rate of mass transfer in terms of Sherwood number at the plate for various values of the parameters involved in the system. The calculated values are shown through figures and tables, discussed numerically and explained physically.

Figures 1 and 2, respectively represent that velocity increases due to increase in Grashof number or modified Grashof number. It is observed from figures 3 that fluid velocity decreases as the Hartmann number increase. Figure 4 depicts that fluid velocity increases with heat source while decreases with heat sink. Figure 5 illustrates that fluid velocity increases due to increase in Eckert number. Figures 6 and 7, respectively show that fluid velocity decreases with increase in Prandtl number or Schmidt number. It is seen from Figure 8 that fluid temperature increases with the increase of heat source parameter while it decreases with the increase of sink. Figure 9 illustrates that fluid temperature increases due to increase in Eckert number. Figures 10 and 11, respectively represent that fluid temperature decreases due to increase in Prandtl number or suction parameter. Figures 12 and 13, respectively represent that the mass concentration decreases with the increase in Schmidt number or suction parameter.

It is seen from the Table 1 that skin-friction coefficient at the plate increases due to increase in Grashof number, modified Grashof number, heat source or Eckert number, while it decreases due to increase in Hartmann number, Prandtl number, Schmidt number or heat sink. Table 2 shows that the Nusselt number at the plate increases due to increase in Prandtl number, heat sink or suction parameter, while it decreases due to increase in Eckert number or heat source. Table 3 depicts that Sherwood number at the plate increases with the increase in Schmidt number or suction parameter.

Table 1. Numerical values of $f'(0)$ **for various values of** Gr, Gm, M, S, $\Pr$, Ec **and** Sc

Gr	Gm	M	S	$\Pr$	Ec	Sc	$f'(0)$
2	1	1	0.5	0.71	0.5	0.22	-0.3796
3	1	1	0.5	0.71	0.5	0.22	0.05
4	1	1	0.5	0.71	0.5	0.22	0.505
2	2	1	0.5	0.71	0.5	0.22	0.129
2	3	1	0.5	0.71	0.5	0.22	0.662
2	1	2	0.5	0.71	0.5	0.22	-0.7993
2	1	3	0.5	0.71	0.5	0.22	-1.136
2	1	1	1	0.71	0.5	0.22	-0.21996
2	1	1	-0.5	0.71	0.5	0.22	-0.5796
2	1	1	-1	0.71	0.5	0.22	-0.7698
2	1	1	0.5	1	0.5	0.22	-0.51018
2	1	1	0.5	7	0.5	0.22	-0.8958
2	1	1	0.5	0.71	0.01	0.22	-0.5104
2	1	1	0.5	0.71	0.2	0.22	-0.441
2	1	1	0.5	0.71	0.5	0.6	-0.4796
2	1	1	0.5	0.71	0.5	1.002	-0.5396

Table 2. Numerical values of $\{-\theta'(0)\}$ **for various values of** S, $\Pr$, Ec **and** V_0

S	$\Pr$	Ec	V_0	$-\theta'(0)$
0.5	0.71	0.5	0.5	0.8151
1	0.71	0.5	0.5	0.5512
-0.5	0.71	0.5	0.5	1.12287
-0.7	0.71	0.5	0.5	1.16
0.5	1	0.5	0.5	0.8491
0.5	7	0.5	0.5	0.8997
0.5	0.71	0.001	0.5	0.8825
0.5	0.71	0.9	0.5	0.761
0.5	0.71	0.5	1	0.9851
0.5	0.71	0.5	1.5	1.149

Table 3. Numerical values of $\{-\phi'(0)\}$ **for various values of** Sc **and** V_0

Sc	V_0	$-\phi'(0)$
0.22	0.5	0.6014
0.3	0.5	0.7163
0.6	0.5	1.071
0.22	1	0.6769
0.22	1.5	0.755

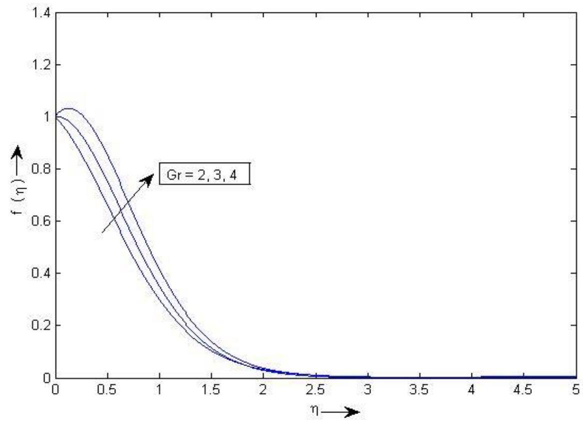

Figure 1. Velocity profiles versus η **for different values of** Gr **when** $Gm = 1, S = 0.5, M = 1,$ $V_0 = 0.5, \Pr = 0.71, Ec = 0.5\, and\ Sc = 0.22.$

