THE DETERMINANT AND THE INVERSE OF NON – SINGULAR 2 X 2 AND 3 X 3 MATRICES USING ADJOINT METHOD

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Abstract- In this paper work we study the derivation of the determinant, the adjoint, and inverse of non – singular 2 x 2 and 3 x 3 matrices. And some computational problems and results were carry out.

Index Terms- Matrix, Determinant, Non - Singular, Adjoint and Inverse.

1.0 INTRODUCTION

The inverse of a 2 x 2 and 3 x 3 non – singular matrix is got by either the adjoint method or using elementary operation, noting that the determinant of the given matrix must be non zero. The determinant is then obtained by subtracting the product of the elements in the ordinary diagonal from the product of the elements in the leading diagonal. The inverse matrix is then found to be equal to the reciprocal of the determinant of the original matrix multiplied by the adjoint matrix. (Ref. Celia, C.W; Nice A.T.F; Elliot.K.F. (1987), Ilori S.A.and Akinyele O. (1986), Lipchitz Seymour. (1988), Meyer, Carl D. (2001), Poole, David (2006), Satursday.E.G. (2012), Stroud.K.A. (2007), Voyevodi N.V.V. (1980 and Stephenson. (1999).

1.1 Some Definitions

1.1.1 Matrix: A matrix can be defined as a set of quantities arranged in a rectangular array of m rows and n columns.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}$$

The individual quantities are called the elements of the matrix of rows and columns is said to be of order m x n.

1.1.2 Determinant Matrix: The determinant of a matrix is a scalar (constant) obtained from the matrix by an appropriate evaluation depending on the order of the matrix. The determinant of matrix A is denoted by $|A|$.

1.1.3 Non - Singular Matrix: This is a square matrix whose determinant is not zero.

1.1.4 Adjoint Matrix: This is the transpose of the cofactors matrix. The adjoint of a matrix A is denoted by (AdjA).

1.1.5 The Inverse Matrix: The inverse of a matrix is another matrix which multiplies the original matrix to give an identity matrix. The inverse of a matrix A is denoted by A^{-1} .

1.2 THE DERIVATION OF THE DETERMINANT AND THE INVERSE OF NON – SINGULAR 2 X 2 AND 3 X 3 MATRICES USING ADJOINT METHOD.

1.2.1 For 2 X 2 Matrix

Let us consider the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

1.2.1.1 The Determinant

$|A| = \text{det}A = ad - bc \neq 0$ 1.3.1.1a

Hence A is non – singular matrix and its has an inverse

1.2.1.2 The Adjoint

$$\text{Adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ 1.3.1.2b

1.2.1.3 The Inverse

$A^{-1} = \frac{1}{|A|}\text{Adj}A$ 1.3.1.3c

Putting equations 1.3.1.1a and 1.3.1.2b into equation 1.3.1.3c we have,

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$ 1.3.1.3d

Hence equation 1.3.1.3d gives the inverse of 2 x 2 non- singular matrix.

1.2.2 For 3 X 3 Matrix

Let consider the matrix

$$B = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

1.2.2.1 The Determinant

$$|B| = \text{det}B = a_1\begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1\begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1\begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

$|B| = a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 \neq 0$ 1.3.2.1a

Let M $=|B| = a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1$ 1.3.2.1b

Hence B is a non-singular matrix and it has inverse.

1.2.2.2 The Adjoint

$$\text{Adj}B = \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - \begin{bmatrix} b_1 & c_1 \\ b_3 & c_3 \end{bmatrix} + \begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \end{bmatrix}$$

$$- \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + \begin{bmatrix} a_1 & c_1 \\ a_3 & c_3 \end{bmatrix} - \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

$$+ \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} - \begin{bmatrix} a_1 & b_1 \\ a_3 & b_3 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$= \begin{bmatrix} b_2c_3 - b_3c_2 & -b_1c_3 + b_3c_1 & b_1c_2 - b_2c_1 \\ -a_2c_3 + a_3c_2 & a_1c_3 - a_3c_1 & -a_1c_2 + a_2c_1 \\ a_2b_3 - a_3b_2 & -a_1b_3 + a_3b_1 & a_1b_2 - a_2b_1 \end{bmatrix} \qquad 1.3.2.2c$$

1.2.2.3 The Inverse

$$B^{-1} = \frac{1}{|B|} \text{AdjB} \qquad 1.3.2.3d$$

Putting equations 1.3.2.1b and 1.3.2.2c into equation 1.3.2.3d we have,

$$= \frac{1}{M}\begin{bmatrix} b_2c_3 - b_3c_2 & -b_1c_3 + b_3c_1 & b_1c_2 - b_2c_1 \\ -a_2c_3 + a_3c_2 & a_1c_3 - a_3c_1 & -a_1c_2 + a_2c_1 \\ a_2b_3 - a_3b_2 & -a_1b_3 + a_3b_1 & a_1b_2 - a_2b_1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{b_2c_3 - b_3c_2}{M} & \frac{-b_1c_3 + b_3c_1}{M} & \frac{b_1c_2 - b_2c_1}{M} \\ \frac{-a_2c_3 + a_3c_2}{M} & \frac{a_1c_3 - a_3c_1}{M} & \frac{-a_1c_2 + a_2c_1}{M} \\ \frac{a_2b_3 - a_3b_2}{M} & \frac{-a_1b_3 + a_3b_1}{M} & \frac{a_1b_2 - a_2b_1}{M} \end{bmatrix} \qquad 1.3.2.3e$$

Hence equation 1.3.2.3e gives the inverse of 3 x 3 non- singular matrix.