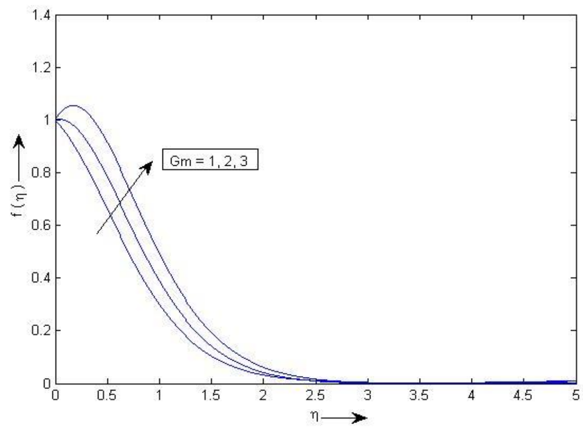

Figure 2. Velocity profiles versus η **for different values of** Gm **when** $Gr = 2, S = 0.5, M = 1,$ $V_0 = 0.5, \Pr = 0.71, Ec = 0.5\, and\ Sc = 0.22.$

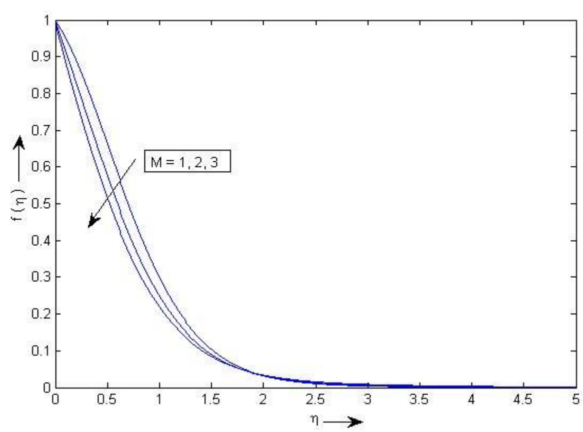

Figure 3. Velocity profiles versus η **for different values of** M **when** $Gr = 2, Gm = 1, S = 0.5,\ V_0 = 0.5, \Pr = 0.71, Ec = 0.5\, and\ Sc = 0.22.$

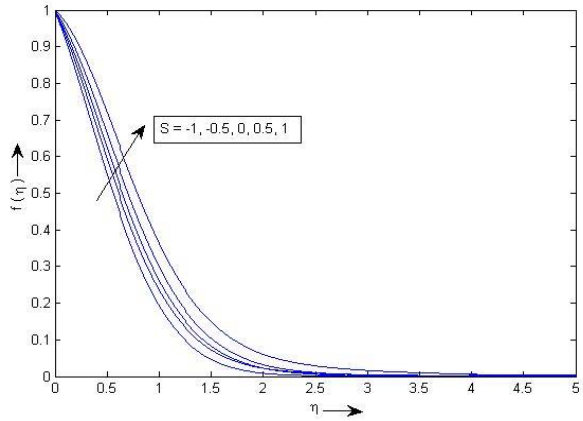

Figure 4. Velocity profiles versus η for different values of S when $Gr = 2, Gm = 1, M = 1, V_0 = 0.5, \Pr = 0.71, Ec = 0.5\, and\ Sc = 0.22.$

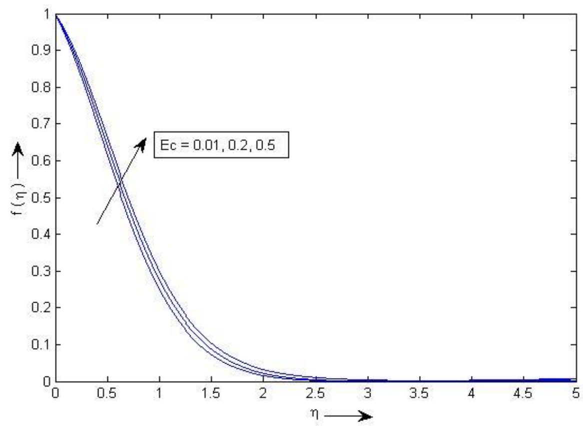

Figure 5. Velocity profiles versus η for different values of Ec when $Gr = 2, Gm = 1, M = 1, V_0 = 0.5, \Pr = 0.71, S = 0.5\, and\ Sc = 0.22.$