1.3 THE COMPUTATION OF DETERMINANT, ADJOINT AND INVERSE OF 2 X 2 AND 3 X 3 MATRICES

1.3.1 Problems of 2 X 2 Matrices.

Some of the problems and results of 2 x 2 matrices are shown in table 1 below.

Table 1

Problem	Matrix	Determinant	Adjoint	Inverse
1	$A = \begin{bmatrix} -3 & 2 \\ -1 & 7 \end{bmatrix}$	19	$\begin{bmatrix} 7 & -2 \\ 1 & -3 \end{bmatrix}$	$\frac{1}{-19}\begin{bmatrix} 7 & -2 \\ 1 & -3 \end{bmatrix}$
2	$B=\begin{bmatrix} -2 & -6 \\ 3 & 5 \end{bmatrix}$	8	$\begin{bmatrix} 5 & 6 \\ -3 & -2 \end{bmatrix}$	$\frac{1}{8}\begin{bmatrix} 5 & 6 \\ -3 & -2 \end{bmatrix}$
3	$C=\begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$	5	$\begin{bmatrix} -4 & -3 \\ -1 & -2 \end{bmatrix}$	$\frac{1}{-5}\begin{bmatrix} -4 & -3 \\ -1 & -2 \end{bmatrix}$
4	$D=\begin{bmatrix} -3 & -5 \\ -2 & 4 \end{bmatrix}$	2	$\begin{bmatrix} 4 & 5 \\ 2 & -3 \end{bmatrix}$	$\frac{1}{-22}\begin{bmatrix} 4 & 5 \\ 2 & -3 \end{bmatrix}$
5	$E=\begin{bmatrix} 11 & 3 \\ 17 & 5 \end{bmatrix}$	4	$\begin{bmatrix} 5 & -17 \\ -3 & 11 \end{bmatrix}$	$\frac{1}{4}\begin{bmatrix} 5 & -17 \\ -3 & 11 \end{bmatrix}$

1.3.2 Problems of 3 X 3 Matrices.

Some of the problems and results of 3 x 3 matrices are shown in table 2 below.

Table 2

Problem	Matrix	Determinant	Adjoint	Inverse
1	$A = \begin{bmatrix} 3 & 2 & 5 \\ 6 & 8 & 4 \\ 7 & 3 & 2 \end{bmatrix}$	-146	$\begin{bmatrix} 4 & 11 & -32 \\ 16 & -29 & 18 \\ -38 & 5 & 12 \end{bmatrix}$	$\frac{1}{-146}\begin{bmatrix} 4 & 11 & -32 \\ 16 & -29 & 18 \\ -38 & 5 & 12 \end{bmatrix}$
2	$B=\begin{bmatrix} 3 & 4 & 5 \\ 7 & 5 & 8 \\ 9 & 2 & 1 \end{bmatrix}$	41	$\begin{bmatrix} -11 & 8 & 2 \\ 65 & -51 & 18 \\ -31 & -30 & 13 \end{bmatrix}$	$\frac{1}{41}\begin{bmatrix} -11 & 8 & 2 \\ 65 & -51 & 18 \\ -31 & -30 & 13 \end{bmatrix}$
3	$C=\begin{bmatrix} 2 & 1 & -1 \\ 1 & -4 & 2 \\ 3 & 2 & -3 \end{bmatrix}$	11	$\begin{bmatrix} 8 & 1 & -2 \\ 9 & -3 & -5 \\ 14 & -1 & -9 \end{bmatrix}$	$\frac{1}{11}\begin{bmatrix} 8 & 1 & -2 \\ 9 & -3 & -5 \\ 14 & -1 & -9 \end{bmatrix}$
4	$D=\begin{bmatrix} 2 & 3 & 2 \\ 1 & 4 & 3 \\ 4 & 1 & 6 \end{bmatrix}$	30	$\begin{bmatrix} 21 & -16 & 1 \\ 6 & 4 & -4 \\ 15 & 10 & 5 \end{bmatrix}$	$\frac{1}{30}\begin{bmatrix} 21 & -16 & 1 \\ 6 & 4 & -4 \\ 15 & 10 & 5 \end{bmatrix}$
5	$E=\begin{bmatrix} 3 & 2 & -8 \\ 4 & 6 & -5 \\ 2 & 7 & 3 \end{bmatrix}$	-13	$\begin{bmatrix} 53 & -62 & 38 \\ -22 & 25 & -17 \\ 16 & -17 & 10 \end{bmatrix}$	$\frac{1}{-13}\begin{bmatrix} 53 & -62 & 38 \\ -22 & 25 & -17 \\ 16 & -17 & 10 \end{bmatrix}$

1.4 CONCLUSION

In conclusion, from the analysis of the computational results in tables 1 and 2 above, we discover that the derivation of the determinant and inverse of non-singular 2 x 2 and 3 x 3 using adjoint method is efficient and accurate when solving matrices problems. And it can also handle matrices of higher orders.

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