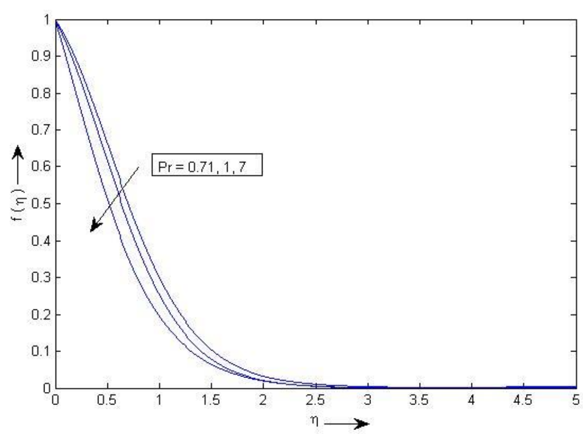

Figure 6. Velocity profiles versus η for different values of $\Pr$ when $Gr = 2, Gm = 1, M = 1, V_0 = 0.5, Ec = 0.5, S = 0.5\, and\ Sc = 0.22.$

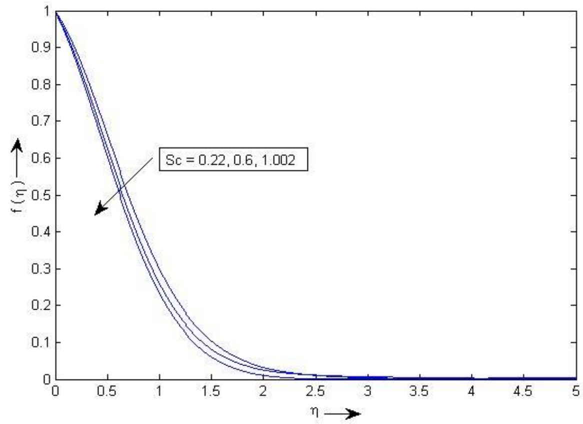

Figure 7. Velocity profiles versus η **for different values of** Sc **when** $Gr = 2, Gm = 1, M = 1,$
$V_0 = 0.5, \Pr = 0.71, S = 0.5 \, and \, Ec = 0.5.$

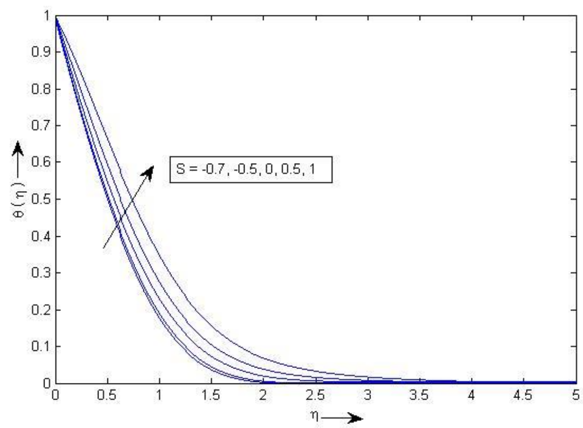

Figure 8. Temperature profiles versus η **for different values of** $\Pr$ **when** $Gr = 2, Gm = 1, M = 1,$
$V_0 = 0.5, \Pr = 0.71, S = 0.5 \, and \, Ec = 0.5.$

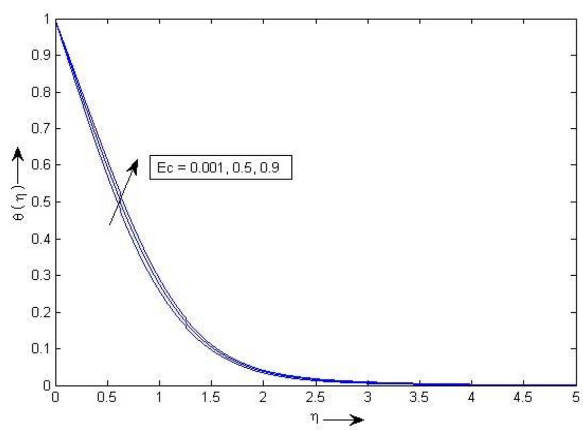

Figure 9. Temperature profiles versus η **for different values of** Ec **when** $Gr = 2, Gm = 1, M = 1,$
$V_0 = 0.5, \Pr = 0.71, S = 0.5 \, and \, Sc = 0.22.$

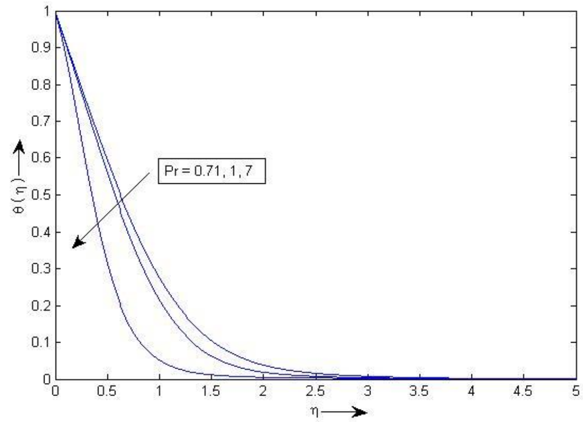

Figure 10. Temperature profiles versus η for different values of $\Pr$ when $Gr = 2, Gm = 1, M = 1, V_0 = 0.5, Ec = 0.5, S = 0.5\, and\, Sc = 0.22.$

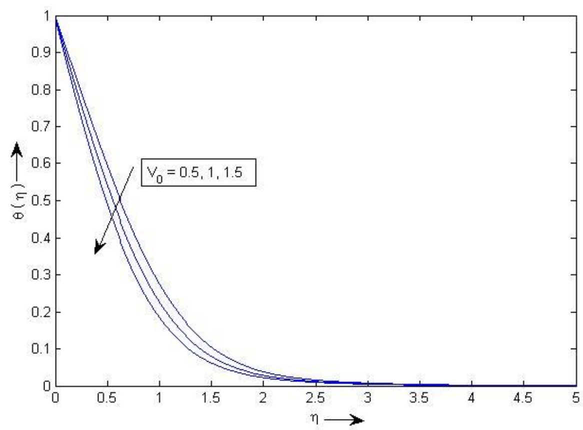

Figure 11. Temperature profiles versus η for different values of V_0 when $Gr = 2, Gm = 1, M = 1, \Pr = 0.71, Ec = 0.5, S = 0.5\, and\, Sc = 0.22.$

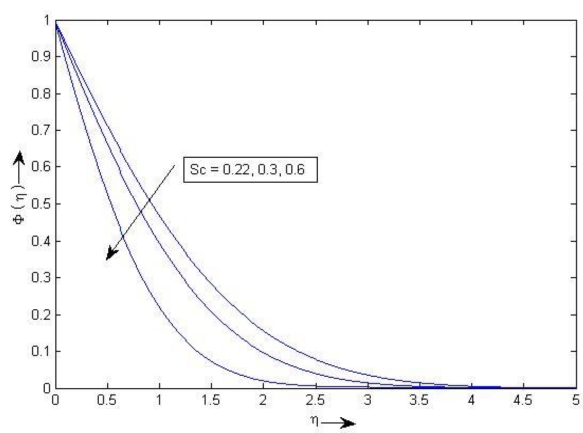

Figure 12. Concentration profiles versus η for different values of Sc when $V_0 = 0.5.$

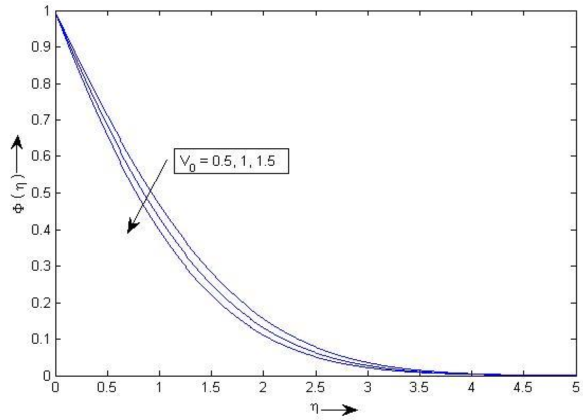

Figure 13. Concentration profiles versus η for different values of V_0 when $Sc = 0.22$.

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